Computing 2D $\delta\underline{R}$ and $\delta\underline{S}$

$$\underline{F} = \underline{R}\,\underline{S}, \quad \underline{R}^T\underline{R} = \underline{I}, \quad \underline{S} = \underline{S}^T \rightarrow \delta\underline{S} = \delta\underline{S}^T$$

$$\underline{R}^T\delta\underline{R} + \delta\underline{R}^T\underline{R} = 0 \rightarrow \underline{R}^T\delta\underline{R} = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}$$

$$\delta\underline{F} = \delta\underline{R}\,\underline{S} + \underline{R}\,\delta\underline{S}$$

$$\underline{R}^T\delta\underline{F} = \underline{R}^T\delta\underline{R}\,\underline{S} + \delta\underline{S} \qquad (*)$$

Let $\underline{R}^T\delta\underline{F} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\underline{S} = \underline{S}^T = \begin{bmatrix} e & f \\ f & g \end{bmatrix}$, $\delta\underline{S} = \begin{bmatrix} y & w \\ w & z \end{bmatrix}$,

then $(*)$ becomes $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix}\begin{bmatrix} e & f \\ f & g \end{bmatrix} + \begin{bmatrix} y & w \\ w & z \end{bmatrix}$, i.e.,

$$\begin{cases} a = xf + y \\ b = xg + w \\ c = -xe + w \\ d = -xf + z \end{cases} \Rightarrow \underbrace{\begin{bmatrix} f & 1 & 0 & 0 \\ g & 0 & 0 & 1 \\ -e & 0 & 0 & 1 \\ -f & 0 & 1 & 0 \end{bmatrix}}_{\underline{A}} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

This system is solvable iff $g + e \neq 0$

i.e. $tr(\underline{S}) \neq 0$ i.e. $tr(\underline{\Sigma}) \neq 0$ i.e. $\sigma_1 + \sigma_2 \neq 0$

After getting x, y, z, w, we have

$$\delta\underline{R} = \underline{R}\begin{bmatrix} 0 & x \\ -x & 0 \end{bmatrix} = \begin{bmatrix} -xR_{12} & xR_{11} \\ -xR_{22} & xR_{21} \end{bmatrix}$$

$$\delta\underline{S} = \begin{bmatrix} y & w \\ w & z \end{bmatrix}$$