

Introduction to Industrial Organization

Dynamic Pricing

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Introduction

- Definition:

A pricing strategy in which businesses set highly flexible prices for products or services based on current market demands.

- Motivations:

- Increased availability of demand data
- Changing prices easily due to new technologies
- Availability of decision-support tools for dynamic pricing

- Goal: Revenue or other considerations

- Where do we see that?

Airlines, retailing, manufacturing, hotels, sport events, ...almost everywhere











Outline

- Revenue Management (RM)
 - ▶ Overview
 - ▶ Market Type NR-I
 - ▶ Market Type R-I-M
- Empirical Examples
 - ▶ Secondary Market for Sports Tickets
 - ▶ Airline Tickets
- [Reference](#): Elmaghraby and Keskinocak (2003), "Dynamic Pricing in the Presence of Inventory Considerations: Research Overview, Current Practices, and Future Directions", *Management Science*.

Revenue Management: Overview

Overview in Revenue Management (RM)

- Quantity-Based Revenue Management
 - ▶ Capacity control: planning, forecasting, and replenishment
 - ▶ Overbooking: static/dynamic; combined with capacity control
- Price-Based Revenue Management
 - ▶ Posted-price mechanisms:
take-it-or-leave-it prices determined by sellers
 - Static posted prices: without enough demand information, high transaction costs associate with changing prices
 - **Dynamic posted prices**: change prices frequently over time.
 - ▶ Price-discovery mechanisms:
prices are determined by bidding process, such as auctions.

Dynamic Posted Prices

- To balance demand and supply, dynamic posted prices are based on factors such as
 - ▶ time of sale
 - ▶ demand information
 - ▶ supply availability
- It has been widely used in industries where the **short-term capacity** (supply) is difficult to change, such as airlines, hotels, electric utilities, or sports events.
- The key factor is the segmentation of the market into multiple classes, e.g. business versus leisure customers.
- In some other industries like retail, where short-term supply is more flexible or where price changes are costly, the focus has been on improving inventory management practices.

Features of Models

- We need to consider some important aspects of modelling:
 1. Replenishment vs. No Replenishment of Inventory (R/NR):
 - For some short-life-cycle products, such as certain fashion apparel, replenishing inventory during the selling season is usually not possible.
 - In the case of NR, the seller needs to decide the prices given a fixed amount of inventory.
 2. Dependent vs. Independent Demand Over Time (D/I):
 - The demand across different periods can be dependent if the product is a durable good or if customers' knowledge about the products plays a role in their decision of purchase.
 - For most nondurable goods, demand is independent over time, such as milk and bread.

Features of Models

- We need to consider some important aspects of modelling:
 - 3. **Myopic** vs. **Strategic** Customers (M/S)
 - Myopic consumers are those make purchasing decisions immediately if the price is below her valuation, without considering future prices.
 - Strategic consumers are those take into account the future path of prices when making purchasing decisions.
 - Consumers are more strategic when facing more expensive and durable purchase.
- In addition to the three characteristics discussed above, some other factors can influence a dynamic pricing policy, such as business rules, cost of implementing price changes, seasonality of and external shocks to demand, and cross-elasticities.

Previous Literature

- The existing pricing-with-inventory literature can be classified into two categories:
 1. NR-I (NR-I-M or NR-I-S):
 - There is no opportunity for inventory replenishment over the selling horizon, and demand is independent over time.
 - It is usually for a short-life cycle product, such as fashion apparel or holiday products, or is at the end of its life cycle (e.g., clearance items).
 2. R-I-M:
 - This category captures most **nondurable products**, such as grocery items, produce, and pharmaceutical products.
 - In these markets, customers purchase a product without regard to future price paths, and make frequent purchases over the selling horizon.

Market NR-I

(No Replenishment and Independent Demand)

Market Type NR-I

- This is the case for no inventory replenishment (NR) and independent demand (I) over time.
- Common assumptions in the models:
 - the firm operates in a market with imperfect competition (e.g., a monopolist).
 - the selling horizon T is finite.
 - the firm has a finite stock of n items and no replenishment option during the selling horizon.
 - investment made in inventory is sunk cost.
 - demand decreases in price.
 - unsold items have a salvage value.
- These assumptions fit well with a large segment of retailing.
- The goal is to price the products to maximize expected (discounted) profits over the (short) selling season.

Market Type NR-I

- One of the most important elements that influences pricing decisions is **demand**.
- How to model the demand? some examples from the literature:
 - N (known to the seller) customers arrive in each period with a reservation price V (unknown, drawn from distribution) (**Lazear, 1986**).
 - demand is a homogeneous (time-invariant) Poisson process with intensity $\lambda(p)$ (**Gallego and van Ryzin, 1994**).
 - demand as a non-homogeneous Poisson process with intensity $\lambda(p, t) = \lambda_t(1 - F_t(p))$ (**Bitran et al., 1998**).
- How about the price pattern over time?
 - ▶ It depends on the model assumptions. However, in general, the prices **decrease** over time.
 - ▶ theoretical models: **Lazear (1986)**, and **Gallego and van Ryzin (1994)**.

Lazear (1986)

- Two-period model
- Buyers have the same reservation price (valuation) drawn from a known distribution function.
- Using dynamic programming, he shows that having two periods (and prices) for selling the good increases expected profits.
- This is mainly because the seller can price the good at a higher price in the first period, and if the good is not sold, he can update his belief about the valuations and drop the price in the second period.
- Extending the two-period results to T periods, Lazear shows that as T increases, the initial price increases, the final price goes to zero, and prices drop by smaller amounts.

Gallego and van Ryzin (1994)

- The continuous-pricing problem with Poisson arrivals.
- They derive optimality conditions and show that
 - At a given point in time, the optimal price decreases as the inventory increases.
 - For a given level of inventory, the optimal price rises if there is more time to sell.
 - More on-hand inventory and/or a longer remaining selling horizon lead to higher expected revenues.
- The differences:
 - ▶ [Lazear \(1986\)](#): for each time period where the good is unsold, the seller can update his belief about customers' valuations.
 - ▶ [Gallego and van Ryzin \(1994\)](#): no demand learning. the decreasing price pattern is due to the decreasing expected profit one can make in the remaining shorter time horizon.

Bridging the Gap

- Nowadays, academic research has been incorporated into the software; however, there is a gap between models needed and literature:
 1. **Multiple products**: products are complements or substitutes.
 2. **Multiple stores**: sellers can move inventory around between stores.
 3. **Salvage value**: current pricing models could be extended such that the salvage value is also a decision variable, depending on the choice of the seller among multiple liquidation channels.
 4. **Initial inventory**: initial inventory decisions should be considered into the optimization problem. (Mantrala and Rao, 2001)
 5. **Strategic customers**: for short-life-cycle products, a customer can observe the pricing policies of a seller over time and try to time her purchases to maximize her expected utility.
 6. **Competitors' pricing decisions**: in a competitive business environment, consumers' purchasing decisions take into account prices offered by competing firms.

Market R-I-M

(Replenishment, Independent Demand, and
Myopic Consumers)

Market Type R-I-M

- For a large portion of the nondurable products sold in retail markets, the question facing managers is how to coordinate **pricing** with **inventory procurement and production decisions**.
- Setting the price of a product too low could lead to stockouts and lost sales at a potentially higher price while waiting for inventory replenishment.
- Conversely, setting the price too high could lead to slow-moving or excess inventory and high holding costs.
- Related literature:
 - Assume that the price for a product is a static single price and is exogenous, so the maximization problem is a typical inventory management problem.
 - Allow price to also be a decision variable and vary from period to period. (We focus on this category)

Related Literature

- Assume that the seller is a **monopolist**, selling a **single product** in a multiperiod setting, and faces a demand that is **not dependent** on sales in previous periods.
- The literature in this category can be divided into three groups based on modeling assumptions
 - I. the seller faces an **uncertain demand**, has **convex** production, holding and ordering costs, and **unlimited** production capacity.
 - II. extends this model by incorporating a **fixed** ordering cost, and **limited** production capacity.
 - III. focuses on models where the seller faces a **deterministic demand**.

Literature I.

- Federgruen and Heching (1999), Thowsen (1975), and Zabel (1970)
- The models address the optimal inventory (y_t) and pricing policy (p_t) of a seller who faces an uncertain demand $D(p_t)$ where prices are periodically changed over time.
- Three types of costs: production, holding, and/or ordering cost.
- They find a base stock list price (BSLP) policy is optimal for a wide range of settings.
- A BSLP policy is defined as follows:
 - If the inventory at the start of period t , x_t , is less than some base stock level y_t^* , produce enough to bring the inventory level up to y_t^* , and charge p_t^* .
 - If $x_t > y_t^*$, produce nothing and offer the product at a discounted price of $p_t^*(x_t)$, where $p_t^*(x_t)$ is decreasing in x_t .

Literature I.

- Zabel (1970) considers a setting where the seller faces a finite selling horizon T .
- Findings:
 - (i) the optimal price p_t^* is a decreasing function of the on-hand inventory y_t .
 - (ii) given an on-hand inventory level of y , the optimal price with t periods left is greater than with $t - 1$ periods left, i.e., $p_t^*(y) > p_{t-1}^*(y)$.
 - (iii) the optimal amount to produce is decreasing in t for any given x , i.e., $y_t^*(x) > y_{t-1}^*(x)$.
 - (iv) the critical level x_t^* is decreasing in t .
- It is interesting to note that results (i) and (ii) are identical to the ones derived by Gallego and van Ryzin (1994) in the non-replenishment settings.

Literature II.

- Thomas (1970) and Chen and Simchi-Levi (2002) allow there to be a fixed component to ordering costs.
- Thomas conjectures, and Chen and Simchi-Levi prove, that an (s, S, p) policy is optimal under some assumptions.
- Under an (s, S, p) policy, whenever the on-hand inventory at the start of period t , x_t goes below s , the seller replenishes up to level S ; if $x_t > s$, the seller orders nothing, and charges price $p(x_t)$.
- An (s, S, p) policy is similar to a BSLP policy, with the exception that the presence of fixed ordering costs implies that there is a maximum inventory level above which it is not profitable to reorder, but instead, better to manage demand uncertainty using flexible pricing and existing inventory.

Literature III.

- Rajan et al. (1992) and Biller et al. (2002) study the use of dynamic pricing in the presence of deterministic demand.
- Rajan et al. (1992) consider a single perishable good, such as fresh produce.
- The deterministic demand for the product is a decreasing function of the age of the product and price.
- Four types of costs: (1) a fixed ordering cost; (2) a constant variable ordering cost; (3) a constant holding cost per unit per time period; and (4) a wastage cost associated with inventory decay over time.
- the optimal price over an order cycle may be increasing, decreasing, or both in t , based on the behavior of the costs over time and the rate at which demand diminishes as the product's age increases.

Bridging the Gap

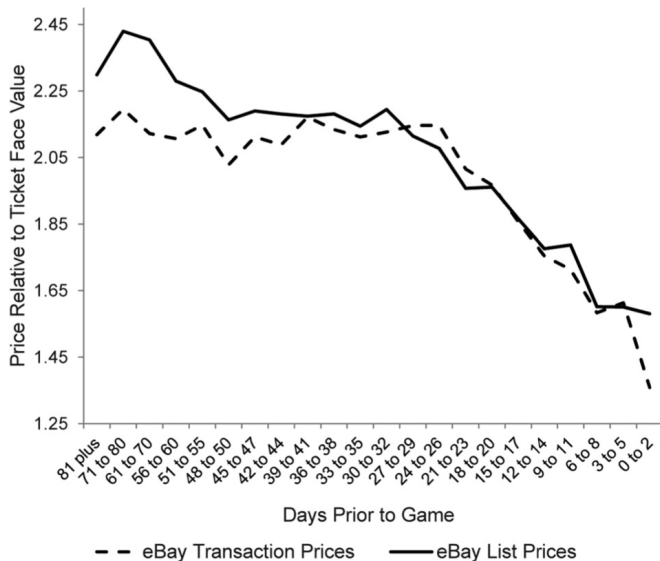
- Comparing the software nowadays and the literature, there are three extensions that must be considered to better capture the decisions and opportunities available to sellers:
 1. **When and how to replenish**: most of the papers assume an exogenous (fixed) periodic replenishment schedule; however, firms can now potentially decide when and how to replenish.
 2. **Multiple products with replenishment**.
 3. **Incorporating business rules**: If the seller faces strategic customers, committing to a rule, such as infrequent and small price drops, might encourage purchases earlier in the time horizon and, hence, increases the seller's profits.

Empirical Examples

Secondary Markets for Sports Tickets

- This is the example from [Sweeting \(2012\)](#), "Dynamic Pricing Behavior in Perishable Goods Markets: Evidence from Secondary Markets for Major League Baseball Tickets"
- Secondary markets: StubHub and eBay
- Dynamic pricing case: perishable product, limited horizon, no replenishment, and myopic/strategic consumers.
- In this paper, he finds that listing prices are [decreasing](#) over time, controlling all other factors which can affect the listing prices.
- The main contribution in this paper is to show that consumers are not strategic in the secondary markets for sports tickets, so the simplest dynamic pricing models describe very accurately.

Average Prices of Tickets on eBay



Theoretical Framework

- A risk-neutral seller i with a single listing to sell at period t , the maximization problem:

$$V_{it} = \max_{p_{it}} p_{it} q_{it}(p_{it}) + [1 - q_{it}(p_{it})] E_t(V_{it+1}),$$

where $q_{it}(p_{it})$ is the probability with which i will sell in period t given his own price p_{it} .

- The optimal price p_{it}^* should satisfy the first-order condition:

$$p_{it}^* = \frac{q_{it}(p_{it}^*) + [1 - q_{it}(p_{it}^*)][\partial E_t(V_{it+1})/\partial p_{it}]}{|\partial q_{it}(p_{it}^*)/\partial p_{it}|} + E_t(V_{it+1}).$$

- In simple dynamic pricing models, $\partial E_t(V_{it+1})/\partial p_{it} = 0$, which indicates that consumers are myopic.

Theoretical Framework

- Therefore, the first-order condition becomes:

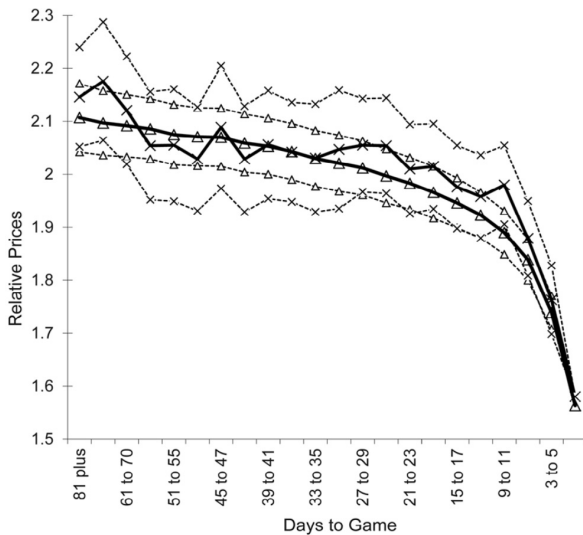
$$p_{it}^* = \frac{q_{it}(p_{it}^*)}{|\partial q_{it}(p_{it}^*)/\partial p_{it}|} + E_t(V_{it+1}).$$

- We can expect that the listing prices are decreasing over time.
- He uses a fixed-effects regression model to capture the price pattern over time:

$$p_{it} = D_t\beta_t^D + F_{it}\beta^F + C_{it}\beta^C + Q_i\beta^Q + FE_i + \epsilon_{it},$$

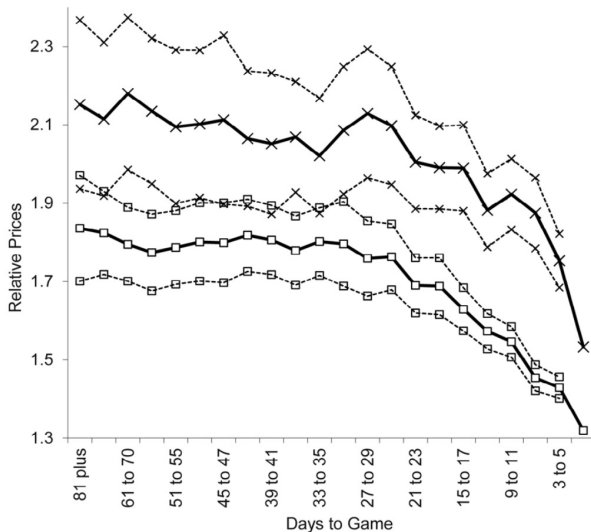
where F are controls for the performance of each team. C represents the number of competing listings. Q includes listing characteristics. D are days-to-go dummies which show on the figure.

Estimated Price Paths for Listed Prices



Note: crosses: eBay; triangles: StubHub

Estimated Price Paths for eBay Transaction Prices



Note: squares: all sales; crosses: fixed-price sales

Regression Results

TABLE 4
WITHIN-SELLER PRICE CHANGES FOR PARTICULAR TYPES OF TICKETS
USING FIXED-PRICE LISTINGS ON eBay

	High Demand	Low Demand	Cheap Seats (≤ \$20)	Expensive Seats (≥ \$45)
Average price 0–2 days prior to game	2.014	.932	1.834	1.431
Selected days-to-go coefficients:				
3–5	.341*** (.097)	.109** (.052)	.184** (.088)	.203*** (.077)
6–8	.683*** (.120)	.184*** (.053)	.400*** (.100)	.381*** (.084)
9–11	.799*** (.120)	.310*** (.048)	.470*** (.092)	.515*** (.073)
15–17	.997*** (.120)	.430*** (.053)	.691*** (.099)	.620*** (.075)
21–23	1.175*** (.150)	.458*** (.054)	.822*** (.130)	.698*** (.081)
30–32	1.263*** (.180)	.509*** (.064)	.868*** (.140)	.745*** (.080)
39–41	1.304*** (.160)	.554*** (.066)	.868*** (.120)	.738*** (.086)
51–55	1.385*** (.180)	.577*** (.068)	.964*** (.120)	.791*** (.085)
81+	1.518*** (.210)	.679*** (.073)	1.019*** (.130)	.821*** (.087)
Observations	84,413	28,158	56,769	49,051
Adjusted R^2	.909	.778	.888	.916

NOTE.—Specifications include seller-game-section-row fixed effects and controls for listing characteristics, competition, and team performance. Robust standard errors clustered on the game are in parentheses.

Airline Tickets

- This example is from [Escobari \(2012\)](#), "Dynamic Pricing, Advance Sales and Aggregate Demand Learning in Airlines".
- This paper empirically studies the dynamic pricing of inventories with uncertain demand over a finite horizon.
- Research questions:
 - for a given inventory of seats, do fares rise as the departure date nears?
 - at a given point prior to the departure date, do fares increase as inventory decreases?
 - do airlines learn about the aggregate demand and adjust their prices as new information about the pattern of sales is revealed?

Dynamic Pricing Problem for Airline Tickets

- Three features of the market:
 - airlines offer tickets in advance, and unsold tickets expire at departure.
 - capacity is also set in advance and can only be modified at a relatively large marginal cost.
 - there is uncertainty about the aggregate demand.
- Hence, airline ticket sales represent an example of dynamic pricing of inventories with uncertain demand over a finite horizon.
- This problem arises in a variety of good and services, such as hotel rooms, cabins on cruise liners, car rentals, and entertainment and sporting events.

Data and Results

- A unique panel data set collected from the online travel agency Expedia.com, which contains prices and seat inventories at the ticket level for 103 days prior to the departure of 228 U.S. domestic flights.
- He uses the data to estimate a pricing equation that is consistent with the theoretical models in [Gallego and van Ryzin \(1994\)](#).
- Empirical results:
 - ▶ the price is lower if there is less time to sell: For every day that passes without sales, the price falls 57.1 cents.
 - ▶ Furthermore, prices increase 7 and 14 days before departure, consistent with the arrival of higher valuation travelers.
 - ▶ one fewer available seat increases fares by 1.53 dollars.
- Consistent with aggregate demand learning and price adjustment, demand shocks have a positive and much larger effect on prices than the positive effect of anticipated sales.

Fares Prior to Departure

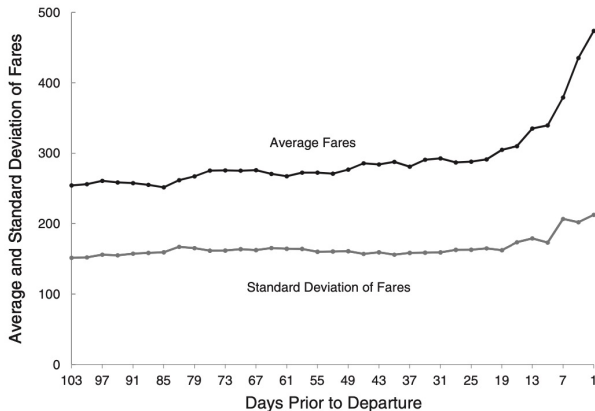


Figure 1
Average and Standard Deviation of Fares

- Average fares appear to increase over time.
- The dispersion of fares across flights is fairly constant, with only a slight increase close to the departure date.

Delta 1588 Atlanta (ATL) to San Jose (SJC)

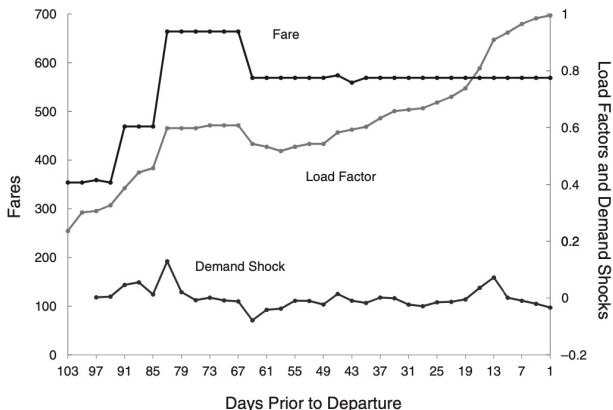


Figure 2
Fares, Load Factors and Demand Shocks (Delta 1588 ATL-SJC)

- load factors refer to actual bookings, and demand shocks are defined as the deviations of actual bookings from the expected bookings.
- Between 85 and 82 days in advance, the load factor increased by 0.14.

Homework 8

- Pick up an industry with the uniform prices, and think about how to implement the dynamic pricing in this industry.
- How does the dynamic pricing help increase the profits?
- Do you expect to face any difficulties? or do you expect any unintended effects?