APPENDIX

A. Formula and Definition

1. Euler Expansion: $e^{j\pm x}=\cos x\pm j\sin x$.
2. Continuous-time Convolution: $x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$
3. Discrete-time Convolution: $x[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]$
4. Continuous-time Fourier Series: $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk(2\pi/T)t}, a_k=\frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$
5. Discrete-time Fourier Series: $x[n]=\sum_{k=\langle N\rangle}a_k e^{jk(2\pi/N)n}, a_k=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk(2\pi/N)n}$
6. Continuous-time Fourier Transform: $x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega, X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$
7. Discrete-time Fourier Transform: $x[n]=\frac{1}{2\pi}\int_{2\pi}X\left(e^{j\omega}\right)e^{j\omega n}d\omega$, $X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}$
8. CT Periodic Signal Fourier Transform: $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$, $X(jw)=2\pi\sum_{k-\infty}^{+\infty}a_k\delta\left(\omega-k\omega_0\right)$
9. CT Aperiodic Signal Parseval Theorem: $E_x=\int_{-\infty}^{\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega<\infty$

B. Convolution Result

1. $(u(t+T)-u(t-T))*(u(t+T)-u(t-T))=(2T-|t|)(u(t+2T)-u(t-2T))$ (Triangular wave!)
2. $e^{-at}u(t)*u(t)=\frac{1-e^{-at}}{a}u(t)$

C. Fourier Transform

1. $\delta\left(t-t_0\right)\xrightarrow{\mathcal{F}}e^{-j\omega t_0}$ $\quad u(t)\xrightarrow{\mathcal{F}}\frac{1}{j\omega}+\pi\delta(\omega)$ $\quad e^{j\omega_0 t}\xrightarrow{\mathcal{F}}2\pi\delta\left(\omega-\omega_0\right)$
2. $\cos\omega_0 t\xrightarrow{\mathcal{F}}\pi\left[\delta\left(\omega-\omega_0\right)+\delta\left(\omega+\omega_0\right)\right]$ $\quad \sin\omega_0 t\xrightarrow{\mathcal{F}}\frac{\pi}{j}\left[\delta\left(\omega-\omega_0\right)-\delta\left(\omega+\omega_0\right)\right]$
3. $x(t)=\begin{cases}1, & |t|<T_1\\ 0, & |t|>T_1\end{cases}\xrightarrow{\mathcal{F}}\frac{2\sin\omega T_1}{\omega}$ $\quad \frac{\sin Wt}{\pi t}\xrightarrow{\mathcal{F}}X(j\omega)=\begin{cases}1, & |\omega|<W\\ 0, & |\omega|>W\end{cases}$
4. $e^{-at}u(t),\operatorname{Re}\{a\}>0\xrightarrow{\mathcal{F}}\frac{1}{a+j\omega}$ $\quad te^{-at}u(t),\operatorname{Re}\{a\}>0\xrightarrow{\mathcal{F}}\frac{1}{(a+j\omega)^2}$
5. $\frac{t^{n-1}}{(n-1)!}e^{-at}u(t),\operatorname{Re}\{a\}>0\xrightarrow{\mathcal{F}}\frac{1}{(a+j\omega)^n}$
6. $\sum_{n=-\infty}^{+\infty}\delta(t-nT)\xrightarrow{\mathcal{F}}\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
7. $\delta\left[n-n_0\right]\xrightarrow{\mathcal{F}}e^{-j\omega n_0}$ $\quad e^{j\omega_0 n}\xrightarrow{\mathcal{F}}2\pi\delta\sum_{k=-\infty}^{+\infty}\left(\omega-\omega_0-2\pi k\right)$
8. $a^n u[n],|a|<1\xrightarrow{\mathcal{F}}\frac{1}{1-ae^{-j\omega}}$ $\quad (n+1)a^n u[n],|a|<1\xrightarrow{\mathcal{F}}\frac{1}{(1-ae^{-j\omega})^2}$

D. Properties of Fourier Transform

1. CT Time Shifting and Frequency Shifting: $x(t-t_0)\xrightarrow{\mathcal{F}}e^{-j\omega t_0}X(j\omega)$ $\quad e^{j\omega_0 t}x(t)\xrightarrow{\mathcal{F}}X\left(j\left(\omega-\omega_0\right)\right)$
2. CT Time and Frequency Scaling: $x(at)\xrightarrow{\mathcal{F}}\frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$
3. CT Differentiation and Integration: $\frac{d}{dt}x(t)\xrightarrow{\mathcal{F}}j\omega X(j\omega)$ $\quad \int_{-\infty}^{t}x(\tau)d\tau\xrightarrow{\mathcal{F}}\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
4. DT Tmie Shifting and Frequency Shifting: $x[n-n_0]\xrightarrow{\mathcal{F}}e^{-j\omega n_0}X(e^{j\omega})$ $\quad e^{j\omega_0 n}x[n]\xrightarrow{\mathcal{F}}X\left(e^{j(\omega-\omega_0)}\right)$
5. DT Difference: $x[n]-x[n-1]\xrightarrow{\mathcal{F}}(1-e^{-jw})X(jw)$ $\quad nx[n]\xrightarrow{\mathcal{F}}j\frac{dX(e^{jw})}{dw}$
6. Differentiation in Frequency : $tx(t)\xrightarrow{\mathcal{F}}j\frac{dX(j\omega)}{d\omega}$ $\quad nx[n]\xrightarrow{\mathcal{F}}j\frac{dX\left(e^{j\omega}\right)}{d\omega}$