Assignment Week 4

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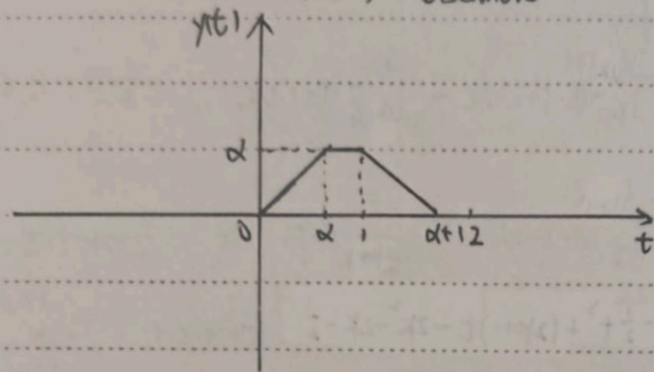
2.10

(a) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \begin{cases} 0, & t<0 \\ \int_0^t d\tau & 0\le t<\alpha \\ \int_{t-\alpha}^t d\tau & \alpha\le t<1 \\ \int_{t-\alpha}^1 d\tau & 1\le t<\alpha+1 \\ 0 & t\ge\alpha+1 \end{cases} = \begin{cases} 0, & t<0 \\ t, & 0\le t<\alpha \\ \alpha, & \alpha\le t<1 \\ \alpha+1-t, & 1\le t<\alpha+1 \\ 0, & t\ge\alpha+1 \end{cases}$

$h(t) = x(t/\alpha) = \begin{cases} 1, & 0\le t\le\alpha \\ 0, & \text{elsewhere} \end{cases}$



(b) $\alpha=1$ 时，$dy(t)/dt$ 由上图可见有 $t=0, 1, 2$ 三处不连续断点

但当$0<\alpha<1$时，由上图可见有 $t=0, \alpha, 1, \alpha+1$ 四处不连续断点

$\therefore \alpha=1$

2.11

(a) $y(t) = x(t)*h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty}[u(\tau-3)-u(\tau-5)]e^{-3(t-\tau)}u(t-\tau)d\tau$

$= \begin{cases} 0 & t<3 \\ \int_3^t e^{-3(t-\tau)}d\tau & 3\le t<5 \\ \int_3^5 e^{-3(t-\tau)}d\tau, & t\ge5 \end{cases} = \begin{cases} 0, & t<3 \\ \frac{1}{3}-\frac{e^{9-3t}}{3}, & 3\le t<5 \\ \frac{e^{15-3t}-e^{9-3t}}{3}, & t\ge5 \end{cases}$

(b) $g(t) = (dx(t)/dt)*h(t) = (\delta(t-3)-\delta(t-5))*h(t) = h(t-3)-h(t-5)$

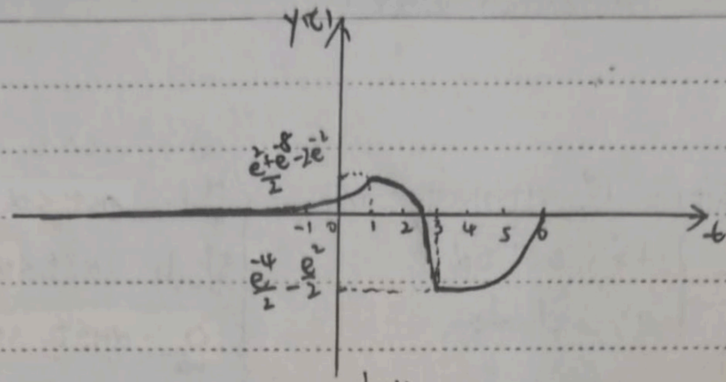
$= e^{9-3t}u(t-3) - e^{15-3t}u(t-5)$

(c) 从结果表达式分析 $y(t) = \int_{-\infty}^{t} g(t)dt$，$g(t) = \frac{dy(t)}{dt}$

2.22

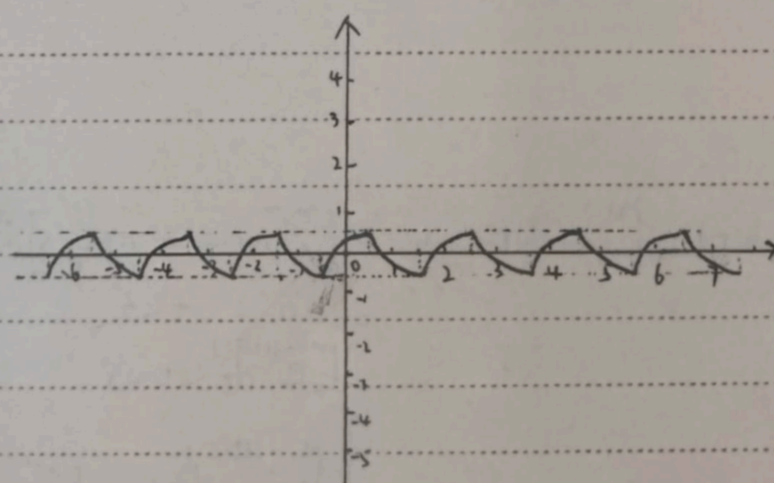
(b) $y(t) = x(t)*h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty}[u(\tau)-2u(\tau-2)+u(\tau-5)]e^{2t-2\tau}u(1+\tau-t)d\tau$

$= \begin{cases} \int_0^2 e^{2t-2\tau}d\tau + \int_2^5 -e^{2t-2\tau}d\tau, & t<1 \\ \int_{t-1}^2 e^{2t-2\tau}d\tau + \int_2^5 -e^{2t-2\tau}d\tau, & 1\le t<3 \\ \int_{t-1}^5 -e^{2t-2\tau}d\tau, & 3\le t<6 \\ 0, & t\ge6 \end{cases} = \begin{cases} \frac{1}{2}e^{2t}+\frac{1}{2}e^{2t-10}-e^{2t-4}, & t<1 \\ \frac{1}{2}e^{2}+\frac{1}{2}e^{2t-10}-e^{2t-4}, & 1\le t<3 \\ \frac{1}{2}e^{2t-10}-\frac{1}{2}e^{2}, & 3\le t<6 \\ 0, & t\ge6 \end{cases}$



(e) $y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \begin{cases} \int_{t-1}^{-\frac{1}{2}+2k} -(\tau-t+1)d\tau + \int_{-\frac{1}{2}+2k}^{t} (\tau-t+1)d\tau, & -\frac{1}{2}+2k \le t \le \frac{1}{2}+2k \ (k\in Z) \\ \int_{t-1}^{\frac{1}{2}+2k} (\tau-t+1)d\tau + \int_{\frac{1}{2}+2k}^{t} -(\tau-t+1)d\tau, & \frac{1}{2}+2k \le t \le \frac{3}{2}+2k \ (k\in Z) \end{cases}$

$= \begin{cases} -t^2+(4k+1)t-4k^2-4k+\frac{1}{4}, & -\frac{1}{2}+2k \le t \le \frac{1}{2}+2k \ (k\in Z) \\ t^2-(4k+3)t+4k^2+6k+\frac{7}{4}, & \frac{1}{2}+2k \le t \le \frac{3}{2}+2k \ (k\in Z) \end{cases}$



2.25 (a) $y[n]=x[n]*h[n]=\sum_{-\infty}^{\infty} x[k]h[n-k]$

$= \sum_{-\infty}^{\infty}\left(3^k u[-k-1]+\left(\frac{1}{3}\right)^k u[k]\right)\left(\frac{1}{4}\right)^{n-k} u[n+3-k]$

$= \begin{cases} \sum_{-\infty}^{n+3} 3^k \cdot \left(\frac{1}{4}\right)^{n-k}, & n\le -4 \\ \sum_{-\infty}^{-1} 3^k \left(\frac{1}{4}\right)^{n-k} + \sum_{0}^{n+3}\left(\frac{1}{3}\right)^k\left(\frac{1}{4}\right)^{n-k}, & n>-4 \end{cases}$

$= \begin{cases} \sum_{-n-3}^{\infty} \left(\frac{1}{12}\right)^k \cdot \left(\frac{1}{4}\right)^n, & n\le -4 \\ \sum_{1}^{\infty} \left(\frac{1}{4}\right)^n \left(\frac{1}{12}\right)^k + \sum_{0}^{n+3}\left(\frac{1}{4}\right)^n\left(\frac{4}{3}\right)^k, & n>-4 \end{cases}$

$= \begin{cases} \frac{12^4}{11} 3^n, & n\le -4 \\ \frac{4^4}{3^{n+3}} - \frac{32}{11\cdot 4^n}, & n>-4 \end{cases}$

(b) take $x_1[n] = 3^n u[-n-1]$, $x_2[n] = (\frac{1}{3})^n u[n]$ $\therefore$ $x = x_1 + x_2$

$y[n] = x[n] * h[n] = (x_1[n] + x_2[n]) * h[n] = x_1[n] * h[n] + x_2[n] * h[n]$

$y_1[n] = x_1[n] * h[n] = \sum_{-\infty}^{\infty} 3^k u[-k-1] (\frac{1}{4})^{n-k} u[n+3-k]$

$$= \begin{cases} \sum_{-\infty}^{n+3} 3^k (\frac{1}{4})^{n-k} , & n \leq -4 \\ \sum_{-\infty}^{-1} 3^k (\frac{1}{4})^{n-k} , & n > -4 \end{cases}$$

$$= \begin{cases} \frac{12^4}{11} 3^n , & n \leq -4 \\ \frac{1}{11 \cdot 4^n} , & n > -4 \end{cases}$$

$y_2[n] = x_2[n] * h[n] = \sum_{-\infty}^{\infty} (\frac{1}{3})^k u[k] (\frac{1}{4})^{n-k} u[n+3-k]$

$$= \begin{cases} 0 , & n \leq -3 \\ \sum_{0}^{n+3} (\frac{1}{3})^k (\frac{1}{4})^{n-k} , & n \geq -3 \end{cases}$$

$$= \begin{cases} 0 , & n \leq -3 \\ \frac{-3}{4^n} + \frac{4^4}{3^{n+3}} , & n > -3 \end{cases}$$

$\therefore$ $y[n] = y_1[n] + y_2[n] = \begin{cases} \frac{12^4}{11} 3^n & n \leq -4 \\ \frac{4^4}{3^{n+3}} - \frac{32}{11 \cdot 4^n} & n \geq -3 \end{cases}$

2.28 (a) Causal and stable

when $n<0$ $h[n] = (\frac{1}{5})^n u[n] = 0$ $\therefore$ causal

$\sum_{-\infty}^{\infty} |h[k]| = \sum_{0}^{\infty} (\frac{1}{5})^k = \frac{5}{4} < \infty$ $\therefore$ stable

(c) neither causal or stable

when $n<0$ $h[n] = (\frac{1}{2})^n u[-n] = (\frac{1}{2})^n \neq 0$ $\therefore$ not causal

$\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{0} (\frac{1}{2})^k = \sum_{0}^{\infty} 2^k = \infty$ $\therefore$ not stable

(e) Causal and unstable

when $n<0$ $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1] = 0$ $\therefore$ causal

$\sum_{-\infty}^{\infty} |h[k]| = \sum_{0}^{1} (\frac{1}{2})^k + \sum_{1}^{\infty} \left| [(1.01)^k + (-\frac{1}{2})^k] \right| = \infty$ $\therefore$ not stable

(g) causal and stable

when $n<0$ $h[n] = n(\frac{1}{3})^n u[n-1] = 0$ $\therefore$ causal

$\sum_{-\infty}^{\infty} |h[k]| = \sum_{1}^{\infty} k(\frac{1}{3})^k = \frac{3}{4} < \infty$ $\therefore$ stable