Introduction to Industrial Organization

Static Oligopoly

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Outline

- Review of Game Theory
- Static Oligopoly Model (Duopoly, and Oligopoly)
  - Cournot model (quantity-based)
  - Bertrand model (price-based)
  - Bertrand model with capacity constraint
    - Small capacities
    - Large capacities (Edgeworth Cycle)

Review of Game Theory

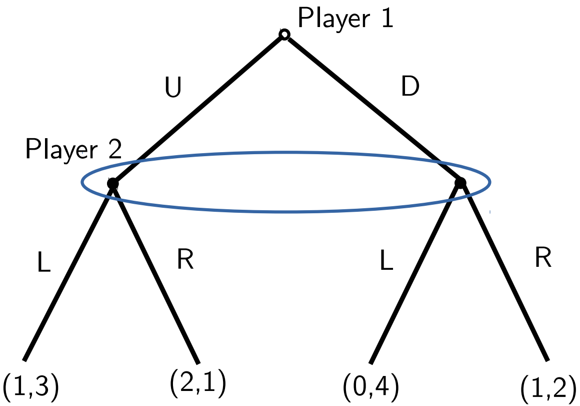
Normal Form

- Normal form representation
  - $i \in N = \{1, 2, \ldots n\}$
  - $s_i \in S_i$
  - $u_i(s_1, s_2, \ldots s_n)$
- Example: two persons game
  - Players: player 1, player 2
  - strategies: s_1, s_2
  - payoffs: $u_1(s_1, s_2), u_2(s_1, s_2)$

		Player 2	
		L	R
Player 1	U	(1,3)	(2,1)
	D	(0,4)	(1,2)

Extensive Form

- Extensive form representation



Best Response Function

- Best Response Strategy

$$s_i \in BR(s_{-i}) \Leftrightarrow u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}),\ \forall s'_i \in S_i$$

- Example:
  - $U = BR_1(L); U = BR_1(R)$
  - $L = BR_2(U); L = BR_2(D)$

		player 2	
		L	R
player 1	U	(1,3)	(2,1)
	D	(0,4)	(1,2)

Nash Equilibrium

- Nash Equilibrium $s^* = \{s_1^*, s_2^*, \dots s_n^*\}$:

$$\forall_i \;\; u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \; \forall s_i \in S_i$$

- Example:
  - $U = BR_1(L); U = BR_1(R)$
  - $L = BR_2(U); L = BR_2(D)$
  - Therefore, $\{U, L\}$ is N.E.

		player 2	
		L	R
player 1	U	(1,3)	(2,1)
	D	(0,4)	(1,2)

Static Oligopoly Model

Static Cournot Model (Duopoly)

- Model settings:
  - Players: 2 identical firms with constant marginal cost c.
  - Strategies:
    - Firm 1 decides q_1;
    - Firm 2 decides q_2.
  - Inverse demand function: $P = a - bQ = a - b(q_1 + q_2)$.
  - Payoffs:

$$\pi_1(q_1, q_2) = \underbrace{(a - b(q_1 + q_2))}_{\text{price}} \cdot q_1 - c \cdot q_1$$

$$\pi_2(q_1, q_2) = (a - b(q_1 + q_2)) \cdot q_2 - c \cdot q_2$$

- Two firms decide the quantities simultaneously, and the price is determined by the market.

Profits Maximization

- Profits maximization problem for firm 1:

$$\max_{q_1} \pi_1 = q_1(a - b(q_1 + q_2)) - c \cdot q_1$$

- First-order condition:

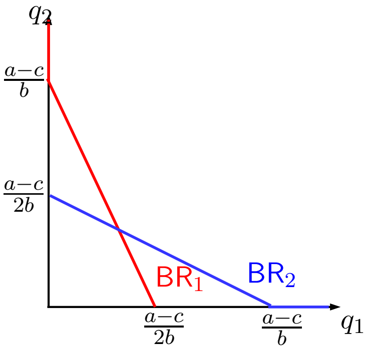
$$\frac{\partial \pi}{\partial q_1} = 0 \;\Rightarrow\; (a - b(q_1 + q_2)) + q_1(-b) - c = 0$$

$$\Rightarrow\; q_1 = \frac{a-c}{2b} - \frac{q_2}{2} \equiv \mathsf{BR}_1(q_2)$$

- Similarly, the best response function for firm 2:

$$q_2 = \mathsf{BR}_2(q_1) \Rightarrow q_2 = \frac{a-c}{2b} - \frac{q_1}{2}$$

Best Response Function



- Two best response functions:

$$\text{BR}_1(q_2) = \begin{cases} \frac{a-c}{2b} - \frac{q_2}{2} & q_2 \leq \frac{a-c}{b} \\ 0 & q_2 > \frac{a-c}{b} \end{cases}$$

$$\text{BR}_2(q_1) = \begin{cases} \frac{a-c}{2b} - \frac{q_1}{2} & q_1 \leq \frac{a-c}{b} \\ 0 & q_1 > \frac{a-c}{b} \end{cases}$$

- By symmetry, the equilibrium

$$q_1^* = q_2^* = \frac{a-c}{3b}.$$

Nash Equilibrium

- Equilibrium price:

$$P^* = a - b\left(\frac{a-c}{3b} + \frac{a-c}{3b}\right) = \frac{1}{3}a + \frac{2}{3}c.$$

- Equilibrium profits:

$$\pi^* = \pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b}.$$

- Question: How about two firms with different marginal costs? Say firm 1 and firm 2 have constant marginal cost c_1 and c_2 respectively. ($c_1 > c_2$) Does the efficient firm provide more quantities? Also, obtain the condition for only one firm existing in the market, and explain it.

Static Cournot Model (Oligopoly, $n > 2$) (I)

- Model settings:
  - Players: $i = 1, 2, ..., n$. n identical firms with constant marginal cost c.
  - Strategies: $\{q_1, q_2, ..., q_n\}$, firm i produces q_i
  - Inverse demand function $P = a - bQ$
  - Payoffs: $\pi_i(q_i, q_{-i})$, $q_{-i} \equiv \{q_1, q_2, ..., q_n\} \setminus \{q_i\}$
- All the firms decide the quantities simultaneously, and the price is determined by the market.
- Profits maximization problem for firm i:

$$\max_{q_i} \underbrace{(a - b(q_1 + q_2 + \cdots + q_n))}_{P} \cdot q_i - c(q_i)$$

Static Cournot Model (Oligopoly, $n > 2$) (II)

- First-order conditions:

$$a - b(q_1 + q_2 + \cdots + q_n) - bq_i - c = 0, \quad \text{for } i = 1, 2, \cdots n$$

- Solve n equations to obtain n variables.
- By symmetric condition: $q_1 = q_2 = \cdots = q_n = q^*$, so

$$\begin{aligned} & a - b(n \cdot q^*) - bq^* - c = 0 \\ \Rightarrow \ & a - c = b(n+1)q^* \\ \Rightarrow \ & q^* = \frac{a-c}{b(n+1)} \end{aligned}$$

- The equilibrium price $P^* = a - b(\frac{n(a-c)}{b(n+1)}) = \frac{a+nc}{n+1}$.

Static Cournot Model (Oligopoly, $n > 2$) (III)

- If $n \to \infty$ (competitive market),

$$P^* = \frac{a + nc}{n + 1} = \frac{\frac{a}{n} + c}{1 + \frac{1}{n}} \to c.$$

- If $n = 1$ (monopoly case),

$$q^* = \frac{a - c}{2b}, \text{ and } P^* = \frac{a + c}{2}.$$

- Question: Obtain the equilibrium for the market with two types of firms. They have different marginal costs: n_1 firms with constant marginal cost c_1 and n_2 firms with constant marginal cost c_2. Do the efficient firms provide more quantities?

Summary of Cournot Model

- In the symmetric linear Cournot model:
  - It converges to perfect competition as the number of firms increases.
  - It represents the monopoly case as the number of firm equal to one.
  - The markup decreases as the number of firms increases.
- In the asymmetric linear Cournot model:
  - A firm's equilibrium profits increase when the firm becomes relatively more efficient than its rivals.
  - A firm with larger market share also has a larger markup.
- Although we typically observe some price-setting in the real world, you can image that the price is determined by the market when sellers commit to bring a certain amount of a product to the market.

Static Bertrand Model (Duopoly) (I)

- Model settings:
  - Players: firm 1, firm 2. Two identical firms with constant marginal costs c.
  - Strategies: firm 1 decides the price P_1, and firm 2 decides the price P_2.
  - Homogeneous products / no search cost $\Rightarrow$ Consumers buy from the firms with the lowest price.
  - Inverse demand function $P(Q) = a - bQ$; demand function $D(P) = \frac{a}{b} - \frac{1}{b}P$.
  - Payoffs:

$$\pi_1(P_1, P_2) = P_1 \cdot D_1(P_1, P_2) - c \cdot D_1(P_1, P_2)$$
$$\pi_2(P_1, P_2) = P_2 \cdot D_2(P_1, P_2) - c \cdot D_2(P_1, P_2),$$

where $D_i(P_1, P_2)$ is the demand for firm i.

Static Bertrand Model (Duopoly) (II)

- The demand for firm 1:

$$D_1(P_1, P_2) = \begin{cases} 0, & \text{if } P_1 > P_2 \\ \frac{1}{2}D(P) & \text{if } P_1 = P_2 \\ D(P_1), & \text{if } P_1 < P_2 \end{cases}$$

- Therefore, the payoffs for firm 1:

$$\begin{aligned} \pi_1(P_1, P_2) =& P_1 \cdot D_1(P_1, P_2) - c \cdot D_1(P_1, P_2) \\ =& \begin{cases} 0, & \text{if } P_1 > P_2 \\ (P_1 - c) \cdot \frac{1}{2}(\frac{a}{b} - \frac{1}{b}P_1), & \text{if } P_1 = P_2 \\ (P_1 - c) \cdot (\frac{a}{b} - \frac{1}{b}P_1), & \text{if } P_1 < P_2 \end{cases} \end{aligned}$$

Static Bertrand Model (Duopoly) (III)

- Similarly, the payoffs for firm 2:

$$\pi_2(P_1, P_2) = \begin{cases} 0, & \text{if } P_2 > P_1 \\ (P_2 - c) \cdot \frac{1}{2}(\frac{a}{b} - \frac{1}{b}P_2), & \text{if } P_2 = P_1 \\ (P_2 - c) \cdot (\frac{a}{b} - \frac{1}{b}P_2), & \text{if } P_2 < P_1 \end{cases}$$

- What is the best response function for firm 1?
  - Idea 1: The price should be set as the monopoly price level if the price by firm 2 is very high.
  - Idea 2: The price should be a little bit lower than that set by firm 2.
  - Idea 3: The profits should be at least positive.

Best Response Functions

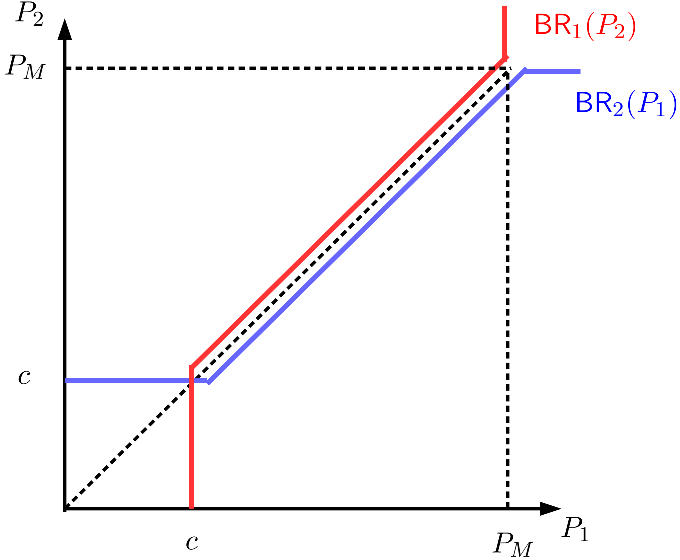
- The best response function for firm 1:

$$
\mathsf{BR}_1(P_2) = \begin{cases} P_M, & \text{if } P_2 > P_M \\ P_2 - \epsilon, & \text{if } P_2 - \epsilon > c \text{ and } P_2 < P_M \\ c, & \text{otherwise} \end{cases}
$$

- Similarly, the best response function for firm 2:

$$
\mathsf{BR}_2(P_1) = \begin{cases} P_M, & \text{if } P_1 > P_M \\ P_1 - \epsilon, & \text{if } P_1 - \epsilon > c \text{ and } P_1 < P_M \\ c, & \text{otherwise} \end{cases}
$$

Figure for Best Response Functions



Nash Equilibrium

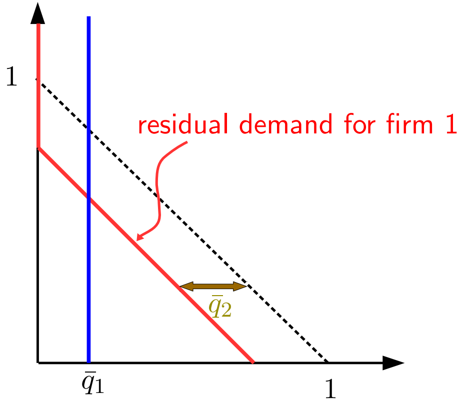
- What is the Nash equilibrium?
  Answer: $P_1^* = P_2^* = c$ (marginal cost)
- Bertrand Paradox: Both firms charge a price equal to the marginal cost.
- Question: If the price can only be set as an integer, and the marginal cost is equal to 0.9, what's the equilibrium outcome?
- How to solve Bertrand Paradox?
  - Price competition with uncertain costs (each firm has private information about its marginal costs)
  - Competition in multiple periods
  - Firms should have capacity constraints.
  - Product differentiation: qualities should be different for different firms.

Static Bertrand Model with Capacity Constraint (I)

- Two types of models:
  - With small capacities
  - With large capacities
- Model settings:
  - Players: two identical firms, firm 1 and firm 2, with zero marginal cost
  - Strategies:
    - Firm 1 decides the price P_1, with the capacity constraint $\bar{q}_1$.
    - Firm 2 decides the price P_2, with the capacity constraint $\bar{q}_2$.
  - Inverse demand function: $P = 1 - Q$.
  - Payoffs: $\pi_1(P_1, P_2)$, $\pi_2(P_1, P_2)$.
  - For small capacities: $\bar{q}_i \in [0, 1/3]$; for large capacities: $\bar{q}_i > 1/3$. (Note: We need to make sure the condition for small capacities in the model! $1/3$ is only the number for this example!)

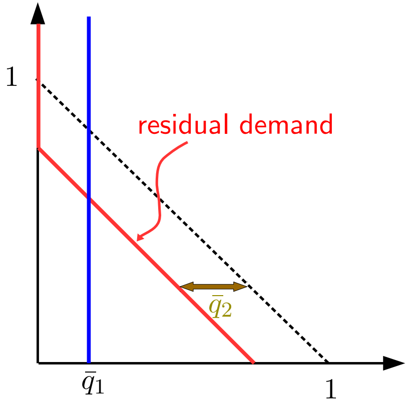
Static Bertrand Model with Capacity Constraint (II)

- Given P_2 and the capacity constraint for firm 2, $\bar{q}_2$, the residual demand function facing firm 1 as follows:



Static Bertrand Model with Capacity Constraint (III)

- Firm 1 also has the capacity constraint $\bar{q}_1$, so the supply curve as the blue line:



Static Bertrand Model with Capacity Constraint (IV)

- The profits maximization problem for firm 1:

$$\max_{P_1} \ (1 - \bar{q}_2 - P_1)P_1$$

$$\text{subject to } 1 - \bar{q}_2 - P_1 \leq \bar{q}_1$$

- If we ignore the constraint, the optimal price $P_1^* = \frac{1-\bar{q}_2}{2}$.
- If $\frac{1-\bar{q}_2}{2} \leq \bar{q}_1$, the capacity constraint is not binding. The optimal price is $P_1^* = \frac{1-\bar{q}_2}{2}$. (However, we will show that it is not a Nash equilibrium! See the model with large capacities!)
- If $\frac{1-\bar{q}_2}{2} > \bar{q}_1$, the capacity constraint is binding. Therefore, the optimal price $P_1^* = 1 - \bar{q}_2 - \bar{q}_1$. This is the case with small capacities.

Static Bertrand Model with Small Capacity (I)

- In small capacities case, we need to show that
  1. $P_1^* = P_2^* = 1 - \bar{q}_1 - \bar{q}_2$ is Nash equilibrium.
  2. In the symmetric case, $\bar{q}_1 = \bar{q}_2 = \bar{q}$, $\bar{q} \leq \frac{1}{3}$ is the condition for small capacities.
- To show the Nash equilibrium:
  - For firm 1: If $P_2 = P_2^*$, $P_1^* = 1 - \bar{q}_1 - \bar{q}_2$ is the best response.
    - If $P_1 > P_1^*$, then $\pi_1(P_1, P_2^*) < \pi_1(P_1^*, P_2^*)$.
    - If $P_1 < P_1^*$, then $\pi_1(P_1, P_2^*) = \bar{q}_1 \cdot P_1 < \bar{q}_1 \cdot P_1^* = \pi_1(P_1^*, P_2^*)$.
  - For firm 2: If $P_1 = P_1^*$, $P_2^* = 1 - \bar{q}_1 - \bar{q}_2$ is the best response.
  - Therefore, $\text{BR}_1(P_2^*) = P_1^*$, and $\text{BR}_2(P_1^*) = P_2^*$.
  - $P_1^* = P_2^* = 1 - \bar{q}_1 - \bar{q}_2$ is Nash equilibrium.

Static Bertrand Model with Small Capacity (II)

- To obtain the condition for small capacities, $\frac{1-\bar{q}_2}{2} > \bar{q}_1$:
  - Under the symmetric case,

$$\bar{q}_1 = \bar{q}_2 = \bar{q} \;\Rightarrow\; \frac{1-\bar{q}}{2} > \bar{q} \;\Rightarrow\; \bar{q} \leq \frac{1}{3} \;\Rightarrow\; \bar{q} \in [0, \frac{1}{3}].$$

  - If the case is not symmetric,

$$\bar{q}_1 \in [0, \frac{1}{3}], \text{ and } \bar{q}_2 \in [0, \frac{1}{3}]$$

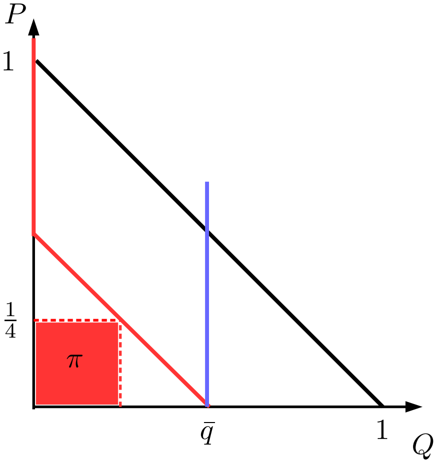
    are the conditions for small capacities.

- Question: If two firms can decide the capacities in the first stage and have price competition in the second stage, what will happen? Example: Two airline companies buy the airplanes first, and they can have price competition based on the fixed capacities.

Static Bertrand Model with Large Capacity (I)

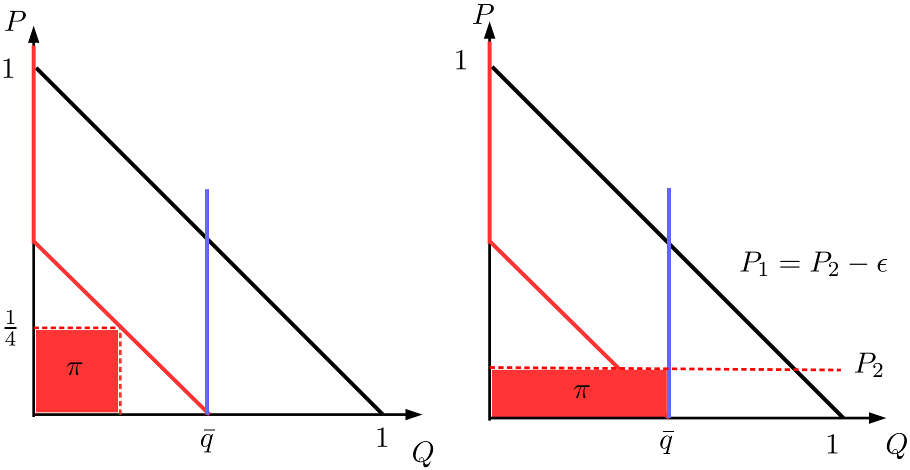
- There is no pure-strategy Nash equilibrium for the case with large capacities.
- Use $\bar{q}_1 = \bar{q}_2 = \bar{q} = \frac{1}{2}$ as example.
- Model settings:
  - Players: two identical firms, firm 1 and firm 2, with zero marginal cost
  - Strategies:
    - Firm 1 decides the price P_1, with the capacity constraint $\bar{q}_1 = 1/2$.
    - Firm 2 decides the price P_2, with the capacity constraint $\bar{q}_2 = 1/2$.
  - Inverse demand function: $P = 1 - Q$.
  - Payoffs: $\pi_1(P_1, P_2)$, $\pi_2(P_1, P_2)$.

Static Bertrand Model with Large Capacity (II)

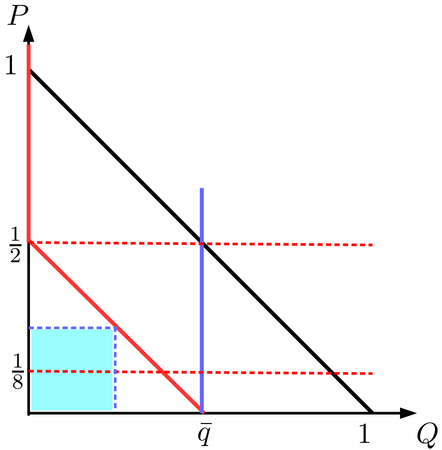


- For firm 1, if $P_2 < P_1$, the optimal price $P_1 = \frac{1}{4}$ based on the residual demand. Profits $\pi_1 = \frac{1}{16}$
- The optimal quantity is smaller than the capacity.
- However, setting $P_1 < P_2$ might get more profits!

Static Bertrand Model with Large Capacity (III)

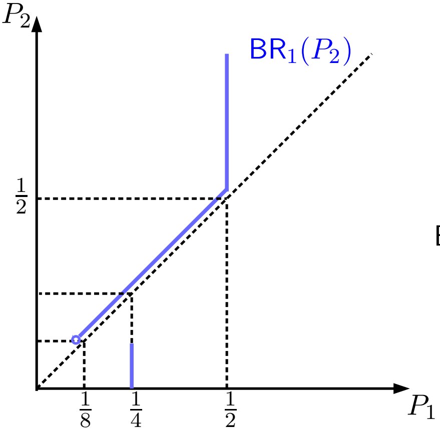


Best Response Functions (I)



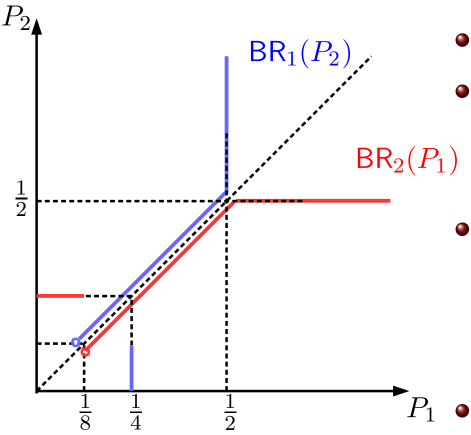
- If $P_2 < \frac{1}{8}$, the best response $\text{BR}_1(P_2) = \frac{1}{4}$.
  - If $P_1 = P_2 - \epsilon < P_2$, then $\pi_1 = P_1 \times \bar{q} < \frac{1}{16}$.
  - If $P_1 > P_2$, the optimal price is $\frac{1}{4}$.
- If $\frac{1}{8} < P_2 \leq \frac{1}{2}$, the best response $\text{BR}_1(P_2) = P_2 - \epsilon$.
- If $P_2 > \frac{1}{2}$, the best response $\text{BR}_1(P_2) = \frac{1}{2}$ because the optimal price for the whole demand is $\frac{1}{2}$.

Best Response Functions (II)



$$\mathsf{BR}_1(P_2) = \begin{cases} \frac{1}{2}, & \text{if } P_2 > \frac{1}{2} \\ P_2 - \epsilon, & \text{if } \frac{1}{8} < P_2 < \frac{1}{2} \\ \frac{1}{4}, & \text{if } P_2 < \frac{1}{8} \end{cases}$$

Equilibrium



- Similarly, we can have $\mathrm{BR}_2(P_1)$.
- No Pure-Strategy Nash equilibrium. (but with mixed-strategy Nash equilibrium)
- If we extend the static model to multiple periods, we can have the equilibrium. (Edgeworth, 1925; Maskin and Tirole, 1988)
- It is called "Edgeworth Cycles".

Homework 2

- What characteristics of a specific industry will you look for to determine whether this industry is better represented by price competition or by quantity competition?
- Which above-mentioned model would you think provides a better first approximation to each of the following industries or markets: (1) the oil refining industry, (2) farmers' markets, (3) cleaning or repairing services.