4.14

4.14 Taking the Fourier transform of both sides of the equation

$$\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = A2^{-2t}u(t)$$

we obtain

$$X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)} = A\{\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\}$$

Taking the inverse Fourier transform of the above equation

$$x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$$

Using Parseval's relation, we have

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi \int_{-\infty}^{\infty} |X(t)|^2 \, dt$$

Using the fact that $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega = 2\pi$, we have

$$\int_{-\infty}^{\infty} |X(t)|^2 \, dt = 1$$

Substituting the previously obtained expression for x(t) in the above equation, we have

$$\int_0^{\infty} [A^2 e^{-2t} + A^2 e^{-4t} - 2A^2 e^{-3t}] dt = 1$$

$$A^2/12 = 1$$

$$\Rightarrow \mathrm{A} = \sqrt{12}$$

We choose A to be $\sqrt{12}$ instead of $-\sqrt{12}$ because we know that x(t) is none negative.

4.25

4.25 (a) Note that y(t)=x(t+1) is a real and even signal. Therefore , $Y(j\omega)$ is also real and even .this implies that $\sphericalangle Y(j\omega)$=0.Also ,since $Y(j\omega) = e^{j\omega} X(j\omega)$,we know that $\sphericalangle Y(j\omega) = -\omega$

(b) we have

$$X(j0) = \int_{-\infty}^{\infty} x(t)dt = 7$$

(c)we have

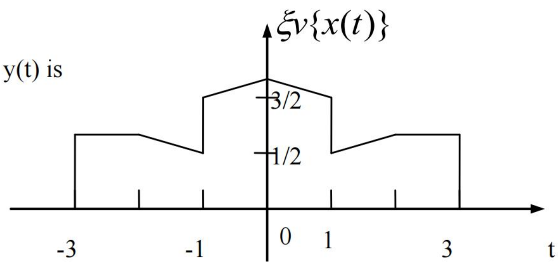
$$\int_{-\infty}^{\infty} x(j\omega)d\omega = 2\pi x(0) = 4\pi$$

(d) Let $Y(j\omega)\frac{2\sin\omega}{\omega}e^{2j\omega}$. The corresponding signal y(t) is

$$y(t) = \{ \begin{matrix} 1, -3 < t < -1 \\ 0, otherwize \end{matrix}$$

Then the given integral is

$$\int_{-\infty}^{\infty} X(j\omega)Y(j\omega)d\omega = 2\pi\{x(t) * y(t)\}_{t=0} = 7\pi$$



(e) $$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= 2\pi \int_{-\infty}^{\infty} |x(t+1)|^2 dt$$

$$= 2\pi \times 2\left[\int_0^1 (t+1)^2 dt + \int_1^2 2^2 dt\right] = 25\frac{\pi}{3}$$

(f)The inverse Fourier transform of $\Re e\{X(j\omega)\}$ is the $\xi v\{x(t)\}$ which is [x(t)+x(-t)]/2.this is as shown is the figure below.

4.31

4.31 (a) We have

$$x(t) = \cos t \xleftrightarrow{FT} X(j\omega) = \pi[\delta(\omega+1) + \delta(\omega-1)]$$

(i) We have

$$h_1(t) = u(t) \xleftrightarrow{FT} H_1(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

Therefore,

$$Y(j\omega) = X(j\omega)H_1(j\omega) = \frac{\pi}{j}[\delta(\omega+1) - \delta(\omega-1)]$$

Taking the inverse Fourier transform ,we obtain

$$y(t) = \sin(t)$$

(ii)We have

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t) \xleftrightarrow{FT} H_2(j\omega) = -2 + \frac{5}{2+j\omega}$$

Therefore,

$$Y(j\omega) = X(j\omega)H_1(j\omega) = \frac{\pi}{j}[\delta(\omega+1) - \delta(\omega-1)]$$

Taking the inverse Fourier transform ,we obtain

$$y(t) = \sin(t)$$

(iii)we have

$$h_3(t)=2te^{-t}u(t) \leftrightarrow H_2(j\omega)=\frac{2}{(1+j\omega)^2}.$$

Therefore,

$$Y(j\omega)=X(j\omega)H_1(j\omega)=\frac{\pi}{j}[\delta(j\omega+1) - \delta(j\omega-1)].$$

Taking the inverse Fourier transform ,we obtain

$$y(t)=\sin(t).$$

(b)An LTI system with impulse response

$$h_4(t)=\frac{1}{2}[h_1(t)+h_2(t)]$$

will have the same response to x(t) =cos(t),we can find other such impulse responses by suitably scaling and linearly combining $h_1(t)$,$h_2(t)$,and $h_3(t)$.

4.33

解：

（a）系统函数

$$H(\mathrm{j}\omega)=\frac{Y(\mathrm{j}\omega)}{X(\mathrm{j}\omega)}=\frac{2}{-\omega^2+6\mathrm{j}\omega+8}=\frac{1}{\mathrm{j}\omega+2}-\frac{1}{\mathrm{j}\omega+4}$$

系统的单位冲激响应为

$$h(t)=\mathscr{F}^{-1}\{H(\mathrm{j}\omega)\}=\mathrm{e}^{-2t}u(t)-\mathrm{e}^{-4t}u(t)$$

（b）

$$X(\mathrm{j}\omega)=\mathscr{F}\{x(t)\}=\frac{1}{(\mathrm{j}\omega+2)^2}$$

$$\begin{aligned}Y(\mathrm{j}\omega)=H(\mathrm{j}\omega)X(\mathrm{j}\omega)&=\frac{2}{-\omega^2+6\mathrm{j}\omega+8}\cdot\frac{1}{(\mathrm{j}\omega+2)^2}\\&=\frac{1}{(\mathrm{j}\omega+2)^3}-\frac{1}{(\mathrm{j}\omega+4)(\mathrm{j}\omega+2)^2}\\&=\frac{1/4}{\mathrm{j}\omega+2}-\frac{1/4}{\mathrm{j}\omega+4}+\frac{-1/2}{(\mathrm{j}\omega+2)^2}+\frac{1}{(\mathrm{j}\omega+2)^3}\end{aligned}$$

$$y(t)=\mathscr{F}^{-1}\{Y(\mathrm{j}\omega)\}=\frac{1}{4}\mathrm{e}^{-2t}u(t)-\frac{1}{4}\mathrm{e}^{-4t}u(t)-\frac{1}{2}t\mathrm{e}^{-2t}u(t)+\frac{1}{2}t^2\mathrm{e}^{-2t}u(t)$$

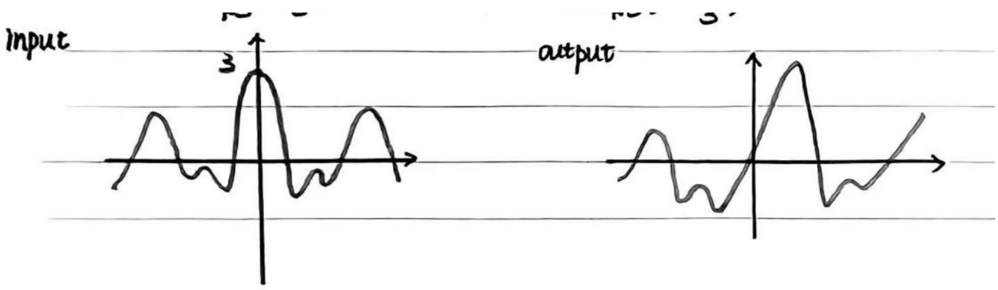
（c）该因果 LTI 系统的系统函数为

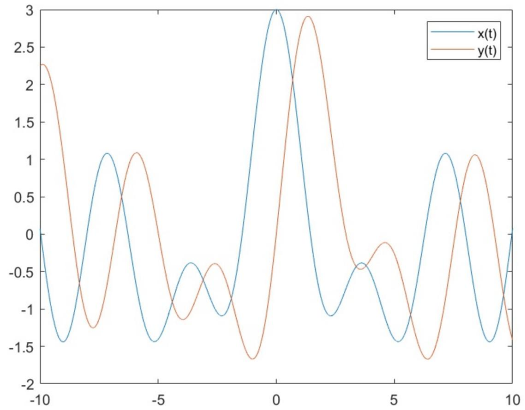
$$H(\mathrm{j}\omega)=\frac{Y(\mathrm{j}\omega)}{X(\mathrm{j}\omega)}=\frac{-2\omega^2-2}{-\omega^2+\sqrt{2}\mathrm{j}\omega+1}=2-\frac{\sqrt{2}-\mathrm{j}\sqrt{2}}{\mathrm{j}\omega+\frac{\sqrt{2}-\mathrm{j}\sqrt{2}}{2}}-\frac{\sqrt{2}+\mathrm{j}\sqrt{2}}{\mathrm{j}\omega+\frac{\sqrt{2}+\mathrm{j}\sqrt{2}}{2}}$$

$$\begin{aligned}h(t)&=\mathscr{F}^{-1}\{H(\mathrm{j}\omega)\}\\&=2\delta(t)-(\sqrt{2}-\mathrm{j}\sqrt{2})\mathrm{e}^{[(-1+\mathrm{j})/\sqrt{2}]t}u(t)-(\sqrt{2}+\mathrm{j}\sqrt{2})\mathrm{e}^{[-(1+\mathrm{j})/\sqrt{2}]t}u(t)\\&=2\delta(t)-\sqrt{2}(1-\mathrm{j})\mathrm{e}^{-\frac{\sqrt{2}}{2}t}\cdot\mathrm{e}^{\mathrm{j}\frac{\sqrt{2}}{2}t}u(t)-\sqrt{2}(1+\mathrm{j})\mathrm{e}^{-\frac{\sqrt{2}}{2}t}\cdot\mathrm{e}^{-\mathrm{j}\frac{\sqrt{2}}{2}t}u(t)\\&=2\delta(t)-\sqrt{2}\mathrm{e}^{-\frac{\sqrt{2}}{2}t}[\mathrm{e}^{\mathrm{j}\frac{\sqrt{2}}{2}t}+\mathrm{e}^{-\mathrm{j}\frac{\sqrt{2}}{2}t}]u(t)+\mathrm{j}\sqrt{2}\mathrm{e}^{-\frac{\sqrt{2}}{2}t}[\mathrm{e}^{\mathrm{j}\frac{\sqrt{2}}{2}t}-\mathrm{e}^{-\mathrm{j}\frac{\sqrt{2}}{2}t}]u(t)\\&=2\delta(t)-2\sqrt{2}\mathrm{e}^{-\frac{\sqrt{2}}{2}t}\cos\left(\frac{\sqrt{2}}{2}t\right)u(t)-2\sqrt{2}\mathrm{e}^{-\frac{\sqrt{2}}{2}t}\sin\left(\frac{\sqrt{2}}{2}t\right)u(t)\end{aligned}$$

4.35

画图示例





解：

（a）

$$|H(j\omega)|=\left|\frac{a-j\omega}{a+j\omega}\right|=\frac{\sqrt{a^2+\omega^2}}{\sqrt{a^2+\omega^2}}=1$$

$$\arg H(j\omega)=-\arctan\frac{\omega}{a}-\arctan\frac{\omega}{a}=-2\arctan\frac{\omega}{a}$$

又因

$$H(j\omega)=-1+\frac{2a}{j\omega+a},$$

则

$$h(t)=\mathscr{F}^{-1}\{H(j\omega)\}=-\delta(t)+2ae^{-at}u(t)$$

（b）因 a=1，则

$$H(j\omega)=\frac{1-j\omega}{1+j\omega},\quad |H(j\omega)|=1,\quad \arg H(j\omega)=-2\arctan\omega$$

即

$$|Y(j\omega)|=|X(j\omega)|,\quad \arg Y(j\omega)=\arg X(j\omega)-2\arctan\omega$$

由于

$$x(t)=\cos\omega_1 t+\cos\omega_2 t+\cos\omega_3 t,\quad \omega_1=\frac{1}{\sqrt{3}},\quad \omega_2=1,\quad \omega_3=\sqrt{3}$$

$$-2\arctan\omega_1=-\frac{\pi}{3},\quad -2\arctan\omega_2=-\frac{\pi}{2},\quad -2\arctan\omega_3=-\frac{2\pi}{3}$$

故

$$y(t)=\cos\left(\frac{t}{\sqrt{3}}-\frac{\pi}{3}\right)+\cos\left(t-\frac{\pi}{2}\right)+\cos\left(\sqrt{3}t-\frac{2\pi}{3}\right)$$