Take Home Final: Due Wed., December 16

Math 270A: Techniques in Scientific Computing

1. FEM Poisson: Use the C++ FEM code on the course webpage (or write your own) to solve the following problems:
$$\Delta u + f(x,y) = 0, \quad (x,y) \in \Omega$$
$$\nabla u \cdot \mathbf{n} = g(x,y), \quad (x,y) \in \partial\Omega$$
where Ω is the given in the file 'mesh_with_holes.dat' and

   a. $f(x,y) = 8\pi^2 \sin(2\pi x)\cos(2\pi y)$, $g(x,y) = \nabla u \cdot \mathbf{n}$ where $u(x,y) = \sin(2\pi x)\cos(2\pi y)$.

   b. $f(x,y) = -4$, $g(x,y) = \nabla u \cdot \mathbf{n}$ where $u(x,y) = x^2 + y^2$.

   c. $f(x,y) = 0$, $g(x,y) = \nabla u \cdot \mathbf{n}$ where $u(x,y) = x + y$. Plot your solutions with the Matlab code on the course webpage.

2. FEM Linear Elasticity: Write a C++ program (you can start from the Poisson code from problem 1) to solve the following problems:
$$\nabla \cdot \sigma + \mathbf{f} = \mathbf{0}, \quad (x,y) \in \Omega$$
$$\mathbf{u}(x,y) = \mathbf{h}(x,y), \quad (x,y) \in \Gamma_h$$
$$\sigma \cdot \mathbf{n} = \mathbf{g}(x,y), \quad (x,y) \in \Gamma_g$$
where $\sigma = 2\mu\epsilon + \lambda \mathrm{Tr}(\epsilon)\mathbf{I}$, $\epsilon = \frac{1}{2}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}}^T\right)$ and

   a. $\Gamma_g = \partial\Omega$, $\mathbf{u}(x,y) = \left(x^2, y^2\right)$ and $\mathbf{f}$ and $\mathbf{g}$ are set accordingly. Use Young's modulus $= 1000$ and Poisson ratio $= .3$.

   b. $\Gamma_h =$ left boundary of $\partial\Omega$, $\Gamma_g =$ remaining portion of $\partial\Omega$, $\mathbf{f} = -9.8$, $\mathbf{h} = \mathbf{0}$ and $\mathbf{g} = \mathbf{0}$. Use Young's modulus $= 10000, 1000, 100$ and Poisson ratio $= .3$. Plot your solutions using the Matlab code from the course webpage.