Assignment Week 11

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7.21

(a) $W_s = \frac{2\pi}{T} = 20000\pi$, $W_m = 5000\pi$ $\Rightarrow$ $W_s > 2W_m = 10000\pi$

so $x(t)$ can be recovered exactly from $x_p(t)$

(b) $W_s = \frac{2\pi}{T} = 20000\pi$, $W_m = 15000\pi$ $\Rightarrow$ $W_s < 2W_m = 30000\pi$

so $x(t)$ can not be recovered exactly from $x_p(t)$

(c) While imaginary part of $X(jw)$ is uncertain, so the sampling theorem can not guarantee whether $x(t)$ can be recovered exactly from $x_p(t)$

(d) As $x(t)$ is real, $X(-jw) = X^*(jw)$. And if $X(jw)=0$, for $w>5000\pi$, then $X(-jw) = X^*(jw) = 0$, $w>5000\pi$ $\Rightarrow$ $X(jw) = 0$, $w<-5000\pi$

then we get $X(jw)=0$ for $|w|>5000\pi$

$\because$ $W_s = \frac{2\pi}{T} = 20000\pi$, $W_m = 5000\pi$. $\Rightarrow$ $W_s > 2W_m = 10000\pi$

so $x(t)$ can be recovered exactly from $x_p(t)$

(e) As $x(t)$ is real , $X(-jw) = X^*(jw)$. If $X(jw)=0$ for $w<-15000\pi$, then $X(-jw)=X^*(jw)=0$ for $w<-15000\pi$ $\Rightarrow$ $X(jw)=0$, $w>15000\pi$

then we get $X(jw)=0$ for $|w|>15000\pi$

$\because$ $W_s = \frac{2\pi}{T} = 20000\pi$, $W_m = 15000\pi$ $\Rightarrow$ $W_s < 2W_m = 30000\pi$

so $x(t)$ can not be recovered exactly from $x_p(t)$

(f) if $X(jw)=0$ for $|w|>W_0$, then $X(jw) * X(jw) = 0$ for $|w|>2W_0$. $\Rightarrow$ $X(jw)=0$ for $|w|>7500\pi$ $\Rightarrow$ $W_s > 2\times7500\pi = 15000\pi$, so $x(t)$ can be recovered exactly from $x_p(t)$

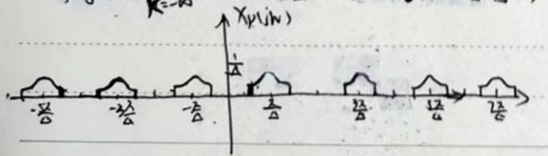
(g) $|X(jw)|=0$ for $w>5000\pi$ $\Rightarrow$ $X(jw)=0$ for $w>5000\pi$ $\Rightarrow$ $W_s > 2\times5000\pi = 10000\pi$

so $x(t)$ can be recovered exactly from $x_p(t)$

7.23

(a) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n\Delta) - \delta(t-(2n+1)\Delta)$, $n \in Z$

$P(jw) = \sum_{k=-\infty}^{\infty} \left[\frac{\pi}{\Delta}\delta(w-k\frac{\pi}{\Delta}) - e^{-jw\Delta}\frac{\pi}{\Delta}\delta(w-k\frac{\pi}{\Delta})\right]$, $k \in Z$



$\Delta < \pi/2W_m \Rightarrow \frac{2\pi}{2\Delta} > 2W_m$

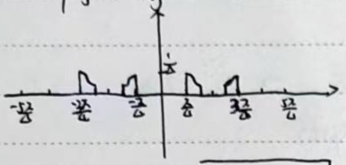
$X_p(jw) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)P(j(w-\theta))d\theta$

$= \frac{1}{2\Delta}\sum_{k=-\infty}^{\infty} X(j(w-k\frac{\pi}{\Delta})) - e^{-jk\pi}X(j(w-k\frac{\pi}{\Delta}))$

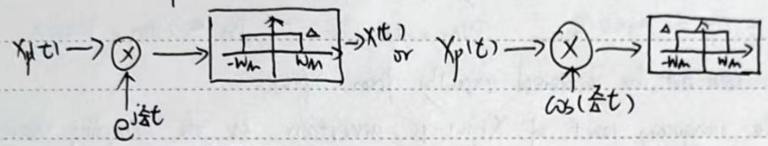
$= \frac{1}{2\Delta}\sum_{k=-\infty}^{\infty} 2X(j(w-(2k+1)\frac{\pi}{\Delta}))$

$= \frac{1}{\Delta}\sum_{k=-\infty}^{\infty} X(j(w-(2k+1)\frac{\pi}{\Delta}))$

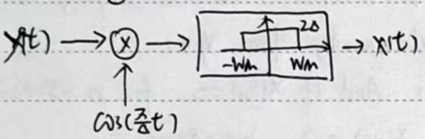
$Y(j\omega) = X_p(j\omega)\, H(j\omega)$



(b)



(c)



(d) in part (a) we can know from figure that $-\frac{\pi}{\Delta} + W_m \le \frac{\pi}{\Delta} - W_m \Rightarrow \Delta \le \frac{\pi}{W_m}$

so the maximum Δ is $\frac{\pi}{W_m}$