Recurrences

1. $T(n) = 4T\left(\frac{n}{2}\right) + 1$

$$= 4\left(4T\left(\frac{n}{4}\right) + 1\right) + 1$$

$$= 4^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} 4^i$$

let $K = \log n$

$$= 4^{\log n} T(1) + \sum_{i=0}^{\log n - 1} 4^i$$

$$= n^2 + 1 + 4 + 16 + \dots + 4^{\log n - 1}$$

$$= n^2 + 1 + 4 + \dots + \frac{1}{4}n^2$$

$$= n^2\left(\frac{1}{4} + \frac{1}{16} + \dots + \frac{1}{n^2} + 1\right)$$

$$\leq n^2 \cdot \frac{4}{3} \in \theta(n^2)$$

2. $T(n) = T(n/2) + n^2$

$$= T(n/4) + (n/2)^2 + n^2$$

$$= T(n/2^k) + \sum_{i=0}^{k-1} \frac{n^2}{2^{i^2}}$$

$$= T(1) + n^2 \sum_{i=0}^{\log n - 1} \frac{1}{2^{i^2}}$$

$$\leq 2n^2 \in \theta(n^2)$$