Assignment Week 9

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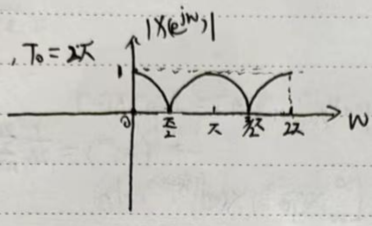
5.2

(a) $x[n] = \delta[n-1] + \delta[n+1]$, $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (\delta[n-1] + \delta[n+1]) e^{-jwn}$

$= e^{-jw} + e^{jw} = 2\cos w$

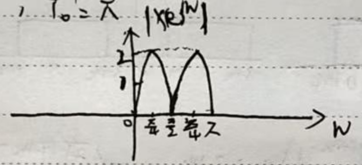
$|X(e^{jw})| = 2|\cos w|$, $T_0 = 2\pi$



(b) $x[n] = \delta[n+2] - \delta[n-2]$, $X(e^{jw}) = \sum_{-\infty}^{\infty} x[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (\delta[n+2] - \delta[n-2]) e^{-jwn}$

$= e^{j2w} - e^{j(-2w)} = 2j\sin 2w$

$|X(e^{jw})| = 2|\sin 2w|$, $T_0' = \pi$



5.5 $\because$ $X(e^{jw}) = \begin{cases} e^{j(-\frac{3}{2}w)}, & 0 \le |w| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le |w| \le \pi \end{cases}$ $\Rightarrow x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{jw}) e^{jwn} dw$

$= \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{j(-\frac{3}{2}w)} e^{jwn} dw$

$= \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{jw(n-\frac{3}{2})} dw$

$= \frac{1}{2\pi j(n-\frac{3}{2})} e^{jw(n-\frac{3}{2})} \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= \frac{1}{2\pi j(n-\frac{3}{2})} \left(e^{j\frac{\pi}{4}(n-\frac{3}{2})} - e^{-j\frac{\pi}{4}(n-\frac{3}{2})}\right)$

$= \frac{1}{2\pi j(n-\frac{3}{2})} 2j \sin(\frac{\pi}{4}n - \frac{3}{8}\pi)$

$= \frac{\sin(\frac{\pi}{4}n - \frac{3}{8}\pi)}{\pi(n-\frac{3}{2})}$

So $x[n] = 0 \Rightarrow \frac{\pi}{4}(n-\frac{3}{2}) = k\pi$ $(k \in \mathbb{Z} \setminus \{0\})$

$\Rightarrow n = 4k + \frac{3}{2}$ $(k \in \mathbb{Z} \setminus \{0\})$ $\Rightarrow$ no integer n satisfies.

when $n \to \infty$ $\lim_{n\to\infty} x[n] = \lim_{n\to\infty} \frac{\sin(\frac{\pi}{4}n - \frac{3}{8}\pi)}{\pi(n-\frac{3}{2})} = 0$, when $n \to -\infty$, the same.

So to get $x[n] = 0$, only we take $n \to \infty$ or $n \to -\infty$.

5.15 take $X(e^{jw}) = \begin{cases} 1, & -w_c+2k\pi \le w \le w_c+2k\pi \\ 0, & \text{else.} \end{cases}$ $(k \in Z)$ $\Rightarrow x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw}) e^{jwn} dw$

$= \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn} dw$

$= \frac{1}{2\pi} \cdot \frac{1}{jn} \cdot e^{jwn}\Big|_{-w_c}^{w_c}$

$= \frac{1}{2\pi jn}(e^{jw_cn} - e^{-jw_cn})$

$= \frac{\sin w_c n}{\pi n}$

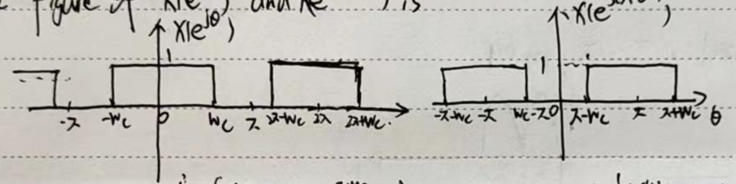
According to Multiplication property: $y[n] = x[n] \cdot x[n]$

$\rightarrow Y(e^{jw}) = \frac{1}{2\pi} X(e^{jw}) * X(e^{jw})$

$\Rightarrow Y(e^{jw}) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta}) X(e^{j(w-\theta)}) d\theta$

$\therefore Y(e^{j\pi}) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta}) X(e^{j(\pi-\theta)}) d\theta$

the figure of $X(e^{j\theta})$ and $X(e^{j(\pi-\theta)})$ is



$\therefore Y(e^{j\pi}) = \begin{cases} \frac{1}{2\pi}\left(\int_{\pi-w_c}^{w_c} d\theta + \int_{2\pi-w_c}^{\pi+w_c} d\theta\right), & w_c \ge \pi - w_c \\ 0, & w_c < \pi - w_c \end{cases} = \begin{cases} \frac{1}{2\pi}(4w_c - 2\pi), & w_c \ge \frac{\pi}{2} \\ 0, & w_c < \frac{\pi}{2} \end{cases}$

$\therefore Y(e^{j\pi}) = \frac{1}{2} \Rightarrow \frac{1}{2\pi}(4w_c - 2\pi) = \frac{1}{2},\ w_c \ge \frac{\pi}{2} \Rightarrow w_c = \frac{3}{4}\pi$

5.21

(a) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (u[n-2] - u[n-6]) e^{-jwn} = \sum_{n=2}^{5} e^{-jwn} = \frac{e^{-j2w}(1-e^{-j4w})}{1-e^{-jw}}$

$= \frac{e^{-j2w} - e^{-j6w}}{1-e^{-jw}}$

(b) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} (\frac{1}{2})^{-n} u[-n-1] e^{-jwn} = \sum_{n=-\infty}^{-1} (\frac{1}{2})^{-n} e^{-jwn} = \sum_{n=1}^{\infty} (\frac{1}{2})^n e^{jwn} = \frac{\frac{1}{2}e^{jw}}{1-\frac{1}{2}e^{jw}}$

$= \frac{e^{jw}}{2-e^{jw}}$

(c) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} (\frac{1}{3})^{|n|} u[-n-2] e^{-jwn} = \sum_{n=-\infty}^{-2} (\frac{1}{3})^{-n} e^{-jwn} = \sum_{n=2}^{\infty} (\frac{1}{3})^n e^{jwn} = \frac{\frac{1}{9}e^{j2w}}{1-\frac{1}{3}e^{jw}}$

$= \frac{e^{j2w}}{9-3e^{jw}}$

(d) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} 2^n \sin(\frac{\pi}{4}n) u[-n] e^{-jwn} = \sum_{n=-\infty}^{0} 2^n \cdot \frac{1}{2j} (e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}) e^{-jwn}$

$$= \frac{1}{2j} \sum_{n=-\infty}^{0} \left[2^n e^{j(\frac{\pi}{4}-w)n} - 2^n e^{-j(\frac{\pi}{4}+w)n} \right]$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left[(\frac{1}{2})^n e^{j(w-\frac{\pi}{4})n} - (\frac{1}{2})^n e^{j(w+\frac{\pi}{4})n} \right]$$

$$= \frac{1}{2j} \left(\frac{1}{1-\frac{1}{2}e^{j(w-\frac{\pi}{4})}} - \frac{1}{1-\frac{1}{2}e^{j(w+\frac{\pi}{4})}} \right)$$

$$= \frac{1}{2j - e^{j(w+\frac{\pi}{4})}} - \frac{1}{2j - e^{j(w+\frac{3}{4}\pi)}}$$

(e) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (\frac{1}{2})^{|n|} \cos(\frac{\pi}{8}(n-1)) e^{-jwn}$

$$= \sum_{n=-\infty}^{-1} \left[(\frac{1}{2})^{-n} (e^{j\frac{\pi}{8}(n-1)} + e^{-j\frac{\pi}{8}(n-1)}) \frac{1}{2} e^{-jwn} \right]$$

$$+ \sum_{n=0}^{\infty} \left[(\frac{1}{2})^n (e^{j\frac{\pi}{8}(n-1)} + e^{-j\frac{\pi}{8}(n-1)}) \frac{1}{2} e^{-jwn} \right]$$

$$= \frac{1}{2} \sum_{n=1}^{\infty} \left[(\frac{1}{2})^n e^{j(w-\frac{\pi}{8})n} \cdot e^{-j\frac{\pi}{8}} + (\frac{1}{2})^n e^{j(w+\frac{\pi}{8})n} \cdot e^{j\frac{\pi}{8}} \right]$$

$$+ \frac{1}{2} \sum_{n=0}^{\infty} \left[(\frac{1}{2})^n e^{j(\frac{\pi}{8}-w)n} \cdot e^{-j\frac{\pi}{8}} + (\frac{1}{2})^n e^{-j(w+\frac{\pi}{8})} e^{j\frac{\pi}{8}} \right]$$

$$= \frac{1}{2} \frac{\frac{1}{2} e^{j(w-\frac{\pi}{8})} \cdot e^{-j\frac{\pi}{8}}}{1-\frac{1}{2}e^{j(w-\frac{\pi}{8})}} + \frac{1}{2} \cdot \frac{\frac{1}{2} e^{j(w+\frac{\pi}{8})} \cdot e^{j\frac{\pi}{8}}}{1-\frac{1}{2}e^{j(w+\frac{\pi}{8})}} + \frac{1}{2} \frac{e^{-j\frac{\pi}{8}}}{1-\frac{1}{2}e^{j(\frac{\pi}{8}-w)}}$$

$$+ \frac{1}{2} \frac{e^{j\frac{\pi}{8}}}{1-\frac{1}{2}e^{-j(w+\frac{\pi}{8})}}$$

$$= \frac{e^{j(w-\frac{\pi}{4})}}{4 - 2e^{j(w-\frac{\pi}{8})}} + \frac{e^{j(w+\frac{\pi}{4})}}{4 - 2e^{j(w+\frac{\pi}{8})}} + \frac{e^{-j\frac{\pi}{8}}}{2 - e^{j(\frac{\pi}{8}-w)}} + \frac{e^{j\frac{\pi}{8}}}{2 - e^{-j(w+\frac{\pi}{8})}}$$

(f) $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = \sum_{n=-3}^{3} n e^{-jwn}$ and $e^{-jw} X(e^{jw}) = \sum_{n=-3}^{3} n e^{-jw(n+1)}$

$$= \sum_{n=-2}^{3} (n-1) e^{-jwn} + 3e^{-j4w}$$

$$\Rightarrow (e^{-jw} - 1) X(e^{jw}) = 3e^{-j4w} - (-3)e^{j3w} - \sum_{n=-2}^{3} e^{-jwn} = 3e^{-j4w} + 3e^{j3w} - \frac{e^{j2w}(1-e^{-j6w})}{1-e^{-jw}}$$

$$= \frac{-4e^{-j4w} + 3e^{j3w} - 4e^{j2w} - 3e^{-j5w}}{1-e^{-jw}}$$

$$\Rightarrow X(e^{jw}) = \frac{3e^{-j5w} + 4e^{j2w} - 4e^{-j4w} - 3e^{j3w}}{(1-e^{-jw})^2}$$

(h) $x[n] = \sin\frac{5\pi}{3}n + \cos\frac{7}{3}\pi n = \frac{1}{2j} e^{j\frac{5\pi}{3}n} - \frac{1}{2j} e^{-j\frac{5}{3}\pi n} + \frac{1}{2} e^{j\frac{7\pi}{3}n} + \frac{1}{2} e^{-j\frac{7\pi}{3}n}$

$\rightarrow X(e^{jw}) = \frac{\pi}{j}\delta(w-\frac{5}{3}\pi) - \frac{\pi}{j}\delta(w+\frac{5\pi}{3}) + \pi\delta(w-\frac{7\pi}{3}) + \pi\delta(w+\frac{7}{3}\pi)$ for $|w|<\pi$