**Problem 5**

Verify that you get the exact solution at the grid nodes if you compute the right hand side an-alytically for the 1D Poisson equation using FEM. Use a uniform grid and Dirichlet boundary conditions at the right of the domain, Neumann at the left.

**Code:**

clear all; close all; clc

% forcing term

f = @(x) 24;

% boundary conditions

u0 = 1;

g = 14;

% non-uniform dx

N = 20;

dx\_average = 1/(N-1);

dx = ones(N-1,1)\*dx\_average;

% initialize mesh

x = zeros(1,N);

x(1) = 0;

for i = 2:N

x(i) = sum(dx(1:i-1));

end

% exact solution

u = zeros(1,N);

for i = 1:N

u(i) = 12\*x(i)^2 - 10\*x(i) + 1;

end

% initial guess

v = zeros(1,N);

v(1) = u0;

% build rhs

rhs = zeros(N-1,1);

for i = 1:N-2

rhs(i) = -f(x(i+1))\*dx\_average;

end

rhs(N-1) = -f(x(N))\*dx\_average/2+ g;

rhs(1) = rhs(1) + u0/dx\_average; % contribution from dirichlet

% build matrix

A = sparse(N-1,N-1);

A(1,1) = A(1,1) + 1/dx\_average; % contribution from dirichlet

for e = 2:N-1

dx\_current = dx\_average;

A(e-1,e-1) = A(e-1,e-1) + 1/dx\_current;

A(e-1,e) = A(e-1,e) - 1/dx\_current;

A(e,e-1) = A(e,e-1) - 1/dx\_current;

A(e,e) = A(e,e) + 1/dx\_current;

end

% solve for solution

unknown = A\rhs;

% build v

v(2:N) = unknown;

% plot

figure()

plot(x,u,'.-',x,v,'o-');

max(abs(u-v))

**Solution plot: u v.s. x**

Dirichlet at x=0: u0 = 1;

Neumann at x=1: g = 14;

N = 20;



The maximum absolute value of numerical solution and exact solution at the grid nodes is round-off

3.5527e-015

i.e.,

We get the exact solution at the grid nodes if you compute the right hand side analytically for the 1D Poisson equation using FEM.