南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

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Properties of Relations

- **Reflexive Relation**: A relation R on a set A is called *reflexive* if $(a, a) \in R$ for every element $a \in A$.

  Irreflexive Relation: A relation R on a set A is called *irreflexive* if $(a, a) \notin R$ for every element $a \in A$.

  Symmetric Relation: A relation R on a set A is called *symmetric* if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$.

  Antisymmetric Relation: A relation R on a set A is called *antisymmetric* if $(b, a) \in R$ and $(a, b) \in R$ implies $a = b$ for all $a, b \in A$.

  Transitive Relation: A relation R on a set A is called *reflexive* if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$ for all $a, b, c \in A$.

Equivalence Relation

- **Definition** A relation R on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

- **Definition** Let R be an equivalence relation on a set A. The set of all elements that are related to an element a of A is called the *equivalence class* of a, denoted by $[a]_R$. When only one relation is considered, we use the notation $[a]$.

$$[a]_R = \{b : (a, b) \in R\}$$

Equivalence Classes and Partitions

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Theorem Let $\{A_1, A_2, \ldots, A_i, \ldots\}$ be a partition of S. Then there is an equivalence relation R on S, that has the sets A_i as its equivalence classes.

Partial Ordering

- **Definition** A relation R on a set S is called a *partial ordering*, or *partial order*, if it is reflexive, antisymmetric, and transitive. A set S together with a partial ordering R is called a *partially ordered set*, or *poset*, denoted by (S, R). Members of S are called *elements of the poset*.

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- **Definition** If $(S, \preccurlyeq)$ is a poset and every two elements of S are comparable, S is called a *totally ordered* or *linearly ordered set*, and $\preccurlyeq$ is called a *total order* or a *linear order*. A totally ordered set is also called a *chain*.

Well-Ordered Set

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Question: Why don't we need a *basic step* here?

Lexicographic Ordering

- **Definition** Given two posets $(A_1, \preccurlyeq_1)$ and $(A_2, \preccurlyeq_2)$, the *lexicographic ordering* on $A_1 \times A_2$ is defined by specifying that (a_1, a_2) is less than (b_1, b_2), i.e., $(a_1, a_2) \prec (b_1, b_2)$, either if $a_1 \prec_1 b_1$ or if $a_1 = b_1$ then $a_2 \prec_2 b_2$.

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  - $\diamond$ *discreet* $\prec$ *discrete*
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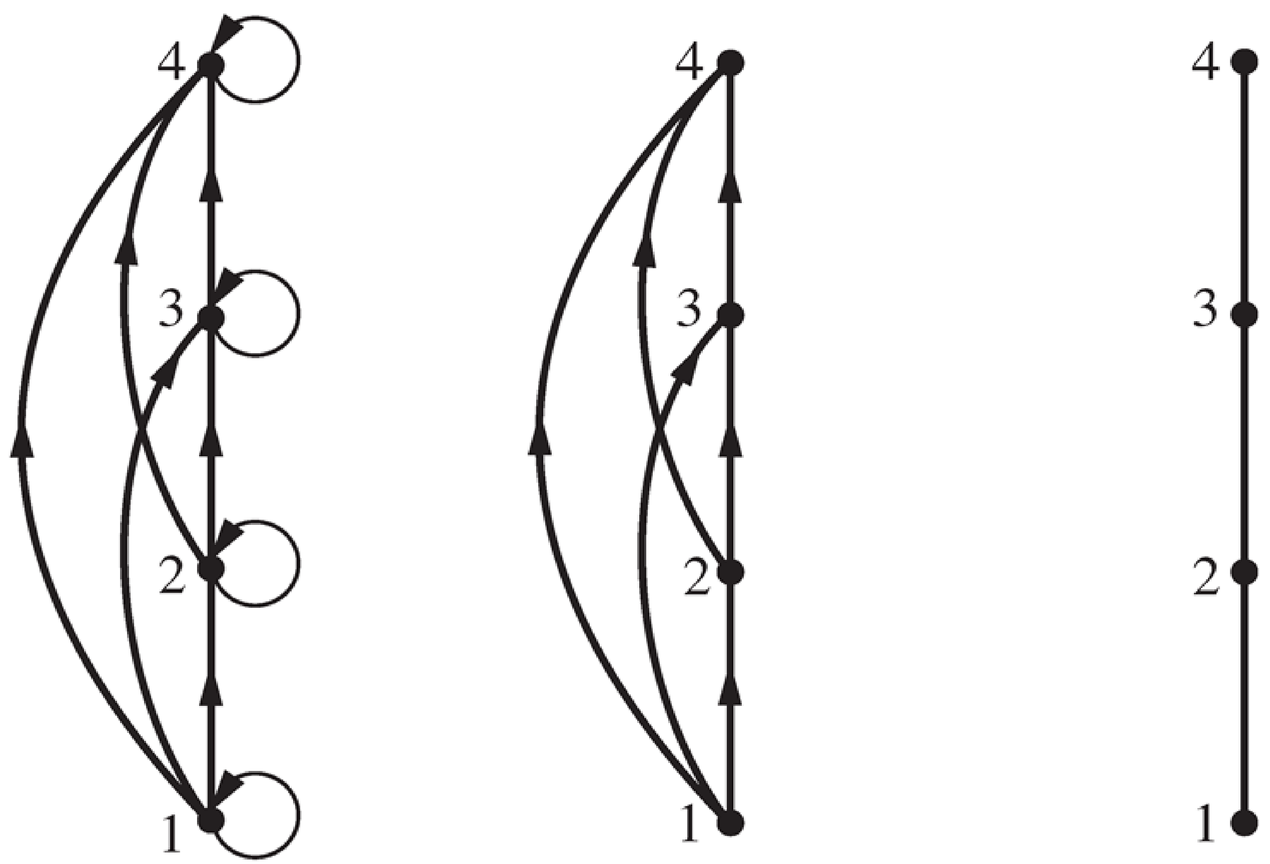
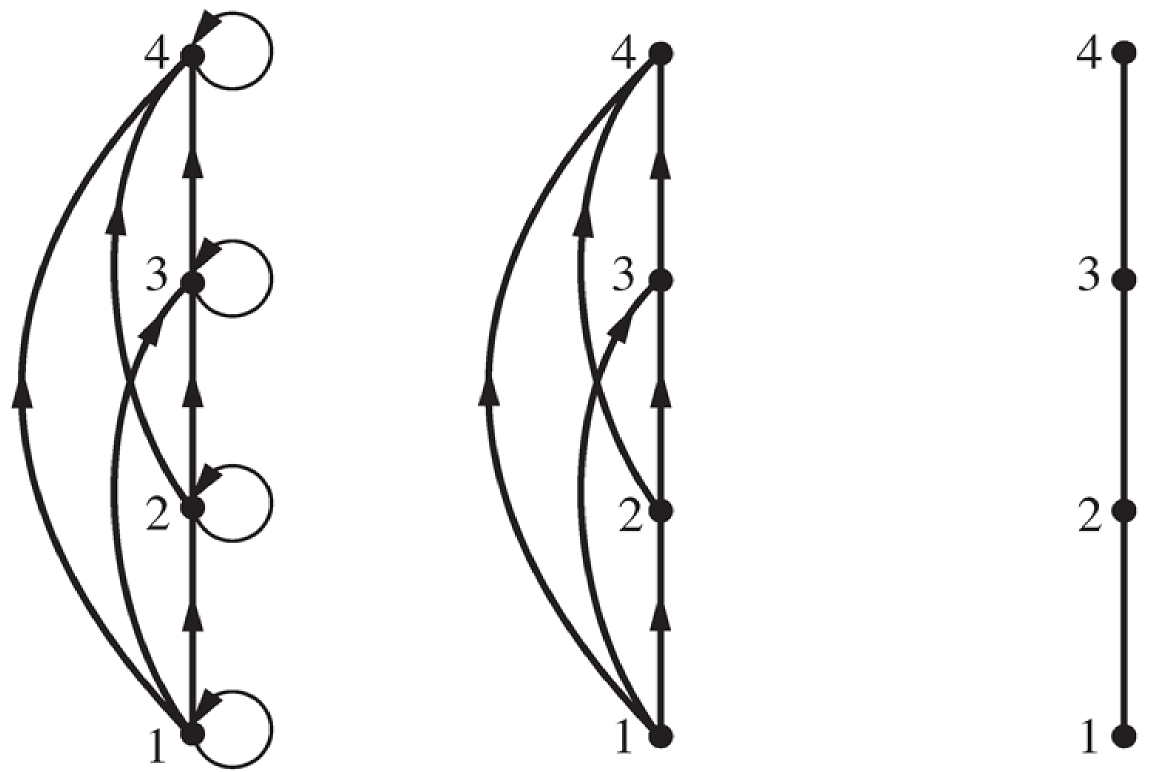


Hasse Diagram

- (a) A partial ordering. The loops are due to the reflexive property

  (b) The edges that must be present due to the transitive property are deleted

  (c) The Hasse diagram for the partial ordering (a)



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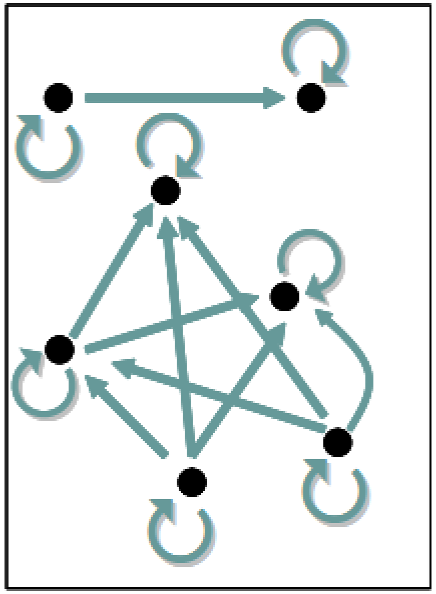
  ◇ Arrange each edge so that its initial vertex is below the terminal vertex. Remove all the arrows, because all edges point upwards toward their terminal vertex.

Hasse Diagram Example



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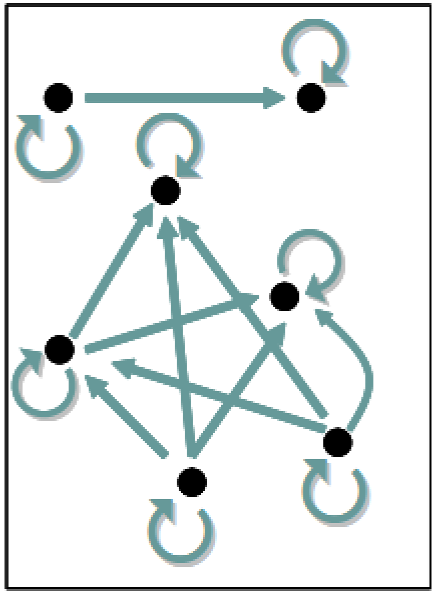

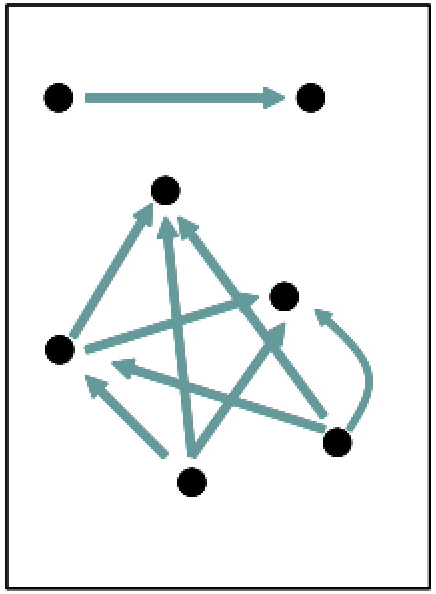

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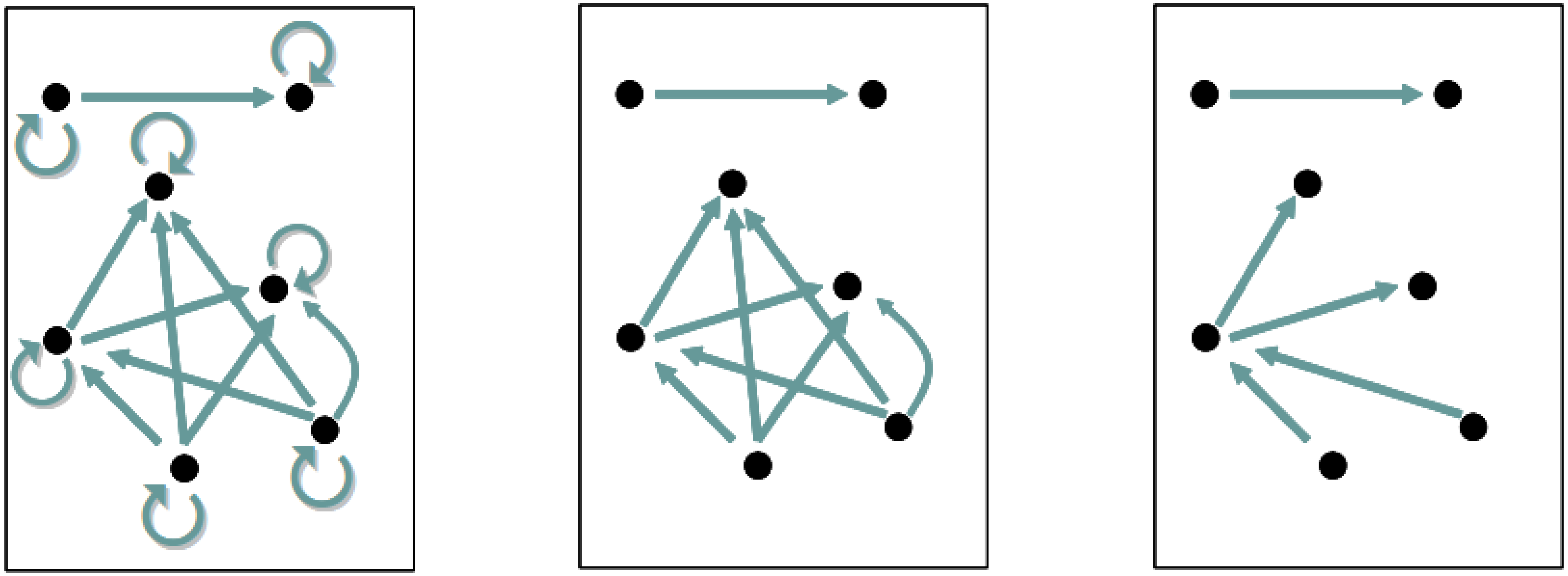

Maximal and Minimal Elements

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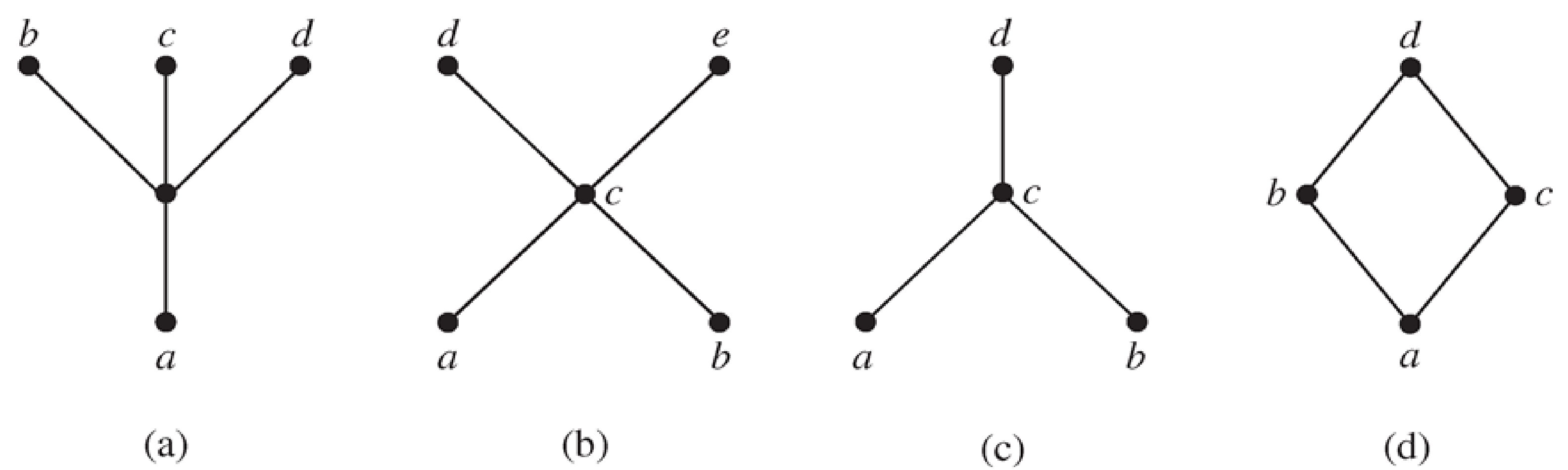

Maximal and Minimal Elements

- **Definition** Let A be a subset of a poset $(S, \preccurlyeq)$.
  - $u \in S$ is called an *upper bound* (resp. *lower bound*) of A if $a \preccurlyeq u$ (resp. $u \preccurlyeq a$) for all $a \in A$.
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Example Find the greatest lower bound and the least upper bound of the sets $\{3, 9, 12\}$ and $\{1, 2, 4, 5, 10\}$, if they exist, in the poset $(\mathbf{Z}^+, |)$.

Lattices

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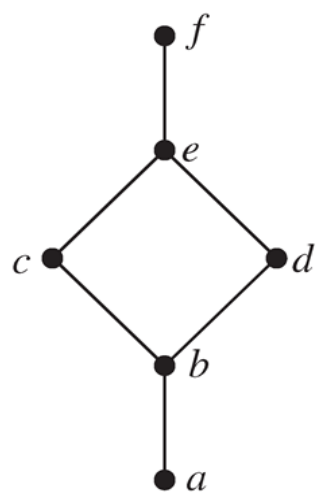

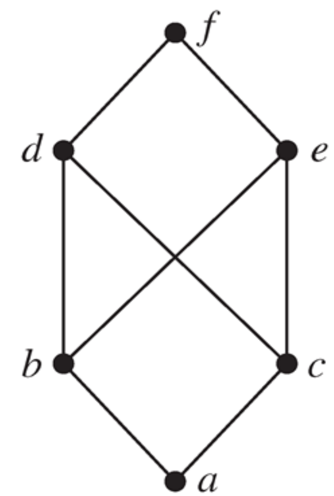

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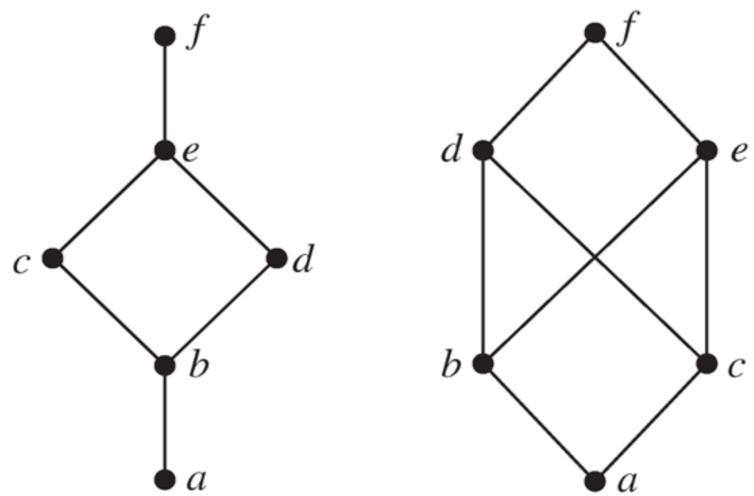

Example Determine whether the posets $(\{1,2,3,4,5\},|)$ and $(\{1,2,4,8,16\},|)$ are lattices.

Topological Sorting

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Topological sorting: Given a partial ordering R, find a total ordering $\preccurlyeq$ such that $a \preccurlyeq b$ whenever $a \mathrel{R} b$. $\preccurlyeq$ is said *compatible with* R.

Topological Sorting for Finite Posets

procedure topological_sort (S: finite poset)

$k := 1$;
while $S \neq \emptyset$
  $a_k :=$ a minimal element of S
  $S := S \setminus \{a_k\}$
  $k := k + 1$
end while
// $\{a_1, a_2, \ldots, a_n\}$ is a compatible total ordering of S

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- Map of some cities in Eastern U.S. with communication lines existing between certain pairs of these cities.

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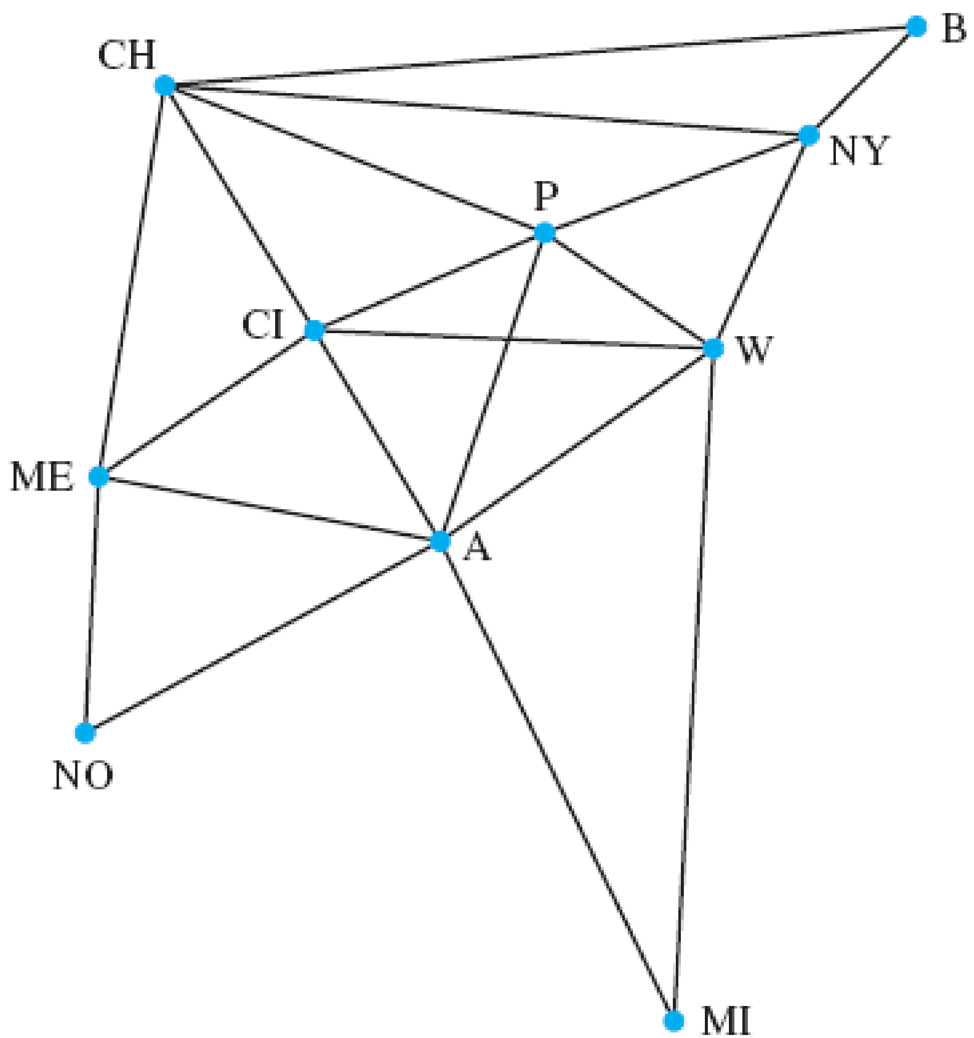

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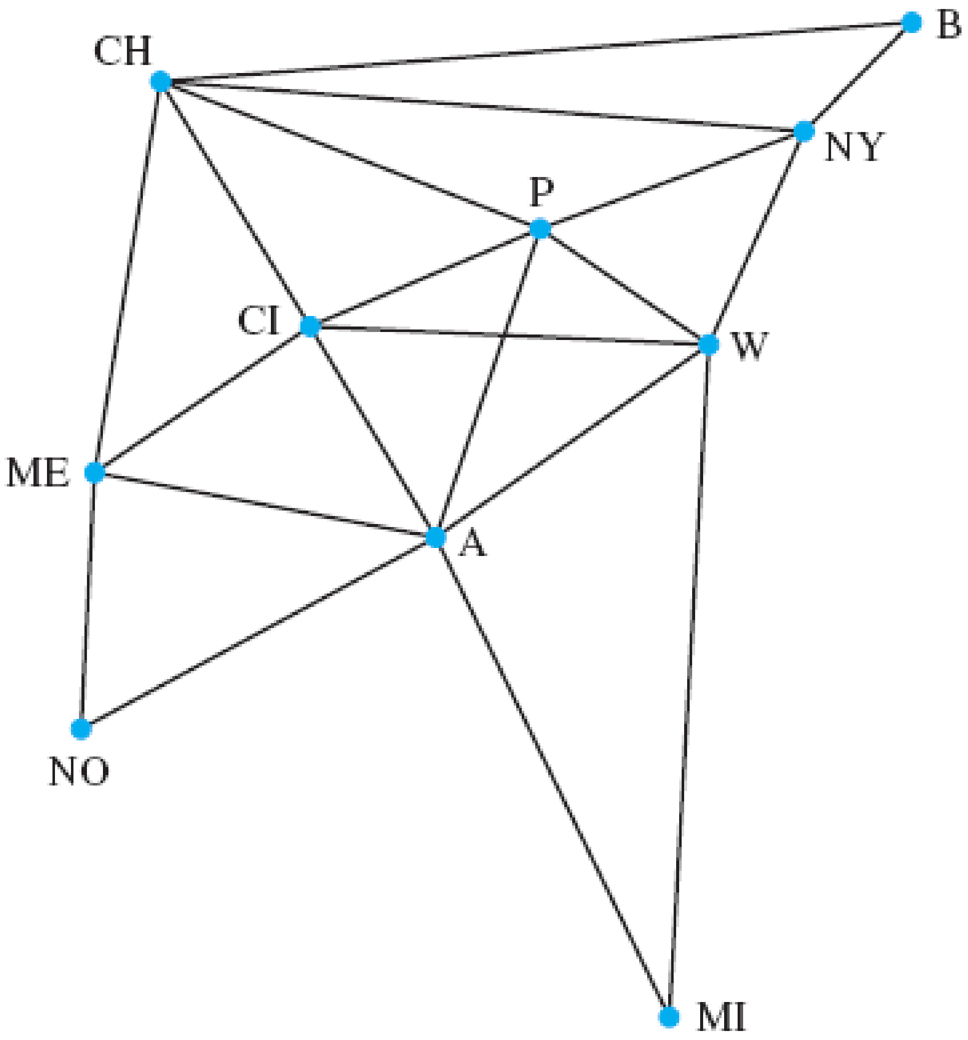

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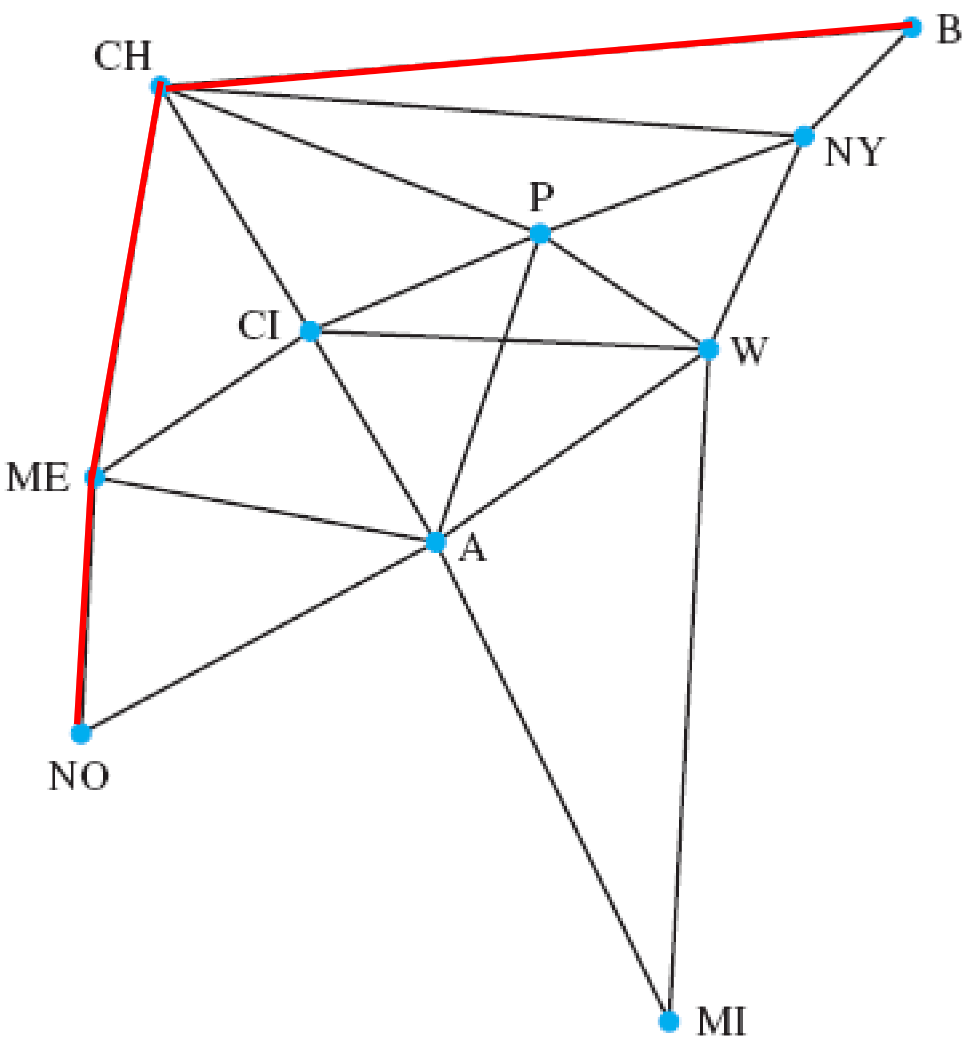

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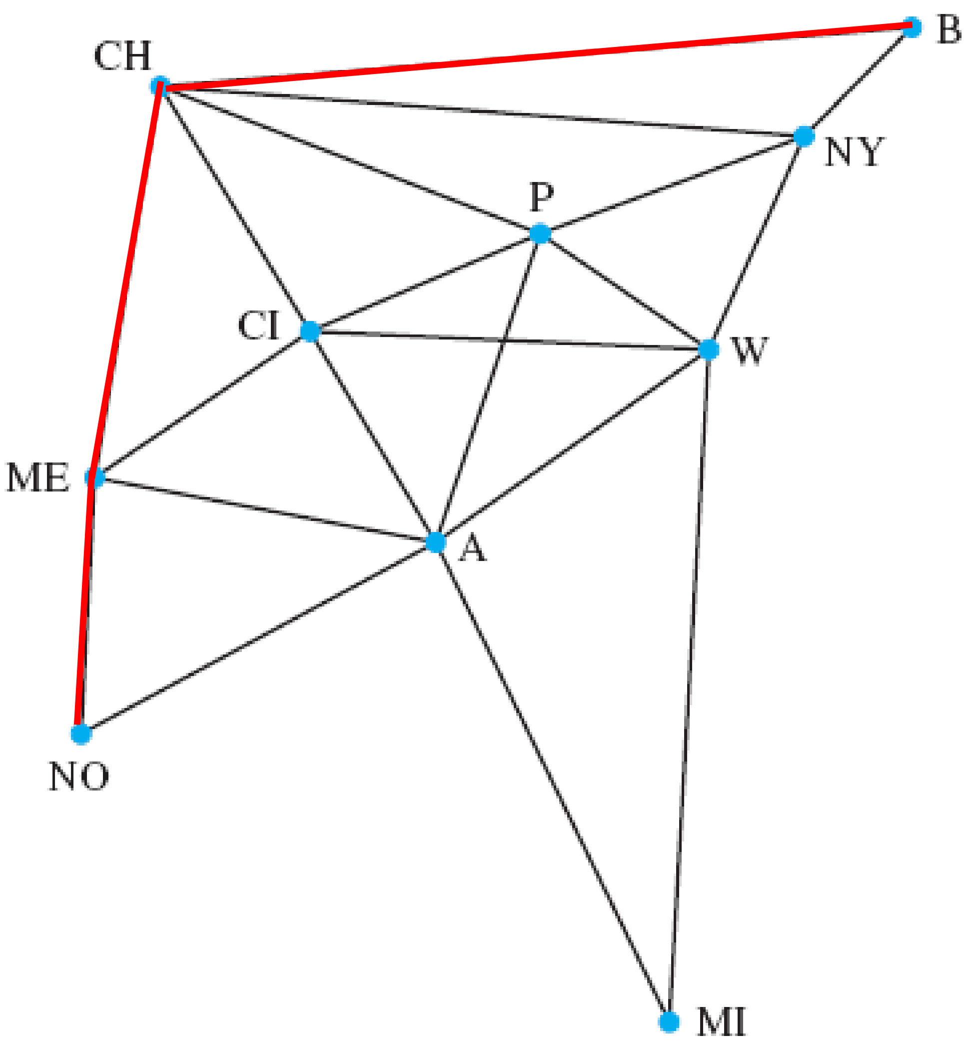

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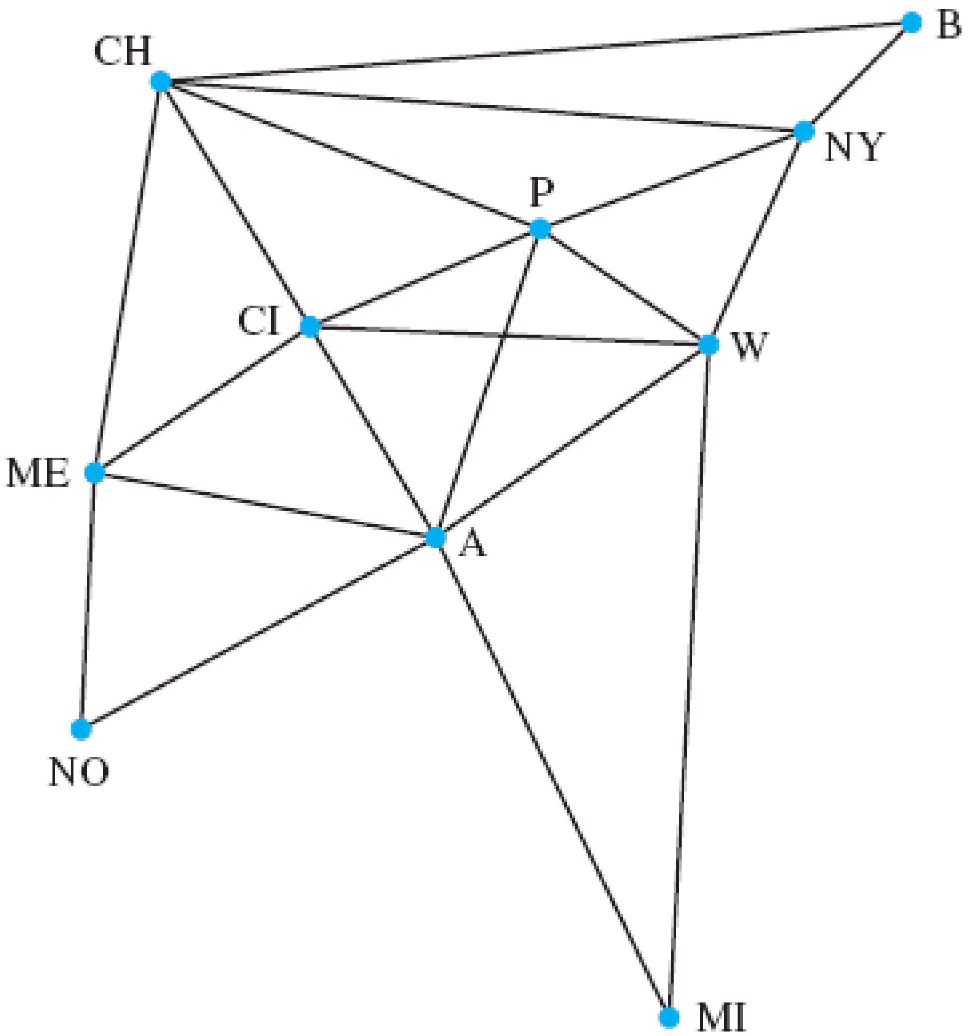

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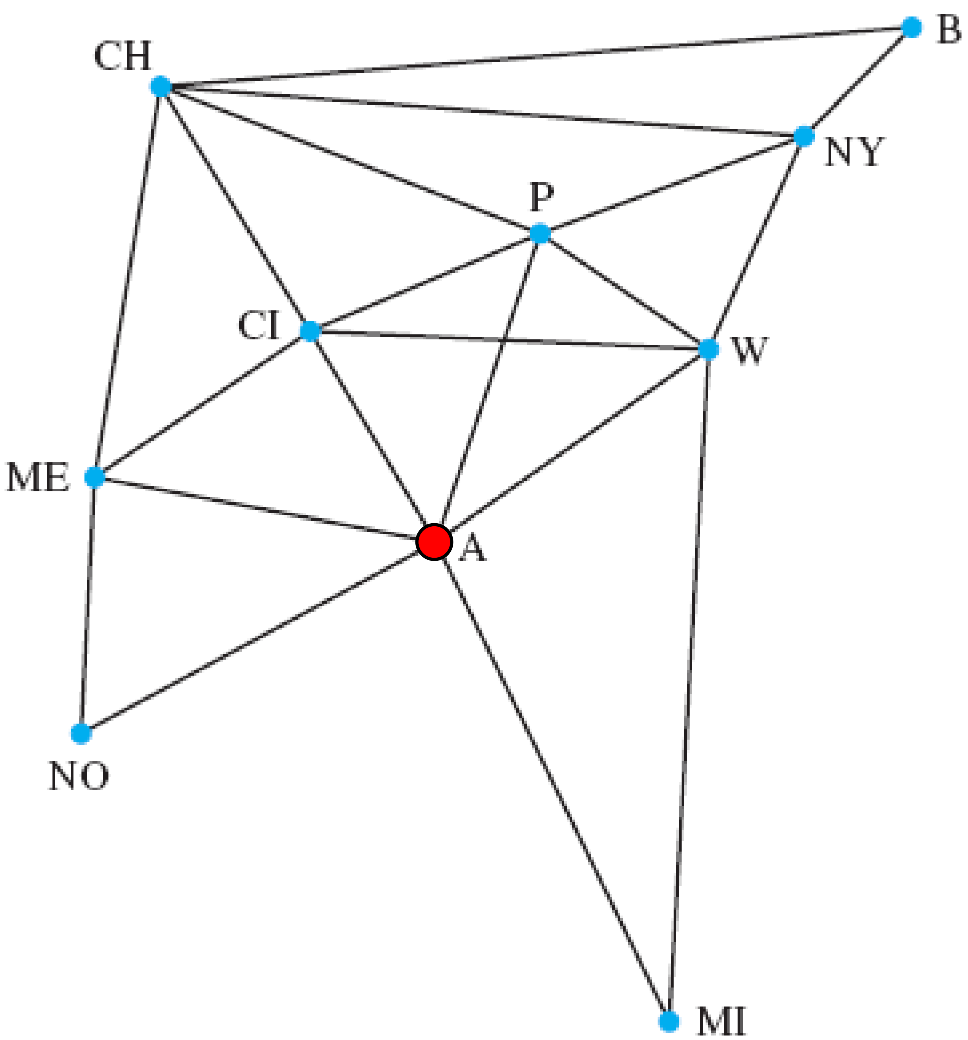

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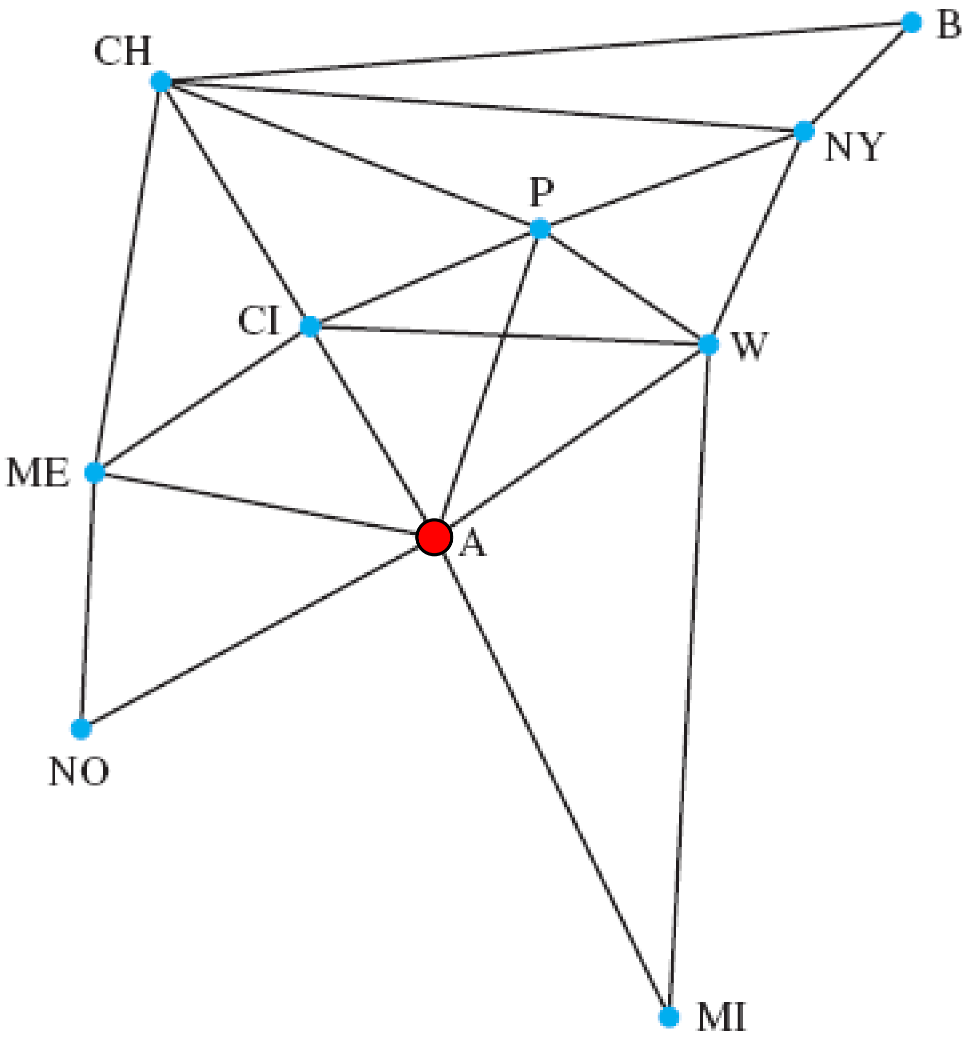

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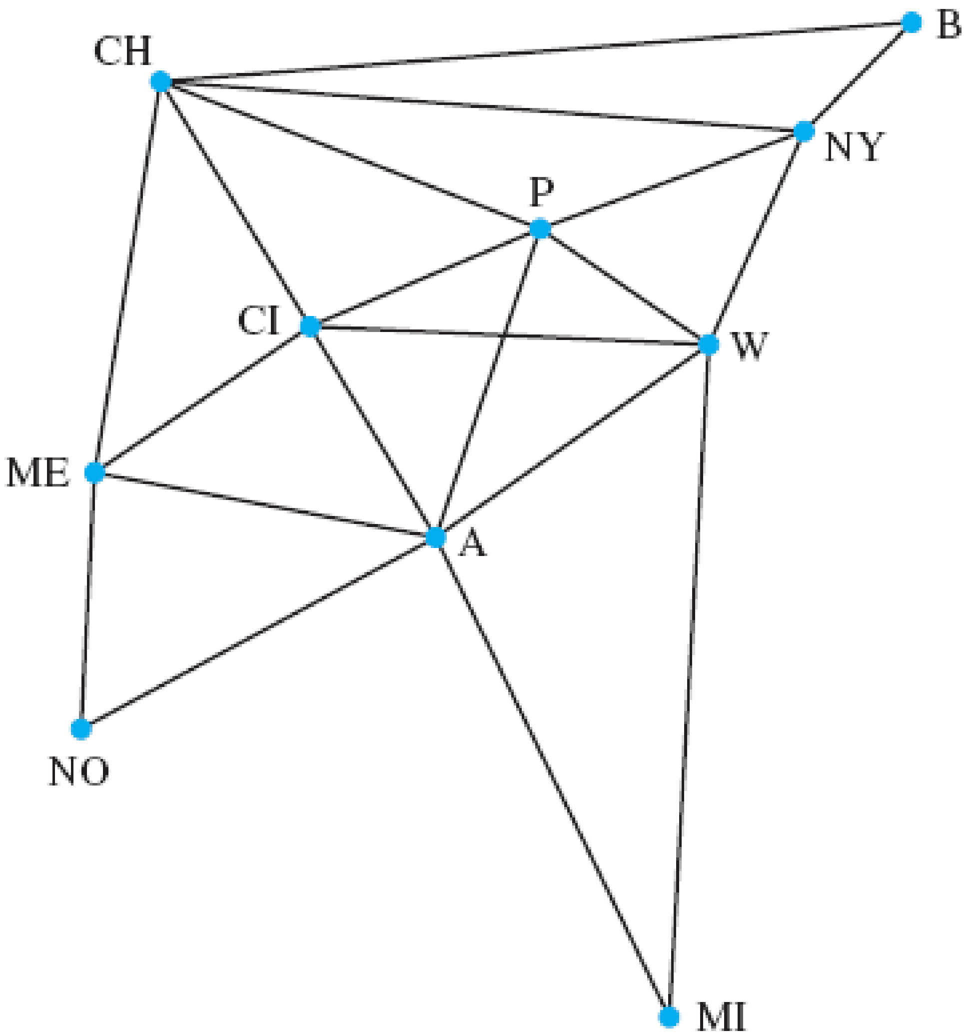

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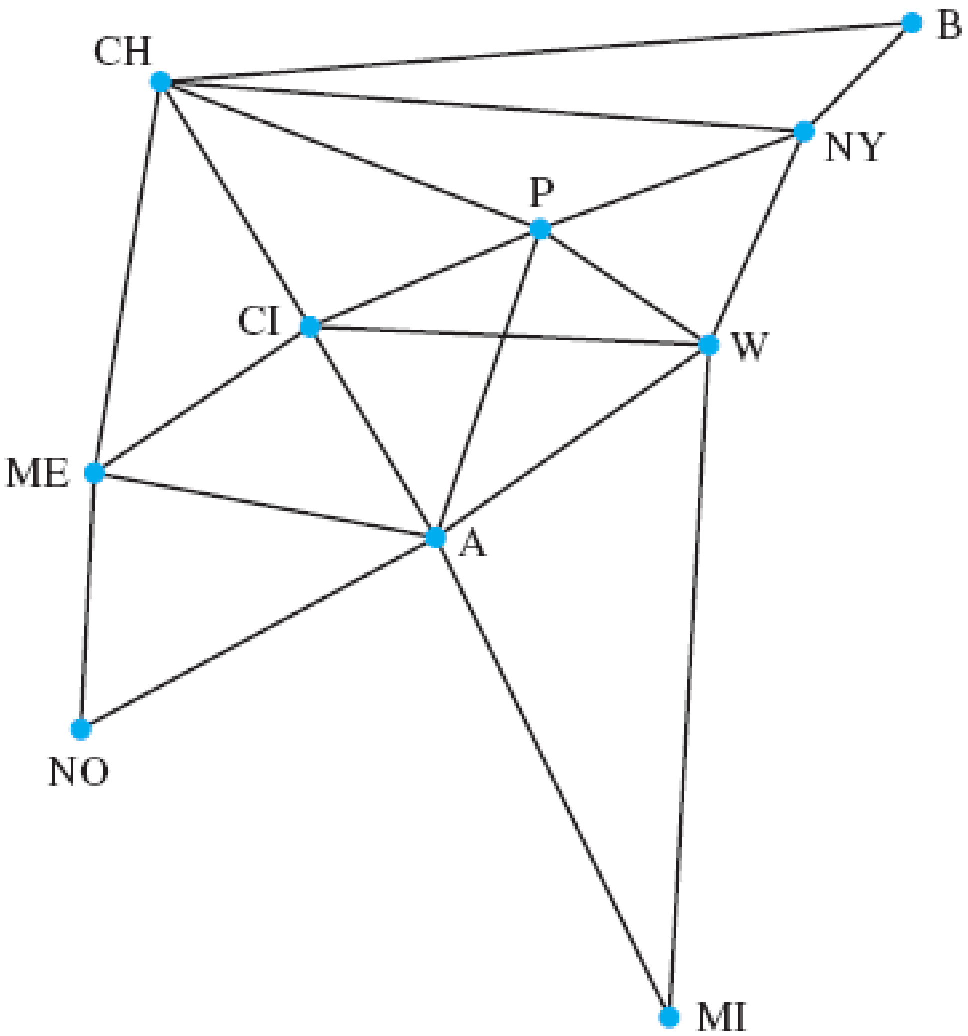

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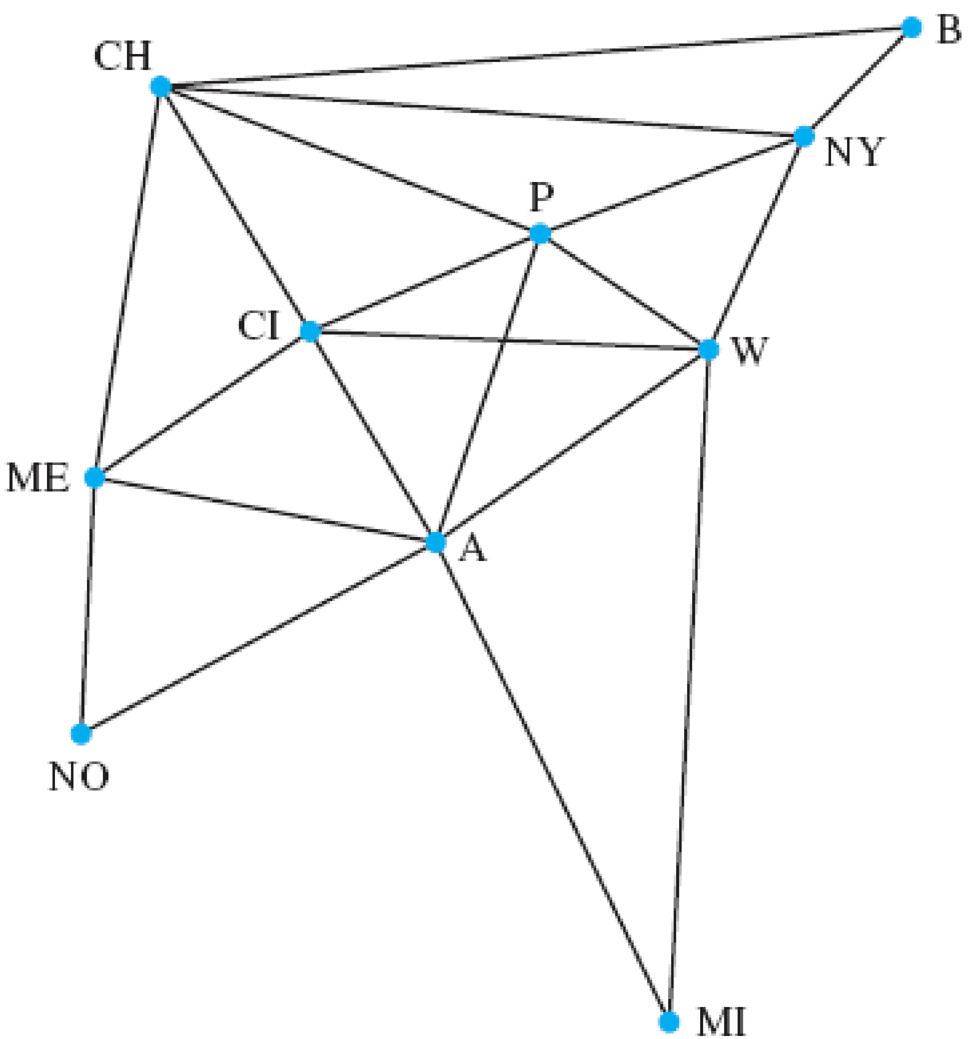

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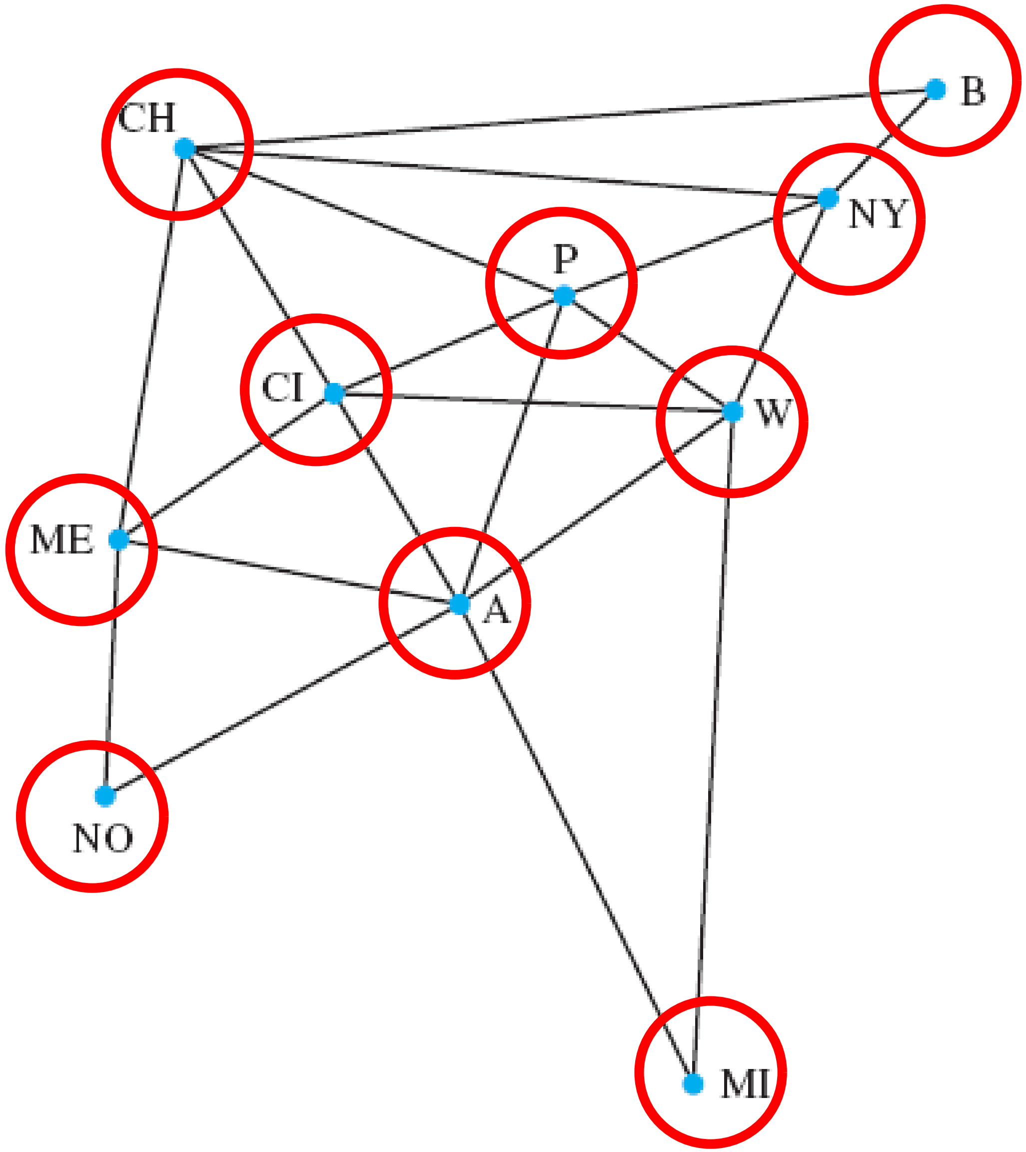

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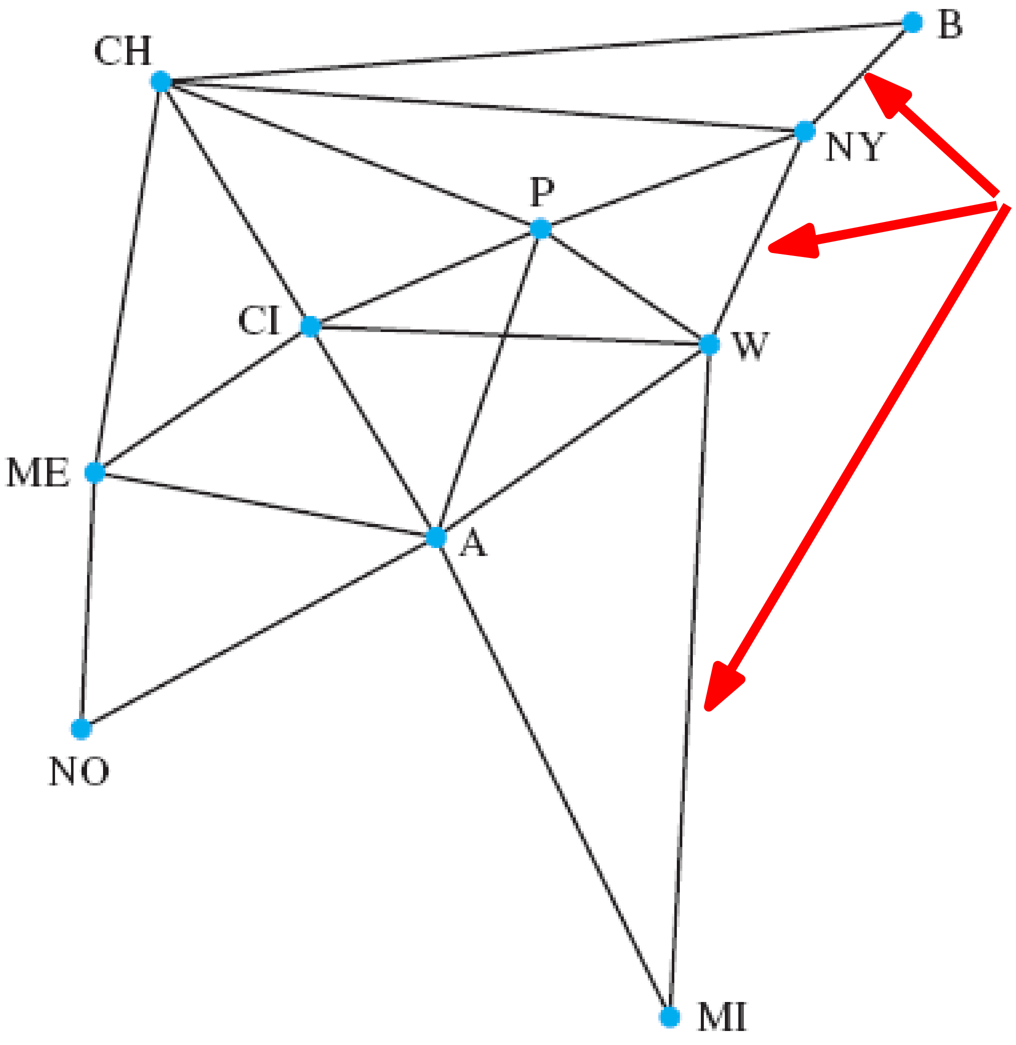

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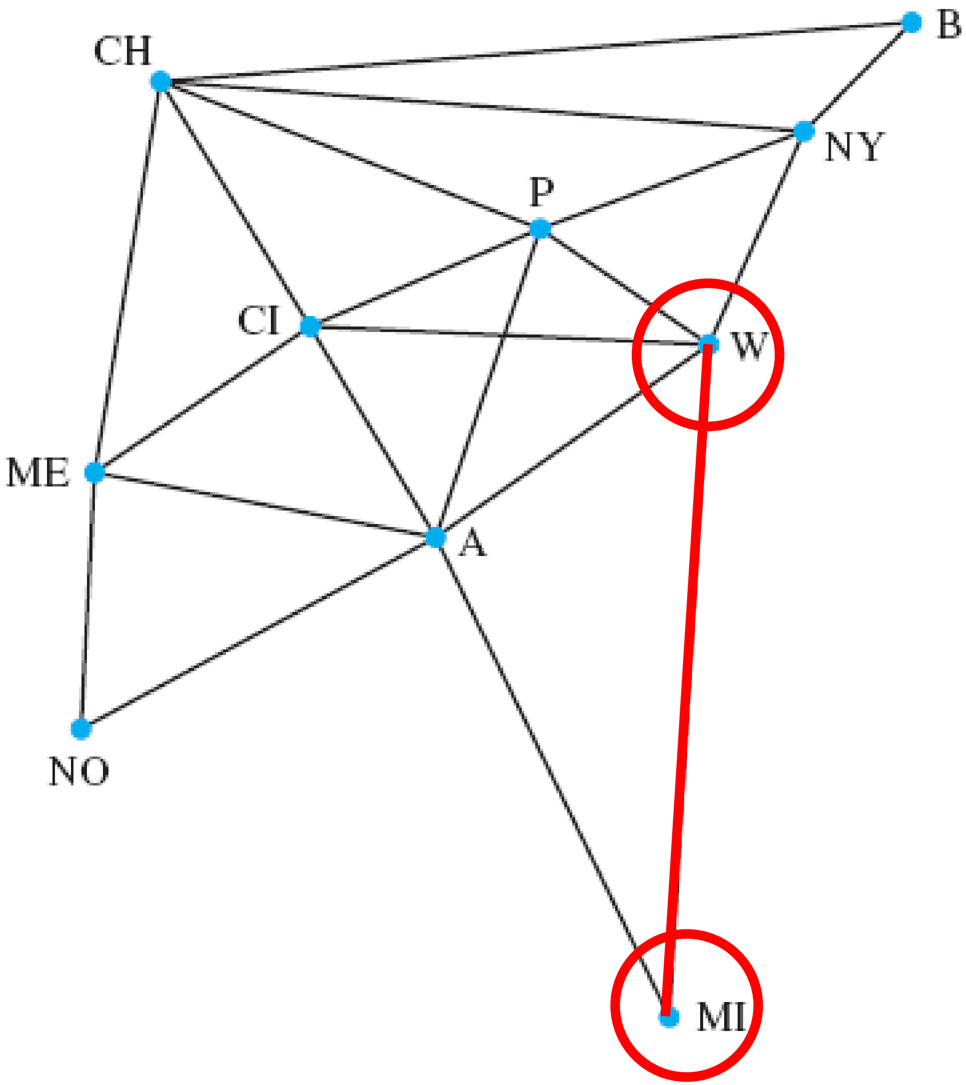

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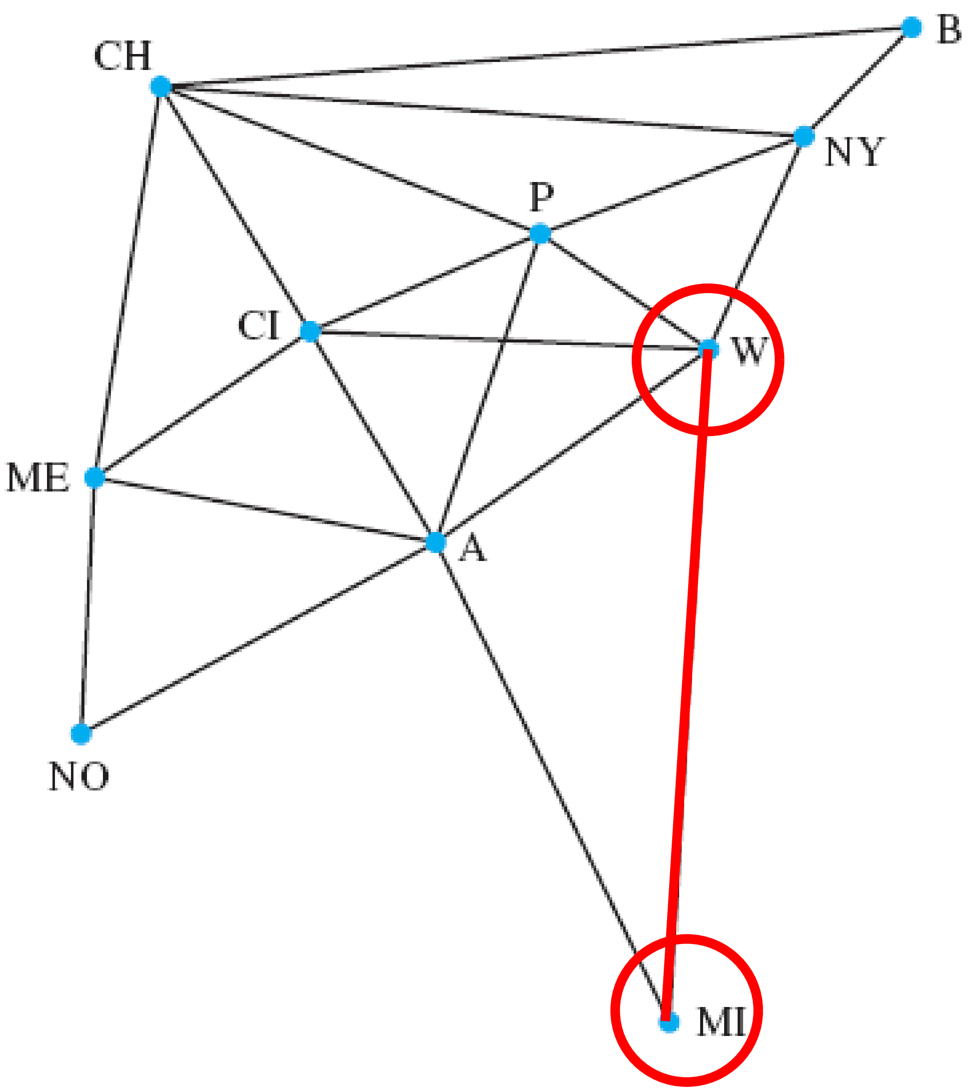

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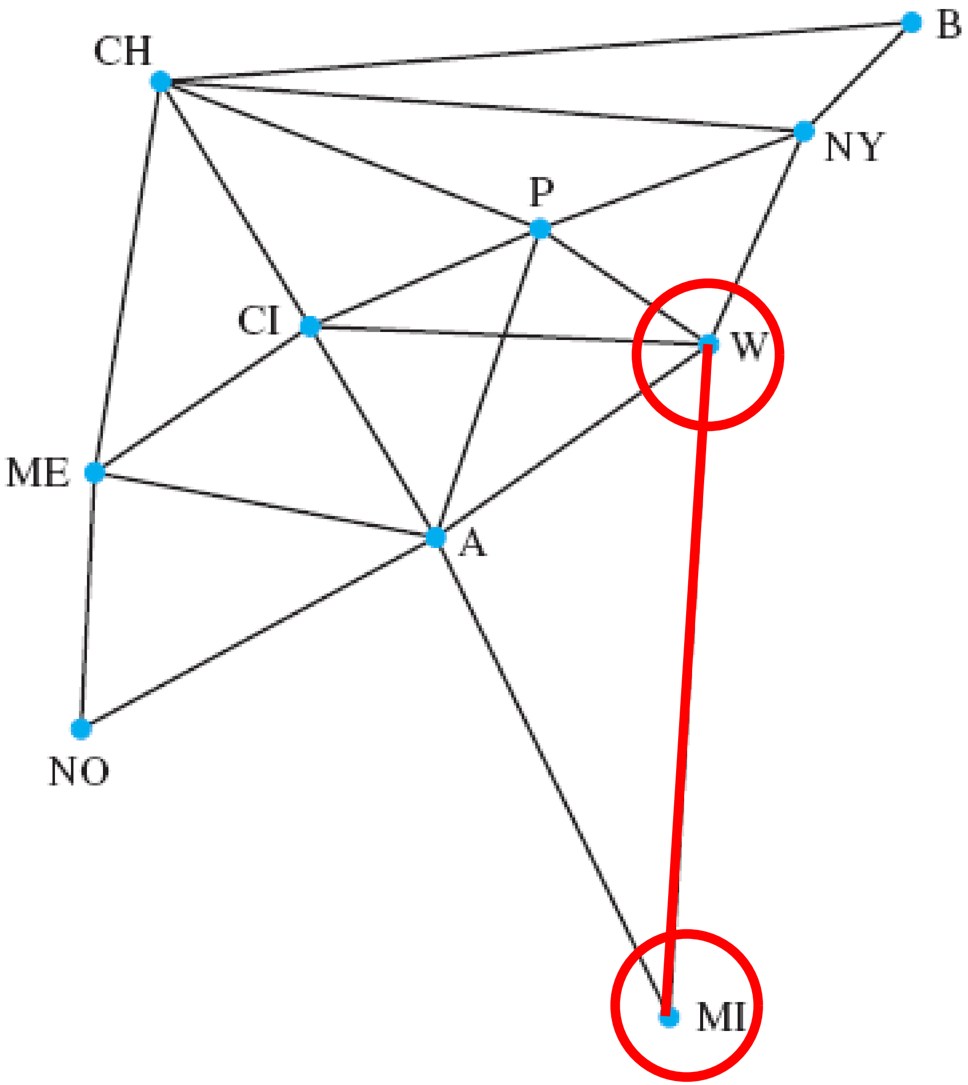

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When a vertex is an endpoint of an edge, we say that the edge and the vertex are incident to each other

More Examples

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  Edges: species have a common ancestor

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How Google models the Internet!

Definition of a Graph

- **Definition**. A *graph* $G = (V, E)$ consists of a nonempty set V of *vertices* (or *nodes*) and a set E of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to be *incident to* (or *connect* its endpoints.

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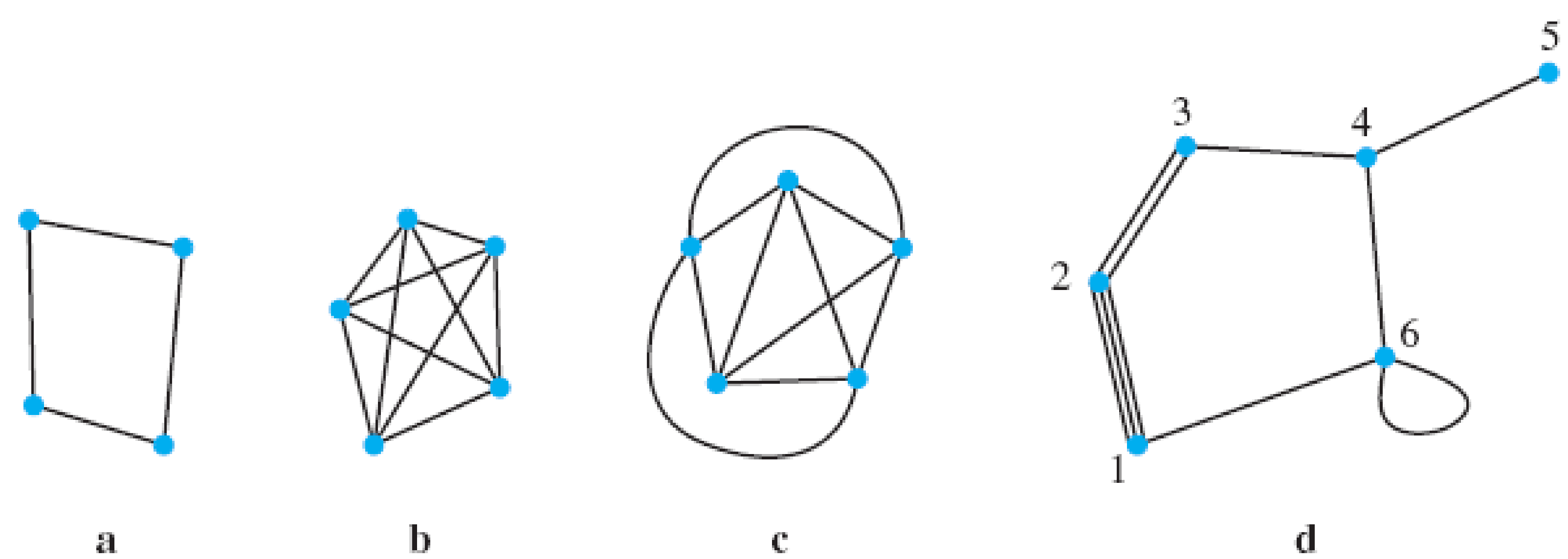

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  Complete graph K_n

  A graph with n vertices that has an edge between each pair of vertices

Graphs

- Graphs and graph theory can be used to model:
  - Computer networks
  - Social networks
  - Communication networks
  - Infromation networks
  - Software design
  - Transportation networks
  - Biological networks

Graph Models

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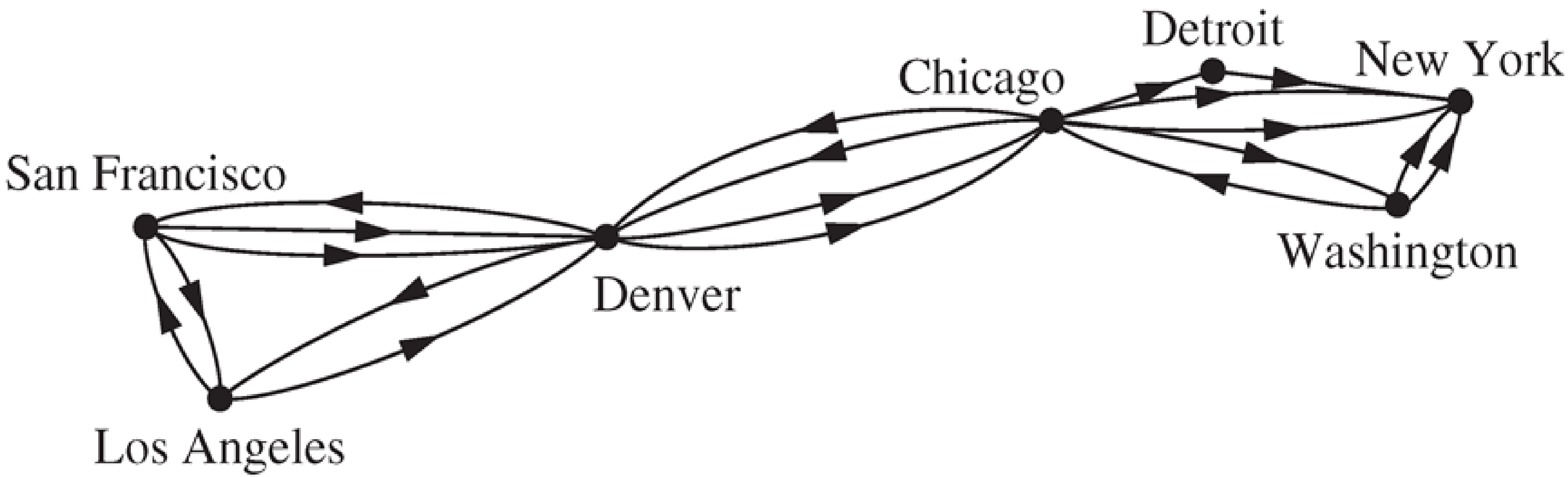

Graph Models

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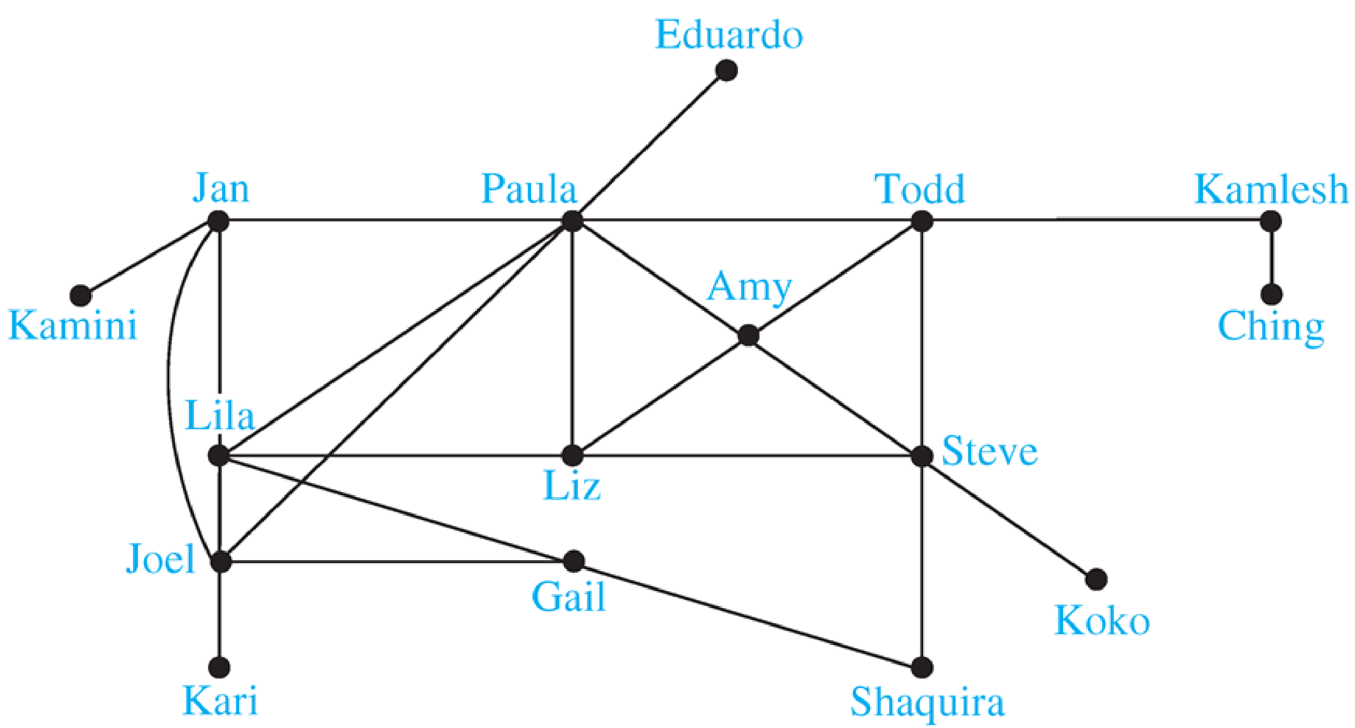

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  Example

  - the Hollywood graph
  - the Erdös number

Undirected Graphs

- **Definition** Two vertices u, v in an undirected graph G are called *adjacent* (or *neighbors*) in G if there is an edge e between u and v. Such an edge e is called *incident* with the vertices u and v and e is said to connect u and v.

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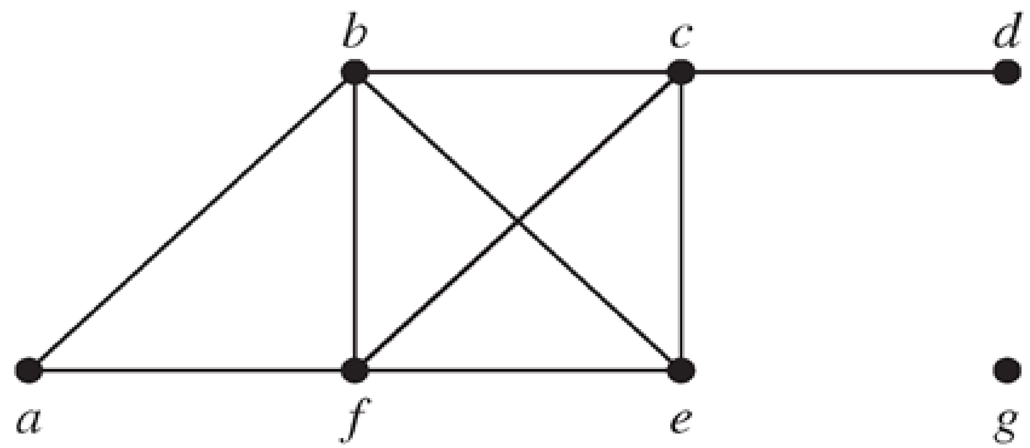
  Definition The *degree of a vertex in an undirected graph* is the number of edges incident with it, except that a loop at a vertex contributes two to the degree of that vertex. The degree of the vertex v is denoted by $\deg(v)$.

Undirected Graphs

- **Example**: What are the degrees and neighborhoods of the vertices in the graph G?



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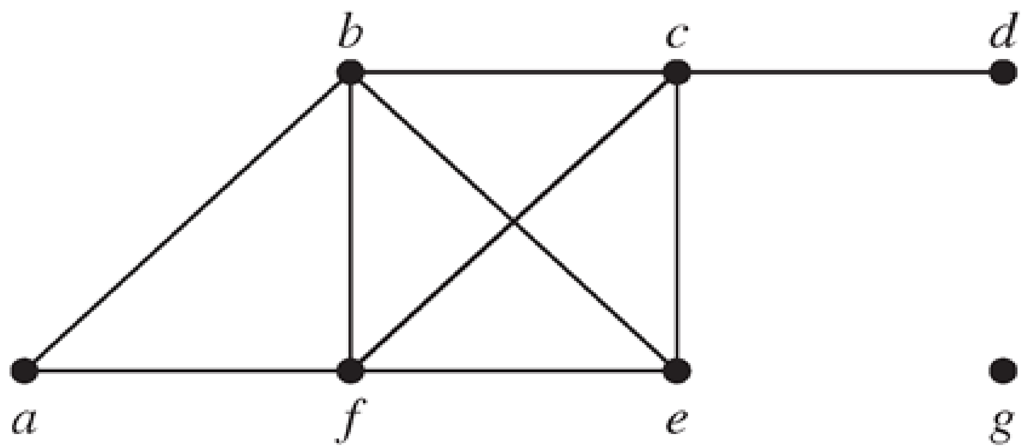

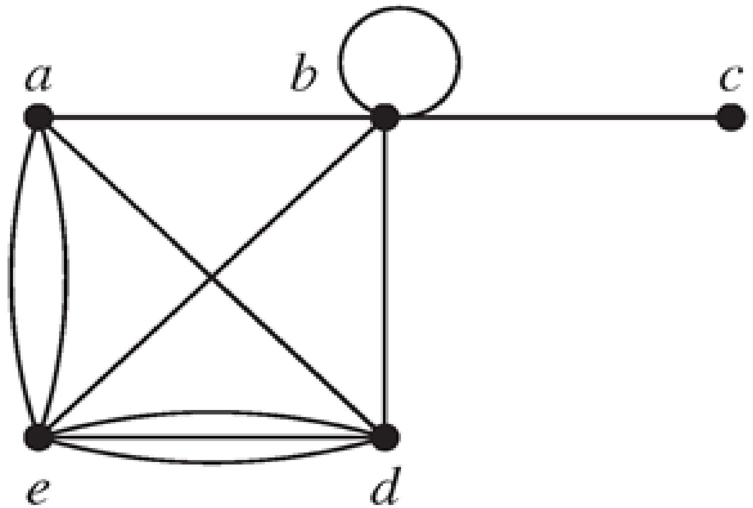

Undirected Graphs

- **Theorem 1** (Handshaking Theorem) If $G = (V, E)$ is an undirected graph with m edges, then

$$2m = \sum_{v \in V} \deg(v)$$

Proof

Undirected Graphs

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  Definition Let (u, v) be an edge in G. Then u is the *initial vertex* of the edge and is *adjacent to v* and v is the *terminal vertex* of this edge and is *adjacent from u*. The initial and terminal vertices of a loop are the same.

Directed Graphs

- **Definition** The *in-degree* of a vertex v, denoted by $\deg^{-}(v)$, is the number of edges which terminate at v. The *out-degree* of v, denoted by $\deg^{+}(v)$, is the number of edges with v as their initial vertex. Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of the vertex.

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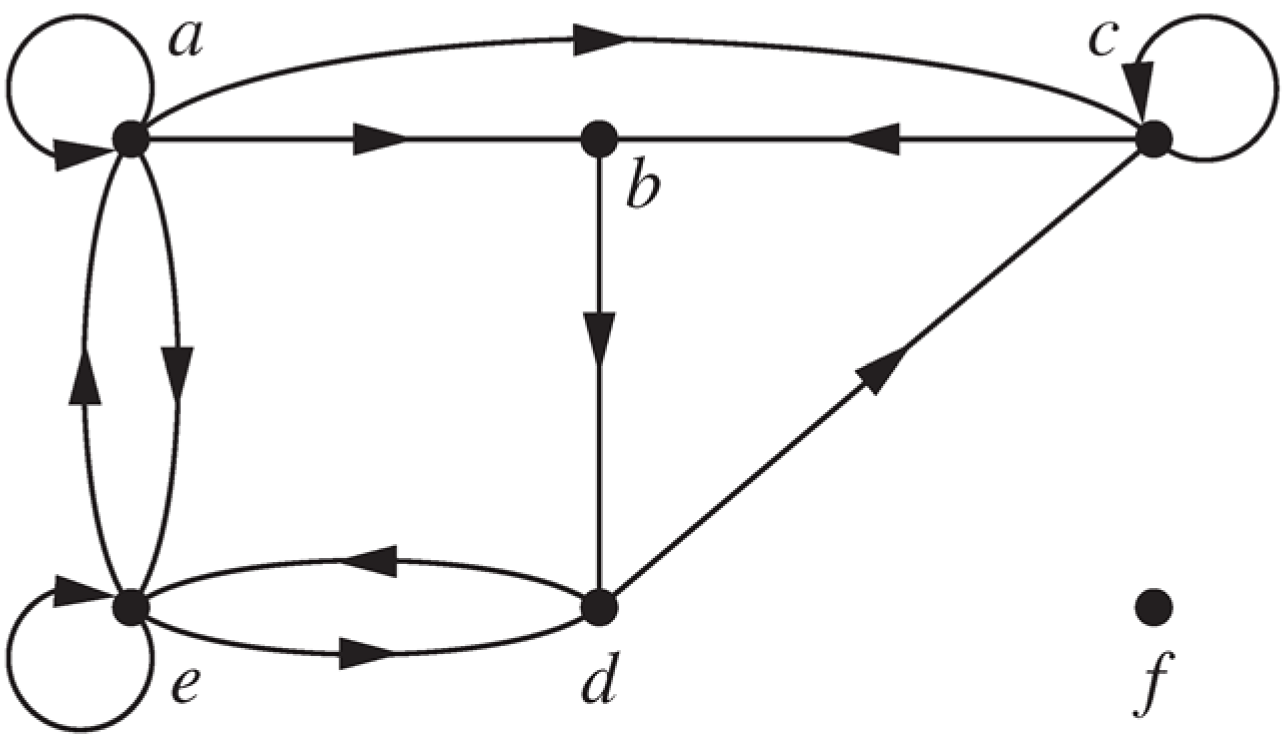

- **Theorem 3** Let $G = (V, E)$ be a graph with directed edges. Then
$$|E| = \sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v)$$

Proof

Complete Graphs

- A *complete graph* on n vertices, denoted by K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices.

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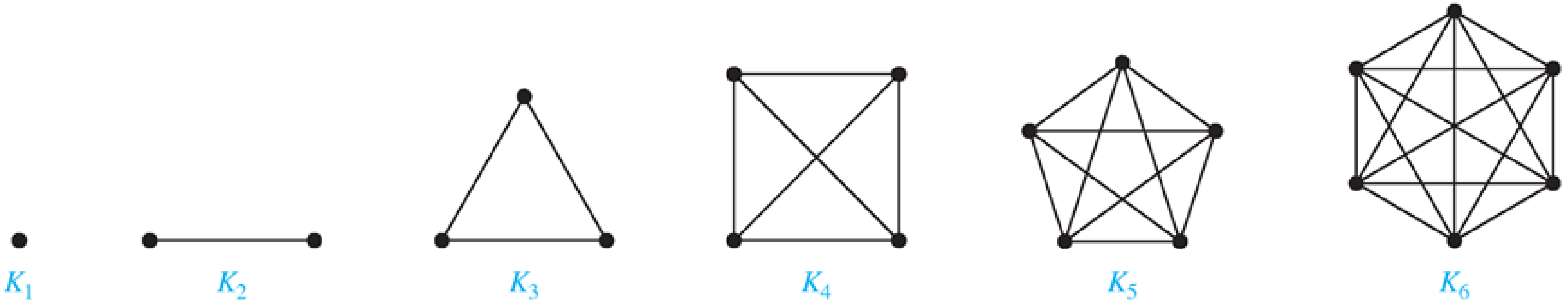

Cycles

- A *cycle* C_n for $n \geq 3$ consists of n vertices $v_1, v_2, \ldots, v_n$, and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}, \{v_n, v_1\}$.

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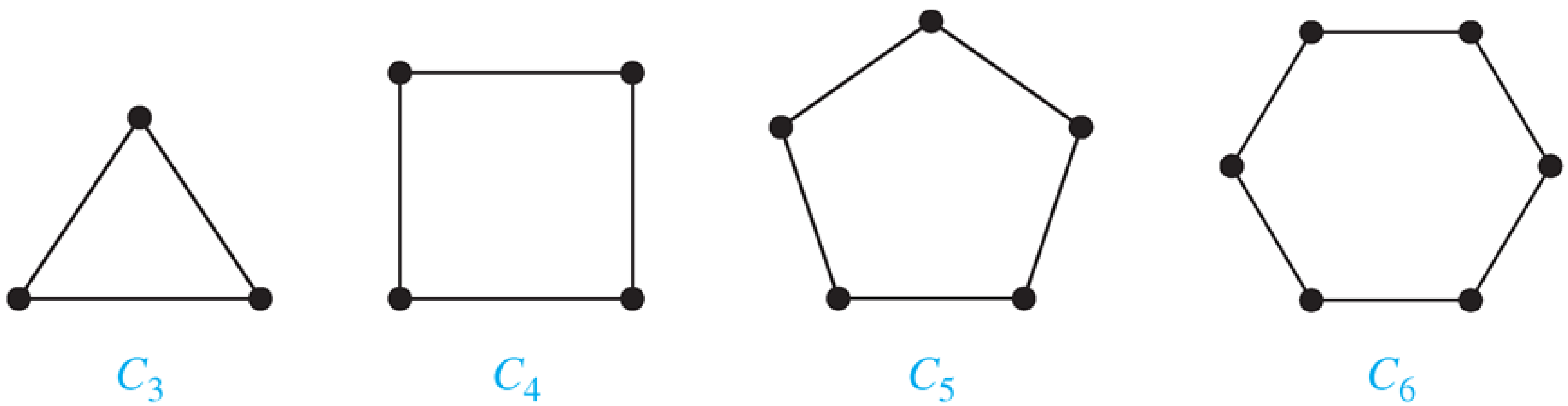

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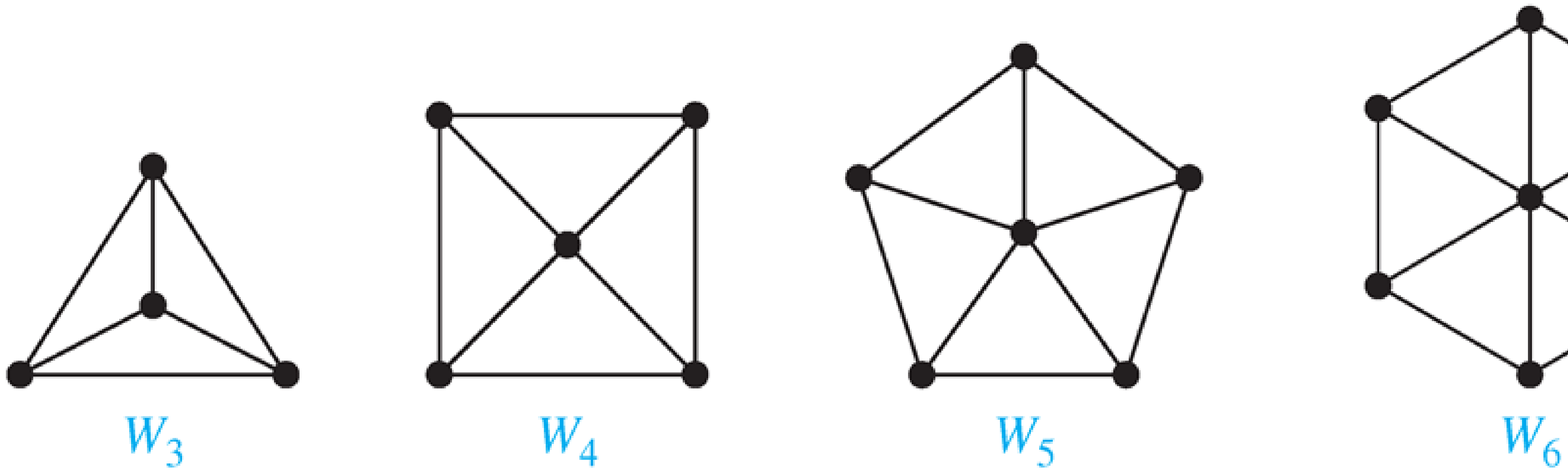

N-dimensional Hypercube

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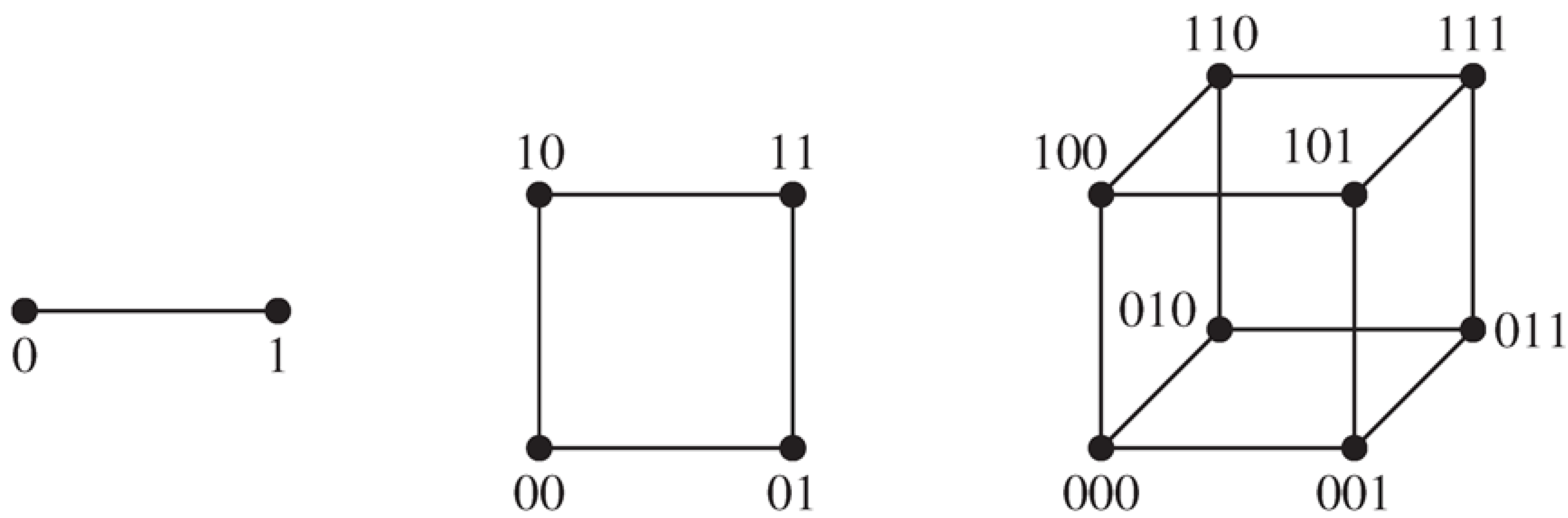

Q_1 Q_2 Q_3

Bipartite Graphs

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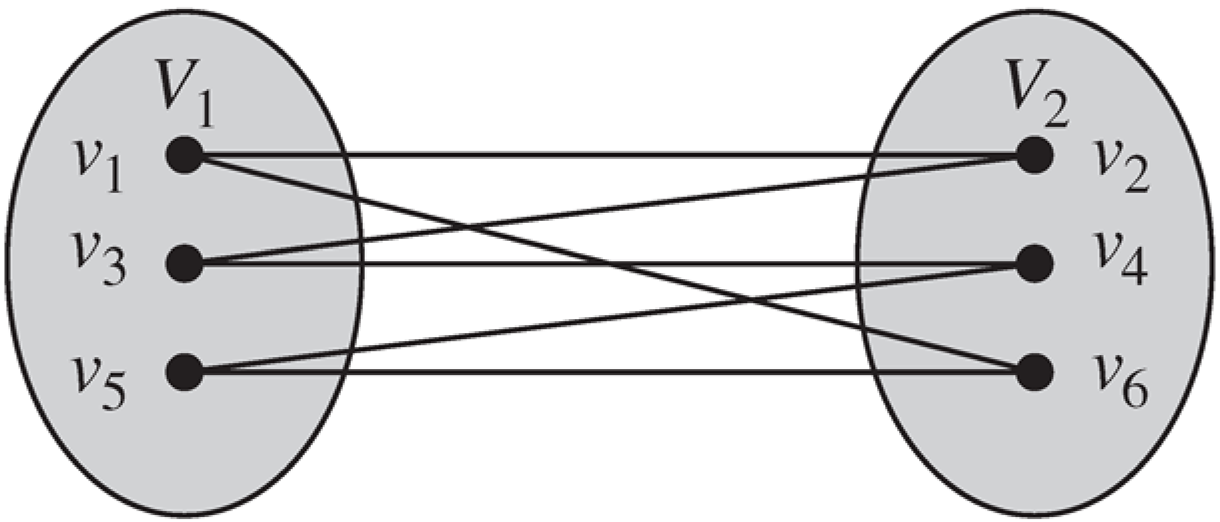
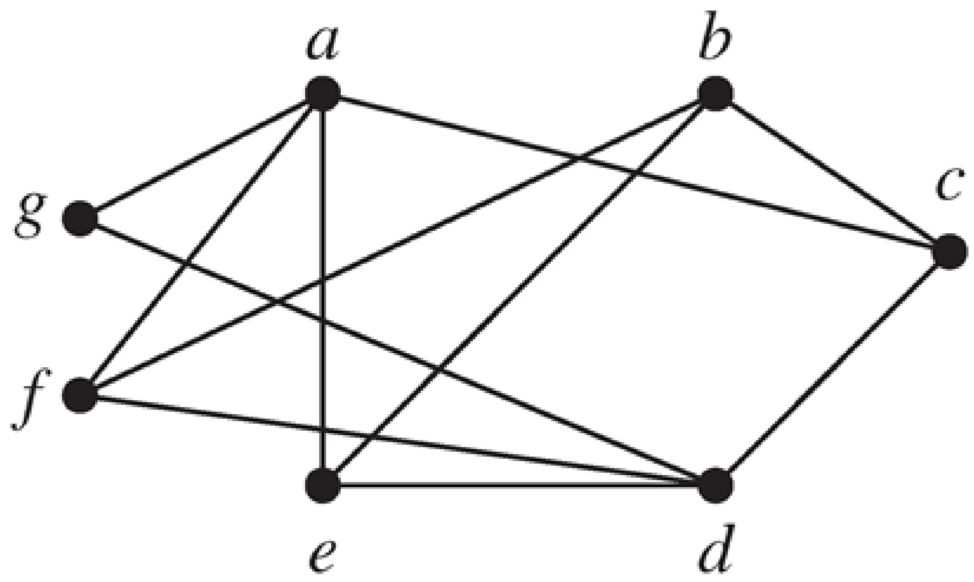


Bipartite Graphs



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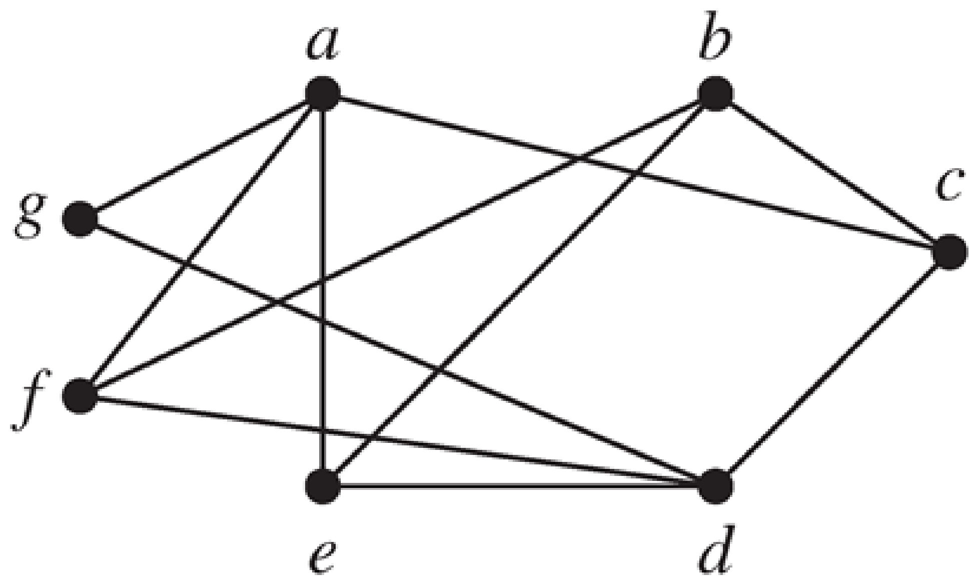
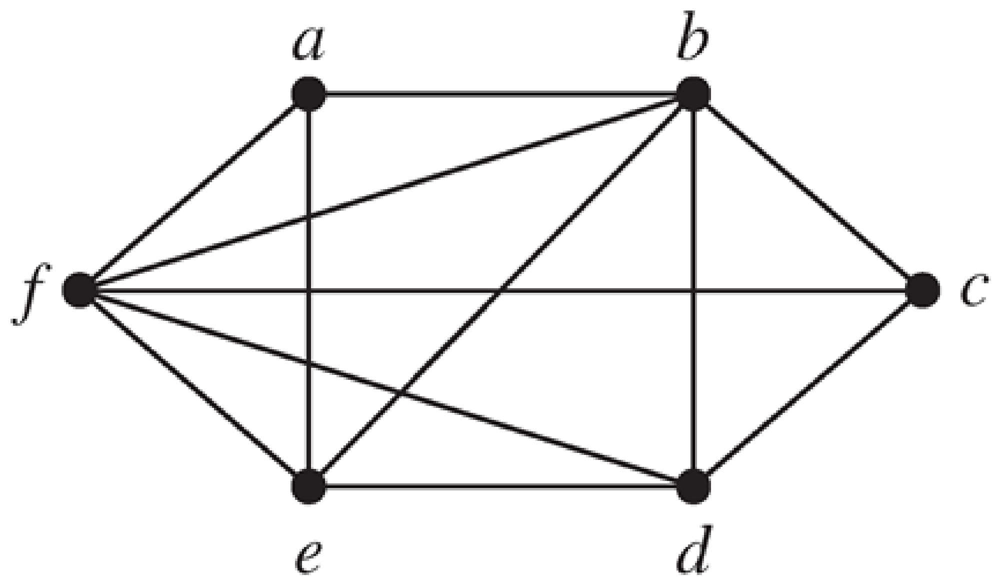
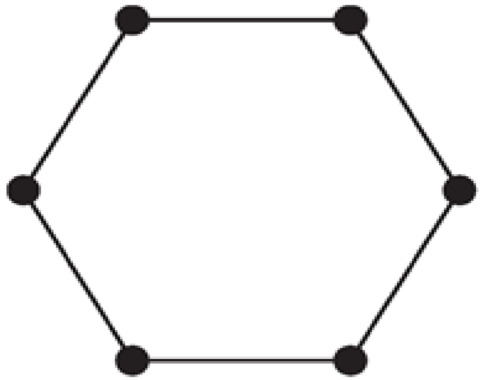




Bipartite Graphs

- **Example** Show that C_6 is bipartite.



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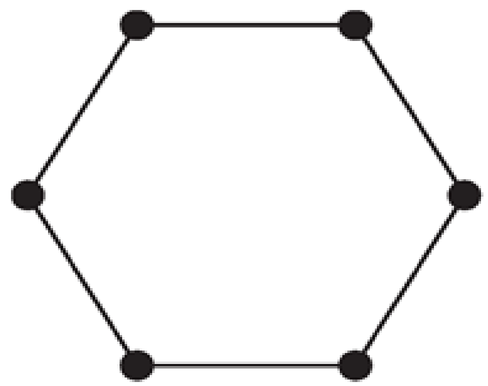
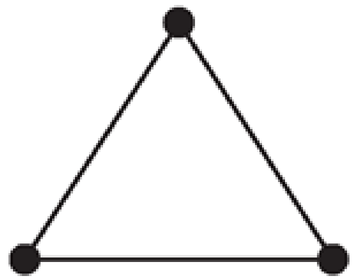


C_6

Example Show that C_3 is not bipartite.



C_3

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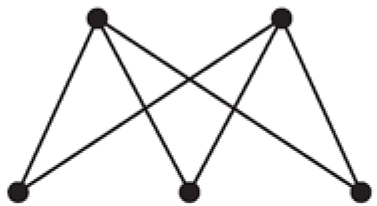
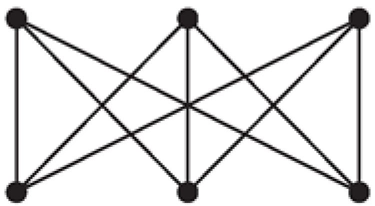


$K_{2,3}$



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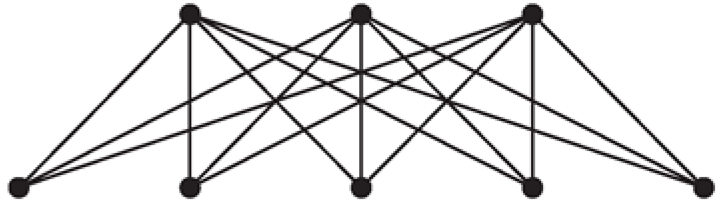
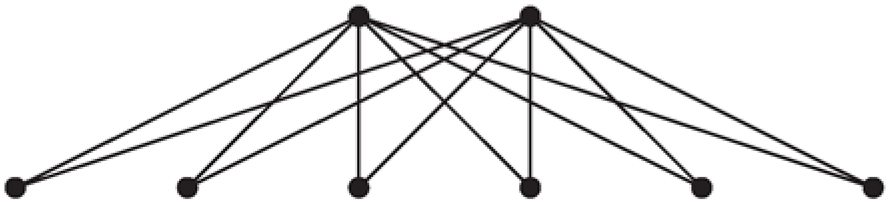


$K_{3,5}$



$K_{2,6}$

Bipartite Graphs and Matchings

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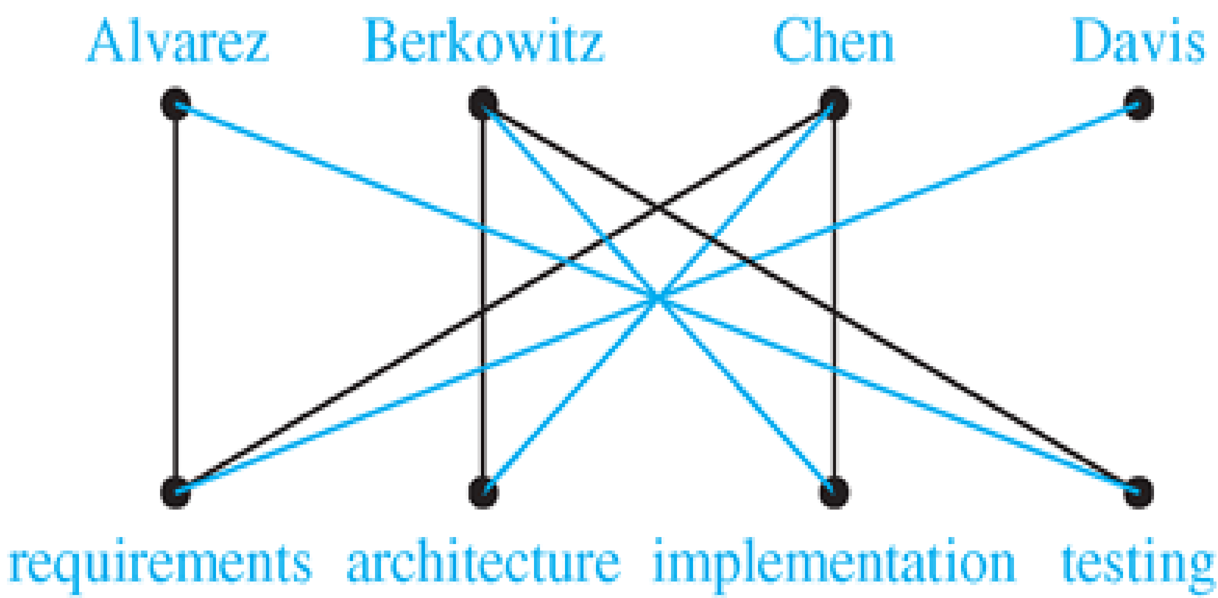

Next Lecture

- graph theory II ...