2.10

2.10 From the given information, we may sketh x(t) and h(t) as show, in Figure S2.10.

(a) With the aid of the plots in Figure S2.10, we can show that $y(t)=x(t)*h(t)$ is as shown in Figure S2.10.

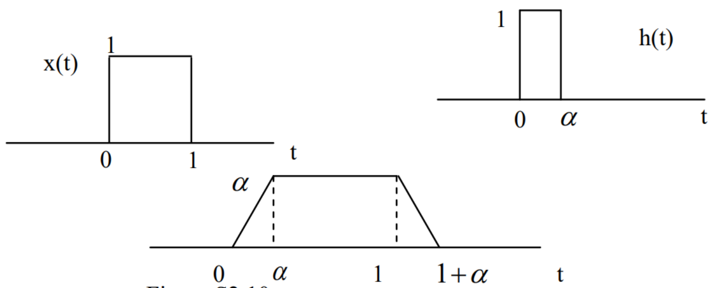


Figure S2.10

Therefore,

$$y(t)=\begin{cases} t, & 0\le t\le \alpha \\ \alpha, & \alpha\le t\le 1 \\ 1+\alpha-t, & 1\le t\le (1+\alpha) \\ 0, & otherwise \end{cases}$$

(b) From the plot of y(t), it is clear that $\frac{dy(t)}{dt}$ has discontinuities at 0, α ,1,and 1+α . If we want $\frac{dy(t)}{dt}$ to have only three discontinuities, then we need to ensure that α =1.

2.11

2.11(a)From the given information, we see that $h(t)$ is non zero only for $0\le t\le\infty$.Therefore,

$$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}h(\tau)x(t-\tau)d\tau$$
$$=\int_{-\infty}^{\infty}e^{-3\tau}(u(t-\tau-3)-u(t-\tau-5)d\tau$$

We can easily show that $(u(t-\tau-3)-u(t-\tau-5))$ is non zero only in the range $(t-5)<\tau<(t-3)$. Therefore, for $t\le 3$, the above integral evaluates to zero .For $3<t\le 5$, the above integral is

$$y(t)=\int_{0}^{t-3}e^{-3\tau}d\tau=\frac{1-e^{-3(t-3)}}{3}$$

For $t>5$, the integral is

$$y(t)=\int_{t-5}^{t-3}e^{-3\tau}d\tau=\frac{(1-e^{-6})e^{-3(t-5)}}{3}=\frac{1}{3}e^{-3t}\left(e^{15}-e^{9}\right)$$

Therefore, the result of this convolution may be expressed as

$$y(t)=\begin{cases} 0, & -\infty<t\le 3 \\ \dfrac{1-e^{-3(t-3)}}{3}, & 3<t\le 5 \\ \dfrac{(1-e^{-6})e^{-3(t-5)}}{3}, & 5<t\le\infty \end{cases}$$

(b) By differentiating x(t) with respect to time we get

$$\frac{dx(t)}{dt}=\delta(t-3)-\delta(t-5)$$

Therefore ,

$$g(t)=\frac{dx(t)}{d(t)}*h(t)=e^{-3(t-3)}u(t-3)-e^{-3(t-5)}u(t-5)$$

(c) From the result of part (a),we m ay compute the derivative of y(t) to be

$$\frac{dy(t)}{dt}=\begin{cases}0, & -\infty < t \le 3 \\ \dfrac{1-e^{-3(t-3)}}{3}, & 3 < t \le 5 \\ \dfrac{(1-e^{-6})e^{-3(t-3)}}{3}, & 5 < t \le \infty\end{cases}$$

This is exactly equal to g(t) .therefore , $g(t)=\frac{dy(t)}{dt}$

2.22

(b) the desire convolution is

y(t)= $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

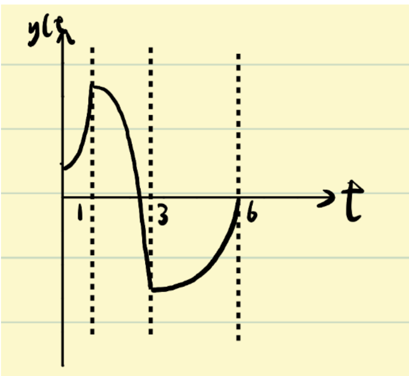
$= \int_0^2 h(t-\tau)d\tau - \int_2^5 h(t-\tau)d\tau$

This may be written as
y(t)=

$$\begin{cases}\int_0^2 e^{2(t-\tau)}d\tau-\int_2^5 e^{2(t-\tau)}d\tau, t <= 1 \\ \int_{t-1}^2 e^{2(t-\tau)}d\tau-\int_2^5 e^{2(t-\tau)}d\tau, 1 <= t <= 3 \\ -\int_{t-1}^5 e^{2(t-\tau)}d\tau, 3 <= t <= 6 \\ 0, 6 < t\end{cases}$$

Therefore y(t)=

$$\begin{cases}(1/2)e^{2t}[1-2e^{-4}+e^{-10}], t <= 1 \\ (1/2)e^{2t}[e^{2-2t}+e^{-10}-2e^{-4}], 1 <= t <= 3 \\ (1/2)e^{2t}[e^{-10}-e^{2-2t}], 3 <= t <= 6 \\ 0, 6 < t\end{cases}$$

Sketch: 

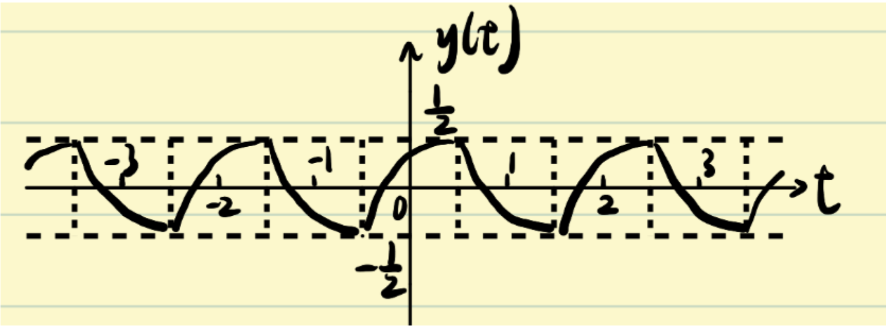
（优秀作业）

(e)

x(t) periodic implies y(t) periodic $\therefore$ determine 1 period only . we have

$$
y(t)=\begin{cases}\int_{t-1}^{-\frac{1}{2}}(t-\tau-1)d\tau+\int_{-\frac{1}{2}}^{t}(1-t+\tau)d\tau=\frac{1}{4}+t-t^2, -\frac{1}{2}<t<\frac{1}{2}\\ \int_{t-1}^{\frac{1}{2}}(1-t+\tau)d\tau+\int_{\frac{1}{2}}^{t}(t-1-\tau)d\tau=t^2-3t+7/4, \frac{1}{2}<t<\frac{3}{2}\end{cases}
$$

The periodic of y(t) is 2.

Sketch: 

(优秀作业)

2.25

参考答案:

$$
\begin{aligned}
y[n]=x[n]*h[n]&=\sum_{k=-\infty}^{\infty}x[k]h[n-k]\\
&=\sum_{k=-\infty}^{-1}3^k\left(\frac{1}{4}\right)^{n-k}u[n-k+3]\\
&\quad+\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n-k}u[n-k+3]\\
&=\frac{1}{12}\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n+k}u[n+k+4]\\
&\quad+\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n-k}u[n-k+3]
\end{aligned}
$$

$$
\frac{1}{12}\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n+k}u[n+k+4]=\begin{cases}\frac{1}{12}\sum_{k=-(n+4)}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n+k}\\ \frac{1}{12}\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n+k}\end{cases}
$$

$$
=\begin{cases}\frac{1}{12}\sum_{k=0}^{\infty}\left(\frac{1}{12}\right)^k\left(\frac{1}{4}\right)^n-\frac{1}{12}\sum_{k=0}^{-(n+5)}\left(\frac{1}{12}\right)^k\left(\frac{1}{4}\right)^n\\ \frac{1}{11}\left(\frac{1}{4}\right)^n\end{cases}
$$

$$
=\begin{cases}\frac{12^4}{11}(3)^n, & n\leqslant-4\\ \frac{1}{11}\left(\frac{1}{4}\right)^n, & n>-4\end{cases}
$$

$$
\sum_{k=0}^{\infty}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n-k}u[n-k+3]=\sum_{k=0}^{n+3}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n-k}
$$

$$
=\left(\frac{1}{4}\right)^n\sum_{k=0}^{n+3}\left(\frac{4}{3}\right)^k=-3\left(\frac{1}{4}\right)^n+\frac{256}{27}\left(\frac{1}{3}\right)^n,\quad n\geqslant-3
$$

故
$$
y[n]=\begin{cases}\frac{12^4}{11}(3)^n, & n\leqslant-4\\ \frac{1}{11}\left(\frac{1}{4}\right)^n-3\left(\frac{1}{4}\right)^n+\frac{256}{27}\left(\frac{1}{3}\right)^n, & n\geqslant-3\end{cases}
$$

优秀作业:

(a)

$$x[n]=\begin{cases}3^n & ,n\le -1\\ (\frac{1}{3})^n & ,n\ge 0\end{cases}$$

$$h[n]=\begin{cases}0 & ,n< -3\\ (\frac{1}{4})^n & ,n\ge -3\end{cases}$$

$$\begin{aligned} y[n] &= \begin{cases}\sum_{i=-3}^{\infty}(\frac{1}{4})^i\cdot 3^{n-i} & ,n\le -4\\ \sum_{i=n+1}^{\infty}(\frac{1}{4})^i\cdot 3^{n-i}+\sum_{i=-3}^{n}(\frac{1}{4})^i\cdot(\frac{1}{3})^{n-i} & ,n\ge -3\end{cases}\\ &= \begin{cases}\frac{20736}{11}\cdot 3^n & ,n\le -4\\ \frac{256}{27}\cdot(\frac{1}{3})^n-\frac{32}{11}\cdot(\frac{1}{4})^n & ,n\ge -3\end{cases}\end{aligned}$$

(b)

divide x into x_1, x_2.

$$x_1[n]=\begin{cases}3^n & ,n\le -1\\ 0 & ,n\ge 0\end{cases}$$

$$x_2[n]=\begin{cases}0 & ,n\le -1\\ (\frac{1}{3})^n & ,n\ge 0\end{cases}$$

assume $y_1[n]=x_1[n]*h[n]$, $y_2[n]=x_2[n]*h[n]$.

$$\begin{aligned} y_1[n] &= \begin{cases}\sum_{i=-3}^{\infty}(\frac{1}{4})^i\cdot 3^{n-i} & ,n\le -4\\ \sum_{i=n+1}^{\infty}(\frac{1}{4})^i\cdot 3^{n-i} & ,n\ge -3\end{cases}\\ &= \begin{cases}\frac{20736}{11}\cdot 3^n & ,n\le -4\\ \frac{1}{11}\cdot(\frac{1}{4})^n & ,n\ge -3\end{cases}\end{aligned}$$

$$\begin{aligned} y_2[n] &= \begin{cases}0 & ,n\le -4\\ \sum_{i=-3}^{n}(\frac{1}{4})^i\cdot(\frac{1}{3})^{n-i} & ,n\ge -3\end{cases}\\ &= \begin{cases}0 & ,n\le -4\\ \frac{256}{27}\cdot(\frac{1}{3})^n-3\cdot(\frac{1}{4})^n & ,n\ge -3\end{cases}\end{aligned}$$

and according to the distributive property of convolution.$y[n]=x[n]*h[n]=x_1[n]*h[n]+x_2[n]*h[n]=y_1[n]+y_2[n]$

$$\begin{aligned} y[n] &= y_1[n]+y_2[n]\\ &= \begin{cases}\frac{20736}{11}\cdot 3^n & ,n\le -4\\ \frac{256}{27}\cdot(\frac{1}{3})^n-\frac{32}{11}\cdot(\frac{1}{4})^n & ,n\ge -3\end{cases}\end{aligned}$$

2.28

2.28 (a) causal because h[n]=0 for n<0 stable because $\sum_{n=0}^{\infty}(\frac{1}{5})^n = \frac{5}{4} < \infty$

(b)not causal because h[n] $\neq 0$ for n<0 stable because $\sum_{n=-2}^{\infty}(0.8)^n = 5 < \infty$

(c)anti- causal because h[n]=0 for n>0 unstable because $\sum_{n=-\infty}^{0}(1/2)^n = \infty$

(d) not causal because h[n] $\neq 0$ for n<0 stable because $\sum_{n=-\infty}^{3}(5)^n = \frac{625}{4} < \infty$

(e) causal because h[n]=0 for n<0 unstable because the second term becomes infinite as n $\rightarrow \infty$.

(f) not causal because h[n] $\neq 0$ for n<0 stable because $\sum_{n=-\infty}^{\infty}|h[n]| = \frac{305}{3} < \infty$

（g）causal because h[n]=0 for n<0. stable because $\sum_{n=\infty}^{\infty}|h[n]| = \overset{3/4}{1} < \infty$