Assignment Week3

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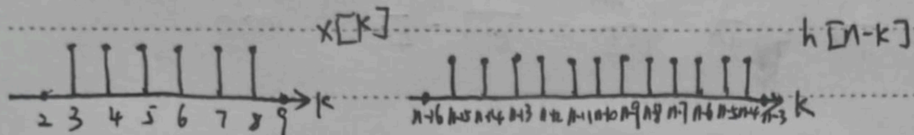
2.4 $y[n] = x[n] * h[n] = \sum_{-\infty}^{\infty} x[k] h[n-k]$



$$\therefore y[n] = \begin{cases} 0, & n \le 6 \\ n-6, & 7 \le n \le 12 \\ 6 & 13 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & n \ge 24 \end{cases}$$

2.6 $y[n] = x[n] * h[n] = \sum_{-\infty}^{\infty} x[k] h[n-k] = \sum_{-\infty}^{\infty} (\frac{1}{3})^{-k} u[-k-1] u[n-k-1]$

$$= \sum_{-\infty}^{\infty} \frac{1}{3} \cdot (\frac{1}{3})^{-k-1} u[-k-1] u[n-k-1]$$

$$= \sum_{-\infty}^{\infty} \frac{1}{3} (\frac{1}{3})^{t} u[t] u[n+t]$$

$$= \begin{cases} \sum_{t=0}^{\infty} \frac{1}{3} (\frac{1}{3})^{t} = \frac{1}{3} \cdot \frac{1}{1-\frac{1}{3}} = \frac{1}{2}, & n \ge 0 \\ \sum_{t=-n}^{\infty} \frac{1}{3} (\frac{1}{3})^{t} = \frac{1}{3} \cdot \frac{(\frac{1}{3})^{-n}}{1-\frac{1}{3}} = \frac{3^{n}}{2}, & n < 0 \end{cases}$$

2.19 (a) As S_1, S_2 are causal LTI, first $y[n-1] = \alpha y[n-2] + \beta w[n-1]$ and

$y[n] = \alpha y[n-1] + \beta w[n] = \alpha y[n-1] + \beta (\frac{1}{2} w[n-1] + x[n]) = \alpha y[n-1] + \frac{\beta}{2} w[n-1] + \beta x[n]$,

then $y[n] = \alpha y[n-1] + \frac{1}{2} (y[n-1] - \alpha y[n-2]) + \beta x[n] = (\alpha + \frac{1}{2}) y[n-1] - \frac{\alpha}{2} y[n-2] + \beta x[n]$

and we know that $y[n] = -\frac{1}{8} y[n-2] + \frac{3}{4} y[n-1] + x[n]$, so $\begin{cases} \alpha + \frac{1}{2} = \frac{3}{4} \\ -\frac{\alpha}{2} = -\frac{1}{8} \\ \beta = 1 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{1}{4} \\ \beta = 1 \end{cases}$

(b) take $x[n] = k\delta[n]$, thus $w[n] = 0$, $n < 0$

so $w[-1] = 0$, $w[0] = \frac{1}{2} w[-1] + x[0] = k$

$w[1] = \frac{1}{2} w[0] + x[1] = \frac{k}{2}$

$w[2] = \frac{1}{2} w[1] + x[2] = \frac{k}{4}$

$\vdots$

$w[n] = \frac{k}{2^n} \Rightarrow h_1[n] = \frac{1}{2^n} u[n]$

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$y[2] = \frac{1}{4} y[1] + w[2] = \frac{k}{4^2}$

$\vdots$

$y[n] = \frac{k}{4^n} \Rightarrow h_2[n] = \frac{1}{4^n} u[n]$

$\therefore$ $h[n]$ of S_1 and S_2 is $h_1[n] * h_2[n]$

$$\Rightarrow h[n] = \sum_{-\infty}^{\infty} h_1[k] \cdot h_2[n-k]$$

$$= \begin{cases} \sum_{k=0}^{n} \frac{1}{2^k} \cdot \frac{1}{4^{n-k}} = \frac{1}{4^n} \cdot \frac{1-2^{n+1}}{1-2} = \frac{1}{2^{n-1}} - \frac{1}{4^n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$= \left(\frac{1}{2^{n-1}} - \frac{1}{4^n}\right) u[n]$$

2.21 (c) $y[n] = x[n] * h[n] = \sum_{-\infty}^{\infty} x[k] \cdot h[n-k] = \sum_{-\infty}^{\infty} \left(-\frac{1}{2}\right)^k u[k-4] \cdot 4^{n-k} u[2-n+k]$

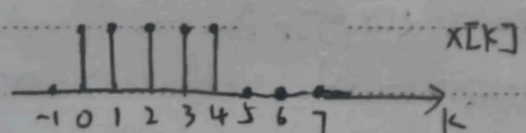
$$= \sum_{-\infty}^{\infty} \left(-\frac{1}{2}\right)^{t+4} u[t] \cdot 4^{n-4-t} u[6-n+t]$$

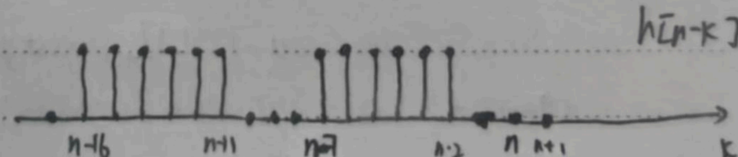
$$= \frac{4^n}{2^{12}} \sum_{-\infty}^{\infty} \left(-\frac{1}{8}\right)^t u[t] u[6-n+t]$$

$$= \begin{cases} \frac{4^n}{2^{12}} \sum_{n-6}^{\infty} \left(-\frac{1}{8}\right)^t, & n \geq 6 \\ \frac{4^n}{2^{12}} \sum_{0}^{\infty} \left(-\frac{1}{8}\right)^t, & n < 6 \end{cases}$$

$$= \begin{cases} \frac{8}{9} \left(-\frac{1}{2}\right)^{n-6}, & n \geq 6 \\ \frac{8}{9} \left(-\frac{1}{2}\right)^{12-2n}, & n < 6 \end{cases}$$

(d) $y[n] = x[n] * h[n] = \sum_{-\infty}^{\infty} x[k] \cdot h[n-k]$





$$\Rightarrow y[n] = \begin{cases} 0, & n < 2 \text{ or } n > 20 \\ n-1, & 2 \leq n \leq 6 \\ 5 & n = 7 \\ 12-n & 8 \leq n \leq 10 \\ 2 & n = 11 \\ n-10 & 12 \leq n \leq 15 \\ 21-n & 16 \leq n \leq 20 \end{cases}$$