3. show $\dfrac{\partial(\sigma_1+\sigma_2+\sigma_3)}{\partial \underline{F}} = \underline{R}$.

$$\delta(\sigma_1+\sigma_2+\sigma_3) = \delta\,(tr(\underline{S}))$$

$$= tr(\delta \underline{S})$$

$$= tr(\delta \underline{R}^T \underline{F} + \underline{R}^T \delta \underline{F})$$

$$= tr(\delta \underline{R}^T \underline{R}\,\underline{S}) + tr(\underline{R}^T \delta \underline{F})$$

$$= \delta R^T_{im} R_{mn} S_{ni} + tr(\underline{R}^T \delta \underline{F})$$

$$= (\underline{R}^T \delta \underline{R})_{ni} S_{ni} + tr(\underline{R}^T \delta \underline{F})$$

(Since $\underline{R}^T \delta \underline{R}$ is skew-symmetric, $\underline{S}$ is symmetric, the ~~first~~ term is 0.)

$$= tr(\underline{R}^T \delta \underline{F})$$

$$= R^T_{ki}\, \delta F_{ik}$$

$$= R_{ij}\, \delta F_{ij}$$

$$= \underline{R} : \delta \underline{F}$$

i.e. $\delta(\sigma_1+\sigma_2+\sigma_3) = \underline{R} : \delta F$

$$\Rightarrow \frac{\partial(\sigma_1+\sigma_2+\sigma_3)}{\partial \underline{F}} = \underline{R}$$