Homework 2, Due Wednesday, June 20

Math 269C: Advanced Numerical Analysis, Numerical Methods for Elliptic Equations

- Consider the interface problem

$$u_{xx} = f, \quad x \in (0,1) \setminus \{\hat{x}\}, \quad \hat{x} \in (0,1)$$

$$[u]|_{\hat{x}} = a$$

$$[u_x]|_{\hat{x}} = b$$

$$u(0) = u_0, \quad u(1) = u_1$$

  Use FEM to solve this problem. Use the weak form you derived in the previous homework. You can use either an embedded grid or you can let the interface elements have a node at the jump. Do a refinement study to show the order of convergence. Use $\hat{x} = .5$. Use $u(x) = \sin(x)$ for $x < \hat{x}$ and $u(x) = \mathrm{e}^x$ for $x > \hat{x}$ as the exact solution. I will be more impressed if you use an embedded grid.

- Write a program that solves the 2D Poisson equation over an arbitrary triangle mesh. Use the mesh that is given on the course webpage.

- Write a program that solves the 2D linear elasticity equations on an arbitrary triangle mesh. Use the mesh that is given on the course webpage.

- Prove that you get the exact solution for mixed Dirichlet/Neumann boundary conditions (i.e. Dirichlet at the right of the domain, Neumann at the left of the domain). Recall that you can do this by showing that the Green's functions associated with grid nodes are in the FEM function space.

- Verify that you get the exact solution at the grid nodes if you compute the right hand side analytically for the 1D Poisson equation using FEM. Use a uniform grid and Dirichlet boundary conditions at the right of the domain, Neumann at the left.

- Prove that the KKT system associated with the embedded 2D Poisson equations with the weak Dirichlet enforcement (discussed in class) is full rank.

- **Extra Credit:** Prove theorem 4.5 in Breass.