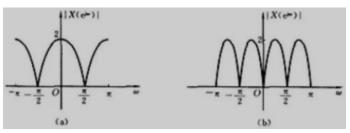
(a)

(a) 
$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} \delta[n-1]e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta[n+1]e^{-j\omega n}$$
 
$$= e^{-j\omega} + e^{j\omega}$$
 
$$= 2\cos\omega$$
 
$$\left|X\left(e^{j\omega}\right)\right| = \left|2\cos\omega\right|$$
 (b)

$$egin{aligned} Xig(e^{j\omega}ig) &= \sum_{n=-\infty}^\infty \delta[n+2] \mathrm{e}^{-j\omega n} - \sum_{n=-\infty}^\infty \delta[n-2] e^{-j\omega n} \ &= e^{j2\omega} - e^{-j2\omega} \ &= 2j\sin2\omega \ ig|Xig(e^{j\omega}ig)ig| = |2\sin2\omega| \end{aligned}$$



Pic. 1

## 5.5 (20)

$$egin{aligned} x[n] &= rac{1}{2\pi} \int_{-rac{4}{\pi}}^{rac{4}{\pi}} 1 \cdot \mathrm{e}^{-jrac{3}{2}\omega} \cdot \mathrm{e}^{j\omega n} \; \mathrm{d}\omega \ &= rac{1}{2\pi} \int_{-rac{\pi}{4}}^{rac{\pi}{4}} \mathrm{e}^{j(n-rac{3}{2})\omega} \mathrm{d}\omega \ &= rac{1}{2\pi} rac{1}{j(n-3/2)} \mathrm{e}^{j(n-rac{3}{2})\omega} igg|_{-rac{\pi}{4}}^{rac{\pi}{4}} \ &= rac{\sin\left[\left(n-rac{3}{2}
ight)rac{\pi}{4}
ight]}{\pi\left(n-rac{3}{2}
ight)} \ &= rac{\sin\left(rac{\pi}{4}n-rac{3\pi}{8}
ight)}{\pi\left(n-rac{3}{2}
ight)} \end{aligned}$$

$$Let\sin\!\left(rac{\pi}{4}n-rac{3\pi}{8}
ight)=0$$
  $Thus\ x[n]=0\ when\ n=4k+rac{3}{2},\ k=\pm 1.\pm 2.\ldots$ 

$$Since \ n \ is \ integer, n 
eq 4k + 3/2 \ x[n] = 0 \ only \ for \ n = \pm \infty$$

3

3

## 5.15 (20)

$$egin{align} Let \ x[n] &= rac{\sin \omega_c n}{\pi n}, \ Xig(e^{j\omega}ig) &= u(\omega + \omega_c) - u(\omega - \omega_c), \ -\pi < \omega \leqslant \pi \ Yig(e^{j\omega}ig) &= rac{1}{2\pi} \int_{2\pi} Xig(e^{j heta}ig) Xig(e^{j(\omega - heta)}ig) d heta \ \end{split}$$

If 
$$2\omega_{\rm c} \leqslant \pi$$
,

$$Yig(e^{j\omega}ig) = egin{cases} -rac{1}{2\pi}|\omega| + rac{\omega_c}{\pi}, & 0\leqslant |\omega|\leqslant 2\omega_c \ 0, & 2\omega_e < |\omega|\leqslant \pi \end{cases}$$

 $Y(e^{j\pi})=0\ not\ conform\ the\ question,$ 

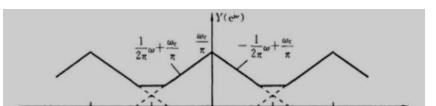
 $thus~2\omega_{
m c}>\pi$ 

As shown in fig.2,

$$Yig(e^{j\pi}ig) = 2igg(-rac{1}{2\pi} imes\pi+rac{\omega_c}{\pi}igg) = -1+rac{2\omega_c}{\pi}$$

$$Since\ Yig(e^{j\pi}ig)=rac{1}{2}$$

Thus 
$$\omega_c=rac{3}{4}\pi$$



0 Pic. 2

(a)

$$x[n] = u[n-2] - u[n-6] \ = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

$$egin{align} Xig(e^{j\omega}ig) &= e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega} \ &= rac{e^{-j2\omega}ig(1-e^{-j4\omega}ig)}{1-e^{-j\omega}} \end{split}$$

(b)

$$u[n] = \left(rac{1}{2}
ight)^n u[n-1] = rac{1}{2} \cdot \left(rac{1}{2}
ight)^{n-1} u[n-1]$$

$$X_1ig(e^{j\omega}ig)=rac{rac{1}{2}e^{-i\omega}}{1-rac{1}{2}e^{-j\omega}}$$

3

$$X(e^{j\omega})=X_1(e^{-j\omega})=rac{rac{1}{2}e^{i\omega}}{1-rac{1}{2}e^{j\omega}}$$

(c)

$$egin{align} x[n] &= \left(rac{1}{3}
ight)^{|n|} u[-n-2] = \left(rac{1}{3}
ight)^{-n} u[-n-2] \ x_1[n] &= \left(rac{1}{3}
ight)^n u[n-2] = rac{1}{9} \cdot \left(rac{1}{3}
ight)^{n-2} u[n-2] \ X_1ig(e^{j\omega}ig) &= rac{rac{1}{9}e^{-j2\omega}}{1-rac{1}{3}e^{-j\omega}} \ \end{array}$$

$$Since \ x[n] = x_1[-n], \ X(e^{j\omega}) = X_1(e^{-j\omega}) = rac{rac{1}{9}e^{j2\omega}}{1 - rac{1}{2}e^{j\omega}}$$

(d)

$$egin{align} x_1[n] &= 2^{-n} \sin\Bigl(-rac{\pi}{4}n\Bigr) u[n] \ &= -igg(rac{1}{2}igg)^n \sin\Bigl(rac{\pi}{4}n\Bigr) u[n] \ &= -rac{1}{2j}igg(rac{1}{2}e^{jrac{\pi}{4}}igg)^n u[n] + rac{1}{2j}igg(rac{1}{2}e^{-jrac{\pi}{4}}igg)^* u[n] \end{aligned}$$

$$X_1ig(e^{j\omega}ig) = -rac{1}{2j}\cdotrac{1}{1-rac{1}{2}e^{jrac{\pi}{4}}e^{-j\omega}} + rac{1}{2j}\cdotrac{1}{1-rac{1}{2}e^{-jrac{\pi}{4}}e^{-j\omega}}$$

Since  $x[n] = x_1[-n]$ ,

$$egin{split} Xig(e^{j\omega}ig) &= X_1ig(e^{-j\omega}ig) \ &= -rac{1}{2j}\cdotrac{1}{1-rac{1}{2}e^{jrac{\pi}{4}}e^{j\omega}} + rac{1}{2j}\cdotrac{1}{1-rac{1}{2}e^{-jrac{\pi}{4}}e^{j\omega}} \ &= rac{-\sqrt{2}e^{j\omega}}{4-2\sqrt{2}e^{j\omega}+e^{j2\omega}} \end{split}$$

(e)

Solution 1

$$egin{align} x_1[n] &= \left(rac{1}{2}
ight)^{|n|}, \ x_2[n] &= \cos\left(rac{\pi}{8}(n-1)
ight) = \cos\left(rac{\pi}{8}n - rac{\pi}{8}
ight) \ X_1ig(\mathrm{e}^{j\omega}ig) &= rac{1-\left(rac{1}{2}
ight)^2}{1-2 imesrac{1}{2}\cos\omega + rac{1}{4}} = rac{3}{5-4\cos\omega} \ X_2ig(\mathrm{e}^{j\omega}ig) &= \pi\mathrm{e}^{-\mathrm{j}rac{\pi}{8}}\delta\Big(\omega - rac{\pi}{8}\Big) + \pi\mathrm{e}^{jrac{\pi}{8}}\delta\Big(\omega + rac{\pi}{8}\Big), \quad -\pi < \omega \leqslant \pi \ Thus \ \end{array}$$

2

Thus

$$egin{aligned} Xig(\mathrm{e}^{j\omega}ig) &= rac{1}{2\pi} \int_{-\pi}^{\pi} X_1ig(\mathrm{e}^{j heta}ig) X_2ig(\mathrm{e}^{j(\omega- heta)}ig) \mathrm{d} heta \ &= rac{rac{3}{2}\mathrm{e}^{-jrac{\pi}{8}}}{5-4\cos(\omega-rac{\pi}{8})} + rac{rac{3}{2}\mathrm{e}^{jrac{\pi}{8}}}{5-4\cos(\omega+rac{\pi}{8})} \end{aligned}$$

## Solution 2

$$egin{aligned} x[n] &= \left(rac{1}{2}
ight)^{|n|} \cos\left(rac{\pi}{8}(n-1)
ight) \ &= \left(rac{1}{2}
ight)^n \cos\left(rac{\pi}{8}n - rac{\pi}{8}
ight) u[n] \ &+ \left(rac{1}{2}
ight)^{-n} \cos\left(-rac{\pi}{8}n + rac{\pi}{8}
ight) u[-n-1] \ x_1[n] &= \left(rac{1}{2}
ight)^n \cos\left(rac{\pi}{8}n - rac{\pi}{8}
ight) u[n] \ x_2[n] &= \left(rac{1}{2}
ight)^n \cos\left(rac{\pi}{8}n + rac{\pi}{8}
ight) u[n-1] \end{aligned}$$

Since

$$egin{aligned} x_1[n] &= rac{1}{2}e^{-jrac{\pi}{8}}igg(rac{1}{2}e^{jrac{\pi}{8}}igg)^nu[n] + rac{1}{2}e^{jrac{\pi}{8}}igg(rac{1}{2}e^{-jrac{\pi}{8}}igg)^nu[n] \ x_2[n] &= rac{1}{2}e^{jrac{\pi}{8}}igg(rac{1}{2}e^{jrac{\pi}{8}}igg)^nu[n-1] + rac{1}{2}e^{-jrac{\pi}{8}}igg(rac{1}{2}e^{-jrac{\pi}{8}}igg)^nu[n-1] \ &= (rac{1}{2}e^{jrac{\pi}{8}})^2(rac{1}{2}e^{jrac{\pi}{8}})^{(n-1)}u[n-1] + (rac{1}{2}e^{-jrac{\pi}{8}})^2(rac{1}{2}e^{-jrac{\pi}{8}})^{(n-1)}u[n-1] \end{aligned}$$

then

$$egin{aligned} X_1(e^{j\omega}) &= rac{rac{1}{2}e^{-jrac{\pi}{8}}}{1-rac{1}{2}e^{jrac{\pi}{8}}e^{-j\omega}} + rac{rac{1}{2}e^{jrac{\pi}{8}}}{1-rac{1}{2}e^{-jrac{\pi}{8}}e^{-j\omega}} \ X_2(e^{j\omega}) &= rac{(rac{1}{2}e^{jrac{\pi}{8}})^2e^{-j\omega}}{1-rac{1}{2}e^{jrac{\pi}{8}}e^{-j\omega}} + rac{(rac{1}{2}e^{-jrac{\pi}{8}})^2e^{-j\omega}}{1-rac{1}{2}e^{-jrac{\pi}{8}}e^{-j\omega}} \end{aligned}$$

Since

$$x[n]=x_1[n]+x_2[-n],$$

then

$$egin{align} Xig(e^{j\omega}ig) &= X_1ig(e^{j\omega}ig) + X_2ig(e^{-j\omega}ig) \ &= rac{rac{3}{2}e^{-jrac{\pi}{8}}}{5 - 4\cosig(\omega - rac{\pi}{8}ig)} + rac{rac{3}{2}e^{jrac{\pi}{8}}}{5 - 4\cosig(\omega + rac{\pi}{8}ig)} \end{array}$$

(f)

$$x[n] = -3\delta[n+3] - 2\delta[n+2] - \delta[n+1] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$$

(h)

$$egin{align} X(e^{j\omega}) &= j\pi \sum_{l=-\infty}^\infty \{\delta(\omega+rac{5\pi}{3}-2\pi l)-\delta(\omega-rac{5\pi}{3}-2\pi l)\} \ &+\pi \sum_{l=-\infty}^\infty \{\delta(\omega+rac{7\pi}{3}-2\pi l)-\delta(\omega-rac{7\pi}{3}-2\pi l)\} \ X(e^{j\omega}) &= j\pi \{\delta(\omega-rac{\pi}{3})-\delta(\omega+rac{\pi}{3})\} \ &+\pi \{\delta(\omega+rac{\pi}{3})+\delta(\omega-rac{\pi}{3})\}, \ -\pi < \omega \leq \pi \ \end{aligned}$$