

Objectives

- Objectives for today:
- Introducing specific vocabulary.
  - Quick revision of quadratic function.
  - Factorising Quadratics.
  - Proving Vieta’s formulas.
  - Carrying out gained knowledge by working out some word problems.

Quick Revision

Forms of Quadratic Function

- $f(x) = ax^2 + bx + c$  is called the **standard form**.
- $f(x) = a(x - x_1)(x - x_2)$  is called the **factored form**, where  $x_1$  and  $x_2$  are the roots of the quadratic function.
- $f(x) = a(x - h)^2 + k$  is called the **vertex form**.

Delta  $\Delta$

$\Delta$  determines tells us how many solutions quadratic equation have:

number of solutions = 
$$\begin{cases} 2 & \text{when } \Delta > 0 \\ 1 & \text{when } \Delta = 0 \\ 0 & \text{when } \Delta < 0 \end{cases}$$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Graph of Quadratic Function

Hi

Figure: Graph of  $f(x) = ax^2|_{\{0,1,0.3,1,0.3,0\}}$

Factorising a Quadratic

Factorising a quadratic means putting it into two brackets, and is useful if you’re trying to draw a graph of a quadratic solve a quadratic equation. It’s pretty easy if  $a = 1$  (in  $ax^2 + bx + c$  form), but can be a real pain otherwise.

In order to factorise a quadratic you should follow steps outlined below:

- ➊ Rearrange the equation into the standard  $ax^2 + bx + c$  form.
- ➋ Write down two brackets:  $(x \quad)(x \quad)$
- ➌ Find two numbers that multiply to give ‘c’ and add or subtract to give ‘b’ (ignoring signs).
- ➍ Put the numbers in brackets and choose their signs.

Myth of Delta  $\Delta$

It’s commonly believed that in order to work out roots of a quadratic function you must count  $\Delta$  and use other previously established formulas. However this is untrue since factorising in many cases is as good or even better than simply counting  $\Delta$ .

Example of Factorisation

Solve  $x^2 + 4x - 21 = 0$  by factorising.

$$x^2 + 4x - 21 = (x \quad)(x \quad)$$

1 and 21 multiply to give 21 - and add or subtract to give 22 and 20.  
3 and 7 multiply to give 21 - and add or subtract to give 10 and 4.

$$x^2 + 4x + 21 = (x + 7)(x - 3)$$

And solving the equation:

$$(x + 7)(x - 3) = 0$$

we get

$$x = -7, \quad x = 3$$

Factorising- Tasks

1. Factorise  $x^2 - x - 12$ .
2. Solve  $x^2 - 8 = 2x$  by factorising.

Proof of Vieta’s Formulas

Let’s prove that:

$$x_1 + x_2 = \frac{-b}{a}$$

When  $\Delta$  is positive we have two roots:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}, \quad x_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

Substituting for  $x_1$  and  $x_2$  respectively, we receive:

$$\begin{aligned} x_1 + x_2 &= \frac{-b - \sqrt{\Delta}}{2a} + \frac{-b + \sqrt{\Delta}}{2a} = \\ &= \frac{(-b - \sqrt{\Delta}) + (-b + \sqrt{\Delta})}{2a} = \frac{-2b}{2a} = \frac{-b}{a} \end{aligned}$$

The same we could do with another pattern, which state that  $x_1x_2 = \frac{c}{a}$ , but proving this is going to be your task in next section.

Vieta’s Formulas- Task

1. Prove that

$$x_1x_2 = \frac{c}{a}$$

Glossary

verb	noun	meaning
add	addition	+
subtract	subtraction	−
multiply	multiplication	·
divide	division	÷
solve	solution	getting answer
substitute	substitution	$t = x^2$

Table: Word Formation

Some Necessary and Useful Vocabulary

- (n.) sign  $\rightarrow +$  or  $-$
- (n.) equation  $\rightarrow something = 0$
- (n.) factor  $\rightarrow$  two multiplied factors give result
- (v.) factorise  $\rightarrow$  putting into brackets
- (n.) coefficient  $\rightarrow$  a constant number i.e.  $a, b, c$  in a pattern  $ax^2 + bx + c$
- (n.) quadratic function  $\rightarrow f(x) = ax^2 + bx + c$
- (n.) root  $\rightarrow \sqrt{sth}$  or solution of quadratic equation
- (n.) formula = pattern