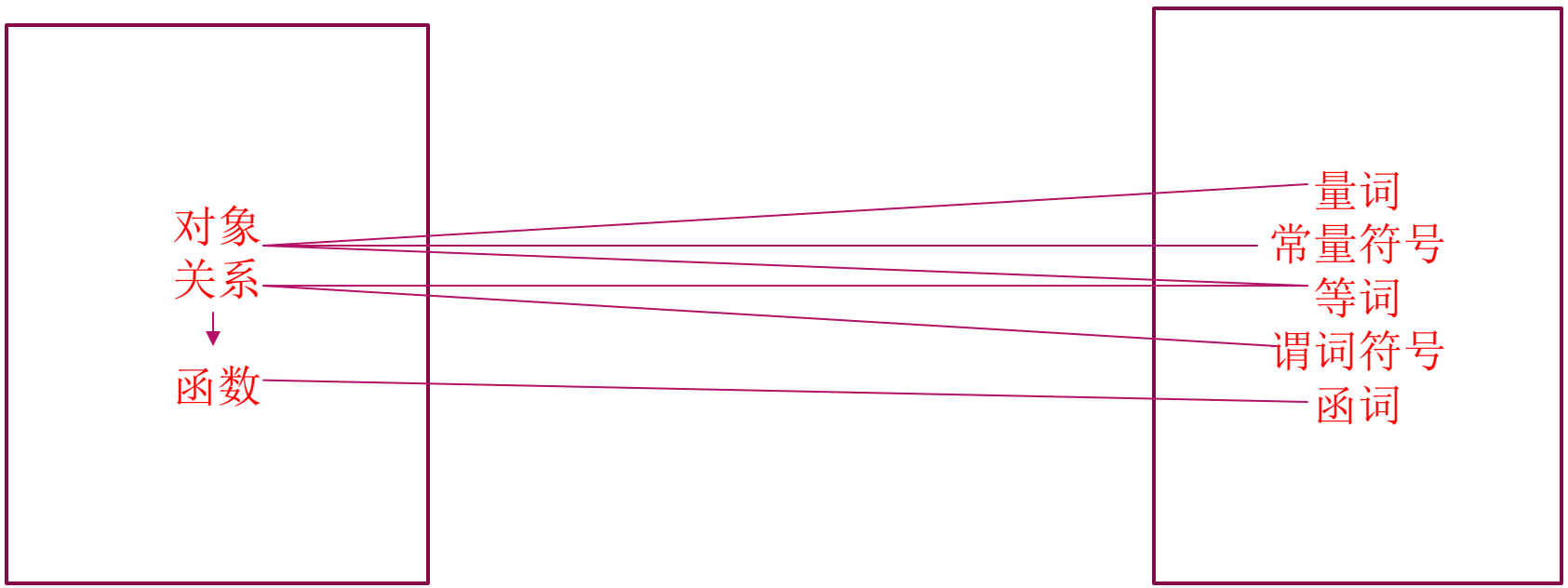
一阶逻辑

一阶逻辑

- 可以进一步分析语句中的成分
- 包含的东西有常量（Constant symbol），谓词符号（Predicate symbol），函数符号（Function symbol），变量（Variable），连词（ $\wedge \vee \rightarrow \leftrightarrow$ ），量词（Quantifiers, $\exists \forall$ ），例如：Father(Mary) = Bob father_of(Mary, Bob)

一阶逻辑（回顾）



一阶逻辑（回顾）

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge\ \vee\ \neg\ \Rightarrow\ \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

一阶逻辑（回顾）

- 与命题逻辑的比较

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts	degree of truth known interval value

一阶逻辑的处理方向

- 退化成命题逻辑
- 一阶逻辑的推理，前向链接，后向链接
- Unify，合取范式，归结证明

Unify的处理过程

p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	$\{x/Jane\}$
$Knows(John, x)$	$Knows(y, OJ)$	$\{x/OJ, y/John\}$
$Knows(John, x)$	$Knows(y, Mother(y))$	$\{y/John, x/Mother(John)\}$
$Knows(John, x)$	$Knows(x, OJ)$	$fail$

变量标准化分离

前向链接

. . . it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

Nono . . . has some missiles, i.e., $\exists x\ Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

. . . all of its missiles were sold to it by Colonel West

$$\forall x\ \ Missile(x) \wedge Owns(Nono, x) \ \Rightarrow\ Sells(West, x, Nono)$$

Missiles are weapons:

$$Missile(x) \Rightarrow Weapon(x)$$

An enemy of America counts as "hostile":

$$Enemy(x, America) \ \Rightarrow\ Hostile(x)$$

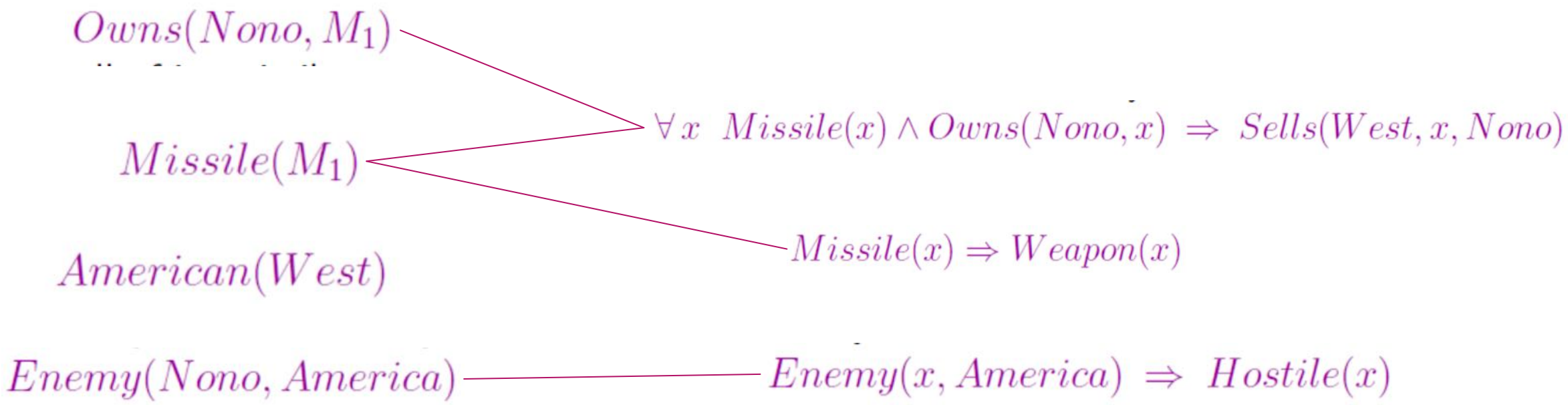
West, who is American . . .

$$American(West)$$

The country Nono, an enemy of America . . .

$$Enemy(Nono, America)$$

已知事实和rule的匹配过程



产生的new集合{ Sells(West,M1,Nono), Weapon(M1), Hostile(Nono)}

更新知识库与rule的匹配过程

$Owns(Nono, M_1)$

$Missile(M_1)$

$American(West)$

$Enemy(Nono, America)$

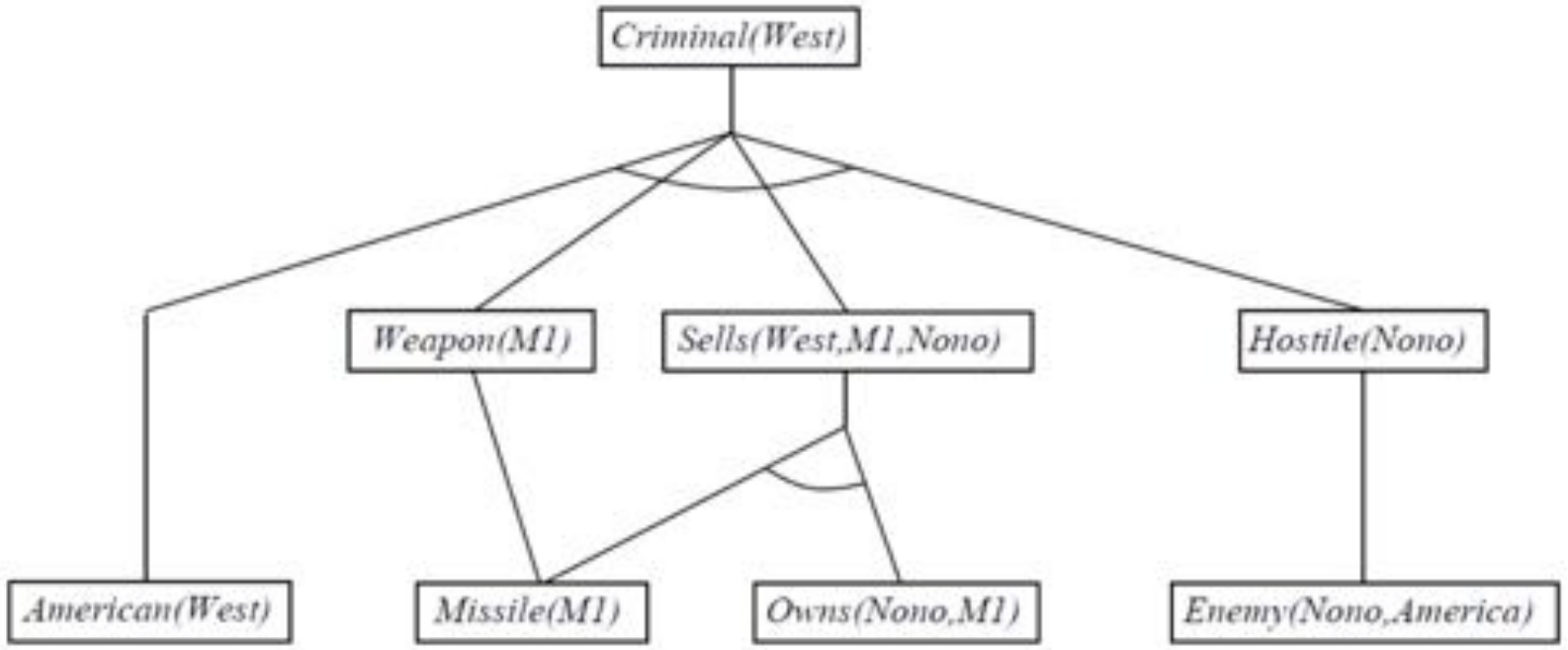
$Weapon(M1)$

$Sells(West,M1,Nono)$

$Hostile(Nono)$

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

前向链接生成树


Criminal(West)
Weapon(M1)
Sells(West,M1,Nono)
Hostile(Nono)
American(West)
Missile(M1)
Owns(Nono,M1)
Enemy(Nono,America)

反向链接：从结果出发深度优先

一阶逻辑转换成合取范式

Everyone who loves all animals is loved by someone:

$$\forall x \ [\forall y \ Animal(y) \Rightarrow Loves(x, y)] \Rightarrow [\exists y \ Loves(y, x)]$$

1. Eliminate biconditionals and implications

$$\forall x \ [\neg \forall y \ \neg Animal(y) \vee Loves(x, y)] \vee [\exists y \ Loves(y, x)]$$

2. Move $\neg$ inwards: $\neg \forall x, p \equiv \exists x \ \neg p$, $\neg \exists x, p \equiv \forall x \ \neg p$:

$$\forall x \ [\exists y \ \neg(\neg Animal(y) \vee Loves(x, y))] \vee [\exists y \ Loves(y, x)]$$
$$\forall x \ [\exists y \ \neg\neg Animal(y) \wedge \neg Loves(x, y)] \vee [\exists y \ Loves(y, x)]$$
$$\forall x \ [\exists y \ Animal(y) \wedge \neg Loves(x, y)] \vee [\exists y \ Loves(y, x)]$$

一阶逻辑转换成合取范式

3. Standardize variables: each quantifier should use a different one

$$\forall x \ [\exists y \ \ Animal(y) \wedge \neg Loves(x, y)] \vee [\exists z \ \ Loves(z, x)]$$

4. Skolemize: a more general form of existential instantiation.
   Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:

$$\forall x \ [Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

5. Drop universal quantifiers:

$$[Animal(F(x)) \wedge \neg Loves(x, F(x))] \vee Loves(G(x), x)$$

6. Distribute $\wedge$ over $\vee$:

$$[Animal(F(x)) \vee Loves(G(x), x)] \wedge [\neg Loves(x, F(x)) \vee Loves(G(x), x)]$$

Skolem函数用来消除存在量词，比如$\forall y(\exists x P(x,y))$,对于任意y存在x使得P(x,y)成立，等价于x是依赖于y的某个值的，意味着存在一种映射关系x=g(y),此时P(x,y)的条件总是满足。所以公式$\forall y(\exists x P(x,y))$可转换为：

$$\forall y(P(g(y), y))$$

举例（West例子的知识库）

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$\forall x \;\; Missile(x) \wedge Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$$

$$\neg Missile(x) \vee \neg Owns(Nono,x) \vee Sells(West,x,Nono)$$

举例（West例子的知识库）

$$Missile(x) \Rightarrow Weapon(x)$$

$$\neg Missile(x) \vee Weapon(x)$$

$$Enemy(x, America) \Rightarrow Hostile(x)$$

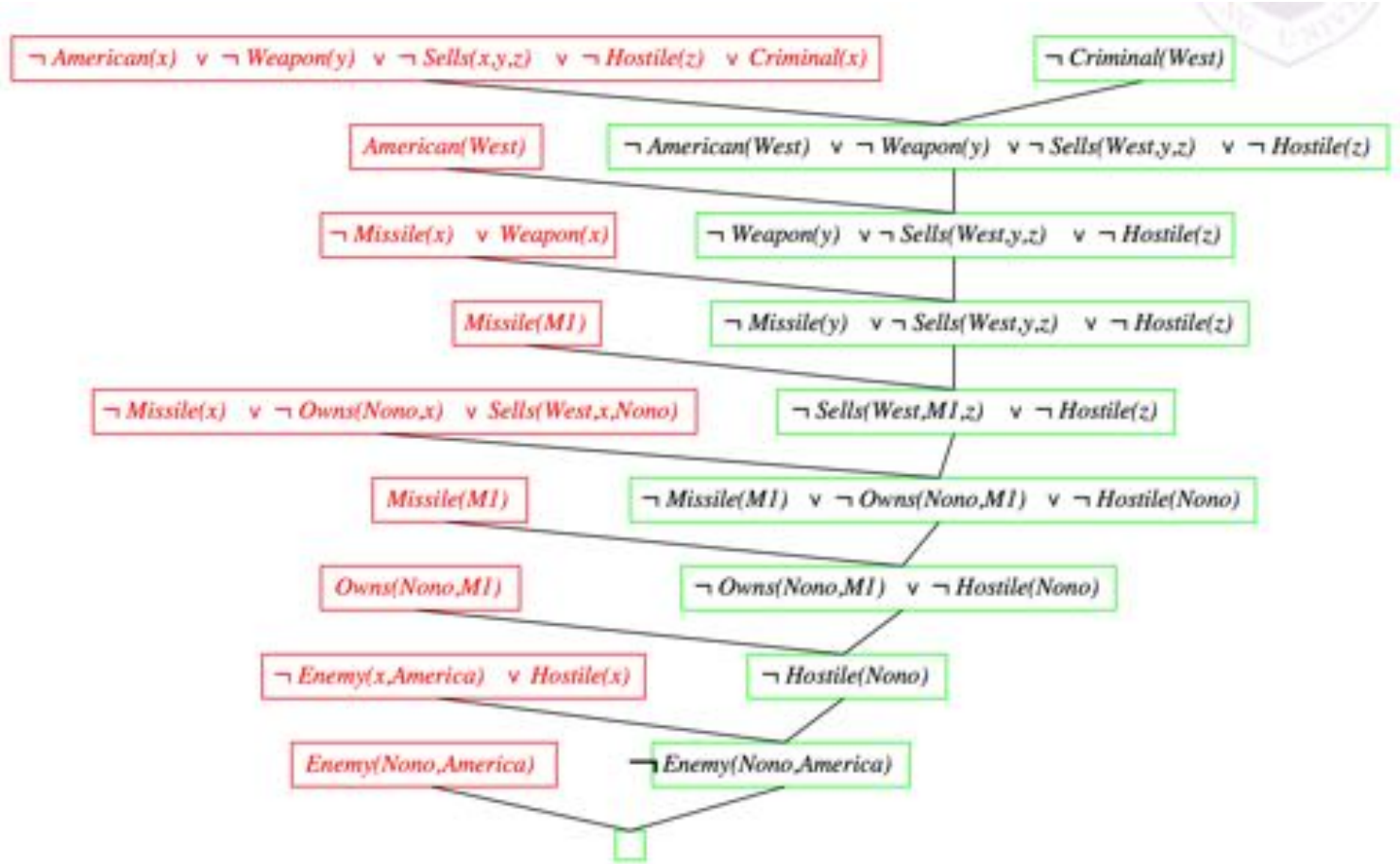
$$\neg Enemy(x, America) \vee Hostile(x)$$

归结证明

- 即证明KB∧$\sim\alpha$ 为false
- 先把α取反
- 转成CNF
- 加入已经转成CNF的知识库中
- 推导出一定为（false）

West例子的归结证明

- 要证明Criminal(West)先生成~ Criminal(West)
- 转成CNF(此处本身已经是了)
- 加入已经转成CNF的知识库中（West的知识库转换CNF的过程见P.15和P.16）
- 推导出结论：即最终一定为（false）



习题

假设任何通过人工智能考试并获奖的人都是快乐的

任何肯学习或幸运的人都可以通过所有的考试

张三不肯学习但是他是幸运的

任何幸运的人都能获奖

求证：张三是快乐的（归结证明）