3.2

Using the Fourier series synthesis eq. (3.95)

$$
\begin{aligned}
x[n] &= a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2} e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4} e^{-j4(2\pi/N)n} \\
&= 1 + e^{j(\pi/4)} e^{j2(2\pi/5)n} + e^{-j(\pi/4)} e^{-j2(2\pi/5)n} \\
&= 1 + 2\cos(\frac{4\pi}{5}n + \frac{\pi}{4}) + 4\cos(\frac{8\pi}{5}n + \frac{\pi}{3}) \\
&= 1 + 2\sin(\frac{4\pi}{5}n + \frac{3\pi}{4}) + 4\sin(\frac{8\pi}{5}n + \frac{5\pi}{6})
\end{aligned}
$$

3.27

Using the Fourier series synthesis eq.(3.38),

$$
\begin{aligned}
x[n] &= a_0 + a_2 e^{j2(2\pi/N)n} + a_{-2} e^{-j2(2\pi/N)n} + a_4 e^{j4(2\pi/N)n} + a_{-4} e^{-j4(2\pi/N)n} \\
&= 2 + 2e^{j\pi/6} e^{j(4\pi/5)n} + 2e^{-j\pi/6} e^{-j(4\pi/5)n} + e^{j\pi/3} e^{j(8\pi/5)n} + e^{-j\pi/3} e^{-j(8\pi/5)n} \\
&= 2 + 4\cos[(4\pi n/5) + \pi/6] + 2\cos[(8\pi n/5) + \pi/3] \\
&= 2 + 4\sin[(4\pi n/5) + 2\pi/3] + 2\sin[(8\pi n/5) + 5\pi/6]
\end{aligned}
$$

3.36

We will first evaluate the frequency response of the system. Consider an input $x[n]$ of the form $e^{j\omega n}$. From the discussion in Section 3.9 we know that the response to this input will be $y[n] = H(e^{j\omega})e^{j\omega n}$. Therefore, substituing these in the given difference equation, we get

$$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}H(e^{j\omega}) = e^{j\omega n}.$$

Therefore,

$$H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}.$$

From eq. (3.131), we know that

$$y[n] = \sum_{k=<N>} a_k H(e^{j2\pi k/N})e^{jk(2\pi/N)n}$$

when the input is $x[n]$. $x[n]$ has the Fourier series coefficients a_k and fundamental frequency $2\pi/N$. Therefore, the Fourier series coefficients of $y[n]$ are $a_k H(e^{j2\pi k/N})$.

(a) Here, $N = 4$ and the nonzero FS coefficients of $x[n]$ are $a_3 = a^*_{-3} = 1/2j$. Therefore, the nonzero FS coefficients of $y[n]$ are

$$b_3 = a_1 H(e^{3j\pi/4}) = \frac{1}{2j(1 - (1/4)e^{-j3\pi/4})}, \qquad b_{-3} = a_{-1}H(e^{-3j\pi/4}) = \frac{-1}{2j(1 - (1/4)e^{j3\pi/4})}.$$

(b) Here, $N = 8$ and the nonzero FS coefficients of $x[n]$ are $a_1 = a_{-1} = 1/2$ and $a_2 = a_{-2} = 1$. Therefore, the nonzero FS coefficients of $y(t)$ are

$$b_1 = a_1 H(e^{j\pi/4}) = \frac{1}{2(1 - (1/4)e^{-j\pi/4})}, \qquad b_{-1} = a_{-1}H(e^{-j\pi/4}) = \frac{1}{2(1 - (1/4)e^{j\pi/4})},$$

$$b_2 = a_2 H(e^{j\pi/2}) = \frac{1}{(1 - (1/4)e^{-j\pi/2})}, \qquad b_{-2} = a_{-2}H(e^{-j\pi/2}) = \frac{1}{(1 - (1/4)e^{j\pi/2})}.$$

3.38

The frequency response of the system may be evaluated as

$$H(e^{j\omega}) = -e^{2j\omega} - e^{j\omega} + 1 + e^{-j\omega} + e^{-2j\omega}.$$

For x[n],N=4 and $w_0 = \pi/2$. the FS coefficients of input x[n] are

a_k=1/4, for all k

Therefore, the FS coefficients of output are

$$b_k = a_k H(e^{jk\omega_0}) = \frac{1}{4}[1 - e^{jk\pi/2} + e^{-jk\pi/2}].$$

3.50

from Table 3.2,we know that if

$$x[n] \overset{FS}{\longleftrightarrow} a_{k,}$$

then ,

$$(-1)^n x[n] = e^{(2\pi/N)(N/2)n} x[n] \overset{FS}{\longleftrightarrow} a_{k-N/2}$$

In this case, N =8. Therefore,

$$(-1)^n x[n \overset{FS}{\longleftrightarrow} a_{k-4}$$

This implies that $x[0]=x[\pm 2]x[\pm 4] = \cdots = 0$.

We are also given that $x[1] = x[5] = \cdots = and = x[3]x[7] = -1$.therefore , one period of x[n] is as shown in Figure S3.50

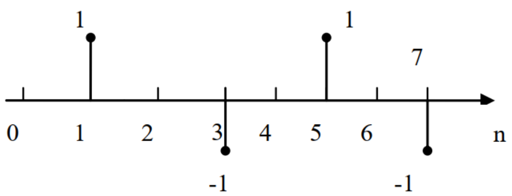

Figure S3.50