of $g(t)$ is 400 Hz and its Nyquist interval is 1/400 seconds.

5.39 (a) With a sampling rate of 8 kHz, the sampling interval is

$$T_s = \frac{1}{8 \times 10^3} = 125\mu s$$

There are 24 voice channels and 1 synchronizing pulse, so the time allotted to each channel is

$$T_{\text{channel}} = \frac{\tau}{25} = 5\mu s$$

(b) If each voice signal is sampled at the Nyquist rate, the sampling rate would be twice the highest frequency component 3.4 kHz, that is, 6.8 kHz. The sampling interval is then

$$T_s = \frac{1}{6.8 \times 10^3} = 147\mu s$$

5.48 The multiplexed signal is

$$s(t) = \sum_{k=1}^{4} [\cos(\omega_a t + \alpha_{k-1}) + \cos(\omega_b t + \beta_{k-1})] m_k(t) \qquad (1)$$

where $\alpha_0 = \beta_0 = 0$. The corresponding output of the product modulator in the coherent detector of the receiver is

$$v_i(t) = s(t)[\cos(\omega_a t + \alpha_{i-1}) + \cos(\omega_b t + \beta_{i-1})] \qquad (2)$$

where $i = 1,2,3,4$. There using Eq. (1) in (2):

$$v_i(t) = \sum_{k=1}^{4} m_k(t)[\cos(\omega_a t + \alpha_{k-1}) + \cos(\omega_b t + \beta_{k-1})] \times [\cos(\omega_a t + \alpha_{i-1}) + \cos(\omega_a t + \beta_{i-1})]$$

Expanding terms:

$$\begin{aligned} v_i(t) = \sum_{k=1}^{4} m_k(t)[&\cos(\omega_a t + \alpha_{k-1})\cos(\omega_a t + \alpha_{i-1}) \\ &+ \cos(\omega_a t + \alpha_{k-1})\cos(\omega_b t + \beta_{i-1}) \\ &+ \cos(\omega_b t + \beta_{k-1})\cos(\omega_a t + \alpha_{i-1}) \\ &+ \cos(\omega_b t + \beta_{k-1})\cos(\omega_b t + \beta_{i-1})] \\ = \frac{1}{2}\sum_{k=1}^{4} m_k(t)[&\cos(\alpha_{k-1} - \alpha_{i-1}) + \cos(\beta_{k-1} - \beta_{i-1})] \\ &+ \cos(2\omega_a t + \alpha_{k-1} + \alpha_{i-1}) + \cos(2\omega_b t + \beta_{k-1} + \beta_{i-1}) \\ &+ \cos((\omega_a + \omega_b)t + \alpha_{k-1} + \beta_{i-1}) \\ &+ \cos((\omega_a - \omega_b)t + \alpha_{k-1} - \beta_{i-1}) \\ &+ \cos((\omega_a + \omega_b)t + \alpha_{i-1} + \beta_{k-1}) \\ &+ \cos((\omega_a - \omega_b)t + \alpha_{i-1} - \beta_{k-1})] \end{aligned}$$

The low-pass filter in the coherent detector removes the six high-frequency components of $v_i(t)$, leaving the output

$$v'_i(t) = \frac{1}{2}\sum_{k=1}^{4} m_k(t)[\cos(\alpha_{k-1} - \alpha_{i-1}) + \cos(\beta_{k-1} - \beta_{i-1})]$$

The requirement on α_k and β_k is therefore

$$\cos(\alpha_{k-1} - \alpha_{i-1}) + \cos(\beta_{k-1} - \beta_{i-1}) = \begin{cases} 2, & i = k \\ 0, & i \neq k \end{cases}$$

where $(i,k) = 1,2,3,4$.