Lemma 1: $h_1(v)$ is admissible.

Proof: Let $\Psi^*(r_i, v)$ be the actual time of type-r_i resources needed by all unarranged jobs of v. Since $\Psi(r_i, v)$ denote the minimum total time of type-r_i resources required to process all unarranged jobs under v, $\Psi^*(r_i, v) \geq \Psi(r_i, v)$. Let $\varsigma^*(r_{ij}, v, l)$ be the actual earliest time that the l-th time interval of r_{ij} in $\alpha(v)$ may be used, where $l = 1, 2, \ldots, \tau(r_{ij}, v)$, $\varepsilon^*(r_{ij}, v, l)$ be the actual latest time that the l-th time interval of r_{ij} in $\alpha(v)$ may be used, and $\Lambda^*(r_{ij}, v)$ denote the actual longest total time that all idle intervals of r_{ij} can be used for unscheduled jobs of v. It is obvious that $\varsigma(r_{ij}, v, l) \leq \varsigma^*(r_{ij}, v, l)$ and $\varepsilon(r_{ij}, v, l) \geq \varepsilon^*(r_{ij}, v, l)$. Hence, $\Lambda(r_{ij}, v) \geq \Lambda^*(r_{ij}, v)$. Let $\varsigma^*(r_{ij}, v)$ be the actual earliest time that an unarranged job can be arranged after the last job in r_{ij} from v. It is obvious that $\varsigma^*(r_{ij}, v) \geq \varsigma(r_{ij}, v)$.

Therefore, we have

$$h^*(v) \geq \max_{i \in z_m} \left\{ \frac{\sum_{1 \leq j \leq C(r_i)} (\varsigma^*(r_{ij}, v) - \Lambda^*(r_{ij}, v)) + \Psi^*(r_i, v)}{C(r_i)} - g(v), 0 \right\}$$

$$\geq \max_{i \in z_m} \left\{ \frac{\sum_{1 \leq j \leq C(r_i)} (\varsigma(r_{ij}, v) - \Lambda(r_{ij}, v)) + \Psi(r_i, v)}{C(r_i)} - g(v), 0 \right\} = h_1(v) .$$

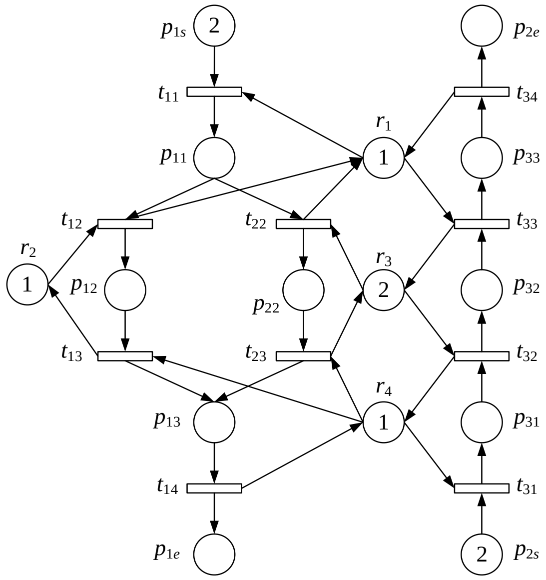

Fig. A. PNS of the FMS in Example a.

Example a: Consider an FMS with $R = \{r_1, r_2, r_3, r_4\}$, where $C(r_1) = C(r_2) = C(r_4) = 1$, and $C(r_3) = 2$. Suppose that the system can process two types of jobs, i.e., $Q = \{1, 2\}$. There are three processing routes, i.e., $\Omega = \{w_1, w_2, w_3\}$, where $w_1 = o_{1s}o_{11}o_{12}o_{13}o_{1e}$ and $w_2 = o_{1s}o_{21}o_{22}o_{23}o_{1e}$ are for type-1 jobs; $w_3 = o_{2s}o_{31}o_{32}o_{33}o_{2e}$ is for type-2 jobs. The processing time of operations is set as $d(o_{11}) = d(o_{21}) = 1$, $d(o_{12}) = 4$, $d(o_{22}) = 2$, $d(o_{13}) = d(o_{23}) = 1$, $d(o_{31}) = 3$, $d(o_{32}) = 2$, and $d(o_{33}) = 1$, respectively. The PNS model is shown in Fig. A.

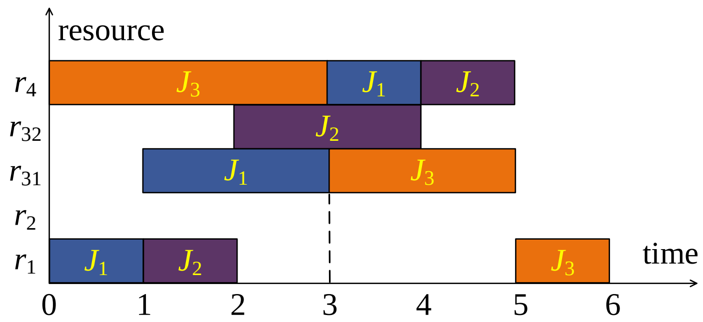

Fig. B. Gantt chart of the schedule in Example a.

Now, consider a schedule in which J_1 and J_2 are both processed on route w_2, and its Gantt chart is shown in Fig. B, where r_{31} and r_{32} denote two different units of type-r_3 resources since the capacity of r_3 is 2, i.e., $C(r_3) = 2$. It can be checked that if J_1 on r_{31} and J_3 on r_4 can exchange their positions directly at time 3, this schedule will satisfy the no-wait constraint. However, in the system considered in this paper, this schedule will lead to a deadlock situation from time 3 and hence is not feasible, because position exchanges of jobs require additional resources or human interventions, which is not allowed in our studied FMSs. This example demonstrates that even in FMSs that satisfy no-wait constraints, deadlocks can occur due to circular waiting for finite resources among different jobs.

***Theorem* 1**: The complexity of HHS is polynomial.

Proof: By Line 17-21, the maximal number of vertexes at the top depth (dep_t) is limited to max_top. Hence, the maximal number of vertexes at each depth in OPEN is limited to max_top. Let α_F be a schedule from the initial state to the final state, and $|\alpha_F|$ denote the length of α_F. Then, the total number of explored vertexes in CLOSED is no more than $max\_top \times |\alpha_F|$, and the total number of unexplored vertexes in OPEN is no more than $max\_top \times height$.

At each iteration of the while-loop from Line 1 to Line 27, a vertex in OPEN is selected and deleted. Since $|\text{OPEN}| \le max\_top \times height$, such a while-loop can be executed at most $max\_top \times height$ times. The complexity of Lines 2-5 is $O(1)$, that of Line 6 is $O(max\_top \times height)$, and that of Lines 7-9 is $O(1)$.

The maximum number of child vertexes that can be generated from any parent vertex is $O(n \times L)$, where n is the number of job types and L is the longest operation sequence length. At each iteration of the nested for-loops from Line 11 to Line 25, a new child vertex is potentially generated and added to OPEN. Thus, these nested loops can generate at most $O(n \times L)$ child vertexes.

The complexity of each iteration within the nested for-loops is dominated by the operations in Lines 13-14. Line 13 (TA algorithm) has complexity $O(L^2\Phi^3|R|)$, where Φ denotes the total number of jobs to be scheduled, and $|R|$ denotes the total number of resources (a detailed proof is provided in [42] of the main manuscript). Line 14 (permutation check) has complexity $O(|\text{OPEN}| + |\text{CLOSED}|) = O(max\_top \times height + max\_top \times |\alpha_F|) = O(max\_top \times (|\alpha_F| + height) = O(max\_top \times |\alpha_F|)$.

Thus, complexity of the nested for-loops from Line 11 to Line 25 is $O((n \times L) \times (L^2\Phi^3|R| + L^2\Phi^3|R| + max\_top \times |\alpha_F|)) = O((n \times L) \times (L^2\Phi^3|R| + max\_top \times |\alpha_F|))$. Since $|\alpha_F| \le \Phi$, so $O((n \times L) \times (L^2\Phi^3|R| + max\_top \times |\alpha_F|)) = O((n \times L) \times (L^2\Phi^3|R|)) = O(nL^3\Phi^3|R|)$.

The total complexity of HHS is $O(\text{HHS}) = O(max\_top \times height \times (max\_top \times height + nL^3\Phi^3|R|) = O(max\_top^2 \times height^2 + max\_top \times height \times nL^3\Phi^3|R|) = O(max\_top \times height \times nL^3\Phi^3|R|)$.

TABLE A

No.	h_1 + HHS		h_1 + HHS + OBE_1		h_1 + HHS + OBE_2	
	MSP	Time	MSP	Time	MSP	Time
T_1	**135**	1.572	140	3.140	**135**	**1.583**
T_2	**169**	**1.983**	172	4.991	**169**	2.026
T_3	**223**	**3.664**	**223**	8.859	**223**	3.841
T_4	340	6.447	**334**	16.967	340	**6.567**
T_5	**423**	13.625	**423**	31.112	**423**	**13.428**
T_6	**670**	33.763	689	74.000	**670**	**33.196**
T_7	**748**	49.921	**748**	101.497	**748**	**48.874**
T_8	**986**	53.173	1033	201.586	**986**	**53.025**
T_9	**1137**	83.172	1163	395.981	**1137**	**80.724**
T_{10}	**1693**	594.58	1752	1263.768	**1693**	**251.601**

TABLE B

No.	h_2 + HHS		h_2 + HHS + OBE_1		h_2 + HHS + OBE_2	
	MSP	Time	MSP	Time	MSP	Time
T_1	**125**	0.058	**125**	0.109	**125**	**0.055**
T_2	172	0.142	**168**	0.304	172	**0.137**
T_3	**233**	0.334	**233**	0.495	**233**	**0.328**
T_4	367	**0.711**	**363**	1.179	367	0.734
T_5	**454**	**1.250**	474	2.194	**454**	1.278
T_6	**702**	**4.134**	737	6.860	**702**	4.177
T_7	**807**	6.034	820	11.465	**807**	**6.001**
T_8	1092	12.679	**1063**	19.569	1092	**12.605**
T_9	**1269**	34.388	1290	51.790	**1269**	**22.656**
T_{10}	**1759**	117.873	1780	123.276	**1759**	**59.764**