

Lemma 1: $h_1(v)$ is admissible.

Proof: Let $\Psi^*(r_i, v)$ be the actual time of type- r_i resources needed by all unarranged jobs of v . Since $\Psi(r_i, v)$ denote the minimum total time of type- r_i resources required to process all unarranged jobs under v , $\Psi^*(r_i, v) \geq \Psi(r_i, v)$. Let $\zeta^*(r_{ij}, v, l)$ be the actual earliest time that the l -th time interval of r_{ij} in $\alpha(v)$ may be used, where $l = 1, 2, \dots, \alpha(r_{ij}, v)$, $\varepsilon^*(r_{ij}, v, l)$ be the actual latest time that the l -th time interval of r_{ij} in $\alpha(v)$ may be used, and $\Lambda^*(r_{ij}, v)$ denote the actual longest total time that all idle intervals of r_{ij} can be used for unscheduled jobs of v . It is obvious that $\zeta(r_{ij}, v, l) \leq \zeta^*(r_{ij}, v, l)$ and $\varepsilon(r_{ij}, v, l) \geq \varepsilon^*(r_{ij}, v, l)$. Hence, $\Lambda(r_{ij}, v) \geq \Lambda^*(r_{ij}, v)$. Let $\zeta^*(r_{ij}, v)$ be the actual earliest time that an unarranged job can be arranged after the last job in r_{ij} from v . It is obvious that $\zeta^*(r_{ij}, v) \geq \zeta(r_{ij}, v)$.

Therefore, we have

$$\begin{aligned} h^*(v) &\geq \max_{i \in \mathcal{C}_m} \left\{ \frac{\sum_{1 \leq j \leq C(r_i)} (\zeta^*(r_{ij}, v) - \Lambda^*(r_{ij}, v)) + \Psi^*(r_i, v)}{C(r_i)} - g(v), 0 \right\} \\ &\geq \max_{i \in \mathcal{C}_m} \left\{ \frac{\sum_{1 \leq j \leq C(r_i)} (\zeta(r_{ij}, v) - \Lambda(r_{ij}, v)) + \Psi(r_i, v)}{C(r_i)} - g(v), 0 \right\} = h_1(v) \end{aligned}$$