Supplementary File for "Petri Net and Hybrid Heuristic Search-based Method for Energy-Minimized Scheduling of Flexible Assembly Systems with Tool Change Processes"

Theorem 1: h_1, h_2, h_3, and h_5 are admissible.

Proof: Let β^* denote the actual optimal schedule from (M, α) to a final vertex, i.e., $M[\beta^* > M_F$. Let $h^*(M, \alpha)$ denote the total energy consumption of β^*. Let $E_W(\alpha, \beta^*)$, $E_O(\alpha, \beta^*)$, $E_I(\alpha, \beta^*)$, $E_H(\alpha, \beta^*)$ and $E_C(\alpha, \beta^*)$ denote the total energy consumption of working, occupied, idle, hold and tool change state from α to $\alpha\beta^*$, respectively. Under E_1, $h^*(M, \alpha) = E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$, while under E_2, $h^*(M, \alpha) = E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$.

Under E_2, $h_1(M, \alpha) = \sum_{p\in P_A\cup P_B}\sum_{j\in \boldsymbol{N}_{M(p)}}(E_R(p, j, M, \alpha) + E_P(p))$ is the minimum working energy consumption of all parts from (M, α) to the end, thus $h_1(M, \alpha) \le E_W(\alpha, \beta^*)$ under E_2. Since $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha)$ is only part of the occupied energy, $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_O(\alpha, \beta^*)$. Since $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha)$ is only part of the idle energy, $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) \le E_I(\alpha, \beta^*)$. Since $E_C(\alpha, \beta^*) \ge 0$, $h_5(M, \alpha) = h_1(M, \alpha) + \sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_2. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_2.

Since $(h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha))$ is only part of the hold energy consumption of all parts from (M, α) to the end, $h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_H(\alpha, \beta^*)$. $h_5(M, \alpha) \le E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_1. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_1. ■

Theorem 2: Both h_2 and h_3 are more informed than h_1. h_5 is the most informed among h_1, h_2, h_3, and h_5.

Proof: Since $\forall(M, \alpha) \in \boldsymbol{R}$ and $\forall r_i \in R'$, $\delta(r_i, M, \alpha) \geq 0$ and $G(r_i, M, \alpha) \geq 0$, $h_2(M, \alpha) = h_1(M, \alpha) + \sum_{r_i \in R'} \delta(r_i, M, \alpha) G(r_i, M, \alpha) \geq h_1(M, \alpha)$. Thus, h_2 is more informed than h_1.

Since $\forall(M, \alpha) \in \boldsymbol{R}$ and $\forall t \in \Pi$, $\kappa(t, M, \alpha) \geq 0$ and $J(t, M, \alpha) \geq 0$, $h_3(M, \alpha) = h_1(M, \alpha) + \sum_{t \in \Pi} \kappa(t, M, \alpha) J(t, M, \alpha) \geq h_1(M, \alpha)$. Thus, h_3 is more informed than h_1.

Since $\forall(M, \alpha) \in \boldsymbol{R}$ and $\forall t \in \Pi$, $\kappa(t, M, \alpha) \geq 0$ and $J(t, M, \alpha) \geq 0$, $h_5(M, \alpha) = h_2(M, \alpha) + \sum_{t \in \Pi} \kappa(t, M, \alpha) J(t, M, \alpha) \geq h_2(M, \alpha)$. Thus, h_5 is more informed than h_2. Since $h_5(M, \alpha) - h_3(M, \alpha) = h_2(M, \alpha) - h_1(M, \alpha) \geq 0$, h_5 is more informed than h_3. Thus, h_5 is the most informed among h_1, h_2, h_3, and h_5. ■

Example 1: For the APNS processing two types of parts in Fig. 1, suppose $M_0 = p_{1s} + p_{2s} + r_1 + r_2 + r_3 + r_4$. The processing time of operations in places p_{11}, p_{12}, p_{13}, p_{21} and p_{22} are all 2. Assume that the tool change time is $\mu = 1$. The Gantt chart of a feasible schedule $\alpha_1 = t_{21}t_{22}t_{11}t_{12}t_{13}t_{14}$ is shown in Fig. 2.

The states of resources in E_1 are illustrated as below. By Fig. 2, the state of r_{11} is tool change during [0, 1) and [3, 4). The state of r_{11} is hold during [1, 3) and [4, 6) since there is a part in it, and idle during [6, 12) since there is not. By Fig. 2, the state of r_{21} is tool change during [6, 7). The state of r_{21} is hold during [7, 9) since there is a part in it, and idle during [0, 6) and [9, 12) since there is not. By Fig. 2, the state of r_{31} is tool change during [9, 10). The state of r_{31} is hold during [10, 12) since there is a part in it, and idle during [0, 9) since there is not. By Fig. 2, the state of r_{41} is tool change during [3, 4). Since t_{13} fires at the time of 9, q_{22} stays in r_{41} during [4, 9). Thus, the state of r_{41} is hold during [4, 9), and idle during [9, 12).

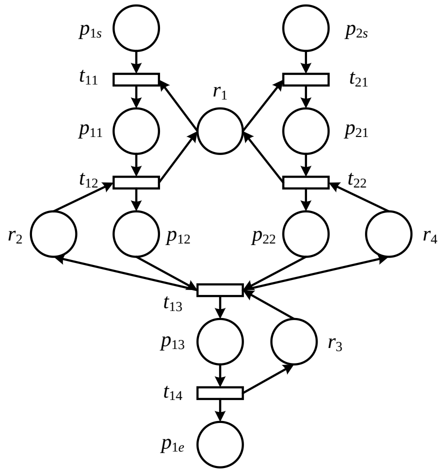

Fig. 1. APNS of FAS in Example 1.

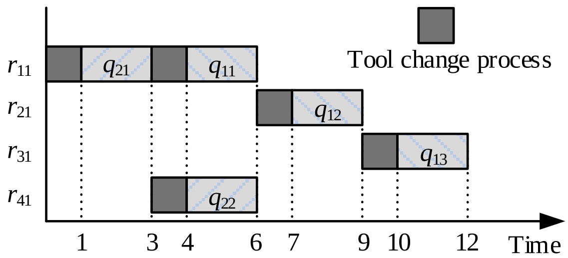

Fig. 2. Gantt chart of schedule α_1.

TABLE I

STATES OF RESOURCES UNDER E_1 AND E_2.

resources	E_1			E_2			
	tool change	hold	idle	tool change	working	occupied	idle
r_{11}	[0, 1), [3, 4)	[1, 3), [4, 6)	[6, 12)	[0, 1), [3, 4)	[1, 3), [4, 6)		[6, 12)
r_{21}	[6, 7)	[7, 9)	[0, 6), [9, 12)	[6, 7)	[7, 9)		[0, 6), [9, 12)
r_{31}	[9, 10)	[10, 12)	[0, 9)	[9, 10)	[10, 12)		[0, 9)
r_{41}	[3, 4)	[4, 9)	[9, 12)	[3, 4)	[4, 6)	[6, 9)	[9, 12)

The states of resources in E_2 are illustrated as below. The idle and tool change states of resources in E_2 are the same as those in E_1 and hence omitted. Since r_{11} is processing a part during [1, 3) and [4, 6), the state of r_{11} is working during [1, 3) and [4, 6). Since r_{21} is processing a part during [7, 9), the state of r_{21} is working during [7, 9). Since r_{31} is processing a part during [10, 12), the state of r_{31} is working during [10, 12). Since r_{41} is processing a part during [4, 6), the state of r_{41} is working during [4, 6). Since the part in r_{41} is finished but has not been moved away during [6, 9), the state of r_{41} is occupied during [6, 9).

The states of resources in E_1 and E_2 are summarized in Table I.

Example 2: For the APNS processing two types of parts in Fig. 1, suppose $M_0 = p_{1s} + p_{2s} + r_1 + r_2 + r_3 + r_4$. The processing time of operations in places p_{11}, p_{12}, p_{13}, p_{21} and p_{22} are 10, 5, 5, 30 and 20, respectively. The energy consumption per unit time in different kinds of states is listed in Table II. For tool change processes, $\mu = 5$ and $e_c = 2$.

TABLE II

ENERGY CONSUMPTION PER UNIT TIME OF RESOURCES IN FIG. 1.

	r_1	r_2	r_3	r_4
Hold state	2.00	10.00	2.00	2.00
Working state	2.00	10.00	2.00	2.00
Occupied state	1.00	5.00	1.00	1.00
Idle state	0.20	1.00	0.20	0.20

Let's calculate $h_4(M_0, \alpha_0)$ under E_1 and E_2. The first part of $h_4(M_0, \alpha_0)$ is $h_1(M_0, \alpha_0)$. $E_R(p_{1s}, 1, M_0, \alpha_0) = E_R(p_{2s}, 1, M_0, \alpha_0) = 0$, $E_P(p_{1s}) = d(p_{11}) \times e_0(r_1) + d(p_{12}) \times e_0(r_2) + d(p_{13}) \times e_0(r_3) = 10 \times 2 + 5 \times 10 + 5 \times 2 = 20 + 50 + 10 = 80$ and $E_P(p_{2s}) = d(p_{21}) \times e_0(r_1) + d(p_{22}) \times e_0(r_4) = 30 \times 2 + 20 \times 2 = 100$. Then $h_1(M_0, \alpha_0) = E_R(p_{1s}, 1, M_0, \alpha_0) + E_P(p_{1s}) + E_R(p_{2s}, 1, M_0, \alpha_0) + E_P(p_{2s}) = 180$.

The rest of $h_4(M_0, \alpha_0)$ under E_1 is calculated as follows. $\Pi = \{t_{12}, t_{22}, t_{13}\}$, for t_{12} and t_{22}, since $M(p_{11}) = M(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. For assembly transition t_{13}, since $p_{12} \in {}^{(o)}t_{13}$, $p_{1s} \in I(p_{12})$ and $M(p_{1s}) = 1 > 0$, $\theta(t_{13}, M_0, \alpha_0) = 1$.

$\Pi = \{t_{12}, t_{13}, t_{22}\}$. Since $|{}^{(o)}t_{12}| = |{}^{(o)}t_{22}| = 1$ and $M_0(p_{11}) = M_0(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. $E_{O1}(t_{13}, M_0, \alpha_0)$ and $E_{O2}(t_{13}, M_0, \alpha_0)$ are calculated as follows. Since $M_0(r_3) > 0$, $T_R(r_3, M_0, \alpha_0) = 0$. $T_O(t_{13}, M_0, \alpha_0) = \max\{T_O(p_{12}, M_0, \alpha_0), T_O(p_{22}, M_0, \alpha_0)\} = \max\{d(p_{11}) + d(p_{12}), d(p_{21}) + d(p_{22})\} = \max\{10 + 5, 30 + 20\} = 50$. Then $E_{O1}(t_{13}, M_0, \alpha_0) = (e_0(r_2) + e_0(r_4)) \times (\max\{T_O(t_{13}, M_0, \alpha_0), T_R(r_3, M_0, \alpha_0)\} - T_O(t_{13}, M_0, \alpha_0)) = (10 + 2) \times (\max\{50, 0\} - 50) = 0$, and $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_0(r_2) + (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{22}, M_0, \alpha_0)) \times e_0(r_4) = (50 - 15) \times 10 + (50 - 50) \times 2 = 350$. Thus $h_4(M_0, \alpha_0) = h_1(M_0, \alpha_0) + \theta(t_{13}, M_0, \alpha_0)(E_{O1}(t_{13}, M_0, \alpha_0) + E_{O2}(t_{13}, M_0, \alpha_0)) = 180 + 1 \times (0 + 350) = 530$ under E_1.

Similarly, under E_2, $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_1(r_2) + (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{22}, M_0, \alpha_0)) \times e_1(r_4) = (50 - 15) \times 5 + (50 - 50) \times 1 = 175$, hence $h_4(M_0, \alpha_0) = 180 + 1 \times (0 + 175) = 355$.

Let's calculate $h_6(M_0, \alpha_0)$ under E_1 and E_2. Since ${}^{(o)}(r_1{}^\bullet) = \varnothing$, $\delta(r_1, M_0, \alpha_0) = 0$. Since ${}^{(o)}(r_2{}^\bullet) = \{p_{11}\}$, ${}^{(o)}(r_4{}^\bullet) = \{p_{21}\}$, ${}^{(o)}(r_3{}^\bullet) = \{p_{12}, p_{22}\}$, and $M(p_{11}) = M(p_{12}) = M(p_{21}) = M(p_{22}) = 0$, $\delta(r_2, M_0, \alpha_0) = \delta(r_3, M_0, \alpha_0) = \delta(r_4, M_0, \alpha_0) = 0$. Then $h_2(M_0, \alpha_0) = h_1(M_0, \alpha_0)$ under both E_1 and E_2. $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) = 530$ under E_1 and $h_6(M_0, \alpha_0) = 355$ under E_2.

$$\boldsymbol{T}(\alpha) = \begin{pmatrix} 35 & 50 & 60 & 70 \\ 0 & 35 & 60 & \end{pmatrix}$$

The energy consumption of a feasible schedule $\alpha = t_{21}t_{22}t_{11}t_{12}t_{13}t_{14}$ is calculated as follows. It can be checked that $M_0[\alpha > M_F$. To calculate the energy consumption during α, matrix $\boldsymbol{T}(\alpha)$ is generated. Let q_{ij} denote the j-th operation for q_i, then the Gantt chart of α is shown in Fig. 3.

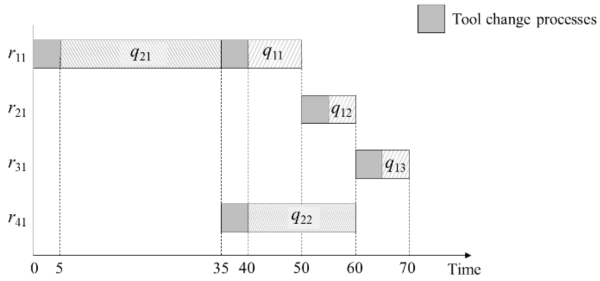

Fig. 3. Gantt chart of schedule α.

Under E_1, $E_H(\alpha) = e_0(r_1)T_H(\alpha, p_{11}) + e_0(r_2)T_H(\alpha, p_{12}) + e_0(r_3)T_H(\alpha, p_{13}) + e_0(r_1)T_H(\alpha, p_{21}) + e_0(r_4)T_H(\alpha, p_{22}) = 2 \times 10 + 10 \times 5 + 2 \times 5 + 2 \times 30 + 2 \times 20 = 180$. From Fig. 3, it is easy to find that the idle time of r_1, r_2, r_3 and r_4 are 20, 60, 60 and 45, respectively. Thus, $E_I(\alpha) = 20 \times 0.2 + 60 \times 1 + 60 \times 0.2 + 45 \times 0.2 = 85$. Tool change processes occur before q_{11}, q_{12}, q_{13}, q_{21} and q_{22}, thus $E_C(\alpha) = 50$. Then $E_1(\alpha) = E_H(\alpha) + E_I(\alpha) + E_C(\alpha) = 180 + 85 + 50 = 315$. Since there is no occupied time during α under E_2, $E_2(\alpha) = E_1(\alpha) = 315$.

Let $h_a^*(M_0, \alpha_0)$ and $h_b^*(M_0, \alpha_0)$ be the actual optimal total energy consumption from (M_0, α_0) to M_F under E_1 and E_2. For a feasible solution α, $E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ and $E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$. Since $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ under E_1 and $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$ under E_2, h_4 and h_6 are not admissible under both E_1 and E_2.

D²WS

Input: An APNS(N, M_0) and *high*, *max_vertexes*, *max_size*, *max_top*;

Output: a feasible transition sequence α;

```
OPEN = {(M0, α0)}; CLOSED = ∅; bottom_depth = 0; top_depth = high; /* initialization */
while (OPEN ≠ ∅) do
    if (φ'(bottom_depth) = 0) /* move the search window down */
        top_depth++; bottom_depth++;
    end if
    select a vertex (M, α) from OPEN with minimum f(M, α) and |α| < top_depth;
    if (φ(|α|) ≥ max_size) /* limit the maximal number of explored vertexes at depth |α| to max_size */
        OPEN = OPEN \ {(M, α)};
        continue;
    end if
    OPEN = OPEN \ {(M, α)}; CLOSED = CLOSED ∪ {(M, α)};
    sort elements in Ξ(M, C) in ascending order according to their A(t, M, α); /*apply DAP to select safe transitions at M*/
    υ(α) = 0; /* υ(α) records the number of successor or son vertexes of (M, α) */
    while (Ξ(M, C) ≠ ∅) do /* generate successor vertexes from M under DAP C */
        let t ∈ Ξ(M, C) be the first transition in Ξ(M, C);
        Ξ(M, C) = Ξ(M, C) \ {t};
        let M[t > M1, and α1 = αt, then compute f(M1, α1);
        if (M1 = MF) /* final marking MF is reached */
            return α1;
        end if
        if (∃ a vertex (M1, α2) in OPEN satisfying B(M1, α1) ≤ B(M1, α2)) /* (M1, α2) is a vertex in OPEN that has identical marking to (M1, α1) but differs in transition sequence. Select one of them by B(M, α) */
            OPEN = (OPEN \ {(M1, α2)}) ∪ {(M1, α1)};
        else if (∄a vertex with M1 in OPEN ∪ CLOSED or ∃ a vertex (M1, α2) in CLOSED satisfying B(M1, α1) < B(M1, α2))
            if (φ(|α1|) < max_size)
                OPEN = OPEN ∪ {(M1, α1)}; υ(α)++;
            end if
            if (φ'(top_depth) ≥ max_top) /* move the search window down */
                discard vertexes at depth bottom_depth;
                top_depth++; bottom_depth++;
            end if
            if (υ(α) ≥ max_vertexes), break; /* limit the maximal number of successor vertexes of (M, α) to max_vertexes */
            end if
        end if-else if
    end while
end while
```

Theorem 3: D^2WS can always output a deadlock-free schedule.

Proof: All vertexes in D^2WS are generated under the DAP C in [25], which prohibit transition firings that lead APNS from safe markings to deadlock. Thus, all markings of generated vertexes are safe. That is to say, from any markings except for M_F, there is at least one enabled transition that can fire and lead to a safe marking.

There are two termination conditions for D^2WS: the final marking is reached or $OPEN = \varnothing$. If D^2WS terminates with the final marking reached, then D^2WS yields a solution. Assume the final marking has not been reached but D^2WS terminates with $OPEN = \varnothing$. Then, all vertexes in the current window (upon termination of the algorithm) are explored. Let (M, α) be an explored vertex with the deepest depth among all the vertexes in the current window. Since $M_F \neq M$, there must be a vertex (M_1, α_1) in the current window generated from (M, α) and $|\alpha_1| = |\alpha| + 1$. (M_1, α_1) will be kept in the current window unless there is a vertex at the depth of $|\alpha_1|$ with marking M_1 or the number of successor vertexes of (M, α) is more than *max_size*. That is a contradiction to the assumption that (M, α) is an explored vertex with the deepest depth among all vertexes in the current window. Thus, D^2WS can always end with the final marking. ■

Theorem 4: The complexity of D^2WS is polynomial.

Proof: By Line 7, the maximal number of explored vertexes at a depth is limited to *max_size*. Hence, the maximal number of explored vertexes at each depth in *CLOSED* is no more than *max_size*. Since the number of enabled transitions under a marking is limited to $|T|$, the maximal number of vertexes at a depth expect *top_depth* in *OPEN* is limited to *max_size* $\times |T|$. The maximal number of vertexes that can stay at *top_depth* is *max_top*. Hence, the maximal number of vertexes at each depth in *OPEN* is limited to $\zeta = \max\{max\_top, max\_size \times |T|\}$. Let α_F be a schedule such that $M_0[\alpha_F > M_F$, and $|\alpha_F|$ denote length of α_F. Then, the total number of explored vertexes in *CLOSED* is no more than *max_size* $\times |\alpha_F|$, and the total number of unexplored vertexes in *OPEN* is no more than $\zeta \times |\alpha_F|$.

At each iteration of the while-loop from Line 2 to Line 35, a vertex in *OPEN* is selected and deleted. Since $|OPEN| \leq \zeta \times |\alpha_F|$, such a while-loop can be executed at most $\zeta \times |\alpha_F|$ times. The complexity of Lines 3-5 is $O(1)$, that of Line 6 is $O(\zeta \times |\alpha_F|)$, that of Lines 7-11 is $O(1)$, and that of Line 13 is $O(1)$. Since $|\Xi(M, C)| \leq |T|$, the complexity of Line 12 is $O(|T| \times \log(|T|))$. At each iteration of the while-loop from Line 14 to Line 34, a transition in $\Xi(M, C)$ is selected and deleted. Thus, such a while-loop can be executed at most $|T|$ times. The complexity of each iteration of the while-loop from Line 14 to Line 34 is dominated by the if-else-if from Line 21 to Line 33. Since $|OPEN| \leq \zeta \times |\alpha_F|$ and $|CLOSED| \leq max\_size \times |\alpha_F|$, the complexity of the if-else-if from Line 21 to Line 33 is $O((\zeta + max\_size) \times |\alpha_F|)$. Thus, complexity of the while-loop from Line 14 to Line 34 is $O(|T| \times (\zeta + max\_size) \times |\alpha_F|)$. $O(D^2WS) = O(\zeta \times |\alpha_F| \times (\zeta \times |\alpha_F| + |T| \times \log(|T|) + |T| \times (\zeta + max\_size) \times |\alpha_F|)) = O(\zeta \times |\alpha_F| \times |T| \times (\log(|T|) + (\zeta + max\_size) \times |\alpha_F|))$. Since $\zeta = \max\{max\_top, max\_size \times |T|\} > |T|$ and $\zeta > max\_size$, $O(D^2WS) = O(\zeta^2 \times |\alpha_F|^2 \times |T|)$. ■

TABLE III

RUNNING TIME (SECOND) OF D^{2}WS FOR IN1~IN20 UNDER E_1.

Instances	ρ_1, ρ_2, ρ_3, ρ_4	h_1	h_2	h_3	h_4	h_5	h_6
IN1	2, 2, 2, 2	0.95	1.08	1.12	1.27	1.26	0.82
IN2	4, 4, 4, 4	3.91	3.78	3.42	2.75	3.90	2.67
IN3	6, 6, 6, 6	7.56	7.61	7.00	5.20	8.30	5.84
IN4	8, 8, 8, 8	11.62	13.29	10.98	7.58	14.32	9.75
IN5	10, 10, 10, 10	16.98	20.79	17.20	11.89	23.32	16.07
IN6	12, 12, 12, 12	22.40	25.94	24.55	15.07	33.44	19.69
IN7	14, 14, 14, 14	31.94	35.42	40.20	21.51	43.57	30.72
IN8	16, 16, 16, 16	42.43	48.70	47.78	27.65	54.73	35.86
IN9	18, 18, 18, 18	56.83	65.74	66.98	37.36	76.90	48.61
IN10	20, 20, 20, 20	70.62	76.03	74.97	48.97	93.71	56.89
IN11	22, 22, 22, 22	85.51	102.86	107.22	63.34	114.71	66.93
IN12	24, 24, 24, 24	108.51	131.31	108.14	69.96	138.43	84.23
IN13	26, 26, 26, 26	129.27	142.30	130.42	88.83	168.45	106.83
IN14	28, 28, 28, 28	146.72	183.74	172.12	95.58	206.91	130.80
IN15	30, 30, 30, 30	186.65	227.01	202.32	117.37	247.98	143.13
IN16	32, 32, 32, 32	210.26	249.87	219.54	134.14	283.89	181.86
IN17	34, 34, 34, 34	242.94	291.56	283.19	168.16	301.12	199.20
IN18	36, 36, 36, 36	276.24	340.01	298.61	181.68	367.91	224.64
IN19	38, 38, 38, 38	300.08	356.31	371.07	208.41	409.15	279.80
IN20	40, 40, 40, 40	351.09	429.54	378.71	248.56	468.30	318.44

TABLE IV

RUNNING TIME (SECOND) OF D^2WS FOR IN1~IN20 UNDER E_2.

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	h_1	h_2	h_3	h_4	h_5	h_6
IN1	2, 2, 2, 2	0.97	1.31	1.26	0.78	1.59	1.02
IN2	4, 4, 4, 4	4.20	3.91	4.31	2.53	5.55	4.55
IN3	6, 6, 6, 6	7.35	8.95	9.68	5.79	9.98	8.71
IN4	8, 8, 8, 8	12.38	16.66	17.05	9.50	20.40	14.44
IN5	10, 10, 10, 10	17.99	24.18	22.16	14.56	26.86	22.67
IN6	12, 12, 12, 12	26.35	34.29	29.97	19.74	42.48	31.25
IN7	14, 14, 14, 14	40.08	49.88	38.99	25.42	55.81	42.32
IN8	16, 16, 16, 16	47.26	59.94	60.64	36.62	65.32	57.73
IN9	18, 18, 18, 18	64.05	83.23	77.65	48.67	75.65	71.16
IN10	20, 20, 20, 20	79.62	99.36	80.08	55.29	100.12	91.99
IN11	22, 22, 22, 22	85.15	119.67	107.77	67.32	140.26	107.76
IN12	24, 24, 24, 24	111.78	151.46	123.74	85.58	179.87	127.65
IN13	26, 26, 26, 26	135.86	175.67	173.13	104.79	204.80	157.59
IN14	28, 28, 28, 28	158.89	209.42	185.76	112.48	232.23	200.17
IN15	30, 30, 30, 30	186.46	255.51	231.20	146.97	298.73	207.15
IN16	32, 32, 32, 32	221.52	290.36	290.64	167.45	318.35	230.58
IN17	34, 34, 34, 34	258.56	325.24	292.09	207.97	363.56	289.46
IN18	36, 36, 36, 36	307.88	374.82	330.88	237.10	422.24	318.46
IN19	38, 38, 38, 38	362.96	451.26	373.39	243.36	488.59	371.65
IN20	40, 40, 40, 40	404.96	530.28	454.56	287.89	602.45	417.53

The parameters of the search windows are randomly distributed in the range of [2, 5] for *max_vertexes*, [2, 7] for *high*, and [2, 5] for *max_size*. *max_top* = 3 × *max_size*. The parameters of search windows used in the first experiment are shown in V.

TABLE V

PARAMETERS OF SEARCH WINDOWS USED IN THE SECOND EXPERIMENT.

Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
*SW*01	3	3	3	9
*SW*02	6	4	3	9
*SW*03	5	4	2	6
*SW*04	2	2	3	9
*SW*05	4	2	3	9
*SW*06	2	2	5	15
*SW*07	3	4	2	6
*SW*08	4	2	2	6
*SW*09	4	5	4	12
*SW*10	4	4	3	9

TABLE VI

SIMULATION RESULTS OF IN41~IN60 UNDER E_1.

Instances	ρ_1, ρ_2, ρ_3, ρ_4	h_1		h_2		h_3		h_4		h_5		h_6	
		BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
IN41	35, 35, 30, 30	25540	26304	25314	25852	25276	27357	24386	24939	25811	27171	24502	25343
IN42	34, 34, 25, 25	23257	23783	22987	23536	23524	25076	22042	22831	23767	24966	22450	22837
IN43	23, 23, 38, 38	25078	25544	24530	25196	25369	26552	23516	24501	25235	26386	23883	24530
IN44	21, 21, 39, 39	24285	25156	24276	24833	24960	26024	23379	24313	24812	25993	23495	24309
IN45	30, 30, 28, 28	22738	23285	22204	23448	22919	24586	21645	22496	23173	24295	21772	22453
IN46	34, 34, 21, 21	21231	22122	21522	21905	21814	23189	20484	21281	21650	22617	20724	21238
IN47	24, 24, 37, 37	24791	25664	24070	24972	25577	26674	23474	24398	25336	26206	23733	24420
IN48	35, 35, 25, 25	23320	24108	23068	23829	23272	25222	22378	22942	23569	24848	22429	23260
IN49	29, 29, 37, 37	26132	26601	25893	26917	27260	27804	25108	26005	26879	28274	25289	26072
IN50	32, 32, 23, 23	21208	22443	21617	22126	22121	22671	20389	21148	21675	23326	20787	21277
IN51	26, 26, 22, 22	19032	19484	18728	19295	19040	19584	17830	18464	19165	20384	18137	18660
IN52	31, 31, 39, 39	27923	28826	27730	28468	28659	30261	26378	27503	28749	29563	26771	27601
IN53	29, 29, 24, 24	20515	21234	20948	21640	21153	22363	19685	20313	21198	22030	20016	20558
IN54	20, 20, 31, 31	20624	21440	20612	21161	21569	22365	19724	20522	21044	22284	20006	20649
IN55	33, 33, 21, 21	21415	21986	20781	21377	21180	22962	20116	20818	20927	22999	20397	20847
IN56	33, 33, 27, 27	23341	24310	23368	24073	23938	24692	22404	23086	23478	24806	22559	23167
IN57	26, 26, 27, 27	20726	21396	20983	21598	20963	22014	19885	20519	21614	22485	20167	20749
IN58	28, 28, 29, 29	22503	23410	22895	23412	22893	24138	21467	22118	23417	24095	21671	22271
IN59	36, 36, 28, 28	25099	25655	24801	25439	25795	26108	24050	24741	25452	27046	24046	24626
IN60	39, 39, 32, 32	27498	28229	27293	28317	28526	29795	26461	27140	27963	29445	26311	27517

TABLE VII

SIMULATION RESULTS OF IN41~IN60 UNDER E_2.

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	h_1		h_2		h_3		h_4		h_5		h_6	
		BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
IN41	35, 35, 30, 30	23761	24207	23713	24130	24122	24655	23596	23879	24117	24475	23426	23861
IN42	34, 34, 25, 25	21809	22053	21712	22044	22180	22446	21363	21728	21911	22269	21482	21797
IN43	23, 23, 38, 38	23674	23970	23522	23823	23706	24270	23199	23613	23729	24132	23262	23542
IN44	21, 21, 39, 39	23237	23681	23346	23778	23394	24067	23072	23383	23496	24003	23023	23443
IN45	30, 30, 28, 28	21187	21629	21287	21713	21367	21944	21176	21384	21501	21922	21028	21377
IN46	34, 34, 21, 21	20398	20695	20133	20524	20343	20806	20079	20306	20458	20858	19996	20308
IN47	24, 24, 37, 37	23521	23812	23407	23724	23494	24122	23335	23524	23358	23992	23127	23363
IN48	35, 35, 25, 25	21895	22455	21972	22352	22034	22668	21816	22043	22272	22732	21803	22148
IN49	29, 29, 37, 37	24972	25291	24972	25398	25163	25688	24462	24903	25071	25546	24584	24911
IN50	32, 32, 23, 23	20424	20825	20292	20732	20494	20855	20034	20306	20268	20752	20048	20304
IN51	26, 26, 22, 22	17760	18027	17746	18013	17915	18317	17531	17753	17772	18201	17535	17704
IN52	31, 31, 39, 39	26217	26655	26358	26764	26350	27009	26046	26384	26706	27206	25945	26264
IN53	29, 29, 24, 24	19450	19830	19590	19893	19878	20158	19424	19605	19760	20083	19302	19584
IN54	20, 20, 31, 31	19607	19936	19490	19885	19828	20292	19459	19755	19696	20163	19433	19733
IN55	33, 33, 21, 21	20061	20276	19964	20223	20164	20499	19751	19937	19965	20503	19624	19973
IN56	33, 33, 27, 27	22281	22662	22109	22655	22347	22777	21846	22104	22322	22575	21701	22029
IN57	26, 26, 27, 27	19559	19840	19669	20008	19886	20416	19545	19728	19728	20158	19455	19683
IN58	28, 28, 29, 29	21177	21424	21121	21502	21528	21943	20802	21212	21150	21721	20868	21154
IN59	36, 36, 28, 28	23578	24082	23513	23860	23751	24242	23213	23470	23696	24099	23116	23424
IN60	39, 39, 32, 32	26038	26483	25944	26321	26361	26881	25680	26063	26228	26662	25792	26097

TABLE VIII

PARAMETERS OF SEARCH WINDOWS USED IN THE THIRD EXPERIMENT.

	Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
Group 1	*GW*01	2	2	2	6
	*GW*02	2	2	3	9
	*GW*03	2	3	3	9
	*GW*04	3	3	3	9
Group 2	*GW*05	3	3	4	12
	*GW*06	3	4	4	12
	*GW*07	4	4	4	12
	*GW*08	4	4	5	15
Group 3	*GW*09	4	5	5	15
	*GW*10	5	5	5	15
	*GW*11	5	5	6	18
	*GW*12	5	6	6	18
Group 4	*GW*13	6	6	6	18
	*GW*14	6	6	7	21
	*GW*15	6	7	7	21
	*GW*16	7	7	7	21

TABLE IX

RESULTS OF NONPARAMETRIC FRIEDMAN'S TESTS (SIGNIFICANCE LEVEL = 0.05).

Data for nonparametric Friedman's tests (significance level = 0.05)	*p*-value
The average results found by six heuristic functions under E_1	0.000
The best results found by six heuristic functions under E_1	0.000
The average results found by six heuristic functions under E_1	0.000
The best results found by six heuristic functions under E_2	0.000
The average results found by h_4 under E_1 using four groups of search windows	0.000
The best results found by h_4 under E_1 using four groups of search windows	0.000
The average results found by h_6 under E_2 using four groups of search windows	0.000
The best results found by h_6 under E_2 using four groups of search windows	0.000

The multiple comparisons method used in this paper can be found in (M. Hollander and D. A. Wolfe, *Nonparametric Statistical Methods*. New York, NY, USA: Wiley, 1973.)

TABLE X

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_1.

h_i	h_j	*p*-value
1	2	0.713
1	3	0.026
1	4	0.000
1	5	0.001
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.000
2	6	0.000
3	4	0.000
3	5	0.920
3	6	0.000
4	5	0.000
4	6	1.000
5	6	0.000

TABLE XI

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_1.

h_i	h_j	*p*-value
1	2	0.990
1	3	0.023
1	4	0.000
1	5	0.777
1	6	0.000
2	3	0.003
2	4	0.000
2	5	0.398
2	6	0.000
3	4	0.000
3	5	0.425
3	6	0.000
4	5	0.000
4	6	0.724
5	6	0.000

TABLE XII

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_2.

h_i	h_j	p-value
1	2	0.065
1	3	0.000
1	4	0.000
1	5	0.000
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.182
2	6	0.000
3	4	0.000
3	5	0.277
3	6	0.000
4	5	0.000
4	6	0.686
5	6	0.000

TABLE XIII

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_2.

h_i	h_j	p-value
1	2	0.944
1	3	0.000
1	4	0.006
1	5	0.547
1	6	0.000
2	3	0.000
2	4	0.079
2	5	0.110
2	6	0.000
3	4	0.000
3	5	0.003
3	6	0.000
4	5	0.000
4	6	0.548
5	6	0.000

TABLE XIV

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_4 UNDER E_1.

Group i	Group j	p-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.000
2	4	0.000
3	4	0.986

TABLE XV

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_4 UNDER E_1.

Group *i*	Group *j*	*p*-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.027
2	4	0.032
3	4	1.000

TABLE XVI

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_6 UNDER E_2.

Group *i*	Group *j*	*p*-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	1.000
2	4	0.004
3	4	0.004

TABLE XVII

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_6 UNDER E_2.

Group *i*	Group *j*	*p*-value
1	2	0.005
1	3	0.107
1	4	0.467
2	3	0.644
2	4	0.000
3	4	0.002

Comparison results of D^{2}WS using search windows in Groups 3 and 4 with A^* using h_5 under E_1 and E_2 are shown in Tables XVIII and XIX, respectively. By Table XVIII and XIX, an out of memory error occurs in A^* with h_5 when $M_0(p_{1s}) = M_0(p_{2s}) = M_0(p_{3s}) = M_0(p_{4s}) = 2$, which validates that A^* cannot handle middle-scale and large-scale instances. Under E_1, D^{2}WS achieves near-optimal solutions (average 5.72% relative error from the optimal solution) while requiring merely 1/2401 of A^*'s average computation time. Under E_2, D^{2}WS achieves near-optimal solutions (average 3.69% relative error from the optimal solution) while requiring merely 1/2667 of A^*'s average computation time.

TABLE XVIII

COMPARISON RESULTS OF D^{2}WS USING SEARCH WINDOWS IN GROUPS 3 AND 4 AND h_4 WITH A^* USING h_5 UNDER E_1

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	D^{2}WS		A^*	
		BST	Average running time (s)	BST	Average running time (s)
IN41	0, 0, 1, 1	811.91	0.000125	757.91	0.001
IN42	1, 1, 0, 0	731.18	0.000125	683.84	0.001
IN43	0, 0, 2, 2	1259.97	0.005250	1172.04	0.091
IN44	2, 2, 0, 0	1112.74	0.006625	1036.88	0.284
IN45	1, 1, 1, 1	1159	0.022875	1080.07	1.480
IN46	0, 0, 3, 3	1631.58	0.047875	1576.92	5.773
IN47	3, 3, 0, 0	1498.42	0.043750	1382.21	28.113
IN48	1, 1, 2, 2	1518.2	0.128875	1500.2	338.704
IN49	2, 2, 1, 1	1462.72	0.152750	1430.86	759.002
IN50	2, 2, 2, 2	1825.76	0.517875	/	/

TABLE XIX

COMPARISON RESULTS OF D^{2}WS USING SEARCH WINDOWS IN GROUPS 3 AND 4 AND h_6 WITH A^* USING h_5 UNDER E_2

Instances	$\rho_1, \rho_2, \rho_3, \rho_4$	D^{2}WS		A^*	
		BST	Average running time (s)	BST	Average running time (s)
IN41	0, 0, 1, 1	778.91	0.000375	754.91	0.001
IN42	1, 1, 0, 0	701.88	0.000250	680.84	0.001
IN43	0, 0, 2, 2	1206.93	0.006625	1165.89	0.169
IN44	2, 2, 0, 0	1066.54	0.007500	1032.88	0.307
IN45	1, 1, 1, 1	1128.83	0.022125	1076.67	2.578
IN46	0, 0, 3, 3	1635.78	0.039625	1567.62	20.838
IN47	3, 3, 0, 0	1414.45	0.045625	1374.91	100.205
IN48	1, 1, 2, 2	1553	0.216000	1487.91	777.749
IN49	2, 2, 1, 1	1427.76	0.259500	/	/
IN50	2, 2, 2, 2	1814.38	0.676750	/	/