

k-means. PCA.

Pixel RNN & GAN

$$p(x) = \prod_{i=1}^n p(x_i | x_1, \dots, x_{i-1})$$

VAE:

$$p_\theta(x) = \int p_\theta(z) p_\theta(x|z) dz$$

encoder: $x \rightarrow z$ decoder: $z \rightarrow y$

$$p_\theta(x) = \int p_\theta(z) p_\theta(x|z) dz$$

\downarrow Gaussian \downarrow NN.

$x \rightarrow q_\phi(z|x) \rightarrow z_{z|x}$ Sample z from $z|x \sim \mathcal{N}(\mu_{z|x}, \Sigma_{z|x})$

$z \rightarrow \mu_{x|z} \rightarrow x_{x|z} \sim \mathcal{N}(\mu_{x|z}, \Sigma_{x|z})$
 \downarrow 10 - variance matrix.

$$\log p_\theta(x^{(i)}) = \mathbb{E}_{z \sim q_\phi(z|x^{(i)})} [\log p_\theta(x^{(i)})]$$

$$= \mathbb{E}_z \left[\log \frac{p_\theta(x^{(i)}|z) p_\theta(z)}{p_\theta(z|x^{(i)})} \right]$$

\downarrow $p(x) = \frac{p(x|z) \cdot p(z)}{p(z|x)}$

$$= \mathbb{E}_z [\log p_\theta(x^{(i)}|z)] - \mathbb{E}_z [\log p_\theta(z|x^{(i)})] + \mathbb{E}_z \left[\log \frac{q_\phi(z|x^{(i)})}{p_\theta(z|x^{(i)})} \right]$$

$$= \mathbb{E}_z [\log p_\theta(x^{(i)}|z)] - D_{KL}(q_\phi(z|x^{(i)}) || p_\theta(z)) + D_{KL}(q_\phi(z|x^{(i)}) || p_\theta(z|x^{(i)}))$$

$\underbrace{\hspace{10em}}_{\text{differentiable}} \quad \text{KL: KL divergence} \quad \downarrow \neq 0$

$q_\phi(z|x^{(i)})$ is intractable.

$$\log p_\theta(x^{(i)}) \approx \mathcal{L}(x^{(i)}, \theta, \phi) \quad \theta^*, \phi^* = \arg \max_{\theta, \phi} \sum_{i=1}^N \mathcal{L}(x^{(i)}, \theta, \phi)$$

GANs:

$$\min_{\theta_g} \max_{\theta_d} \left[\mathbb{E}_{x \sim p_{\text{data}}} \log D_\theta(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_\theta(G_\theta(z))) \right]$$

Discriminator output for real data x .

Discriminator output for generated fake data $G_\theta(z)$.

Gradient ascent.

$$\max_{\theta_d} \left[\mathbb{E}_{x \sim p_{\text{data}}} \log D_\theta(x) + \mathbb{E}_{z \sim p(z)} \log (1 - D_\theta(G_\theta(z))) \right]$$

Gradient descent.

$$\min_{\theta_g} \mathbb{E}_{z \sim p(z)} \log (1 - D_\theta(G_\theta(z)))$$

instead:

Gradient ascent.

$$\max_{\theta_g} \mathbb{E}_{z \sim p(z)} \log (D_\theta(G_\theta(z)))$$

raise the rate that discriminator has error on classification.

