1.

$I_1(\underline{F}) = tr(\underline{F}^T\underline{F})$, $J = |\underline{F}|$

a. show $\sigma_1 + \sigma_2 = \sqrt{I_1(\underline{F}) + 2J}$

$I_1(\underline{F}) = tr(\underline{F}^T\underline{F}) = tr(\underline{V}\,\underline{\Sigma}^2\underline{V}^T) = tr(\underline{V}\,\underline{\Sigma}^2\,\underline{V}^{-1})$

$= V_{il}\,\Sigma^2_{lk}\,V^{-1}_{ki} = (V^{-1}V)_{kl}\,\Sigma^2_{lk} = \delta_{kl}\,\Sigma^2_{lk}$

$= \Sigma^2_{kk} = tr(\underline{\Sigma}^2) = \sigma_1^2 + \sigma_2^2$

$J = |\underline{U}\,\underline{\Sigma}\,\underline{V}^T| = |\underline{\Sigma}| = \sigma_1\sigma_2$

$\sqrt{I_1(\underline{F}) + 2J} = \sqrt{\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2} = \sigma_1 + \sigma_2$

b. show $\sigma_1 - \sigma_2 = \sqrt{I_1(\underline{F}) - 2J}$

$\sqrt{I_1(\underline{F}) - 2J} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2} = \sqrt{(\sigma_1 - \sigma_2)^2}$

since $\sigma_1 \geq |\sigma_2| \geq \sigma_2$, $\sigma_1 - \sigma_2 \geq 0$

i.e. $\sqrt{I_1(\underline{F}) - 2J} = \sigma_1 - \sigma_2$