Assignment Week 7

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4.5 $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2[u(w+3)-u(w-3)]e^{j(-\frac{3}{2}w+\pi)} \cdot e^{jwt}dw$

$$= \frac{1}{2\pi}\int_{-3}^{3} -2\, e^{j(wt-\frac{3}{2}w)}dw$$

$$= \frac{1}{2\pi} \frac{-2}{j(t-\frac{3}{2})} e^{j(t-\frac{3}{2})w}\Big|_{-3}^{3}$$

$$= \frac{-2\sin(3t-\frac{9}{2})}{\pi(t-\frac{3}{2})}$$

$$x(t)=0 \Rightarrow \begin{cases} 3t-\frac{9}{2} = k\pi \\ t-\frac{3}{2} \neq 0 \end{cases} \Rightarrow t = \frac{k}{3}\pi + \frac{3}{2} \quad (k\neq 0)\ k\in Z$$

4.21 (b) $X(jw) = \int_{-\infty}^{\infty} e^{-3|t|}\sin 2t\, e^{-jwt}dt = \int_{-\infty}^{0} \frac{1}{2j}e^{3t}(e^{j2t}-e^{-j2t})e^{-jwt}dt$

$$+\int_{0}^{\infty}\frac{1}{2j}e^{-3t}(e^{j2t}-e^{-j2t})e^{-jwt}dt$$

$$= \frac{1}{2j}\cdot\frac{1}{3+2j-wj}e^{(3+2j-wj)t}\Big|_{-\infty}^{0} - \frac{1}{2j}\cdot\frac{1}{3-2j-wj}e^{(3+2j-wj)t}\Big|_{-\infty}^{0}$$

$$+\frac{1}{2j}\cdot\frac{1}{-3+2j-jw}e^{(-3+2j-wj)t}\Big|_{0}^{\infty} - \frac{1}{2j}\frac{1}{-3-2j-jw}e^{(-3-2j-wj)t}\Big|_{0}^{\infty}$$

$$= \frac{1}{2j}\cdot\frac{1}{3+2j-wj} - \frac{1}{2j}\cdot\frac{1}{3-2j-wj} + \frac{1}{2j}\frac{1}{3-2j+jw}$$

$$-\frac{1}{2j}\cdot\frac{1}{3+2j+jw} = \frac{3}{j}\frac{1}{9+(2-w)^2} - \frac{3}{j}\frac{1}{9+(2+w)^2}$$

$$= \frac{3j}{9+(2+w)^2} - \frac{3j}{9+(2-w)^2}$$

(g) $X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt = \int_{-2}^{-1} -e^{-jwt}dt + \int_{-1}^{1} te^{-jwt}dt + \int_{1}^{2} e^{-jwt}dt$

$$= \frac{1}{jw}e^{-jwt}\Big|_{-2}^{-1} - \left[\frac{1}{(jw)^2}+\frac{t}{jw}\right]e^{-jwt}\Big|_{-1}^{1} + \frac{1}{-jw}e^{-jwt}\Big|_{1}^{2}$$

$$= \frac{1}{-jw}(e^{2jw}+e^{-2jw}) + \frac{1}{(jw)^2}(e^{jw}-e^{-jw})$$

$$= \frac{2j}{w}\cos 2w - \frac{2j}{w^2}\sin w$$

(h) $x(t) = 2\sum_{k=-\infty}^{\infty}\delta(t-2k) + \sum_{k=-\infty}^{\infty}\delta[(t-1)-2k]$, as $x(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT) \longleftrightarrow X(jw) = \sum_{k=-\infty}^{\infty}\frac{2\pi}{T}\delta(w-\frac{k2\pi}{T})$

and $x(t-t_0) \longleftrightarrow e^{-jwt_0}X(jw)$

$$\Rightarrow X(jw) = (2+e^{-jw})\sum_{k=-\infty}^{\infty}\pi\,\delta(w-k\pi)$$

4.22 (c) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(j\omega)|e^{j\angle X(j\omega)}e^{j\omega t}dt$

$$= \frac{1}{2\pi}\int_{-1}^{0} -\omega e^{-3j\omega+j\omega t}d\omega + \frac{1}{2\pi}\int_0^1 \omega e^{-3j\omega + j\omega t}d\omega$$

$$= \frac{-1}{2\pi}\left(\frac{-1}{[j(t-3)]^2} + \frac{\omega}{j(t-3)}\right)e^{j(t-3)\omega}\Big|_{-1}^{0}$$

$$+ \frac{1}{2\pi}\left(\frac{-1}{[j(t-3)]^2}+\frac{\omega}{j(t-3)}\right)e^{j(t-3)\omega}\Big|_0^1$$

$$= \frac{1}{2\pi}\left(\frac{-1}{(t-3)^2} + \left[\frac{1}{(t-3)^2} - \frac{1}{j(t-3)}\right]e^{-j(t-3)}\right)$$

$$+\frac{1}{2\pi}\left(\left[\frac{1}{(t-3)^2}+\frac{1}{j(t-3)}\right]e^{j(t-3)} - \frac{1}{(t-3)^2}\right)$$

$$= \frac{1}{2\pi}\left(\frac{-2}{(t-3)^2} + \frac{2\cos(t-3)}{(t-3)^2} + \frac{2\sin(t-3)}{t-3}\right)$$

$$= \frac{1}{\pi}\left[\frac{\cos(t-3)-1}{(t-3)^2} + \frac{\sin(t-3)}{t-3}\right]$$

(e) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\Big[\int_{-3}^{-2} -e^{j\omega t}d\omega + \int_{-2}^{-1}(\omega+1)e^{j\omega t}d\omega + \int_1^2(\omega-1)e^{j\omega t}d\omega + \int_2^3 e^{j\omega t}d\omega\Big]$

$$= \frac{1}{2\pi}\left(\frac{-1}{jt}e^{j\omega t}\Big|_{-3}^{-2} + \left(\frac{1}{jt} - \frac{1}{(jt)^2}+\frac{\omega}{jt}\right)e^{j\omega t}\Big|_{-2}^{-1}\right.$$

$$\left. + \left(\frac{-1}{jt}-\frac{1}{(jt)^2}+\frac{\omega}{jt}\right)e^{j\omega t}\Big|_1^2 + \frac{1}{jt}e^{j\omega t}\Big|_2^3\right)$$

$$= \frac{1}{2\pi}\Big(-\cancel{\frac{e^{-j2t}}{jt}} + \frac{e^{-j3t}}{jt} + \cancel{\frac{e^{-jt}}{jt}} + \frac{e^{-jt}}{t^2} - \cancel{\frac{e^{-jt}}{jt}} - \cancel{\frac{e^{-j2t}}{jt}} - \frac{e^{-j2t}}{t^2} + \cancel{\frac{2}{jt}}e^{-j2t}$$

$$- \cancel{\frac{e^{j2t}}{jt}} + \frac{e^{j2t}}{t^2} + \cancel{\frac{2}{jt}}e^{j2t} + \cancel{\frac{e^{jt}}{jt}} - \frac{e^{jt}}{t^2} - \cancel{\frac{e^{jt}}{jt}} + \frac{e^{j3t}}{jt} - \cancel{\frac{e^{j2t}}{jt}}\Big)$$

$$= \frac{1}{2\pi}\left[\frac{1}{jt}(e^{j3t}+e^{-j3t}) + \frac{1}{t^2}(e^{j2t}-e^{-j2t}+e^{-jt}-e^{jt})\right]$$

$$= \frac{1}{2\pi}\left(\frac{2\cos 3t}{jt} + \frac{2j\sin 2t - 2j\sin t}{t^2}\right)$$

$$= \frac{1}{j\pi t}\left(\cos 3t + \frac{\sin t - \sin 2t}{t}\right)$$

4.27 (a) $X(j\omega) = \int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt = \int_1^2 e^{-j\omega t}dt + \int_2^3 -e^{-j\omega t}dt$

$$= \frac{1}{-j\omega}e^{-j\omega t}\Big|_1^2 + \frac{1}{j\omega}e^{-j\omega t}\Big|_2^3 = -\frac{e^{-j2\omega}}{j\omega} + \frac{e^{-j\omega}}{j\omega} + \frac{e^{-j3\omega}}{j\omega} - \frac{e^{-j2\omega}}{j\omega}$$

$$= \frac{1}{j\omega}(e^{-j\omega}+e^{-j3\omega}-2e^{-j2\omega})$$

(b) $a_k = \frac{1}{T}\int_T \tilde{x}(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_1^3 x(t)e^{-jk\frac{2\pi}{T}t}dt = \frac{1}{T}\left(\int_1^2 e^{-jk\frac{2\pi}{T}t}dt + \int_2^3 -e^{-jk\frac{2\pi}{T}t}dt\right)$

$$= \frac{1}{T}\left(\frac{T}{-jk2\pi}e^{-jk\frac{2\pi}{T}t}\Big|_1^2 + \frac{T}{jk2\pi}e^{-jk\frac{2\pi}{T}t}\Big|_2^3\right)$$

$$= \frac{1}{2jk\pi}\left(e^{-jk\frac{2\pi}{T}} + e^{-jk\frac{6\pi}{T}} - 2e^{-jk\frac{4\pi}{T}}\right)$$

and if we take $\omega = \frac{2\pi k}{T}$ in $X(j\omega)$, then

$X(j\frac{2\pi k}{T}) = \frac{T}{2jk\pi}\left(e^{-jk\frac{2\pi}{T}}+e^{-jk\frac{6\pi}{T}}-2e^{-jk\frac{4\pi}{T}}\right) = Ta_k$, so $a_k = \frac{1}{T}X(j\frac{2\pi k}{T})$