A4

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1.a)

0	13
1	
2	~~10~~ 20 → 10
3	
4	17

$h(17) = 19 \bmod 5 = 4$

$h(10) = 12 \bmod 5 = 2$

$h(20) = 22 \bmod 5 = 2$

$h(13) = 15 \bmod 5 = 0$

b)

0	13
1	
2	10
3	20
4	17

$h(17) = 19 \bmod 5 = 4$

$h(10) = 12 \bmod 5 = 2$

$h(20) = 22 \bmod 5 = 2$

2 filled, next empty block $2 \Rightarrow 3$

$h(13) = 15 \bmod 5 = 0$

c)

0	13
1	
2	10
3	20
4	17

$h(17) = 19 \bmod 5 = 4$

$h(10) = 12 \bmod 5 = 2$

$h_1(20) = 22 \bmod 5 = 2$

$h_1(20) + h_2(20) = 2 + 1 + (20 \bmod 4) = 3$

$h_1(13) = 15 \bmod 5 = 0$

d)

0	13
1	
2	10
3	17
4	20

$h_1(17) = 19 \bmod 5 = 4$

$h_1(10) = 12 \bmod 5 = 2$

$h_1(20) = 22 \bmod 5 = 2$

Kick 10 out

$h_2(10) = \lfloor 10/5 \rfloor = 2$

Kick 20 out

$h_2(20) = \lfloor 20/5 \rfloor = 4$

Kick 17 out

$h_2(17) = \lfloor 17/5 \rfloor = 3$

$h_1(13) = 15 \bmod 5 = 0$

2. $h(191) = \lfloor 191/16 \rfloor + (191 \bmod 16) = 26$, (11010)

$h(142) = \lfloor 142/16 \rfloor + (142 \bmod 16) = 22$, (10110)

$h(192) = \lfloor 192/16 \rfloor + (192 \bmod 16) = 12$, (01100)

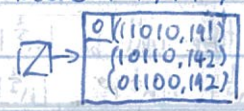
$h(248) = \lfloor 248/16 \rfloor + (248 \bmod 16) = 23$, (10111)

$h(217) = \lfloor 217/16 \rfloor + (217 \bmod 16) = 22$, (10110)

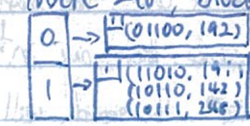
$h(95) = \lfloor 95/16 \rfloor + (95 \bmod 16) = 20$, (10100)

Initial: d=0

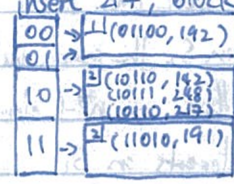
insert 191, 142, 192



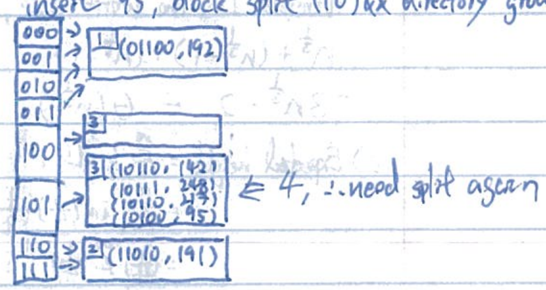
insert 248, block split (/) && directory grow



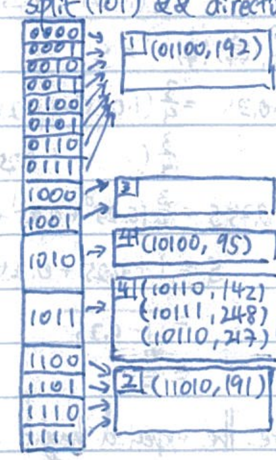
insert 217, block split (1) && directory grow



insert 95, block split (10) && directory grow

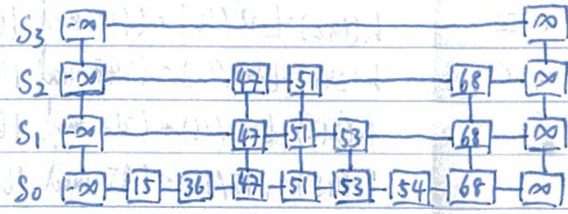


split (101) && directory grow



3.a) $\boxed{T,}\boxed{T,}\boxed{H,H,T,}\boxed{H,T,}\boxed{H,H,T,}\boxed{H,H,T,}\boxed{T,}$ H,T, H,H ...

54 15 51 53 47 68 36



b) In S_2, there will be $\lfloor n^{\frac{1}{3}} \rfloor + 2$ elements.

$\{-\infty, \lfloor n^{\frac{2}{3}} \rfloor, 2\lfloor n^{\frac{2}{3}} \rfloor, \dots, \lfloor n^{\frac{1}{3}} \rfloor \lfloor n^{\frac{2}{3}} \rfloor, \infty\}$

In S_1, there will be $\lfloor n^{\frac{2}{3}} \rfloor + 2$ elements

$\{-\infty, \lfloor n^{\frac{1}{3}} \rfloor, 2\lfloor n^{\frac{1}{3}} \rfloor, \dots, \lfloor n^{\frac{2}{3}} \rfloor \lfloor n^{\frac{1}{3}} \rfloor, \infty\}$

In the worst case, the search will move $n^{\frac{1}{3}}$ times in S_2, since there are $n^{\frac{1}{3}}$ elements between ∞ and $-\infty$. The search will move $n^{\frac{1}{3}}-1$ times at S_1 since there are $n^{\frac{1}{3}}-1$ elements in S_1 between adjacent elements in S_2. The search will move $n^{\frac{1}{3}}-1$ times at S_0 since there are $n^{\frac{1}{3}}-1$ elements in S_0 between adjective elements in S_1.

Therefore In the worst case, the runtime will be

$n^{\frac{1}{3}} + (n^{\frac{1}{3}}-1) + (n^{\frac{1}{3}}-1)$

$= 3n^{\frac{1}{3}} - 2 \in \Theta(n^{\frac{1}{3}})$ ∴ Proved

c) Expected height $= \sum_{i=1}^{\infty}$ height i × probability at height i

$= \sum_{i=1}^{\infty} i \times (\frac{3}{4})(\frac{1}{4})^{i-1}$

$= (\frac{3}{4}) \sum_{i=1}^{\infty} (\frac{1}{4})^{i-1} \cdot i$

$= \frac{3}{4}(1 \cdot 0.25^0 + 2 \cdot 0.25^1 + 3 \cdot 0.25^2 + \dots)$

Let $\sum_{i=1}^{\infty} i (\frac{3}{4})(\frac{1}{4})^{i-1} = S$

$S - S \times 0.25 = \frac{3}{4}(1 \cdot 0.25^0 + 2 \cdot 0.25^1 + 3 \cdot 0.25^2 + \dots) - \frac{3}{4}(\quad 1 \cdot 0.25^1 + 2 \cdot 0.25^2 + \dots)$

$0.75S = \frac{3}{4}(0.25^0 + 0.25^1 + 0.25^2 + \dots)$

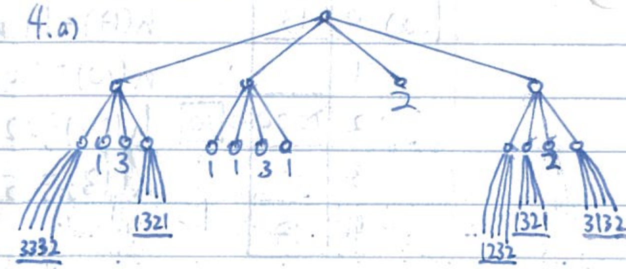
$S = 1 + 0.25^1 + 0.25^2 + 0.25^3 + \dots$

$= \frac{1}{1-0.25}$

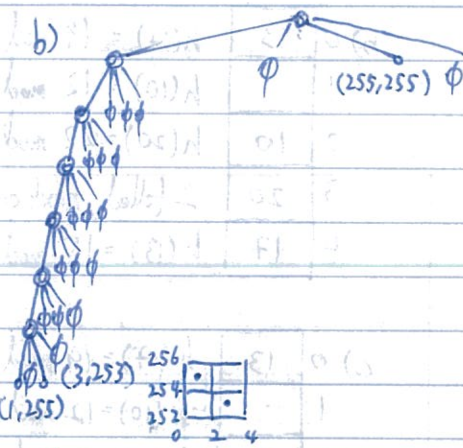
$= \frac{4}{3}$

Therefore the expected height is $\frac{4}{3}$.

4.a)



b)



256	h = 0
256	h = 1
128	h = 2
64	h = 3
32	h = 4
16	h = 5
8	h = 6
4	h = 7

∴ Three points are {(1,255), (3,253), (255,255)}

5.b) Pseudo Code:

printTree

} $O(1)$

for it from 0 to $\frac{n}{2}$
    map insert } $\frac{1}{2} n\log n$

printTree($\frac{n}{2}$) } $T(\frac{n}{2})$

} $O(1)$

for it from $\frac{n}{2}+1$ to n
    map insert } $\frac{1}{2} n\log n$

printTree($\frac{n}{2}$) } $T(\frac{n}{2})$

main

} $O(1)$

for i from 1 to n
    map insert } $n\log n$

printTree($\frac{n}{2}$) } $T(n)$

5.b) According to the pseducode, The runtime for this program will be $O(1)+O(n\log n)+T(n)$

Where $T(n) = \begin{cases} \frac{1}{2}n\log n + T(\frac{n}{2}) + \frac{1}{2}n\log n + T(\frac{n}{2}) = 2T(\frac{n}{2}) + n\log n, & \text{if } n>1 \\ 1 & \text{, if } n\geq 1 \end{cases}$

$\therefore T(n) \in \Theta(n\log^2 n) \in O(n)$ // reference from piazza post @328.

Therefore the total runtime will be $O(1)+O(n\log n)+O(n)$ which will be $O(n\log n)$

The running time is $O(n\log n)$!