

**Supplementary File for “Minimizing Energy Consumption of Flexible Assembly Systems  
with Tool Change Processes Using Hybrid Heuristic Search”**

*Theorem 1:*  $h_1, h_2, h_3$ , and  $h_5$  are admissible.

*Proof:* Let  $\beta^*$  denote the actual optimal schedule from  $(M, \alpha)$  to a final vertex, i.e.,  $M[\beta^*] > M_F$ . Let  $h^*(M, \alpha)$  denote the total energy consumption of  $\beta^*$ . Let  $E_W(\alpha, \beta^*)$ ,  $E_O(\alpha, \beta^*)$ ,  $E_I(\alpha, \beta^*)$ ,  $E_H(\alpha, \beta^*)$  and  $E_C(\alpha, \beta^*)$  denote the total energy consumption of working, occupied, idle, hold and tool change state from  $\alpha$  to  $\alpha\beta^*$ , respectively. Under  $E_1$ ,  $h^*(M, \alpha) = E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$ , while under  $E_2$ ,  $h^*(M, \alpha) = E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$ .

Under  $E_2$ ,  $h_1(M, \alpha) = \sum_{p \in P_A \cup P_B} \sum_{j \in N_{M(p)}} (E_R(p, j, M, \alpha) + E_P(p))$  is the minimum working energy consumption of all parts from  $(M, \alpha)$  to the end, thus  $h_1(M, \alpha) \leq E_W(\alpha, \beta^*)$  under  $E_2$ . Since  $\sum_{t \in \Pi} \kappa(t, M, \alpha)J(t, M, \alpha)$  is only part of the occupied energy,  $\sum_{t \in \Pi} \kappa(t, M, \alpha)J(t, M, \alpha) \leq E_O(\alpha, \beta^*)$ . Since  $\sum_{r_i \in R} \delta(r_i, M, \alpha)G(r_i, M, \alpha)$  is only part of the idle energy,  $\sum_{r_i \in R} \delta(r_i, M, \alpha)G(r_i, M, \alpha) \leq E_I(\alpha, \beta^*)$ . Since  $E_C(\alpha, \beta^*) \geq 0$ ,  $h_5(M, \alpha) = h_1(M, \alpha) + \sum_{r_i \in R} \delta(r_i, M, \alpha)G(r_i, M, \alpha) + \sum_{t \in \Pi} \kappa(t, M, \alpha)J(t, M, \alpha) \leq E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$ , hence  $h_5(M, \alpha)$  is admissible under  $E_2$ . By the definitions of  $h_1(M, \alpha)$ - $h_5(M, \alpha)$ ,  $h_2(M, \alpha) \leq h_5(M, \alpha)$  and  $h_1(M, \alpha) \leq h_3(M, \alpha) \leq h_5(M, \alpha)$ , hence  $h_1(M, \alpha)$ ,  $h_2(M, \alpha)$  and  $h_3(M, \alpha)$  are admissible under  $E_2$ .

Since  $(h_1(M, \alpha) + \sum_{t \in \Pi} \kappa(t, M, \alpha)J(t, M, \alpha))$  is only part of the hold energy consumption of all parts from  $(M, \alpha)$  to the end,  $h_1(M, \alpha) + \sum_{t \in \Pi} \kappa(t, M, \alpha)J(t, M, \alpha) \leq E_H(\alpha, \beta^*)$ .  $h_5(M, \alpha) \leq E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$ , hence  $h_5(M, \alpha)$  is admissible under  $E_1$ . By the definitions of  $h_1(M, \alpha)$ - $h_5(M, \alpha)$ ,  $h_2(M, \alpha) \leq h_5(M, \alpha)$  and  $h_1(M, \alpha) \leq h_3(M, \alpha) \leq h_5(M, \alpha)$ , hence  $h_1(M, \alpha)$ ,  $h_2(M, \alpha)$  and  $h_3(M, \alpha)$  are admissible under  $E_1$ . ■

**Example 1:** For the APNS processing two types of parts in Fig. 1, suppose  $M_0 = p_{1s} + p_{2s} + r_1 + r_2 + r_3 + r_4$ . The processing time of operations in places  $p_{11}$ ,  $p_{12}$ ,  $p_{13}$ ,  $p_{21}$  and  $p_{22}$  are 10, 5, 5, 30 and 20, respectively. The energy consumption per unit time in different kinds of states is listed in Table I. For tool change processes,  $\mu = 5$  and  $e_c = 2$ .

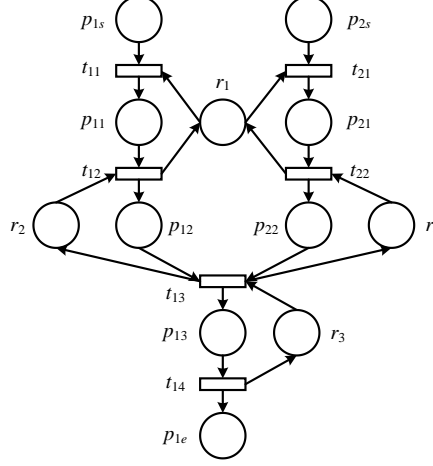


Fig. 1. APNS of FAS in Example 1.

TABLE I  
ENERGY CONSUMPTION PER UNIT TIME OF RESOURCES IN FIG. 1.

	$r_1$	$r_2$	$r_3$	$r_4$
Hold state	2.00	10.00	2.00	2.00
Working state	2.00	10.00	2.00	2.00
Occupied state	1.00	5.00	1.00	1.00
Idle state	0.20	1.00	0.20	0.20

Let's calculate  $h_4(M_0, \alpha_0)$  under  $E_1$  and  $E_2$ . The first part of  $h_4(M_0, \alpha_0)$  is  $h_1(M_0, \alpha_0)$ .  $E_R(p_{1s}, 1, M_0, \alpha_0) = E_R(p_{2s}, 1, M_0, \alpha_0) = 0$ ,  $E_P(p_{1s}) = d(p_{11}) \times e_0(r_1) + d(p_{12}) \times e_0(r_2) + d(p_{13}) \times e_0(r_3) = 10 \times 2 + 5 \times 10 + 5 \times 2 = 20 + 50 + 10 = 80$  and  $E_P(p_{2s}) = d(p_{21}) \times e_0(r_1) + d(p_{22}) \times e_0(r_4) = 30 \times 2 + 20 \times 2 = 100$ . Then  $h_1(M_0, \alpha_0) = E_R(p_{1s}, 1, M_0, \alpha_0) + E_P(p_{1s}) + E_R(p_{2s}, 1, M_0, \alpha_0) + E_P(p_{2s}) = 180$ .

The rest of  $h_4(M_0, \alpha_0)$  under  $E_1$  is calculated as follows.  $\Pi = \{t_{12}, t_{22}, t_{13}\}$ , for  $t_{12}$  and  $t_{22}$ , since  $M(p_{11}) = M(p_{21}) = 0$ ,  $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$ . For assembly transition  $t_{13}$ , since  $p_{12} \in {}^{(o)}t_{13}$ ,  $p_{1s} \in I(p_{12})$  and  $M(p_{1s}) = 1 > 0$ ,  $\theta(t_{13}, M_0, \alpha_0) = 1$ .

$\Pi = \{t_{12}, t_{13}, t_{22}\}$ . Since  $|{}^{(o)}t_{12}| = |{}^{(o)}t_{22}| = 1$  and  $M_0(p_{11}) = M_0(p_{21}) = 0$ ,  $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$ .  $E_{O1}(t_{13}, M_0, \alpha_0)$  and  $E_{O2}(t_{13}, M_0, \alpha_0)$  are calculated as follows. Since  $M_0(r_3) > 0$ ,  $T_R(r_3, M_0, \alpha_0) = 0$ .  $T_O(t_{13}, M_0, \alpha_0) = \max\{T_O(p_{12}, M_0, \alpha_0), T_O(p_{22}, M_0, \alpha_0)\} = \max\{d(p_{11}) + d(p_{12}), d(p_{21}) + d(p_{22})\} = \max\{10 + 5, 30 + 20\} = 50$ . Then  $E_{O1}(t_{13}, M_0, \alpha_0) = (e_0(r_2) + e_0(r_4)) \times (\max\{T_O(t_{13}, M_0, \alpha_0), T_R(r_3, M_0, \alpha_0)\} - T_O(t_{13}, M_0, \alpha_0)) = (10 + 2) \times (\max\{50, 0\} - 50) = 0$ , and  $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_0(r_2) + (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{22}, M_0, \alpha_0)) \times e_0(r_4) = (50 - 15) \times 10 + (50 - 50) \times 2 = 350$ . Thus  $h_4(M_0, \alpha_0) = h_1(M_0, \alpha_0) + \theta(t_{13}, M_0, \alpha_0)(E_{O1}(t_{13}, M_0, \alpha_0) + E_{O2}(t_{13}, M_0, \alpha_0)) = 180 + 1 \times (0 + 350) = 530$  under  $E_1$ .

Similarly, under  $E_2$ ,  $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_1(r_2) + (T_O(t_{13}, M_0, \alpha_0) -$

$T_O(p_{22}, M_0, \alpha_0) \times e_1(r_4) = (50 - 15) \times 5 + (50 - 50) \times 1 = 175$ , hence  $h_4(M_0, \alpha_0) = 180 + 1 \times (0 + 175) = 355$ .

Let's calculate  $h_6(M_0, \alpha_0)$  under  $E_1$  and  $E_2$ . Since  ${}^{(o)}(r_1^\bullet) = \emptyset$ ,  $\delta(r_1, M_0, \alpha_0) = 0$ . Since  ${}^{(o)}(r_2^\bullet) = \{p_{11}\}$ ,  ${}^{(o)}(r_4^\bullet) = \{p_{21}\}$ ,  ${}^{(o)}(r_3^\bullet) = \{p_{12}, p_{22}\}$ , and  $M(p_{11}) = M(p_{12}) = M(p_{21}) = M(p_{22}) = 0$ ,  $\delta(r_2, M_0, \alpha_0) = \delta(r_3, M_0, \alpha_0) = \delta(r_4, M_0, \alpha_0) = 0$ . Then  $h_2(M_0, \alpha_0) = h_1(M_0, \alpha_0)$  under both  $E_1$  and  $E_2$ .  $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) = 530$  under  $E_1$  and  $h_6(M_0, \alpha_0) = 355$  under  $E_2$ .

$$\mathbf{T}(\alpha) = \begin{pmatrix} 35 & 50 & 60 & 70 \\ 0 & 35 & 60 & \end{pmatrix}$$

The energy consumption of a feasible schedule  $\alpha = t_{21}t_{22}t_{11}t_{12}t_{13}t_{14}$  is calculated as follows. It can be checked that  $M_0[\alpha > M_F$ . To calculate the energy consumption during  $\alpha$ , matrix  $\mathbf{T}(\alpha)$  is generated. Let  $q_{ij}$  denote the  $j$ -th operation for  $q_i$ , then the Gantt chart of  $\alpha$  is shown in Fig. 2.

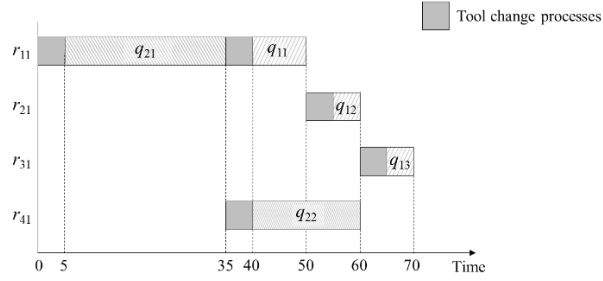


Fig. 2. Gantt chart of schedule  $\alpha$

Under  $E_1$ ,  $E_H(\alpha) = e_0(r_1)T_H(\alpha, p_{11}) + e_0(r_2)T_H(\alpha, p_{12}) + e_0(r_3)T_H(\alpha, p_{13}) + e_0(r_1)T_H(\alpha, p_{21}) + e_0(r_4)T_H(\alpha, p_{22}) = 2 \times 10 + 10 \times 5 + 2 \times 5 + 2 \times 30 + 2 \times 20 = 180$ . From Fig. 2, it is easy to find that the idle time of  $r_1$ ,  $r_2$ ,  $r_3$  and  $r_4$  are 20, 60, 60 and 45, respectively. Thus,  $E_I(\alpha) = 20 \times 0.2 + 60 \times 1 + 60 \times 0.2 + 45 \times 0.2 = 85$ . Tool change processes occur before  $q_{11}$ ,  $q_{12}$ ,  $q_{13}$ ,  $q_{21}$  and  $q_{22}$ , thus  $E_C(\alpha) = 50$ . Then  $E_1(\alpha) = E_H(\alpha) + E_I(\alpha) + E_C(\alpha) = 180 + 85 + 50 = 315$ . Since there is no occupied time during  $\alpha$  under  $E_2$ ,  $E_2(\alpha) = E_1(\alpha) = 315$ .

Let  $h_a^*(M_0, \alpha_0)$  and  $h_b^*(M_0, \alpha_0)$  be the actual optimal total energy consumption from  $(M_0, \alpha_0)$  to  $M_F$  under  $E_1$  and  $E_2$ . For a feasible solution  $\alpha$ ,  $E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$  and  $E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$ . Since  $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$  under  $E_1$  and  $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$  under  $E_2$ ,  $h_4$  and  $h_6$  are not admissible under both  $E_1$  and  $E_2$ .

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**D<sup>2</sup>WS**

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**Input:** An APNS( $N, M_0$ ) and  $high, max\_vertexes, max\_size, max\_top$ ;

**Output:** a feasible transition sequence  $\alpha$ ;

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1:  $OPEN = \{(M_0, \alpha_0)\}$ ;  $CLOSED = \emptyset$ ;  $bottom\_depth = 0$ ;  $top\_depth = high$ ; /* initialization */
2: while ( $OPEN \neq \emptyset$ ) do
3:   if ( $\varphi(bottom\_depth) = 0$ ) /* move the search window down */
4:      $top\_depth++$ ;  $bottom\_depth++$ ;
5:   end if
6:   select a vertex  $(M, \alpha)$  from  $OPEN$  with minimum  $f(M, \alpha)$  and  $|\alpha| < top\_depth$ ;
7:   if ( $\varphi(|\alpha|) \geq max\_size$ ) /* limit the maximal number of explored vertexes at depth  $|\alpha|$  to  $max\_size$  */
8:      $OPEN = OPEN \setminus \{(M, \alpha)\}$ ;
9:     continue;
10:  end if
11:   $OPEN = OPEN \setminus \{(M, \alpha)\}$ ;  $CLOSED = CLOSED \cup \{(M, \alpha)\}$ ;
12:  sort elements in  $\Xi(M, C)$  in ascending order according to their  $A(t, M, \alpha)$ ; /* apply DAP to select safe transitions at  $M$  */
13:   $\nu(\alpha) = 0$ ; /*  $\nu(\alpha)$  records the number of successor or son vertexes of  $(M, \alpha)$  */
14:  while ( $\Xi(M, C) \neq \emptyset$ ) do /* generate successor vertexes from  $M$  under DAP  $C$  */
15:    let  $t \in \Xi(M, C)$  be the first transition in  $\Xi(M, C)$ ;
16:     $\Xi(M, C) = \Xi(M, C) \setminus \{t\}$ ;
17:    let  $M[t] > M_1$ , and  $\alpha_1 = \alpha t$ ;
18:    if ( $M_1 = M_F$ ) /* final marking  $M_F$  is reached */
19:      return  $\alpha_1$ ;
20:    end if
21:    if ( $\exists$  a vertex  $(M_1, \alpha_2)$  in  $OPEN$  satisfying  $B(M_1, \alpha_1) \leq B(M_1, \alpha_2)$ ) /* keep a promising vertex by  $B(M, \alpha)$  */
22:       $OPEN = (OPEN \setminus \{(M_1, \alpha_2)\}) \cup \{(M_1, \alpha_1)\}$ ;
23:    else if ( $\nexists$  a vertex with  $M_1$  in  $OPEN \cup CLOSED$  or  $\exists$  a vertex  $(M_1, \alpha_2)$  in  $CLOSED$  satisfying  $B(M_1, \alpha_1) < B(M_1, \alpha_2)$ )
24:      if ( $\varphi(|\alpha_1|) < max\_size$ )
25:         $OPEN = OPEN \cup \{(M_1, \alpha_1)\}$ ;  $\nu(\alpha)++$ ;
26:      end if
27:      if ( $\varphi(top\_depth) \geq max\_top$ ) /* move the search window down */
28:        discard vertexes at depth  $bottom\_depth$ ;
29:         $top\_depth++$ ;  $bottom\_depth++$ ;
30:      end if
31:      if ( $\nu(\alpha) \geq max\_vertexes$ ), break; /* limit the maximal number of successor vertexes of  $(M, \alpha)$  to  $max\_vertexes$  */
32:    end if
33:  end if-else if
34: end while
35: end while
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*Theorem 2:* D<sup>2</sup>WS can always output a deadlock-free schedule.

*Proof:* All vertexes in D<sup>2</sup>WS are generated under the DAP  $C$  in [22], which prohibit transition firings that lead APNS from safe markings to deadlock. Thus, all markings of generated vertexes are safe. That is to say, from any markings except for  $M_F$ , there is at least one enabled transition that can fire and lead to a safe marking.

There are two termination conditions for D<sup>2</sup>WS: the final marking is reached or  $OPEN = \emptyset$ . If D<sup>2</sup>WS terminates with the final marking reached, then D<sup>2</sup>WS yields a solution. Assume the final marking has not been reached but D<sup>2</sup>WS terminates with  $OPEN = \emptyset$ . Then, all vertexes in the current window (upon termination of the algorithm) are explored. Let  $(M, \alpha)$  be an explored vertex with the deepest depth among all the vertexes in the current window. Since  $M_F \neq M$ , there must be a vertex  $(M_1, \alpha_1)$  in the current window generated from  $(M, \alpha)$  and  $|\alpha_1| = |\alpha| + 1$ .  $(M_1, \alpha_1)$  will be kept in the current window unless there is a vertex at the depth of  $|\alpha_1|$  with marking  $M_1$  or the number of successor vertexes of  $(M, \alpha)$  is more than  $max\_size$ . That is a contradiction to the assumption that  $(M, \alpha)$  is an explored vertex with the deepest depth among all vertexes in the current window. Thus, D<sup>2</sup>WS can always end with the final marking. ■

*Theorem 3:* The complexity of D<sup>2</sup>WS is polynomial.

*Proof:* By Line 7, the maximal number of explored vertexes at a depth is limited to  $max\_size$ . Hence, the maximal number of explored vertexes at each depth in *CLOSED* is no more than  $max\_size$ . Since the number of enabled transitions under a marking is limited to  $|T|$ , the maximal number of vertexes at a depth expect  $top\_depth$  in *OPEN* is limited to  $max\_size \times |T|$ . The maximal number of vertexes that can stay at  $top\_depth$  is  $max\_top$ . Hence, the maximal number of vertexes at each depth in *OPEN* is limited to  $\zeta = \max\{max\_top, max\_size \times |T|\}$ . Let  $\alpha_F$  be a schedule such that  $M_0[\alpha_F > M_F]$ , and  $|\alpha_F|$  denote length of  $\alpha_F$ . Then, the total number of explored vertexes in *CLOSED* is no more than  $max\_size \times |\alpha_F|$ , and the total number of unexplored vertexes in *OPEN* is no more than  $\zeta \times |\alpha_F|$ .

At each iteration of the while-loop from Line 2 to Line 35, a vertex in *OPEN* is selected and deleted. Since  $|OPEN| \leq \zeta \times |\alpha_F|$ , such a while-loop can be executed at most  $\zeta \times |\alpha_F|$  times. The complexity of Lines 3-5 is  $O(1)$ , that of Line 6 is  $O(\zeta \times |\alpha_F|)$ , that of Lines 7-11 is  $O(1)$ , and that of Line 13 is  $O(1)$ . Since  $|\Xi(M, C)| \leq |T|$ , the complexity of Line 12 is  $O(|T| \times \log(|T|))$ . At each iteration of the while-loop from Line 14 to Line 34, a transition in  $\Xi(M, C)$  is selected and deleted. Thus, such a while-loop can be executed at most  $|T|$  times. The complexity of each iteration of the while-loop from Line 14 to Line 34 is dominated by the if-else-if from Line 21 to Line 33. Since  $|OPEN| \leq \zeta \times |\alpha_F|$  and  $|CLOSED| \leq max\_size \times |\alpha_F|$ , the complexity of the if-else-if from Line 21 to Line 33 is  $O((\zeta + max\_size) \times |\alpha_F|)$ . Thus, complexity of the while-loop from Line 14 to Line 34 is  $O(|T| \times (\zeta + max\_size) \times |\alpha_F|)$ .  $O(D^2WS) = O(\zeta \times |\alpha_F| \times (\zeta \times |\alpha_F| + |T| \times \log(|T|) + |T| \times (\zeta + max\_size) \times |\alpha_F|)) = O(\zeta \times |\alpha_F| \times |T| \times (\log(|T|) + (\zeta + max\_size) \times |\alpha_F|))$ . Since  $\zeta = \max\{max\_top, max\_size \times |T|\} > |T|$  and  $\zeta > max\_size$ ,  $O(D^2WS) = O(\zeta^2 \times |\alpha_F|^2 \times |T|)$ . ■

TABLE II  
RUN TIME (SECOND) OF D<sup>2</sup>WS FOR IN1~IN20 UNDER  $E_1$ .

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	$h_1$	$h_2$	$h_3$	$h_4$	$h_5$	$h_6$
IN1	2, 2, 2, 2	0.95	1.08	1.12	1.27	1.26	0.82
IN2	4, 4, 4, 4	3.91	3.78	3.42	2.75	3.90	2.67
IN3	6, 6, 6, 6	7.56	7.61	7.00	5.20	8.30	5.84
IN4	8, 8, 8, 8	11.62	13.29	10.98	7.58	14.32	9.75
IN5	10, 10, 10, 10	16.98	20.79	17.20	11.89	23.32	16.07
IN6	12, 12, 12, 12	22.40	25.94	24.55	15.07	33.44	19.69
IN7	14, 14, 14, 14	31.94	35.42	40.20	21.51	43.57	30.72
IN8	16, 16, 16, 16	42.43	48.70	47.78	27.65	54.73	35.86
IN9	18, 18, 18, 18	56.83	65.74	66.98	37.36	76.90	48.61
IN10	20, 20, 20, 20	70.62	76.03	74.97	48.97	93.71	56.89
IN11	22, 22, 22, 22	85.51	102.86	107.22	63.34	114.71	66.93
IN12	24, 24, 24, 24	108.51	131.31	108.14	69.96	138.43	84.23
IN13	26, 26, 26, 26	129.27	142.30	130.42	88.83	168.45	106.83
IN14	28, 28, 28, 28	146.72	183.74	172.12	95.58	206.91	130.80
IN15	30, 30, 30, 30	186.65	227.01	202.32	117.37	247.98	143.13
IN16	32, 32, 32, 32	210.26	249.87	219.54	134.14	283.89	181.86
IN17	34, 34, 34, 34	242.94	291.56	283.19	168.16	301.12	199.20
IN18	36, 36, 36, 36	276.24	340.01	298.61	181.68	367.91	224.64
IN19	38, 38, 38, 38	300.08	356.31	371.07	208.41	409.15	279.80
IN20	40, 40, 40, 40	351.09	429.54	378.71	248.56	468.30	318.44

TABLE III  
RUN TIME (SECOND) OF D<sup>2</sup>WS FOR IN1~IN20 UNDER  $E_2$ .

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	$h_1$	$h_2$	$h_3$	$h_4$	$h_5$	$h_6$
IN1	2, 2, 2, 2	0.97	1.31	1.26	0.78	1.59	1.02
IN2	4, 4, 4, 4	4.20	3.91	4.31	2.53	5.55	4.55
IN3	6, 6, 6, 6	7.35	8.95	9.68	5.79	9.98	8.71
IN4	8, 8, 8, 8	12.38	16.66	17.05	9.50	20.40	14.44
IN5	10, 10, 10, 10	17.99	24.18	22.16	14.56	26.86	22.67
IN6	12, 12, 12, 12	26.35	34.29	29.97	19.74	42.48	31.25
IN7	14, 14, 14, 14	40.08	49.88	38.99	25.42	55.81	42.32
IN8	16, 16, 16, 16	47.26	59.94	60.64	36.62	65.32	57.73
IN9	18, 18, 18, 18	64.05	83.23	77.65	48.67	75.65	71.16
IN10	20, 20, 20, 20	79.62	99.36	80.08	55.29	100.12	91.99
IN11	22, 22, 22, 22	85.15	119.67	107.77	67.32	140.26	107.76
IN12	24, 24, 24, 24	111.78	151.46	123.74	85.58	179.87	127.65
IN13	26, 26, 26, 26	135.86	175.67	173.13	104.79	204.80	157.59
IN14	28, 28, 28, 28	158.89	209.42	185.76	112.48	232.23	200.17
IN15	30, 30, 30, 30	186.46	255.51	231.20	146.97	298.73	207.15
IN16	32, 32, 32, 32	221.52	290.36	290.64	167.45	318.35	230.58
IN17	34, 34, 34, 34	258.56	325.24	292.09	207.97	363.56	289.46
IN18	36, 36, 36, 36	307.88	374.82	330.88	237.10	422.24	318.46
IN19	38, 38, 38, 38	362.96	451.26	373.39	243.36	488.59	371.65
IN20	40, 40, 40, 40	404.96	530.28	454.56	287.89	602.45	417.53



The parameters of the search windows are randomly distributed in the range of  $[2, 5]$  for  $max\_vertexes$ ,  $[2, 7]$  for  $high$ , and  $[2, 5]$  for  $max\_size$ .  $max\_top = 3 \times max\_size$ . The parameters of search windows used in the first experiment are shown in IV.

TABLE IV  
PARAMETERS OF SEARCH WINDOWS USED IN THE SECOND EXPERIMENT.

Search windows	<i>high</i>	<i>max_vertexes</i>	<i>max_size</i>	<i>max_top</i>
<i>SW01</i>	3	3	3	9
<i>SW02</i>	6	4	3	9
<i>SW03</i>	5	4	2	6
<i>SW04</i>	2	2	3	9
<i>SW05</i>	4	2	3	9
<i>SW06</i>	2	2	5	15
<i>SW07</i>	3	4	2	6
<i>SW08</i>	4	2	2	6
<i>SW09</i>	4	5	4	12
<i>SW10</i>	4	4	3	9

TABLE V  
SIMULATION RESULTS OF IN41~IN60 UNDER  $E_1$ .

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	$h_1$		$h_2$		$h_3$		$h_4$		$h_5$		$h_6$	
		BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
IN41	35, 35, 30, 30	25540	26304	25314	25852	25276	27357	24386	24939	25811	27171	24502	25343
IN42	34, 34, 25, 25	23257	23783	22987	23536	23524	25076	22042	22831	23767	24966	22450	22837
IN43	23, 23, 38, 38	25078	25544	24530	25196	25369	26552	23516	24501	25235	26386	23883	24530
IN44	21, 21, 39, 39	24285	25156	24276	24833	24960	26024	23379	24313	24812	25993	23495	24309
IN45	30, 30, 28, 28	22738	23285	22204	23448	22919	24586	21645	22496	23173	24295	21772	22453
IN46	34, 34, 21, 21	21231	22122	21522	21905	21814	23189	20484	21281	21650	22617	20724	21238
IN47	24, 24, 37, 37	24791	25664	24070	24972	25577	26674	23474	24398	25336	26206	23733	24420
IN48	35, 35, 25, 25	23320	24108	23068	23829	23272	25222	22378	22942	23569	24848	22429	23260
IN49	29, 29, 37, 37	26132	26601	25893	26917	27260	27804	25108	26005	26879	28274	25289	26072
IN50	32, 32, 23, 23	21208	22443	21617	22126	22121	22671	20389	21148	21675	23326	20787	21277
IN51	26, 26, 22, 22	19032	19484	18728	19295	19040	19584	17830	18464	19165	20384	18137	18660
IN52	31, 31, 39, 39	27923	28826	27730	28468	28659	30261	26378	27503	28749	29563	26771	27601
IN53	29, 29, 24, 24	20515	21234	20948	21640	21153	22363	19685	20313	21198	22030	20016	20558
IN54	20, 20, 31, 31	20624	21440	20612	21161	21569	22365	19724	20522	21044	22284	20006	20649
IN55	33, 33, 21, 21	21415	21986	20781	21377	21180	22962	20116	20818	20927	22999	20397	20847
IN56	33, 33, 27, 27	23341	24310	23368	24073	23938	24692	22404	23086	23478	24806	22559	23167
IN57	26, 26, 27, 27	20726	21396	20983	21598	20963	22014	19885	20519	21614	22485	20167	20749
IN58	28, 28, 29, 29	22503	23410	22895	23412	22893	24138	21467	22118	23417	24095	21671	22271
IN59	36, 36, 28, 28	25099	25655	24801	25439	25795	26108	24050	24741	25452	27046	24046	24626
IN60	39, 39, 32, 32	27498	28229	27293	28317	28526	29795	26461	27140	27963	29445	26311	27517

TABLE VI  
SIMULATION RESULTS OF IN41~IN60 UNDER  $E_2$ .

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	$h_1$		$h_2$		$h_3$		$h_4$		$h_5$		$h_6$	
		BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
IN41	35, 35, 30, 30	23761	24207	23713	24130	24122	24655	23596	23879	24117	24475	23426	23861
IN42	34, 34, 25, 25	21809	22053	21712	22044	22180	22446	21363	21728	21911	22269	21482	21797
IN43	23, 23, 38, 38	23674	23970	23522	23823	23706	24270	23199	23613	23729	24132	23262	23542
IN44	21, 21, 39, 39	23237	23681	23346	23778	23394	24067	23072	23383	23496	24003	23023	23443
IN45	30, 30, 28, 28	21187	21629	21287	21713	21367	21944	21176	21384	21501	21922	21028	21377
IN46	34, 34, 21, 21	20398	20695	20133	20524	20343	20806	20079	20306	20458	20858	19996	20308
IN47	24, 24, 37, 37	23521	23812	23407	23724	23494	24122	23335	23524	23358	23992	23127	23363
IN48	35, 35, 25, 25	21895	22455	21972	22352	22034	22668	21816	22043	22272	22732	21803	22148
IN49	29, 29, 37, 37	24972	25291	24972	25398	25163	25688	24462	24903	25071	25546	24584	24911
IN50	32, 32, 23, 23	20424	20825	20292	20732	20494	20855	20034	20306	20268	20752	20048	20304
IN51	26, 26, 22, 22	17760	18027	17746	18013	17915	18317	17531	17753	17772	18201	17535	17704
IN52	31, 31, 39, 39	26217	26655	26358	26764	26350	27009	26046	26384	26706	27206	25945	26264
IN53	29, 29, 24, 24	19450	19830	19590	19893	19878	20158	19424	19605	19760	20083	19302	19584
IN54	20, 20, 31, 31	19607	19936	19490	19885	19828	20292	19459	19755	19696	20163	19433	19733
IN55	33, 33, 21, 21	20061	20276	19964	20223	20164	20499	19751	19937	19965	20503	19624	19973
IN56	33, 33, 27, 27	22281	22662	22109	22655	22347	22777	21846	22104	22322	22575	21701	22029
IN57	26, 26, 27, 27	19559	19840	19669	20008	19886	20416	19545	19728	19728	20158	19455	19683
IN58	28, 28, 29, 29	21177	21424	21121	21502	21528	21943	20802	21212	21150	21721	20868	21154
IN59	36, 36, 28, 28	23578	24082	23513	23860	23751	24242	23213	23470	23696	24099	23116	23424
IN60	39, 39, 32, 32	26038	26483	25944	26321	26361	26881	25680	26063	26228	26662	25792	26097

TABLE VII  
PARAMETERS OF SEARCH WINDOWS USED IN THE THIRD EXPERIMENT.

	Search windows	<i>high</i>	<i>max_vertexes</i>	<i>max_size</i>	<i>max_top</i>
Group 1	<i>GW01</i>	2	2	2	6
	<i>GW02</i>	2	2	3	9
	<i>GW03</i>	2	3	3	9
	<i>GW04</i>	3	3	3	9
Group 2	<i>GW05</i>	3	3	4	12
	<i>GW06</i>	3	4	4	12
	<i>GW07</i>	4	4	4	12
	<i>GW08</i>	4	4	5	15
Group 3	<i>GW09</i>	4	5	5	15
	<i>GW10</i>	5	5	5	15
	<i>GW11</i>	5	5	6	18
	<i>GW12</i>	5	6	6	18
Group 4	<i>GW13</i>	6	6	6	18
	<i>GW14</i>	6	6	7	21
	<i>GW15</i>	6	7	7	21
	<i>GW16</i>	7	7	7	21

TABLE VIII

RESULTS OF NONPARAMETRIC FRIEDMAN'S TESTS (SIGNIFICANCE LEVEL = 0.05).

Data for nonparametric Friedman's tests (significance level = 0.05)	$p$ -value
The average results found by six heuristic functions under $E_1$	0.000
The best results found by six heuristic functions under $E_1$	0.000
The average results found by six heuristic functions under $E_1$	0.000
The best results found by six heuristic functions under $E_2$	0.000
The average results found by $h_4$ under $E_1$ using four groups of search windows	0.000
The best results found by $h_4$ under $E_1$ using four groups of search windows	0.000
The average results found by $h_6$ under $E_2$ using four groups of search windows	0.000
The best results found by $h_6$ under $E_2$ using four groups of search windows	0.000

The multiple comparisons method used in this paper can be found in (M. Hollander and D. A. Wolfe, *Nonparametric Statistical Methods*. New York, NY, USA: Wiley, 1973.)

TABLE IX

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER  $E_1$ .

$h_i$	$h_j$	$p$ -value
1	2	0.713
1	3	0.026
1	4	0.000
1	5	0.001
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.000
2	6	0.000
3	4	0.000
3	5	0.920
3	6	0.000
4	5	0.000
4	6	1.000
5	6	0.000

TABLE X

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER  $E_1$ .

$h_i$	$h_j$	$p$ -value
1	2	0.990
1	3	0.023
1	4	0.000
1	5	0.777
1	6	0.000
2	3	0.003
2	4	0.000
2	5	0.398
2	6	0.000
3	4	0.000
3	5	0.425
3	6	0.000
4	5	0.000
4	6	0.724
5	6	0.000

TABLE XI

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER  $E_2$ .

$h_i$	$h_j$	$p$ -value
1	2	0.065
1	3	0.000
1	4	0.000
1	5	0.000
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.182
2	6	0.000
3	4	0.000
3	5	0.277
3	6	0.000
4	5	0.000
4	6	0.686
5	6	0.000

TABLE XII

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER  $E_2$ .

$h_i$	$h_j$	$p$ -value
1	2	0.944
1	3	0.000
1	4	0.006
1	5	0.547
1	6	0.000
2	3	0.000
2	4	0.079
2	5	0.110
2	6	0.000
3	4	0.000
3	5	0.003
3	6	0.000
4	5	0.000
4	6	0.548
5	6	0.000

TABLE XIII

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND  $h_4$  UNDER  $E_1$ .

Group $i$	Group $j$	$p$ -value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.000
2	4	0.000
3	4	0.986

TABLE XIV

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND  $h_4$  UNDER

$E_1.$		
Group $i$	Group $j$	$p$ -value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.027
2	4	0.032
3	4	1.000

TABLE XV

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND  $h_6$  UNDER

$E_2.$		
Group $i$	Group $j$	$p$ -value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	1.000
2	4	0.004
3	4	0.004

TABLE XVI

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND  $h_6$  UNDER

$E_2.$		
Group $i$	Group $j$	$p$ -value
1	2	0.005
1	3	0.107
1	4	0.467
2	3	0.644
2	4	0.000
3	4	0.002