4.5

$$x(t) = -\frac{2\sin(3(t-3/2))}{\pi(t-3/2)}$$

$$t = \frac{k\pi}{3} + \frac{3}{2}$$

4.21

(b) the given signal is

$$x(t) = e^{-3t}\sin(2t)\,u(t) + e^{3t}\sin(2t)\,u(-t)$$

we have

$$x_1(t) = e^{-3t}\sin(2t)\,u(t) \overset{FS}{\leftrightarrow} X_1(j\omega) = \left(\frac{1}{2j}\right)\left(\frac{1}{3-j2+j\omega} - \frac{1}{3+j2+j\omega}\right) = \frac{2}{(3+j\omega)^2+4}$$

$$x_2(t) = e^{3t}\sin(2t)\,u(-t) = -x_1(-t) \overset{FS}{\leftrightarrow} -X_1(-j\omega) = X_2(j\omega)$$

$$X_2(j\omega) = -\left(\frac{1}{2j}\right)\left(\frac{1}{3-j2-j\omega} - \frac{1}{3+j2-j\omega}\right) = -\frac{2}{(3-j\omega)^2+4}$$

Therefore

$$X(j\omega) = X_1(j\omega) + X_2(j\omega) = \frac{2}{(3+j\omega)^2+4} - \frac{2}{(3-j\omega)^2+4} = \frac{3j}{9+(\omega+2)^2} - \frac{3j}{9+(\omega-2)^2}$$

(g) the given signal is

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = t[u(t) - u(t-1)] + [u(t-1) - u(t-2)]$$

$$x_2(t) = -x_1(-t)$$

we have

$$X_1(j\omega) = \int_{-\infty}^{+\infty} x_1(t)e^{-j\omega t}dt = \frac{j\omega e^{-2j\omega} + e^{-j\omega} - 1}{\omega^2}$$

$$X_2(j\omega) = -X_1(-j\omega) = \frac{j\omega e^{2j\omega} - e^{j\omega} + 1}{\omega^2}$$

Therefore

$$X(j\omega) = X_1(j\omega) + X_2(j\omega) = \frac{j\omega(e^{2j\omega} + e^{-2j\omega}) + (e^{-j\omega} - e^{j\omega})}{\omega^2} = \frac{2j}{\omega^2}[\omega\cos(2\omega) - \sin(\omega)]$$

(h)

If

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k),$$

then

$$x(t) = 2x_1(t) + x_1(t-1).$$

Therefore,

$$X(j\omega) = X_1(j\omega)[2 + e^{-\omega}] = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - k\pi)[2 + (-1)^k].$$

4.22

(c)

$$X(j\omega) = |\omega|[u(\omega+1) - u(\omega-1)]e^{-j3\omega}$$

we have:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-1}^{0} -\omega e^{(jt-j3)\omega}d\omega + \frac{1}{2\pi}\int_{0}^{1} \omega e^{(jt-j3)\omega}d\omega$$

$$x(t) = \frac{1}{\pi}\left[\frac{\cos(t-3)}{(t-3)^2} + \frac{\sin(t-3)}{t-3} - \frac{1}{(t-3)^2}\right]$$

(e)

$$x(t) = \frac{\cos 3t}{j\pi t} + \frac{\sin t - \sin 2t}{j\pi t^2}.$$

4.27

(a)

The Fourier transform $X(j\omega)$ is

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{1}^{2} e^{-j\omega t}dt - \int_{2}^{3} e^{-j\omega t}dt \\ &= 2\frac{\sin(\omega/2)}{\omega}\{1 - e^{-j\omega}\}e^{-j3\omega/2} \end{aligned}$$

(b)

The Fourier series coefficients a_k are

$$\begin{aligned} a_k &= \frac{1}{T}\int_{<T>} \tilde{x}(t)e^{-j\frac{2\pi}{T}kt} \\ &= \frac{1}{2}\{\int_{1}^{2} e^{-j\frac{2\pi}{T}kt}dt - \int_{2}^{3} e^{-j\frac{2\pi}{T}kt}dt\} \\ &= \frac{\sin(k\pi/2)}{k\pi}\{1 - e^{-jk\pi}\}e^{-j3k\pi/2} \end{aligned}$$

Comparing the answers to parts (a) and (b), it is clear that

$$a_k = \frac{1}{T}X(j\frac{2\pi k}{T}),$$

where $T = 2$.