Assignment Week12

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4.50 a) $X_2(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_2[n]e^{-j\Omega n} = \sum_{k=-\infty}^{\infty} x[k]e^{-j\Omega kq} = X(e^{j\Omega q})$

$\therefore X_0(e^{j\Omega}) = X_2(e^{j\Omega})H_0(e^{j\Omega}) = X(e^{j\Omega q})H_0(e^{j\Omega})$

if $x[n] = \sin(\frac{3\pi}{4}n)/\pi n$, then $X(e^{j\Omega}) = \begin{cases} 1, & |\Omega| < \frac{3}{4}\pi \\ 0, & \frac{3}{4}\pi \le |\Omega| < \pi \end{cases}$, 2π periodic

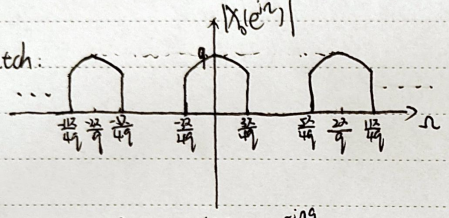
$H_0(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h_0[n]e^{-j\Omega n} = \frac{1-e^{-j\Omega q}}{1-e^{-j\Omega}}$, $\therefore |X(e^{j\Omega q})| = \begin{cases} 1, & |\Omega| < \frac{3}{4q}\pi \\ 0, & \frac{3\pi}{4q} \le |\Omega| < \frac{\pi}{q} \end{cases}$ 2π periodic, $|H_0(e^{j\Omega})| = \left|\frac{1-e^{-j\Omega q}}{1-e^{-j\Omega}}\right|$

$\Rightarrow |X_0(e^{j\Omega})| = |X(e^{j\Omega q})| \cdot |H_0(e^{j\Omega})|$

$= \left|\frac{1-e^{-j\Omega q}}{1-e^{-j\Omega}}\right| \cdot \left|\frac{e^{j\frac{\Omega q}{2}}}{e^{j\frac{\Omega}{2}}}\right|$

$= \left|\frac{\sin\frac{\Omega q}{2}}{\sin\frac{\Omega}{2}}\right|$

Sketch:



b) $H(e^{j\Omega}) = \begin{cases} \sum_{n=-\infty}^{\infty} h_0[n]e^{-j\Omega n} = \frac{1-e^{-j\Omega q}}{1-e^{-j\Omega}}, & |\Omega| < \frac{\pi}{q} \\ 0, & \frac{\pi}{q} \le |\Omega| < \frac{2\pi}{q} \end{cases}$

i. $H(e^{j\Omega}) = \begin{cases} \frac{1-e^{-j2\Omega}}{1-e^{-j\Omega}}, & |\Omega| < \frac{3}{8}\pi \\ 0, & \frac{3}{8}\pi \le |\Omega| < \frac{5}{8}\pi \end{cases}$

ii. $H(e^{j\Omega}) = \begin{cases} \frac{1-e^{-j4\Omega}}{1-e^{-j\Omega}}, & |\Omega| < \frac{3}{16}\pi \\ 0, & \frac{3}{16}\pi \le |\Omega| < \frac{5}{16}\pi \end{cases}$

4.51 As $0.9 < |G(jw)| < 1.1$, for $100\pi < w < 200\pi$, $\Omega_a = 100\pi T_s$, $\Omega_b = 200\pi T_s$, And

$0.9 < |H_0(e^{j\Omega})| = \left|\int_0^{T_s} e^{-jwt}dt\right| = \left|\frac{2\sin(\frac{T_s}{2}w)}{w}\right| < 1.1$, at $100\pi < w < 200\pi$

$\Rightarrow \begin{cases} \frac{2\sin(50\pi T_s)}{100\pi T_s} < 1.1 \\ \frac{2\sin(100\pi T_s)}{200\pi T_s} > 0.9 \end{cases}$ $\Rightarrow 200\pi T_s < \frac{2\sin(110\pi T_s)}{0.9} < \frac{2}{0.9}$ $\Rightarrow 0.9(100\pi T_s) < \sin(100\pi T_s)$

$\Rightarrow 0 < 100\pi T_s < 0.78668$

$\Rightarrow T_{s\max} = 0.0025$

$\Rightarrow \Omega_a = 0.25\pi$, $\Omega_b = 0.5\pi$

for $H_{a(jw)}$: $w_{1\min} = 200\pi$, $w_{2\max} = \frac{\pi}{2T_s} = 400\pi$

for $H_{c(jw)}$: $w_{3\min} = 200\pi$, $w_{4\max} = \frac{2\pi}{T_s} - 200\pi = 600\pi$