Assignment Week 14

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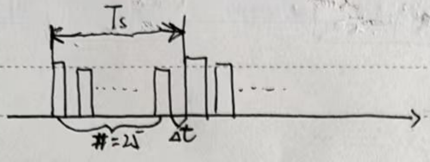
5.39.

(a) $T_s = \frac{1}{f_s} = \frac{1}{8000} = 1.25 \times 10^{-4}$ s. the total num of pulse within a sample period is $24+1=25$



so the spacing between successive pulses $\Delta t = \frac{T_s}{25} - 1\mu s$
$= 5\mu s - 1\mu s$
$= 4\ \mu s$

(b) by sampling theorem, under Nyquist rate sampling, $f_s = 2f_m = 6.8$ kHz, then $T_s = \frac{1}{f_s} = 1.47\underline{06} \times 10^{-4}$ s

then the spacing between successive pulses $\Delta t = \frac{T_s}{25} - 1\mu s = 5.882\mu s - 1\mu s = 4.882\ \mu s$

5.48 we need to ensure $C_i \perp C_j$ ($C_i = \cos(w_a t + \alpha_i) + \cos(w_b t + \beta_i)$, when $i=0$, $\alpha_i = \beta_i = 0$)

$$C_i C_j = [\cos(w_a t + \alpha_i) + \cos(w_b t + \beta_i)][\cos(w_a t + \alpha_j) + \cos(w_b t + \beta_j)]$$

$$= \cos(w_a t + \alpha_i)\cos(w_a t + \alpha_j) + \cos(w_a t + \alpha_i)\cos(w_b t + \beta_j) + \cos(w_b t + \beta_i)\cos(w_a t + \alpha_j) + \cos(w_b t + \beta_i)\cos(w_b t + \beta_j)$$

$$= \frac{1}{2}\cos(\alpha_i - \alpha_j) + \frac{1}{2}\cos(\beta_i - \beta_j) + \frac{1}{2}\cos(2w_a t + \alpha_i + \alpha_j) + \frac{1}{2}\cos((w_a - w_b)t + \alpha_i - \beta_j) + \frac{1}{2}\cos((w_a + w_b)t + \alpha_i + \beta_j)$$

$$+ \frac{1}{2}\cos((w_b - w_a)t + \beta_i - \alpha_j) + \frac{1}{2}\cos((w_b + w_a)t + \beta_i + \alpha_j) + \frac{1}{2}\cos(2w_b t + \beta_i + \beta_j)$$

to ensure $C_i \perp C_j$, the DC component of $C_i C_j$ should be zero

$$\Rightarrow \frac{1}{2}\cos(\alpha_i - \alpha_j) + \frac{1}{2}\cos(\beta_i - \beta_j) = 0$$

so 1° $C_0 \perp C_1 \Rightarrow \cos\alpha_1 + \cos\beta_1 = 0$

2° $C_0 \perp C_2$, $C_1 \perp C_2 \Rightarrow \begin{cases} \cos\alpha_2 + \cos\beta_2 = 0 \\ \cos(\alpha_2 - \alpha_1) + \cos(\beta_2 - \beta_1) = 0 \end{cases}$

3° $C_0 \perp C_3$, $C_1 \perp C_3$, $C_2 \perp C_3 \Rightarrow \begin{cases} \cos\alpha_3 + \cos\beta_3 = 0 \\ \cos(\alpha_3 - \alpha_1) + \cos(\beta_3 - \beta_1) = 0 \\ \cos(\alpha_3 - \alpha_2) + \cos(\beta_3 - \beta_2) = 0 \end{cases}$

More explicitly, if we take $\alpha_1 = \beta_1 = \frac{\pi}{2}$, then $\begin{cases} \cos\alpha_2 + \cos\beta_2 = 0 \\ \sin\alpha_2 + \sin\beta_2 = 0 \end{cases} \Rightarrow$ take $\alpha_2 = \frac{\pi}{4}$, $\beta_2 = \frac{5}{4}\pi$

then $\begin{cases} \cos\alpha_3 + \cos\beta_3 = 0 \\ \sin\alpha_3 + \sin\beta_2 = 0 \\ \cos\alpha_3 - \cos\beta_3 + \sin\alpha_3 - \sin\beta_3 = 0 \end{cases} \Rightarrow$ take $\alpha_3 = \frac{3}{4}\pi$, $\beta_3 = \frac{7}{4}\pi$