Mathematical Statistics Assignment 3

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Q3.1 Let $X_1, ..., X_n \overset{\text{iid}}{\sim} U[\theta_1, \theta_2]$. Find the MLEs of θ_1 and θ_2.

Sovle: The joint desity of $\boldsymbol{x} = (X_1, ..., X_n)^T$ is

$$f(\boldsymbol{x}; \theta_1, \theta_2) = \begin{cases} \dfrac{1}{(\theta_2 - \theta_1)^n} & , \text{if } \theta_1 \leqslant x_i \leqslant \theta_2, i = 1, ..., n, \\ 0 & , \text{elsewhere.} \end{cases}$$

Then the likehood function is

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i; \theta_1, \theta_2) = \begin{cases} \displaystyle\prod_{i=1}^{n} \frac{1}{(\theta_2 - \theta_1)} = \frac{1}{(\theta_2 - \theta_1)^n} & , \theta_1 \leqslant x_{(1)} \text{ and } \theta_2 \geqslant x_{(n)} \\ 0 & , \text{elsewhere.} \end{cases}$$

Because $\theta_2 - \theta_1 \geqslant x_{(n)} - x_{(1)}$, and $L(\theta_1, \theta_2)$ is a monotone and decreasing function of $\theta_2 - \theta_1$ when $\theta_2 - \theta_1 \in [x_{(n)} - x_{(1)}, \infty)$. Thus $\hat{\boldsymbol{\theta}} = \widehat{(\theta_2 - \theta_1)} = x_{(n)} - x_{(1)}$ is the MLE of θ_1 and θ_2.

Q3.17. Supposet that X follows a geometeric distribution,

$$\Pr(X = k) = p(1-p)^{k-1}, k = 1, 2. ...$$

where $0 \leqslant p \leqslant 1$, and assume an i.i.d. sample fo size n.

a) Find the moment of estimator of p

b) Find the MLE of p.

c) Let p have a uniform prior distribution on $[0, 1]$. What is the posterior distribution of p? What is the posterior mean?

Q3.18.