PHYS234 TUT Notes

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1 Fourier Series / Fourier Transformations

1.1 Fourier Series

i: complex unit
τ: period

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{i2\pi nt}{\tau}} \tag{1}$$

$$C_n = \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t) e^{\frac{-i2\pi nt}{\tau}} dt \tag{2}$$

<u>Ex</u>:

1. $f(x) = 1$, find the F.S.

$$\begin{aligned}
\tau &= 2 \\
f(x) &= \sum_{n=-\infty}^{\infty} C_n e^{\frac{i2\pi nx}{\tau}} \\
C_n &= \frac{1}{2} \int_{-1}^{1} 1 * e^{\frac{-i2\pi nx}{2}} dx \\
&= \frac{1}{\pi n} sin(\pi n) \qquad n \neq 0 \\
&\int_{-1}^{1} 1dx = 2 \qquad n = 0
\end{aligned}$$

$$f(x) = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{sin(\pi n)}{\pi n} e^{\frac{i2\imath nx}{\tau}}$$

2. $f(x) = 1 - x^2 \qquad -1 \leq x \leq 1, f(x)$ period: 2, Find F.S.

$$\begin{aligned}
f(x) &= \sum_{n=-\infty}^{\infty} C_n e^{\frac{i2\pi nx}{\tau}} \qquad \tau = 2 \\
C_n &= \frac{1}{2} \int_{-1}^{1} (1 - x^2) e^{\frac{-i2\pi nx}{2}} dx \\
&= \frac{1}{2} \frac{4sin(\pi n) - 4\pi ncos(\pi n)}{(\pi n)^3} \qquad n \neq 0 \\
&\int_{-1}^{1} (1 - x^2) dx = 0 \qquad n = 0
\end{aligned}$$

<u>Fourier Series:</u>

* need $f(x)$ to be periodical $x \in (-\infty, \infty)$
* caution about singularity of C_n's
* More "flat", the messier F.S.
* follow the formula

1.2 Fourier Transformations

$$f(\nu) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \nu} dx$$

$$f(x) = \int_{-\infty}^{\infty} f(\nu) e^{2\pi i x \nu} d\nu \qquad \text{Reverse}$$

ν : frequency

x : time

<u>Ex:</u> $f(x) = e^{-\|x\|}$

$$\begin{aligned} f(\nu) &= \int_{-\infty}^{\infty} e^{-\|x\|} e^{-2\pi i x \nu} dx \\ &= \int_{-\infty}^{\infty} e^{x} e^{-2\pi i x \nu} dx + \int_{-\infty}^{\infty} e^{-x} e^{-2\pi i x \nu} dx \\ &= \frac{2}{1 + (e\pi\nu)^2} \end{aligned}$$