Assignment 5

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P397 Ex.37

Q: How many functions are there from the set {1, 2, ...,n}, where n is a postive integer, to the set{0,1}

a) that are one-to-one?

b) that assign 0 to both 1 and n?

b) that assign 1 to exactly one of the positive integers less than n?

Solve:

a) The

P398 Ex. 50

Q: How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

Solve:

P398 Ex. 62

Q: Suppose that p and q are prime numbers and that n = pq. Use the principle of inclusion–exclusion to find the number of positive integers not exceeding n that are relativelyprime to n.

P398 Ex. 64

Q: Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.

P405 Ex. 10

Let $(x_i,\ y_i) = 1,\ 2,\ 3,\ 4,\ 5$ be a set of five distinct points with integer coordinates in the xy plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.

P406 Ex. 40

Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

P413 Ex. 13

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

Solve:

$A_n^n \times A_{n+1}^n = n! * (n+1)!$

P422 Ex. 24

Show that if p is a prime and k is an integer such that $1 \leqslant k \leqslant p-1$, then p divides $\binom{p}{k}$.

P422 Ex. 27

P525 Ex. 12

Find the solution to $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$

for $n = 3, 4, 5, \ldots$, with $a_0 = 3$, $a_1 = 6$, and $a_2 = 0$.

Solve:

$x^3 + 2x^2 + x + 2 = 0$

$\Rightarrow (x-2)(x^2-1) = 0$

$\Rightarrow x = 2, x = -1, x = 1$

So, $a_n = \alpha_1 \bullet 2^n + \alpha_2 \bullet (-1)^n + \alpha_3$

Then,

$a_0 = 3 = \alpha_1 + \alpha_2 + \alpha_3$

$a_1 = 6 = 2\alpha_1 - \alpha_2 + \alpha_3$

$a_2 = 0 = 4\alpha_1 + \alpha_2 + \alpha_3$

$\Rightarrow \alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 2$

$\Rightarrow a_n = 2^n + 2$

P525 Ex. 28

a) Find all solutions of the recurrence relation

$a_n = 2a_{n-1} + 2n^2$

b) Find the solution of the recurrence relation in part (a) with initial condition $a_1 = 4$.

Solve:

Firstly,

$a_n = 2a_{n-1}$

$\Rightarrow a_n^{(h)} = \alpha_1 \bullet 2^n$

and

$a_n^{(p)} = C \bullet 2n^2 (C \, \text{is constant})$

Then,

$C \bullet 2n^2 = 2 \bullet C \bullet 2(n-1)^2 + 2(n-1)^2$

$\Rightarrow C = 4/7$

$\Rightarrow a_n =$

P526 Ex. 44

Let A_n be the $n \times n$ matrix with 2s on its main diagonal, 1s in all positions next to a diagonal element, and 0s everywhere else. Find a recurrence relation for d_n, the determinant of A_n. Solve this recurrence relation to find a formula for d_n.

P550 Ex. 22

Give a combinatorial interpretation of the coefficient of x^6 in the expansion $(1 + x + x^2 + x^3 + ...)^n$. Use this interpretation to find this number

P551 Ex. 42

Use generating functions to prove Pascal's identity:

C(n, r) = C(n − 1, r) + C(n − 1, r − 1) when n and r are positive integers with r < n. [Hint: Use the identity

(1 + x)n = (1 + x)n−1 + x(1 + x)n−1.]

P583 Ex. 47

Q: How many relations are there on a set with n elements that are

Solve:

a) symmetric: $2^{C_n^2+n}$

b) antisymmetric: $2^n \times 3^{C_n^2}$

d) irreflexive:

e) reflexive and symmetric

Ref: https://math.stackexchange.com/questions/

606803/number-of-relations-on-a-set-with-n-elements

P607 Ex. 20

Let R be the relation that contains the pair (a, b) if a and b are cities such that there is a direct non-stop airline flight from a to b. When is (a, b) in

a)R^2 ? **b)** R^3? **c)** R^*?

Solve:

a)

P607 Ex. 22

Suppose that the relation R is reflexive. Show that R^* is reflexive.

Prove:

P607 Ex. 23

Suppose that the relation R is symmetric. Show that R^* is symmetric

P607 Ex. 24

Suppose that the relation R is irreflexive. Is the relation R^2 necessarily irreflexive?

P615 Ex. 16

Q: Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad{=}bc$. Show that R is an equivalence relation.

Prove:

P616 Ex. 40

Q: a) What is the equivalence class of (1, 2) with respect to the equivalence relation in Exercise 16?

b) Give an interpretation of the equivalence classes for the equivalence relation R in Exercise 16. [Hint: Look at the ratio a/b corresponding to (a, b).]

Solve:

a)

b)

P630 Ex. 6

Q: Which of these are posets?

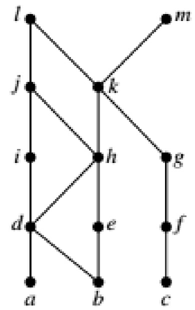
a) $(\mathbf{Z}, =)$ **b)** $(\mathbf{Z}, \neq)$ **c)** $(\mathbf{Z}, \geq)$ **d)** $(\mathbf{Z}, \tilde{})$

Slove:

a and c.

P631 Ex. 32

Q: Answer these questions for the partial order represented by this Hasse diagram.



Solve:

a) Find the maximal elements.

l, m

b) Find the minimal elements.

a, b, c

d) Is there a greatest element?

No.

e) Find all upper bounds of $\{a,b,c\}$

f) Find the least upper bound of {a, b, c}, if it exists.

g)Find all lower bounds of {f, g, h}

h)Find the greatest lower bound of {f, g, h}, if it exists.