ENGI 7854/9804 Image processing and applications
Spring 2020 - Memorial University

Assignment 2: Due on July 17, 2020

Total Marks : 100

Submit an electronic copy of your assignment to the appropriate dropbox folder.

Overview

The objective of this assignment is to design a set of filters in the frequency domain to filter out periodic noise. You will first add periodic noise to an image (in the frequency domain) and display the results. Then, you will design band-reject filters (Ideal, Butterworth, and Gaussian) to remove the noise (in frequency domain) and display the results.

The band-reject frequency domain filters for *Ideal*, *Butterworth*, and *Gaussian* are modelled as follows:

Ideal band-reject filter

$$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Butterworth band-reject filter

$$H(u,v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2}\right]^{2n}} \tag{2}$$

Gaussian band-reject filter

$$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2} \tag{3}$$

where, D is the distance $D(u,v)$ from the center of the filter, D_0 is the cutoff frequency, W is the width of the band, and n is the order of the Butterworth filter.

Perform the following steps. Include all the resulting images and MATLAB code with your report.

Questions

Download the test image *moonlanding.png* from D2L located in the *Assignment 2* folder and save it in the MATLAB working directory.

1. (5 marks) Convert to the frequency domain and center the spectrum. Calculate the maximum value of the frequency spectrum (use MATLAB command $FS_{max} = \max(FS(:))$, where FS is the frequency spectrum and FS_{max} is its maximum value).

2. (5 marks) Display the frequency spectrum obtained in Question 1 as an image.

3. (25 marks) Using the centered frequency spectrum obtained in Question 1, manually introduce noise at eight points. The points *must* be located on the circle at a distance of 100 from the center point of the image and having 45° angle increments at the four cardinal and intercardinal directions (N,S,E,W,NE,NW,SE,SW). Find the 3×3 neighborhoods of these eight elements and set their values to $FS_{max}/10$ where FS_{max} was obtained in Question 1. Display the resultant spectrum as an image.

4. (5 marks) Convert the frequency domain representation in Question 2 to the spatial domain, scale it and display the result. Observe the result of adding noise, and comment briefly on the noisy image.

5. (5 marks) Now, using the image from Question 4 (which is corrupted with periodic noise), obtain the frequency domain representation, center the spectrum and display it. Comment on whether or not it looks like the frequency spectrum you manually created in Question 3.

6. (30 marks) Assuming the cutoff frequency D_0=100 and the width W= 8, design:

   (a) Ideal

   (b) Butterworth with order 4 (i.e. n=4)

   (c) Gaussian

   band-reject filters to remove the periodic noise you introduced in the frequency domain. The filters should be of the same size as the original image. Display the three designed filters as three separate binary images.

7. (10 marks) Implement the band reject filters you developed in Question 6 to remove the noise you also introduced using element-wise multiplication. Display the results in the frequency domain for each filter individually.

8. (10 marks) Convert the three frequency domain representations of the filtered image found in Question 7 to the spatial domain, scale them and display the results. Compare the resultant images:

   (a) with the noisy image in Question 4.

   (b) with the original image, *moonlanding.png*.

9. (5 marks) Comment on the differences in the performance of the three band-reject filters used, with respect to their ability to recover the original image.