Assignment Week10

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5.23 (a) $X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n] = (-1)+1+2+1+1+2+1+(-1) = 6$

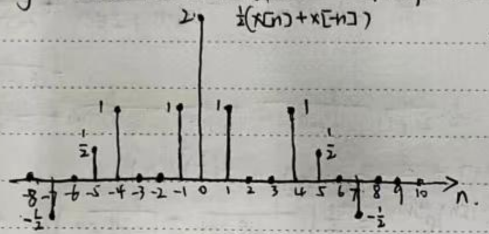
(b) $x[n+2]$ is even and real $\Rightarrow \measuredangle(e^{j2w}X(e^{jw})) = 0 \Rightarrow \measuredangle X(e^{jw}) = e^{-j2w}$

(c) $\int_{-\pi}^{\pi} X(e^{jw})dw = \frac{2\pi}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})e^{jw0}dw = 2\pi x[0] = 4\pi$

(d) $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\pi n} = \sum_{n=-\infty}^{\infty}(-1)^n x[n] = (2+2) - [(-1)+1+1+1+1+(-1)]$

$= 4 - 2 = 2$

(e) the signal whose Fourier transform is $Re\{X(w)\}$ is $\frac{1}{2}(x[n]+x[-n])$



$$= \begin{cases} -\frac{1}{2}, & n=\pm 7 \\ \frac{1}{2}, & n=\pm 5 \\ 1, & n=\pm 4, \pm 1 \\ 0, & \text{else} \\ 2, & n=0 \end{cases}$$

(f) (i) $\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 2\pi\sum_{n=-\infty}^{\infty}|x[n]|^2 = 2\pi(1+1+4+1+1+4+1+1)$

$= 28\pi$

(ii) $\int_{-\pi}^{\pi}\left|\frac{dX(e^{jw})}{dw}\right|^2 dw = 2\pi\sum_{n=-\infty}^{\infty}|nx[n]|^2 = 2\pi(9+1+1+9+64+25+49)$

$= 316\pi$

5.29 (a) $H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} = \sum_{n=-\infty}^{\infty}(\frac{1}{2})^n u[n]e^{-jwn} = \sum_{n=0}^{\infty}(\frac{1}{2})^n e^{-jwn} = \frac{1}{1-\frac{1}{2}e^{-jw}}$

(i) $X(e^{jw}) = \sum_{n=-\infty}^{\infty}(\frac{3}{4})^n u[n]e^{-jwn} = \sum_{n=0}^{\infty}(\frac{3}{4})^n e^{-jwn} = \frac{1}{1-\frac{3}{4}e^{-jw}}$

$\Rightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1}{(1-\frac{3}{4}e^{-jw})(1-\frac{1}{2}e^{-jw})} = \frac{3}{1-\frac{3}{4}e^{-jw}} - \frac{2}{1-\frac{1}{2}e^{-jw}}$

$\Rightarrow y[n] = 3(\frac{3}{4})^n u[n] - 2(\frac{1}{2})^n u[n]$

(ii) $X(e^{jw}) = \sum_{n=-\infty}^{\infty}(n+1)(\frac{1}{4})^n u[n]e^{-jwn} = \sum_{n=0}^{\infty}(n+1)(\frac{1}{4})^n e^{-jwn} = \frac{1}{(1-\frac{1}{4}e^{-jw})^2}$

$\Rightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1}{(1-\frac{1}{2}e^{-jw})(1-\frac{1}{4}e^{-jw})^2} = \frac{4}{1-\frac{1}{2}e^{-jw}} - \frac{2}{1-\frac{1}{4}e^{-jw}} - \frac{1}{(1-\frac{1}{4}e^{-jw})^2}$

$\Rightarrow y[n] = 4(\frac{1}{2})^n u[n] - 2(\frac{1}{4})^n u[n] - (n+1)(\frac{1}{4})^n u[n]$

(iii) $X(e^{jw}) = \mathcal{F}\{e^{j\pi n}\} = \sum_{l=-\infty}^{\infty} 2\pi\delta(w-\pi-2\pi l)$

$\Rightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \sum_{l=-\infty}^{\infty} 2\pi\delta(w-\pi-2\pi l)\frac{1}{1-\frac{1}{2}e^{-jw}} = \frac{2}{3}\sum_{l=-\infty}^{\infty} 2\pi\delta(w-\pi-2\pi l)$

$\Rightarrow y[n] = \frac{2}{3}(-1)^n$

(b) $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left[\left(\frac{1}{2}\right)^n \cos\frac{\pi n}{2}\right]u[n]e^{-j\omega n} = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \frac{1}{2}\left(e^{j\frac{\pi}{2}n}+e^{-j\frac{\pi}{2}n}\right)e^{-j\omega n}$

$= \frac{1}{2}\sum_{n=0}^{\infty}\left\{\left[\frac{1}{2}e^{j(\frac{\pi}{2}-\omega)}\right]^n+\left[\frac{1}{2}e^{-j(\omega+\frac{\pi}{2})}\right]^n\right\}$

$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}e^{j(\frac{\pi}{2}-\omega)}}+\frac{1}{1-\frac{1}{2}e^{-j(\omega+\frac{\pi}{2})}}\right)$

(i) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n u[n]e^{-j\omega n} = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n e^{-j\omega n} = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{2}\frac{1}{(1-\frac{1}{2}e^{j(\frac{\pi}{2}-\omega)})(1-\frac{1}{2}e^{-j\omega})} + \frac{1}{2}\frac{1}{(1-\frac{1}{2}e^{-j(\omega+\frac{\pi}{2})})(1-\frac{1}{2}e^{-j\omega})}$

$= \frac{j}{2(j-1)}\frac{1}{1-\frac{1}{2}e^{j(\frac{\pi}{2}-\omega)}} - \frac{1}{2(j-1)}\frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{-j}{2(-j-1)}\frac{1}{1-\frac{1}{2}e^{-j(\omega+\frac{\pi}{2})}} - \frac{1}{2(-j-1)}\frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$= \frac{j}{2(j-1)}\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{2}}e^{-j\omega}} + \frac{j}{2(j+1)}\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{2}}e^{-j\omega}} + \frac{1}{2}\frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$\Rightarrow y[n] = \frac{j}{2(j-1)}\left(\frac{j}{2}\right)^n u[n] + \frac{j}{2(j+1)}\left(-\frac{j}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^n u[n]$

(ii) $X(e^{j\omega}) = \frac{1}{2}\sum_{l=-\infty}^{\infty}2\pi\delta(\omega-\frac{\pi}{2}-2\pi l) + \frac{1}{2}\sum_{-\infty}^{\infty}2\pi\delta(\omega+\frac{\pi}{2}-2\pi l) = \pi\sum_{l=-\infty}^{\infty}\left[\delta(\omega-\frac{\pi}{2}-2\pi l)+\delta(\omega+\frac{\pi}{2}-2\pi l)\right]$

$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{\pi}{2}\left(\frac{1}{1-\frac{1}{2}e^{j\frac{\pi}{2}}e^{-j\omega}}+\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{2}}e^{-j\omega}}\right)\sum_{l=-\infty}^{\infty}\left[\delta(\omega-\frac{\pi}{2}-2\pi l)+\delta(\omega+\frac{\pi}{2}-2\pi l)\right]$

$= \frac{\pi}{2}\cdot\frac{8}{3}\sum_{l=-\infty}^{\infty}\left[\delta(\omega-\frac{\pi}{2}-2\pi l)+\delta(\omega+\frac{\pi}{2}-2\pi l)\right]$

$= \frac{4}{3}\cdot\pi\sum_{l=-\infty}^{\infty}\left[\delta(\omega-\frac{\pi}{2}-2\pi l)+\delta(\omega+\frac{\pi}{2}-2\pi l)\right]$

$\Rightarrow y[n] = \frac{4}{3}\cos\frac{\pi}{2}n$

(c) $Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = 3e^{j5\omega}+4e^{-j5\omega}+e^{j4\omega}-e^{j3\omega}-2e^{-j3\omega}-3e^{j2\omega}+e^{j\omega}+6e^{-j\omega}+1$

$\Rightarrow y[n] = 3\delta[n+5]+4\delta[n-5]+\delta[n+4]-\delta[n+3]-2\delta[n-3]-3\delta[n+2]+\delta[n+1]$
$+6\delta[n-1]$
$+\delta[n]$

5.33 (a) take $x[n]=\delta[n]$. do Fourier Transform both sides:

$H(e^{j\omega}) + \frac{1}{2}e^{-j\omega}H(e^{j\omega}) = 1$

$\Rightarrow H(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$

(b) (i) $X(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \Rightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{(1-\frac{1}{2}e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})}$

$= \frac{1}{2}\frac{1}{1-\frac{1}{2}e^{-j\omega}} + \frac{1}{2}\frac{1}{1+\frac{1}{2}e^{-j\omega}}$

$\Rightarrow y[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(-\frac{1}{2}\right)^n u[n]$

(ii) $X(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}} \Rightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2}$

$\Rightarrow y[n] = (n+1)\left(-\frac{1}{2}\right)^n u[n]$

(iii) $X(e^{jw}) = 1 + \frac{1}{2}e^{-jw} \Rightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = 1$

$\Rightarrow y[n] = \delta[n]$

(iv) $X(e^{jw}) = 1 - \frac{1}{2}e^{-jw} \Rightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1-\frac{1}{2}e^{-jw}}{1+\frac{1}{2}e^{-jw}} = \frac{2}{1+\frac{1}{2}e^{-jw}} - 1$

$\Rightarrow y[n] = 2(-\frac{1}{2})^n u[n] - \delta[n]$

(C) (i) $Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1-\frac{1}{4}e^{-jw}}{(1+\frac{1}{2}e^{-jw})^2} = \frac{3}{2}\frac{1}{(1+\frac{1}{2}e^{-jw})^2} - \frac{1}{2}\frac{1}{1+\frac{1}{2}e^{-jw}}$

$\Rightarrow y[n] = \frac{3}{2}(n+1)(-\frac{1}{2})^n u[n] - \frac{1}{2}(-\frac{1}{2})^n u[n]$

(ii) $Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}} \Rightarrow y[n] = (\frac{1}{4})^n u[n]$

(iii) $Y(e^{jw}) = \frac{1}{(1-\frac{1}{4}e^{-jw})(1+\frac{1}{2}e^{-jw})^2} = \frac{1}{9}\frac{1}{1-\frac{1}{4}e^{-jw}} + \frac{2}{9}\frac{1}{1+\frac{1}{2}e^{-jw}} + \frac{2}{3}\frac{1}{(1+\frac{1}{2}e^{-jw})^2}$

$\Rightarrow y[n] = \frac{1}{9}(\frac{1}{4})^n u[n] + \frac{2}{9}(-\frac{1}{2})^n u[n] + \frac{2}{3}(n+1)(-\frac{1}{2})^n u[n]$

(iv) $Y(e^{jw}) = \frac{1+2e^{-3jw}}{1+\frac{1}{2}e^{-jw}} = \frac{4e^{-2jw}+2e^{-3jw}-(8e^{-jw}+4e^{-2jw})+(16+8e^{-jw})-15}{1+\frac{1}{2}e^{-jw}}$

$= 4e^{-2jw} - 8e^{-jw} + 16 - \frac{15}{1+\frac{1}{2}e^{-jw}}$

$\Rightarrow y[n] = 4\delta[n-2] - 8\delta[n-1] + 16\delta[n] - 15(-\frac{1}{2})^n u[n]$