4.50. Consider interpolating a signal $x[n]$ by repeating each value q times as depicted in Fig. P4.50. That is, we define $x_o[n] = x[\text{floor}(\frac{n}{q})]$ where floor(z) is the integer less than or equal to z. Letting $x_z[n]$ be derived from $x[n]$ by inserting $q-1$ zeros between each value of $x[n]$, that is,

$$x_z[n] = \begin{cases} x[\frac{n}{q}], & \frac{n}{q} \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

We may then write $x_o[n] = x_z[n] * h_o[n]$, where $h_o[n]$ is:

$$h_o[n] = \begin{cases} 1, & 0 \le n \le q-1 \\ 0, & \text{otherwise} \end{cases}$$

Note that this is the discrete-time analog of the zero-order hold. The interpolation process is completed by passing $x_o[n]$ through a filter with frequency response $H(e^{j\Omega})$.

$$X_z(e^{j\Omega}) = X(e^{j\Omega q})$$

(a) Express $X_o(e^{j\Omega})$ in terms of $X(e^{j\Omega})$ and $H_o(e^{j\Omega})$. Sketch $|X_o(e^{j\Omega})|$ if $x[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n}$.

$$X_o(e^{j\Omega}) = X(e^{j\Omega q})H_o(e^{j\Omega})$$

$$x[n] = \frac{\sin(\frac{3\pi}{4}n)}{\pi n} \xleftrightarrow{DTFT} X(e^{j\Omega}) = \begin{cases} 1 & |\Omega| < \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} \le |\Omega| < \pi, \quad 2\pi \text{ periodic} \end{cases}$$

$$|X_o(e^{j\Omega})| = |X(e^{j\Omega q})| \left| \frac{\sin(\Omega\frac{q}{2})}{\sin(\frac{\Omega}{2})} \right|$$

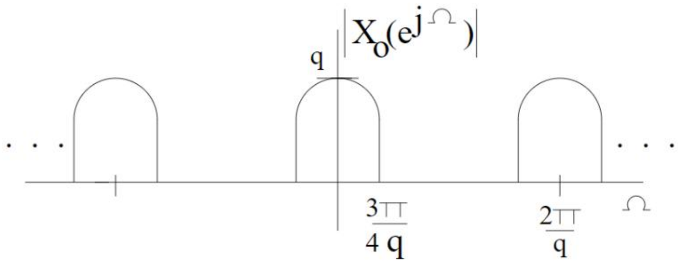

Figure P4.50. (a) Sketch of $|X_o(e^{j\Omega})|$

(b) Assume $X(e^{j\Omega})$ is as shown in Fig. P4.49. Specify the constraints on $H(e^{j\Omega})$ so that ideal interpolation is obtained for the following cases.

For ideal interpolation, discard components other than those centered at multiples of 2π. Also, some correction is needed to correct for magnitude and phase distortion.

$$H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})}{\sin(\Omega\frac{q}{2})} e^{j\Omega\frac{q}{2}} & |\Omega| < \frac{W}{q} \\ 0 & \frac{W}{q} \le |\Omega| < 2\pi - \frac{W}{q}, \quad 2\pi \text{ periodic} \end{cases}$$

(i) $q = 2, \;\; W = \frac{3\pi}{4}$

$$H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})e^{j\Omega}}{\sin(\Omega)} & |\Omega| < \frac{3\pi}{8} \\ 0 & \frac{3\pi}{8} \le |\Omega| < \frac{13\pi}{8}, \quad 2\pi \text{ periodic} \end{cases}$$

(ii) $q = 4, \;\; W = \frac{3\pi}{4}$

$$H(e^{j\Omega}) = \begin{cases} \frac{\sin(\frac{\Omega}{2})}{\sin(2\Omega)} e^{j2\Omega} & |\Omega| < \frac{3\pi}{16} \\ 0 & \frac{3\pi}{16} \le |\Omega| < \frac{29\pi}{16}, \quad 2\pi \text{ periodic} \end{cases}$$

4.51. The system shown in Fig. P4.51 is used to implement a bandpass filter. The discrete-time filter $H(e^{j\Omega})$ has frequency response on $-\pi < \Omega \leq \pi$

$$H(e^{j\Omega}) = \begin{cases} 1, & \Omega_a \leq |\Omega| \leq \Omega_b \\ 0, & \text{otherwise} \end{cases}$$

Find the sampling interval T_s, Ω_a, Ω_b, W_1, W_2, W_3, and W_4, so that the equivalent continuous-time frequency response $G(j\omega)$ satisfies

$$0.9 < |G(j\omega)| < 1.1, \quad \text{for } 100\pi < \omega < 200\pi$$

$$G(j\omega) = 0 \quad \text{elsewhere}$$

In solving this problem, choose W_1 and W_3 as small as possible and choose T_s, W_2 and W_4 as large as possible.

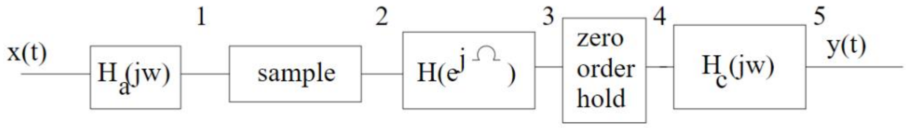

Figure P4.51. Graph of the system

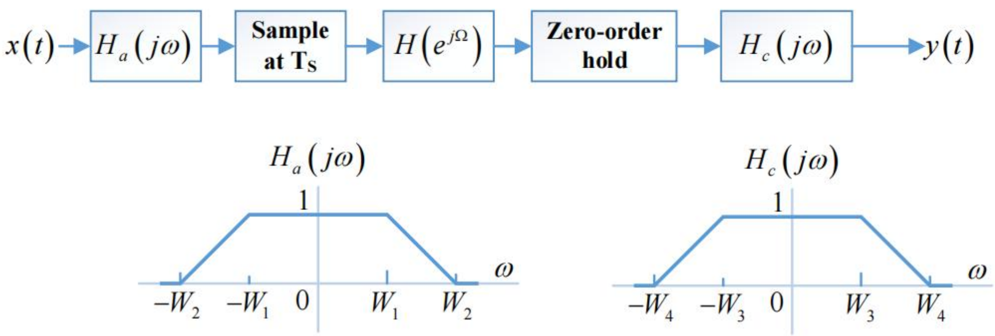

FIGURE P4.51

(3) Passband:

$$\begin{aligned}
100\pi < \omega &< 200\pi \\
\text{Thus} & \\
\Omega_a &= 100\pi T_s \\
\Omega_b &= 200\pi T_s \\
(4)\ |H_o(j\omega)| &= \left|\frac{2\sin(\omega\frac{T_s}{2})}{\omega}\right| \\
\text{at } \omega = 100\pi & \\
\frac{2\sin(50\pi T_s)}{100\pi T_s} &< 1.1 \\
\text{at } \omega = 200\pi & \\
\frac{2\sin(100\pi T_s)}{200\pi T_s} &> 0.9 \\
\text{implies:} & \\
T_s(100\pi) &< 0.785 \\
\max T_s &= 0.0025 \\
(5)\ \min W_3 &= 200\pi \\
\max W_4 &= \frac{2\pi}{T_s} - 200\pi = 600\pi \\
(3)\ \Omega_a &= 0.25\pi \\
\Omega_b &= 0.5\pi \\
(1) \text{ and } (2) & \\
\min W_1 &= 200\pi \\
\max W_2 &= \frac{1}{2}\frac{2\pi}{T_s} = 400\pi, \text{ No overlap.}
\end{aligned}$$