Week 5

3.3 （20：6 * 2）

$$\omega_0 = 2\pi/6 = \pi/3$$

$$a_0 = 2,$$

$$a_2 = a_{-2} = 1/2,$$

$$a_5^* = a_{-5} = 2j$$

3.21 （20：1 * 4）

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{j\omega_0 t}$$

$$= \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{\mathrm{j}k(2\pi/8)t}$$

$$= a_1 \mathrm{e}^{j(2\pi/8)t} + a_{-1}\mathrm{e}^{-j(2\pi/8)t} + a_5 \mathrm{e}^{j5(2\pi/8)t} + a_{-5}\mathrm{e}^{-j5(2\pi/8)t}$$

$$= \mathrm{j}\mathrm{e}^{\mathrm{j}(\pi/4)t} - \mathrm{j}\mathrm{e}^{-j(\pi/4)t} + 2\mathrm{e}^{j5(\pi/4)t} + 2\mathrm{e}^{-j5(\pi/4)t}$$

$$= -2\sin\left(\tfrac{\pi}{4}t\right) + 4\cos\left(\tfrac{5\pi}{4}t\right)$$

$$= 2\cos\left(\tfrac{\pi}{4}t + \tfrac{\pi}{2}\right) + 4\cos\left(\tfrac{5\pi}{4}t\right)$$

3.22 （20：（3 * 2）* 2）

pic. (b)

$$a_0 = \tfrac{1}{T}\int_T x(t)\mathrm{d}t = \tfrac{1}{6}\left[\int_{-2}^{-1}(t+2)\mathrm{d}t + \int_{-1}^{1}\ \mathrm{d}t + \int_1^2 (2-t)\mathrm{d}t\right] = \tfrac{1}{2}$$

$$a_k = \tfrac{1}{T}\int_T x(t)\mathrm{e}^{-jk(\pi/3)t}\ \mathrm{d}t = \tfrac{6}{k^2\pi^2}\sin\tfrac{k\pi}{2}\sin\tfrac{k\pi}{6}, \quad k \neq 0$$

$$a_k = \begin{cases} 0, & k \quad \text{even} \\ \frac{6}{\pi^2 k^2}\sin\left(\frac{\pi k}{2}\right)\sin\left(\frac{\pi k}{6}\right), & k \quad \text{odd} \end{cases}$$

$$x(t) = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{12}{k^2\pi^2}\sin\frac{k\pi}{2}\sin\frac{k\pi}{6}\cos\frac{k\pi}{3}t$$

pic.(d)

$$a_0 = \tfrac{1}{T}\int_T x(t)\mathrm{d}t = \tfrac{1}{2}\int_0^2 [\delta(t) - 2\delta(t-1)]\mathrm{d}t = -\tfrac{1}{2}$$

$$a_t = \tfrac{1}{2} - \mathrm{e}^{jk\pi} = \tfrac{1}{2} - (-1)^k, \quad k \neq 0$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty}\left[\tfrac{1}{2} - (-1)^k\right]\mathrm{e}^{jk\pi t}$$

pic. (f)

$$a_0 = \tfrac{1}{T}\int_T x(t)\mathrm{d}t = \tfrac{1}{3}\int_0^1 2\ \mathrm{d}t + \tfrac{1}{3}\int_1^2\ \mathrm{d}t = 1$$

$$a_k = \tfrac{1}{T}\int_T x(t)\mathrm{e}^{-jk(2\pi/3)t}\ \mathrm{d}t$$

$$= \tfrac{1}{\mathrm{j}k\pi}\left(1 - \mathrm{e}^{-j2k\pi/3}\right) + \tfrac{\frac{1}{2}}{\mathrm{j}k\pi}\left(\mathrm{e}^{-j2k\pi/3} - \mathrm{e}^{-j4k\pi/3}\right)$$

$$= \tfrac{2}{k\pi}\mathrm{e}^{-jk\pi/3}\sin\tfrac{k\pi}{3} + \tfrac{1}{k\pi}\mathrm{e}^{-jk\pi}\sin\tfrac{k\pi}{3}$$

$$= \tfrac{\sin(k\pi/3)}{k\pi/3}\left(\tfrac{2}{3}\mathrm{e}^{-jk\pi/3} + \tfrac{1}{3}\mathrm{e}^{-jk\pi}\right), \quad k \neq 0$$

$$x(t) = \sum_{t=-\infty}^{\infty} a_k \mathrm{e}^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \tfrac{\sin(k\pi/3)}{k\pi/3}\left(\tfrac{2}{3}\mathrm{e}^{-jk\pi/3} + \tfrac{1}{3}\mathrm{e}^{-jk\pi}\right)\mathrm{e}^{jk(2\pi/3)t}$$

3.24（20：4 * 4）

(1)

$$a_0 = \frac{1}{T}\int_T x(t)\mathrm{d}t = \frac{1}{2}\int_0^1 t\,\mathrm{d}t + \frac{1}{2}\int_1^2 (2-t)\mathrm{d}t = 1/2$$

(2)

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

$$b_k = \frac{1}{2}\int_0^1 e^{-j\pi kt}dt - \frac{1}{2}\int_1^2 e^{-j\pi kt}dt$$

$$= \frac{1}{j\pi k}\left[1 - e^{-j\pi k}\right]$$

(3)

$$a_k = \frac{1}{jk\pi}b_k = -\frac{1}{\pi^2 k^2}\left\{1 - e^{-jk\pi}\right\}$$

3.25（20：4 * 4）

(a) The nonzero FS coefficients of x(t) are a1=a$_{-1}$=1/2.

(b) The nonzero FS coefficients of x(t) are b1=b_{-1}^{*}=1/2.

(c)Using the multiplication property, we know that

$$z(t) = x(t)y(t) \xleftrightarrow{FS} c_k = \sum_{l=-\infty}^{\infty} a_l b_{k=l}$$

Therefore,

$$c_k = a_k * b_k = \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2]$$

This implies that the nonzero Fourier series coefficients of z(t) are c2=c_{-2}^{*}=(1/4j)

(d)

$$z(t) = cos(4\pi t)sin(4\pi t) = \frac{1}{2}sin(8\pi t) = -\frac{1}{4}j[e^{j2(4\pi)t} - e^{-j2(4\pi)t}]$$

the nonzero Fourier series coefficients of z(t) are c2=c_{-2}^{*}=(1/4j)