Discrete Math Assignment 6

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P650 Ex.11: Let G be a simple graph. Show that the relation R on the set of vertices of G such that uRv if and only if there is an edge associated to $\{u,\ v\}$ is a symmetric, irreflexive relation on G.

Proof:

If uRv, there is an edge associated with $\{u, v\}$. Since this is undirected, this edge can be presented as vRu. Thus R is symmetric. No loop in simple graph, which mean R is irreflexive.

QED.

P668 Ex. 64: Show that if G is a simple graph with n vertices and e edges, then $e \leqslant v^2/4$.

Proof:

Let the partition be

P668 Ex. 66: Describe an algorithm to decide whether a graph is bipartite based on the fact that a graph is bipartite if and only if it is possible to color its vertices two different colors so that no two vertices of the same color are adjacent.

Solve:

We start by coloring any vertex red. Then we color all the vertices adjacent to this vertex blue. Then we color all the vertices red, then color all the vertices adjacent to red vertices blue. If we ever are in the position of trying to color a vertex with the color opposite to the color it already has, then we stop and know that the graph is not bipartite. If the process terminates before all the vertices have been colored, then we color some uncolor vertex red and begin the process again. Eventually we will have either colored all the vertices or stopped and decided that the graph is not biparitite.

P676 Ex. 28: What is the sum of the entries in a row of the adjacency matrix for an undicrected graph? For a directed graph?

P676 Ex. 29 What is the sum of the entries in a column of the adjacency

matrix for an undirected graph? For a directed graph?

P676 Ex. 32 Find an adjacency matrix for each of these graphs.

a) K_n b) C_n c) W_n d) $K_{m,n}$ e) Q_n

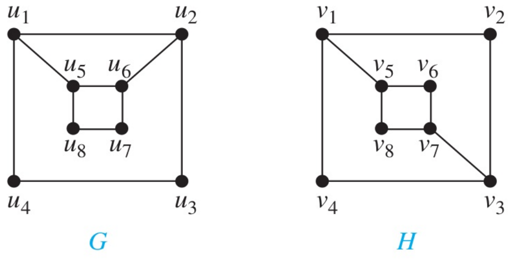
P677 Ex. 45 Show that isomorphism of simple graphs is an equiva-

lence relation.

P677 Ex. 52

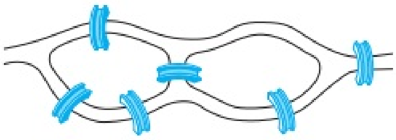
Show that if G is a self-complementary simple graph with v vertices, then v $\equiv$ 0 or 1 (mod 4).

P690 Ex. 20 Use paths either to show that these graphs are not isomorphic or to find an isomorphism between these graphs.



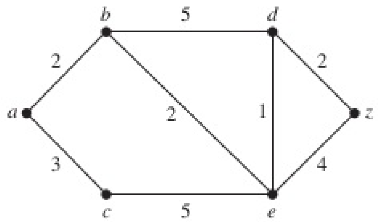
P691 Ex. 28 Show that every connected graph with n vertices has at least n-1 edges.

P704 Ex. 10 Can someone cross an the bridges shown in this map exactly once and returen to the starting point?



P716 Ex. 2 Find the length of a shortest path between a and z in the given weighted graph

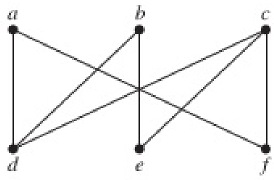
2.



P716 Ex. 14 Explain how to find a path with the least nubmer of edges between two vertices in an undirected graph by considering it as a shortest path problesm in a weighted graph.

P725 Ex. 6 Determine whether the given graph is planar. If so, draw it so that no edges cross.

6.



P726 Ex. 17 Suppose that a connected planar simple graph with e edges and v vertices contains no simple circuits of length 4 or less. Show that $e \leqslant (5/3)v - (10/3)$ if $v \geqslant 4$.

P756 Ex. 16 Which complete bipartite graphs $K_{m,n}$, where are m and n are positive integers, are trees?

P757 Ex. 46 How many vertices, leaves and internal verices does the rooted Fibonacci tree T_n are defined recursively in the following way. T_1 and T_2 are both the rooted tree consisting of a single vertex, and for $n = 3, 4......$, the rooted tree T_n is constructed from a root with T_{n-1} as its left subtree and T_{n-2} as its right subtree.

P784 Ex. 24 What is the value of each of these postfix expressions?

a) $-$ $*$ **2** / **8 4 3**

b) $\uparrow$ $-$ $*$ **3 3** $*$ **4 2 5**

c) $+$ $-$ $\uparrow$ **3 2** $\uparrow$ **2 3** / **6** $-$ **4 2**

d) $*$ $+$ **3** $+$ **3** $\uparrow$ **3** $+$ **3 3 3**

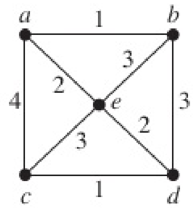
P795 Ex. 11 How many different spanning trees does each of these simple graphs have?

a) K_3 b) K_4 c)K_5

P795 Ex. 12 How many noinsomorphic spanning trees does each of these simple graphs have?

a) K_3 b) K_4 c)K_5

P802 Ex. 2 Use Prim's algorithm to find a minimum spanning tree for the given weighted



graph.

P802 Ex. 6 Use Kruskal's algorithm to find minimum spanning tree for the weighted graph Ex.2