Introduction to Industrial Organization

Monopolistic Competition

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Outline

- Product Differentiation
- Theoretical models:
    - Bertrand model with differentiated products
    - Hotelling model
    - Salop circular model

Product Differentiation

Monopolistic Competition

- Monopolistic competition exhibits the following four features:
  1. There is a large number of firms, each producing a single variety of a differentiated product.
  2. Each firm is negligible, in the sense that firms do not interact directly through strategic interdependence but only indirectly through aggregate demand effects.
  3. There are no entry or exit barriers so that economic profits are zero.
  4. Each firm faces a downward-sloping demand curve and therefore enjoys market power.
- The first three features are related to perfect competition, and the last one is related to monopoly.
- Examples: the markets for restaurants, clothing, shoes, and service industries in large cities.

Product Differentiation

- Types of product differentiation:
  - Horizontal:
    - Definition: when the two prices are equal, some consumers prefer one product and other consumers prefer the other product.
    - Example: Pepsi and Coca Cola.
  - Vertical:
    - Definition: when the two prices are equal, all the consumers prefer the same product.
    - Example: BMW and Ford.
  - Mixed:
    - Example: hair-cutting, clothing,

Bertrand Model with Differentiated Products

Bertrand Model with Differentiated Products

- Model settings:
  - Two firms, firm 1 and firm 2, with marginal cost c.
  - Two kinds of products are produced separately by firm 1 and firm 2.
  - Demand function:

$$q_1 = a - b_1 p_1 + b_2 p_2;$$
$$q_2 = a - b_1 p_2 + b_2 p_1,$$

    where $b_1 > b_2 > 0$.
  - Two firms decide the prices, p_1 and p_2 simultaneously.
- The profits maximization problem for firm 1:

$$\max_{p_1} \pi_1 = (p_1 - c)(a - b_1 p_1 + b_2 p_2).$$

Bertrand Model with Differentiated Products

- First-order conditions:

$$a - b_1 p_1 + b_2 p_2 - b_1 p_1 + b_1 c = 0$$

- So the best response function for firm 1:

$$p_1 = \frac{a + cb_1 + b_2 p_2}{2b_1} \equiv \mathsf{BR}(p_2)$$

- Similarly, the best response function for firm 2:

$$p_2 = \mathsf{BR}(p_1) = \frac{a + cb_1 + b_2 p_1}{2b_1}.$$

Equilibrium

- In equilibrium, $p_1 = p_2 = p^*$, so

$$p^* = \frac{a + cb_1}{2b_1 - b_2}.$$

- Equilibrium quantities:

$$q_1 = q_2 = q^* = \frac{ab_1 + cb_1 b_2 - cb_1^2}{2b_1 - b_2}$$

- If $a > c(b_1 - b_2)$, then $q^* > 0$.
- Example: If $b_1 = 2, b_2 = 1, a = 5$ and $c = 1$, then $p^* = \frac{7}{3} > c$. $q^* = \frac{8}{3} > 0$.
- Firms have the incentives to differentiate their products.

Hotelling Model

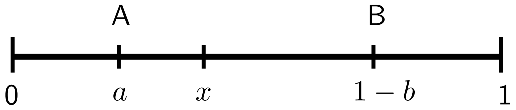
Model Settings I

- One street with the length equal to 1.
- Consumers locate on the street **uniformly**. For instance, there are 100 consumers, and then 50 consumers are in $[0, \frac{1}{2}]$.
- Two firms, firm A and firm B, produce the **homogeneous** product with constant marginal cost c.
- Firm A locates at a, and firm B locates at $1 - b$.

Model Settings II

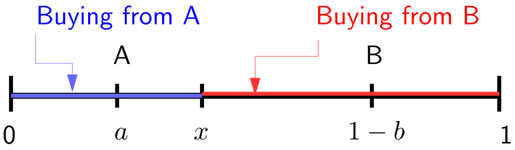
- Prices set by firm A and firm B are P_A and P_B.
- Consumers' utilities:
  - only one unit demand with utility u.
  - prices: P_A or P_B.
  - transportation cost: if the distance is d, then the cost is td^2.
  - For instance, the utility of consumer at x buying the product from firm A and firm B:

$$u - P_A - t(x-a)^2;$$
$$u - P_B - t(1-b-x)^2.$$



Model Settings III

- Two stage game:
  - Stage 1: firms A and B choose the location, $\{a, b\}$, simultaneously.
  - Stage 2: Two firms decide the prices, P_A and P_B, simultaneously.
- In the second stage:
  - Given the location, $\{a, b\}$, two firms decide the prices, , P_A and P_B.
  - To find out a consumer at x who is indifferent between firm A and firm B.



Second Stage

- Obtain x from:

$$u - P_A - t(x-a)^2 = u - P_B - t(1-b-x)^2.$$

- Then

$$x = a + \frac{1-a-b}{2} + \frac{P_B - P_A}{2t(1-a-b)}.$$

- The demand function for two firms:

$$q_A(P_A, P_B) = a + \frac{1-a-b}{2} + \frac{P_B - P_A}{2t(1-a-b)};$$

$$q_B(P_A, P_B) = b + \frac{1-a-b}{2} + \frac{P_A - P_B}{2t(1-a-b)}.$$

- Product differentiation along the street.

Second Stage

- Note: an example when $a = 0$, and $b = 0$
  - If $P_A = P_B$: then $q_A = \frac{1}{2}$ and $q_B = \frac{1}{2}$.
  - If $P_A > P_B$: then

$$q_A = \frac{1}{2} + \frac{P_B - P_A}{2t} < \frac{1}{2};$$
$$q_B = \frac{1}{2} + \frac{P_A - P_B}{2t} > \frac{1}{2}.$$

- The profits maximization problem for firm A:

$$\begin{aligned}\max_{P_A}\ \pi_A &= q_A(P_A, P_B)(P_A - c)\\ &= \left(a + \frac{1-a-b}{2} + \frac{P_B - P_A}{2t(1-a-b)}\right)(P_A - c)\end{aligned}$$

Second Stage

- First-order condition:

$$P_A = at(1-b-a) + \frac{t(1-a-b)^2}{2} + \frac{P_B + c}{2} \equiv \mathsf{BR}(P_B).$$

- Similarly, the best response function for firm B:

$$P_B = bt(1-b-a) + \frac{t(1-a-b)^2}{2} + \frac{P_B + c}{2} \equiv \mathsf{BR}(P_A).$$

- In equilibrium:

$$P_A^* = c + t(1-b-a)\left(1 + \frac{a-b}{3}\right);$$
$$P_B^* = c + t(1-b-a)\left(1 + \frac{b-a}{3}\right).$$

Note for Equilibrium

- Example: If $a = b = 0$, then

$$P_A^* = P_B^* = c + t$$

- If $a = b = \frac{1}{2}$ (two firms locate at the middle point),

$$P_A^* = P_B^* = c.$$

  In this case, two products are homogeneous, so price is equal to marginal cost.

- If $a > b$, then $P_A^* > P_B^*$.

First Stage

- The profits maximization problem for firm A and firm B:

$$\max_a \pi_A(a,b) = \left(P_A^*(a,b) - c\right) q_A\left(P_A^*(a,b), P_B^*(a,b)\right).$$

$$\max_b \pi_B(a,b) = \left(P_B^*(a,b) - c\right) q_B\left(P_A^*(a,b), P_B^*(a,b)\right).$$

- To simplify the problem: we only consider the case $a = b$, so

$$P_A^* = c + t(1-a-b) = c + t(1-2a);\ \ P_B^* = c + t(1-2a)$$

and

$$q_A(P_A^*, P_B^*) = q_B(P_A^*, P_B^*) = \frac{1}{2}.$$

Equilibrium

- In the first stage:
  - The profits maximization problem for firm A becomes:

$$\max_{a} \pi_A(a,b) = \frac{1}{2}(c + t(1-2a) - c) = \frac{1}{2}t(1-2a)$$

  - We need to find the optimal a^* from $[0, \frac{1}{2}]$, so the optimal location $a^* = 0$.
  - The equilibrium location strategies $a^* = b^* = 0$.
    $\Rightarrow$ Maximal Differentiation Principle!
- Extension:
  - To the case without the assumption $a = b$.
  - To the case that two firms decide the location sequentially.
  - Drawback: hard to discuss the number of firms more than 2.

Other Cases

- Question: If two firms can not decide the price in the second stage, say $P_A = P_B = \bar{P}$, what will be the Nash equilibrium for the simultaneous location game?
- Answer: $a^* = b^* = 1/2$. Two firms locate at the center. Implication: they offer the same products.
- Question: From a social point of view, how should they locate their firms to minimize the travel cost of consumers?
- Answer: $a^* = b^* = 1/4$.

Salop Circular Model

Salop Circular Model

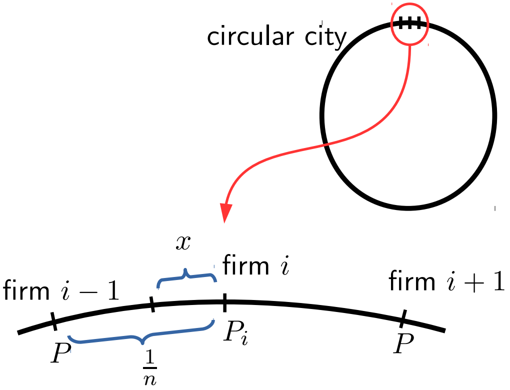
- Model settings:
  - A circle street with circumference equal to 1.
  - Consumers locate on the circle **uniformly**.
  - n firms in the market.
  - Firms are identical with cost function $C(q) = F + cq$.
  - Consumers' utilities:

$$u - P - t|d|,$$

    where $t|d|$ is the linear transportation cost for consumers.
- Two-stage game:
  - In the first stage: all the firms decide the location simultaneously.
  - In the second stage: all the firms decide the price simultaneously.

Maximal Differentiation Principle

- Based on maximal differentiation principle, n firms should locate uniformly on the circle with distance $\frac{1}{n}$ between each of them.
- Given the location, how do they decide the prices?



Demand

- Given price P by other firms and price P_i by firm i, obtain the consumer who are indifferent between firm i and firm $i-1$:

$$u - P_i - tx = u - P - (\frac{1}{n} - x)t$$
$$\Rightarrow \; x = -\frac{P_i - P}{2t} + \frac{1}{2n}.$$

- The demand for firm i:

$$q_i(P_i, P) = 2x = \frac{1}{n} - \frac{P_i - P}{t}.$$

- Note:
  - If $P_i > P$, then $q_i < \frac{1}{n}$.
  - If $P_i = P$, then $q_i = \frac{1}{n}$.
  - If $P_i < P$, then $q_i > \frac{1}{n}$.

Profits Maximization Problem

- The profits maximization problem for firm i:

$$\begin{aligned}\max_{P_i}\ \pi_i &= (P_i - c)q_i(P_i, P) - F \\ &= (P_i - c)\left(\frac{1}{n} - \frac{P_i - P}{t}\right) - F\end{aligned}$$

- First-order condition:

$$\begin{aligned}&\left(\frac{1}{n} - \frac{P_i - P}{t}\right) + (-\frac{1}{t})(p_i - c) = 0 \\ \Rightarrow\ & P_i = \frac{t}{2n} + \frac{P + c}{2}.\end{aligned}$$

- In equilibrium, $P_i = P = P^*$, so $P^* = c + \frac{t}{n}$, and $q^* = \frac{1}{n}$.

Equilibrium

- Implication of $P^* = c + \frac{t}{n}$:
  - If $n \to \infty$, $P^* \to c$. If there are two many firms in the market, market is competitive.
  - If there is no transportation cost, $t = 0$, then $P^* = c$. Because of homogeneous product, the equilibrium price should be equal to the marginal cost.
  - If $t > 0$, then $P^* > c$. Product differentiation makes the price greater than the marginal cost.

Free Entry Case

- If more firms are free to enter into the market, what's the equilibrium number of firms?
- Use the zero profit condition to obtain the equilibrium number of firms n^*.
- The profits for each firm:

$$\pi_i = \left(c + \frac{t}{n} - c\right)\frac{1}{n} - F = \frac{t}{n^2} - F$$

- By zero profit condition:

$$\frac{t}{n^2} - F = 0 \;\Rightarrow\; n^* = \sqrt{\frac{t}{F}}.$$

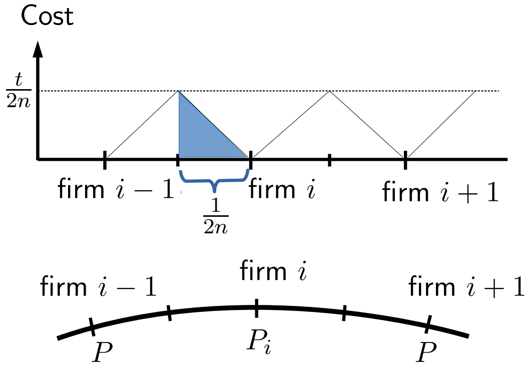
Free Entry Case

- The equilibrium price:

$$P^* = c + \frac{t}{n^*} = c + \sqrt{tF}.$$

- Note:
  - If F increases, then n^* decreases and P^* increases.
  - If t increases, then n^* increases and P^* increases.
- If we consider both the fixed costs for firms and the transportation costs for consumers, what is the "first-best" allocation?

Social Planner's Problem



- Transportation costs: $\left(\frac{1}{2n}\right)\left(\frac{t}{2n}\right)\left(\frac{1}{2}\right) \times 2 \times n = \frac{t}{4n}$.

Social Planner's Problem

- Total costs include:
  - Transportation costs: $\frac{t}{4n}$.
  - Fixed costs: nF.
- To minimize the total costs:

$$\min_{n} \ \frac{t}{4n} + nF$$

- First-order condition:

$$-\frac{t}{4n^2} + F = 0 \ \Rightarrow \ n^{**} = \sqrt{\frac{t}{4F}} = \frac{1}{2}\sqrt{\frac{t}{F}} < n^*$$

- Summary:
  - The first-best number of firms: $n^{**} = \frac{1}{2}\sqrt{\frac{t}{F}}$.
  - Free-entry number of firms: $n^* = \sqrt{\frac{t}{F}}$.

Homework 4

- Please provide an example of product differentiation, including horizontal and vertical differentiation if possible, and answer the following questions:
  - Obtain the product prices and compare them. Do you find any differences among those with horizontal (vertical) differentiation?
  - If possible, obtain the market share of those products. Based on the prices and market shares (quantities), do you find any interesting phenomenon?
  - In your example, do you think that the new firm attempt to increase product differentiation? horizontal or vertical? Explain that.