Assignment Week 6

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3.2 $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} = 1 + e^{j\frac{\pi}{4}} e^{2j\omega_0 n} + e^{-j\frac{\pi}{4}} \cdot e^{-2j\omega_0 n} + 2e^{j\frac{\pi}{3}} \cdot e^{4j\omega_0 n} + 2e^{-j\frac{\pi}{3}} \cdot e^{-4j\omega_0 n}$

$= 1 + 2\cos(2\omega_0 n + \frac{\pi}{4}) + 4\cos(4\omega_0 n + \frac{\pi}{3})$

$= 1 + 2\cos(\frac{4\pi}{5} n + \frac{\pi}{4}) + 4\cos(\frac{8\pi}{5} n + \frac{\pi}{3})$

$= 1 + 2\sin(\frac{4}{5}\pi n + \frac{3}{4}\pi) + 4\sin(\frac{8}{5}\pi n + \frac{5}{6}\pi)$

3.27 $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} = 2 + 2e^{j\frac{\pi}{6}} e^{2j\omega_0 n} + 2e^{-j\frac{\pi}{6}} \cdot e^{-2j\omega_0 n} + e^{j\frac{\pi}{3}} e^{4j\omega_0 n} + e^{-j\frac{\pi}{3}} \cdot e^{-4j\omega_0 n}$

$= 2 + 4\cos(2\omega_0 n + \frac{\pi}{6}) + 2\cos(4\omega_0 n + \frac{\pi}{3})$

$= 2 + 4\sin(\frac{4\pi}{5} n + \frac{2}{3}\pi) + 2\sin(\frac{8\pi}{5} n + \frac{5}{6}\pi)$

3.36 (a) take $x[n] = e^{j\omega n} \Rightarrow y[n] = H(e^{j\omega}) e^{j\omega n}$

then $H(e^{j\omega}) e^{j\omega n} - \frac{1}{4} H(e^{j\omega}) e^{j\omega(n-1)} = e^{j\omega n}$

$\Rightarrow H(e^{j\omega}) = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$

and when $x[n] = \sin(\frac{3}{4}\pi n) \Rightarrow N = 8,\ \omega_0 = \frac{\pi}{4}$ $x[n] = \frac{1}{2j}(e^{3j\omega_0 n} - e^{-3j\omega_0 n})$

$\Rightarrow a_3 = a_{-3}^* = \frac{1}{2j}$

$\therefore b_3 = H(e^{j3\omega_0}) \frac{1}{2j} = \frac{1}{2j - \frac{j}{2} e^{-3j\omega_0}} = \frac{1}{2j - \frac{j}{2} e^{-\frac{3\pi}{4} j}}$, $b_{-3} = H(e^{-j3\omega_0}) \frac{-1}{2j} = \frac{1}{-2j + \frac{j}{2} e^{3j\omega_0}} = \frac{1}{-2j + \frac{j}{2} e^{\frac{3}{4}\pi j}}$

so the non-zero Fourier series coefficients of $y[n]$ are $\begin{cases} b_3 = \frac{1}{2j - \frac{j}{2} e^{-\frac{3\pi}{4} j}} \\ b_{-3} = \frac{1}{-2j + \frac{j}{2} e^{\frac{3\pi}{4} j}} \end{cases}$

(b) when $x[n] = \cos\frac{\pi}{4} n + 2\cos\frac{\pi}{2} n \Rightarrow N = 8,\ \omega_0 = \frac{\pi}{4},\ x[n] = \frac{1}{2}(e^{j\omega_0 n} + e^{-j\omega_0 n})$ $+ (e^{2j\omega_0 n} + e^{-2j\omega_0 n})$

$\Rightarrow a_1 = a_{-1} = \frac{1}{2},\quad a_2 = a_{-2} = 1$

$\Rightarrow b_1 = H(e^{j\omega_0}) \cdot \frac{1}{2} = \frac{1}{2 - \frac{1}{2} e^{-\frac{\pi}{4} j}}$, $b_{-1} = H(e^{-j\omega_0}) \cdot \frac{1}{2} = \frac{1}{2 - \frac{1}{2} e^{\frac{\pi}{4} j}}$

$b_2 = H(e^{2j\omega_0}) \cdot 1 = \frac{1}{1 - \frac{1}{4} e^{-\frac{\pi}{2} j}}$, $b_{-2} = H(e^{-2j\omega_0}) \cdot 1 = \frac{1}{1 - \frac{1}{4} e^{\frac{\pi}{2} j}}$

3.38 $H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n] e^{-jwn} = -e^{2jw} - e^{jw} + 1 + e^{-jw} + e^{-2jw}$

$$x[n] = \begin{cases} 1, & n=4k \\ 0, & \text{else} \end{cases} \Rightarrow N = 4, \quad w_0 = \frac{\pi}{2}$$

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jkw_0 n} = \frac{1}{4}$$

$$\therefore b_k = H(e^{jkw_0}) a_k = \frac{1}{4}\left(-e^{jk\pi} - e^{\frac{jk\pi}{2}} + 1 + e^{-\frac{jk\pi}{2}} + e^{-jk\pi}\right)$$

$$= \frac{1}{4}\left(1 - 2j\sin(k\pi) - 2j\sin\left(\frac{k}{2}\pi\right)\right)$$

$$= \frac{1}{4} - \frac{j}{2}\sin\frac{k}{2}\pi$$

3.50 $w_0 = \frac{2\pi}{8} = \frac{\pi}{4}$, by shift property. $x[n] \xleftrightarrow{FS} a_k \quad x[n]e^{j4w_0 n} \xleftrightarrow{FS} a_{k-4}$

$x[n] = \sum_{k=\langle N\rangle} a_k e^{jkw_0 n}$, $x[n]e^{j4w_0 n} = (-1)^n x[n] = \sum_{k=\langle N\rangle} a_{k-4} e^{jkw_0 n}$. as $a_k = -a_{k-4}$

$\Rightarrow x[n] = (-1)^{n+1} x[n] \Rightarrow$ when $n = 2m$, $x[2m] = -x[2m] \Rightarrow x[2m] = 0$

as $x[2n+1] = (-1)^n$. so one period of $x[n]$ is