PHYS234 Notes

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1 History

A conservative revolutionary In about 1908, Planck convert to the view that the quantum of action representsan irreducible phenomenon beyond the understanding of classical physics

Einstein in 1905

1. photoelectric effect
2. dissertation, proving the existence of atoms
3. Brownian motion
4. special relativity
5. $E = mc^2$

Johann Jakob Balmer's formula

$$v = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Rutherford atom model was unstable in classical physics

Niels Bohr - grandfather of quantum physics

1. solve the stability problem of Rutherford's model
2. classical physics could not apply inside the atom
3. orbits have something to do with the Planck - Einstein quantum relation of the light photon ($E = hv$).

Bohr derived Balmer's formula

Bohr's model of atom

1. Electrons in atoms orbit the nucleus
2. Electrons can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting EM radiation with a frequency v given by the energy gap of the levels according to the Planck relation:
$$\Delta E = E_2 - E_1 = hv$$

angular momentum L is restricted to be an integer multiple fo a fixed unit

$$L = mvr = \frac{nh}{2\pi} = nh$$

where $n = 1, 2, 3, \ldots$ is called the principal quantum number.
he mixed classical and quantum physics to get

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Important applications of QT in 20$^{\text{th}}$ century

1. Invention of transistors
2. Invention of lasers
3. Invention of STM
4. ...

2 Chapter 1 The Wave Function and The Schrödinger Equation

2.1 de Broglie's matter wave

$$p = \frac{h}{\lambda} \tag{1}$$

This equation is valid for electrons, ions, photons and anyother $\Rightarrow$ every particle

Paul Langevin de Broglie's thesis is the first feeble ray of light on the worst of our plys enigmas

$$\begin{aligned} L &= n\hbar \qquad (n = 1, 2, 3, \ldots) \\ L_1 &= \hbar \neq 0 \Rightarrow r_0 = 0.527\mathring{A} \\ 2\pi r &= n\lambda \qquad (n = 1, 2, 3, \ldots) \end{aligned}$$

angular momentum:

$$L = rp = \frac{n\lambda}{2\pi} * \frac{h}{r} = n\frac{h}{2\pi} = n\hbar$$

2.2 Schrödinger Equation

$$F = -\frac{\partial V}{\partial x} \qquad v - potential \tag{2}$$

Classical phys:

$$
\begin{aligned}
&F = ma \qquad (Newtonian 2nd law) \\
&F = m\frac{d^2x}{dt^2} \\
&\Downarrow \\
&X = X(t) \\
&\Downarrow \\
&v = \frac{dx}{dt}; p = mv = m\frac{dx}{dt} \\
&T = \frac{1}{2}mv^2 = \frac{1}{2}m(\frac{dx}{dt})^2 \\
&E = T + V = \frac{p^2}{2m} + V
\end{aligned}
$$

Quantum mechanics:

$$E \rightarrow i\hbar\frac{\partial}{\partial t}; p \rightarrow \underset{\sim}{p} = i\hbar\nabla$$

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi(x,t) \tag{3}$$

It is a postulate only! no proof

$$\Psi(x,t) - \text{The Wave function} \Rightarrow \text{to describe physics properties}$$

Consider free particles, from classical mechanics

$$E = \frac{1}{2}m\vec{v}^2 = \frac{(m\vec{v}^2)}{2m} = \frac{\vec{p}^2}{2m} \tag{4}$$

de Broglie's hypothesis:

$$
\begin{aligned}
E &= hv \\
\lambda &= \frac{h}{p} \\
\omega &= 2\pi v \qquad \text{angular frequency} \\
\|\vec{k}\| &= \frac{2\pi}{\lambda} \qquad \text{wave vector} \\
\hbar &= \frac{h}{2\vec{v}}
\end{aligned}
$$

$$E = \hbar\omega \tag{5}$$

$$\vec{p} = \hbar\vec{k} \tag{6}$$

particle motion can be described as a classical plane wave:

$$\begin{aligned}
\Psi(\vec{Y},t) =& \Psi_0 e^{i(\vec{k}\vec{Y}-\omega t)} \\
=& \Psi_0 e^{\frac{i(\vec{p}\vec{Y}-Et)}{\hbar}} \\
\frac{\partial \Psi}{\partial t} =& -\frac{iE\Psi_0}{\hbar} e^{\frac{i(\vec{p}\vec{Y}-Et)}{\hbar}} \\
\nabla =& \frac{i\vec{p}\Psi_0}{\hbar} e^{\frac{i\vec{p}\vec{r}-Et}{\hbar}} \\
\nabla^2\Psi =& -\frac{\vec{p}^2}{\hbar^2}\Psi e^{\frac{i(\vec{p}\vec{Y}-Et)}{\hbar}}
\end{aligned}$$

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi \tag{7}$$

$$-i\hbar\nabla\Psi = \vec{p}\Psi \tag{8}$$

$$-\hbar^2\nabla^2\Psi = \vec{p}^2\Psi \tag{9}$$

using Eq (4), we get

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2\nabla^2}{2m}\Psi \tag{10}$$

For particles in a potential $V(\vec{Y})$

$$E = \frac{\vec{p}^2}{2m} + V(\vec{Y}) \tag{11}$$

$$i\hbar\frac{\partial\Psi(\vec{Y},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{Y})\right)\Psi(\vec{Y},t) \tag{12}$$

2.3 Statistical Interpretation of the wave function $\Psi(\vec{Y},t)$

$$F = ma \to X(t) \to v = \frac{dx}{dt}, p = m\frac{dx}{dt}$$

Schrödinger Eq $\to \Psi(\vec{Y},t) \to$ quantum state of the system $\to$ physics properties of the system
The wave in QM is not a wave in physical space, it is a wave in an abastract mathematical space
for free particles

$$\begin{aligned}
E =& \frac{p^2}{2m} \Rightarrow E = \hbar\omega, p = \frac{h}{\lambda} = \hbar k \\
\omega =& \frac{\hbar k^2}{2m} \Rightarrow v_g \text{ group velosity of the wave} = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \frac{p}{m} = v \text{ classical velosity} \\
\frac{d^2\omega}{dk^2} =& \frac{\hbar}{m} > 0 \Rightarrow \text{The wave packet is diverging}
\end{aligned}$$

There is something wrong in de Broglie's hypothsis

2.3.1 Born's Stat Interpretation

It states that: The probability of finding a particle described by the wave function $\Psi(x,t)$ in the region, x to $x+dx$ is given by $\rho(x,t)dx = \|\Psi(x,t)\|^2 dx$ $\rho(x,t) = \|\Psi(x,t)\|^2 = \Psi(x,t)^*\Psi(x,t)$