5.5 (10 * 2)

(a)

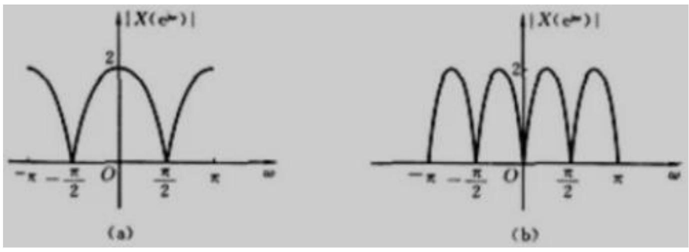
$$X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}\delta[n-1]e^{-j\omega n}+\sum_{n=-\infty}^{\infty}\delta[n+1]e^{-j\omega n}$$
$$=\mathrm{e}^{-j\omega}+\mathrm{e}^{j\omega}$$
$$=2\cos\omega$$
$$\left|X\left(e^{j\omega}\right)\right|=|2\cos\omega|$$

3

(b)

$$X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}\delta[n+2]\mathrm{e}^{-j\omega n}-\sum_{n=-\infty}^{\infty}\delta[n-2]e^{-j\omega n}$$
$$=e^{j2\omega}-e^{-j2\omega}$$
$$=2j\sin 2\omega$$
$$\left|X\left(e^{j\omega}\right)\right|=|2\sin 2\omega|$$

3



Pic. 1

5.5 (20)

$$x[n]=\frac{1}{2\pi}\int_{-\frac{4}{\pi}}^{\frac{4}{\pi}}1\cdot\mathrm{e}^{-j\frac{3}{2}\omega}\cdot\mathrm{e}^{j\omega n}\,\mathrm{d}\omega$$
$$=\frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\mathrm{e}^{j\left(n-\frac{3}{2}\right)\omega}\mathrm{d}\omega$$
$$=\frac{1}{2\pi}\frac{1}{j(n-3/2)}\mathrm{e}^{j\left(n-\frac{3}{2}\right)\omega}\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$
$$=\frac{\sin\left[\left(n-\frac{3}{2}\right)\frac{\pi}{4}\right]}{\pi\left(n-\frac{3}{2}\right)}$$
$$=\frac{\sin\left(\frac{\pi}{4}n-\frac{3\pi}{8}\right)}{\pi\left(n-\frac{3}{2}\right)}$$

3

$$Let\sin\left(\frac{\pi}{4}n-\frac{3\pi}{8}\right)=0$$
$$Thus\ x[n]=0\ when\ n=4k+\frac{3}{2},\ k=\pm1.\pm2...$$
$$Since\ n\ is\ integer, n\neq 4k+3/2$$
$$x[n]=0\ only\ for\ n=\pm\infty$$

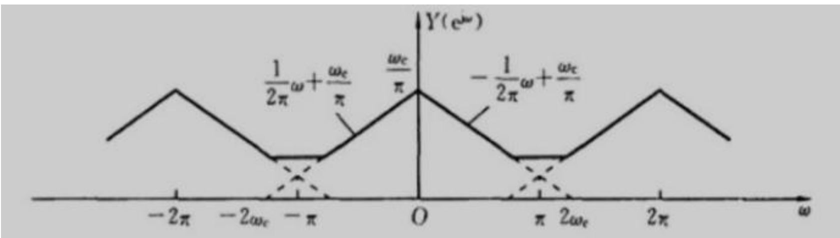
3

5.15 (20)

$$Let\ x[n]=\frac{\sin\omega_c n}{\pi n},$$
$$X\left(e^{j\omega}\right)=u(\omega+\omega_c)-u(\omega-\omega_c),\ -\pi<\omega\leqslant\pi$$
$$Y\left(e^{j\omega}\right)=\frac{1}{2\pi}\int_{2\pi}X\left(e^{j\theta}\right)X\left(e^{j(\omega-\theta)}\right)d\theta$$

$$If\ 2\omega_c\leqslant\pi,$$
$$Y\left(e^{j\omega}\right)=\begin{cases}-\frac{1}{2\pi}|\omega|+\frac{\omega_c}{\pi}, & 0\leqslant|\omega|\leqslant 2\omega_c\\ 0, & 2\omega_e<|\omega|\leqslant\pi\end{cases}$$
$$Y\left(e^{j\pi}\right)=0\ not\ conform\ the\ question,$$
$$thus\ 2\omega_c>\pi$$
$$As\ shown\ in\ fig.2,$$
$$Y\left(e^{j\pi}\right)=2\left(-\frac{1}{2\pi}\times\pi+\frac{\omega_c}{\pi}\right)=-1+\frac{2\omega_c}{\pi}$$
$$Since\ Y\left(e^{j\pi}\right)=\frac{1}{2}$$
$$Thus\ \omega_c=\frac{3}{4}\pi$$

3



Pic. 2

5.21 (5 * 7)

(a)

$$\begin{aligned}x[n]&=u[n-2]-u[n-6]\\&=\delta[n-2]+\delta[n-3]+\delta[n-4]+\delta[n-5]\end{aligned}$$

$$\begin{aligned}X\left(e^{j\omega}\right)&=e^{-j2\omega}+e^{-j3\omega}+e^{-j4\omega}+e^{-j5\omega}\\&=\frac{e^{-j2\omega}\left(1-e^{-j4\omega}\right)}{1-e^{-j\omega}}\end{aligned}$$

2

(b)

$$x_1[n]=\left(\frac{1}{2}\right)^n u[n-1]=\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

$$X_1\left(e^{j\omega}\right)=\frac{\frac{1}{2}e^{-i\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

$$X(e^{j\omega}) = X_1(e^{-j\omega}) = \frac{\frac{1}{2}e^{i\omega}}{1-\frac{1}{2}e^{j\omega}}$$ $\boxed{2}$

(c)

$$x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2] = \left(\frac{1}{3}\right)^{-n} u[-n-2]$$
$$x_1[n] = \left(\frac{1}{3}\right)^{n} u[n-2] = \frac{1}{9}\cdot\left(\frac{1}{3}\right)^{n-2} u[n-2]$$
$$X_1\left(e^{j\omega}\right) = \frac{\frac{1}{9}e^{-j2\omega}}{1-\frac{1}{3}e^{-j\omega}}$$
$$Since\ x[n] = x_1[-n],\ X(e^{j\omega}) = X_1(e^{-j\omega}) = \frac{\frac{1}{9}e^{j2\omega}}{1-\frac{1}{3}e^{j\omega}}$$ $\boxed{2}$

(d)

$$\begin{aligned}x_1[n] &= 2^{-n}\sin\left(-\frac{\pi}{4}n\right)u[n]\\ &= -\left(\frac{1}{2}\right)^{n}\sin\left(\frac{\pi}{4}n\right)u[n]\\ &= -\frac{1}{2j}\left(\frac{1}{2}e^{j\frac{\pi}{4}}\right)^{n}u[n] + \frac{1}{2j}\left(\frac{1}{2}e^{-j\frac{\pi}{4}}\right)^{*}u[n]\end{aligned}$$

$$X_1\left(e^{j\omega}\right) = -\frac{1}{2j}\cdot\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{4}}e^{-j\omega}} + \frac{1}{2j}\cdot\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j\omega}}$$
$$Since\ x[n] = x_1[-n],$$
$$\begin{aligned}X\left(e^{j\omega}\right) &= X_1\left(e^{-j\omega}\right)\\ &= -\frac{1}{2j}\cdot\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{4}}e^{j\omega}} + \frac{1}{2j}\cdot\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}e^{j\omega}}\\ &= \frac{-\sqrt{2}e^{j\omega}}{4-2\sqrt{2}e^{j\omega}+e^{j2\omega}}\end{aligned}$$ $\boxed{2}$

(e)

Solution 1

$$x_1[n] = \left(\frac{1}{2}\right)^{|n|},$$
$$x_2[n] = \cos\left(\frac{\pi}{8}(n-1)\right) = \cos\left(\frac{\pi}{8}n-\frac{\pi}{8}\right)$$
$$X_1\left(\mathrm{e}^{j\omega}\right) = \frac{1-\left(\frac{1}{2}\right)^2}{1-2\times\frac{1}{2}\cos\omega+\frac{1}{4}} = \frac{3}{5-4\cos\omega}$$
$$X_2\left(\mathrm{e}^{j\omega}\right) = \pi\mathrm{e}^{-\mathrm{j}\frac{\pi}{8}}\delta\left(\omega-\frac{\pi}{8}\right) + \pi\mathrm{e}^{j\frac{\pi}{8}}\delta\left(\omega+\frac{\pi}{8}\right),\ -\pi<\omega\leqslant\pi$$
$$Thus$$

Thus

$$X\left(\mathrm{e}^{j\omega}\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X_1\left(\mathrm{e}^{j\theta}\right)X_2\left(\mathrm{e}^{j(\omega-\theta)}\right)\mathrm{d}\theta$$
$$=\frac{\frac{3}{2}\mathrm{e}^{-j\frac{\pi}{8}}}{5-4\cos\left(\omega-\frac{\pi}{8}\right)}+\frac{\frac{3}{2}\mathrm{e}^{j\frac{\pi}{8}}}{5-4\cos\left(\omega+\frac{\pi}{8}\right)}$$

Solution 2

$$x[n]=\left(\frac{1}{2}\right)^{|n|}\cos\left(\frac{\pi}{8}(n-1)\right)$$
$$=\left(\frac{1}{2}\right)^{n}\cos\left(\frac{\pi}{8}n-\frac{\pi}{8}\right)u[n]$$
$$+\left(\frac{1}{2}\right)^{-n}\cos\left(-\frac{\pi}{8}n+\frac{\pi}{8}\right)u[-n-1]$$
$$x_1[n]=\left(\frac{1}{2}\right)^{n}\cos\left(\frac{\pi}{8}n-\frac{\pi}{8}\right)u[n]$$
$$x_2[n]=\left(\frac{1}{2}\right)^{n}\cos\left(\frac{\pi}{8}n+\frac{\pi}{8}\right)u[n-1]$$

Since

$$x_1[n]=\frac{1}{2}e^{-j\frac{\pi}{8}}\left(\frac{1}{2}e^{j\frac{\pi}{8}}\right)^{n}u[n]+\frac{1}{2}e^{j\frac{\pi}{8}}\left(\frac{1}{2}e^{-j\frac{\pi}{8}}\right)^{n}u[n]$$
$$x_2[n]=\frac{1}{2}e^{j\frac{\pi}{8}}\left(\frac{1}{2}e^{j\frac{\pi}{8}}\right)^{n}u[n-1]+\frac{1}{2}e^{-j\frac{\pi}{8}}\left(\frac{1}{2}e^{-j\frac{\pi}{8}}\right)^{n}u[n-1]$$
$$=(\frac{1}{2}e^{j\frac{\pi}{8}})^2(\frac{1}{2}e^{j\frac{\pi}{8}})^{(n-1)}u[n-1]+(\frac{1}{2}e^{-j\frac{\pi}{8}})^2(\frac{1}{2}e^{-j\frac{\pi}{8}})^{(n-1)}u[n-1]$$

then

$$X_1(e^{j\omega})=\frac{\frac{1}{2}e^{-j\frac{\pi}{8}}}{1-\frac{1}{2}e^{j\frac{\pi}{8}}e^{-j\omega}}+\frac{\frac{1}{2}e^{j\frac{\pi}{8}}}{1-\frac{1}{2}e^{-j\frac{\pi}{8}}e^{-j\omega}}$$
$$X_2(e^{j\omega})=\frac{(\frac{1}{2}e^{j\frac{\pi}{8}})^2e^{-j\omega}}{1-\frac{1}{2}e^{j\frac{\pi}{8}}e^{-j\omega}}+\frac{(\frac{1}{2}e^{-j\frac{\pi}{8}})^2e^{-j\omega}}{1-\frac{1}{2}e^{-j\frac{\pi}{8}}e^{-j\omega}}$$

Since

$x[n]=x_1[n]+x_2[-n],$

then

$$X\left(e^{j\omega}\right)=X_1\left(e^{j\omega}\right)+X_2\left(e^{-j\omega}\right)$$
$$=\frac{\frac{3}{2}e^{-j\frac{\pi}{8}}}{5-4\cos\left(\omega-\frac{\pi}{8}\right)}+\frac{\frac{3}{2}e^{j\frac{\pi}{8}}}{5-4\cos\left(\omega+\frac{\pi}{8}\right)}$$

$\boxed{2}$

(f)

$$x[n]=-3\delta[n+3]-2\delta[n+2]-\delta[n+1]+\delta[n-1]+2\delta[n-2]+3\delta[n-3]$$

$$
\begin{aligned}
X(e^{j\omega}) &= -3e^{j3\omega} - 2e^{j2\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-j2\omega} + 3e^{-j3\omega} \\
&= -6\cos 3\omega - 4\cos 2\omega - 2\cos\omega
\end{aligned}
$$

2

(h)

$$
\begin{aligned}
X(e^{j\omega}) = j\pi \sum_{l=-\infty}^{\infty} &\{\delta(\omega + \frac{5\pi}{3} - 2\pi l) - \delta(\omega - \frac{5\pi}{3} - 2\pi l)\} \\
&+ \pi \sum_{l=-\infty}^{\infty} \{\delta(\omega + \frac{7\pi}{3} - 2\pi l) - \delta(\omega - \frac{7\pi}{3} - 2\pi l)\}
\end{aligned}
$$

$$
\begin{aligned}
X(e^{j\omega}) = j\pi&\{\delta(\omega - \frac{\pi}{3}) - \delta(\omega + \frac{\pi}{3})\} \\
&+ \pi\{\delta(\omega + \frac{\pi}{3}) + \delta(\omega - \frac{\pi}{3})\}, \ -\pi < \omega \le \pi
\end{aligned}
$$

2