

# Lecture 36

## Review for Final Examination

Part I: Chapters 1 – 4 & Appendix B

Part II: Chapter 5



# 1). 2018 Final Examination

- **Time and Date**

19:00 – 22:00,

25 June (Monday) 2018, 3 hours

- **Venue**

The First Teaching Building, Room 107  
for Class I; Room 110 for Class II

- **Range**

Chapters 1–5, Appendix B

- **Assessment**

Final Examination (50%)

## 2). Please Bring Your Calculators

- Candidates taking examinations that permit the use of calculators may use any calculator which fulfils the following criteria:
  - (a) it should be self-contained, silent, battery-operated and pocket-sized.
  - (b) it should have numeral-display facilities only.
  - (c) it should be used only for the purposes of calculation.

## **2). Please Bring Your Calculators (Cont'd)**

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- It is the candidate's responsibility to ensure that the calculator operates satisfactorily.
- The candidate must record the name and type of the calculator on the front page of the examination scripts.

### 3). Please Bring Two Pens/Pencils

- When one pen does not work, you can use another one.

## 4). The Range of the Examination

I BF

- Chapter 1 excludes §1.11–§1.12
- Chapter 2 excludes Theorem 2.1 on p.74, §2.5.2, §2.5.3 and §2.5.4
- Chapters 3
- Appendix B excludes B.3
  
- Chapter 4
- Chapter 5

## 5). The Distribution and Marks of the Questions in the Examination

- There is 1(1,1) question in Chapter 1(2, 3) with a total of 15(10, 10) Marks.
- There is 1 in Chapter 4 with a total of 10 Marks.
- There are 3 questions in Chapter 5 with a total of 45 Marks, where MPT (10), UMPT (10), LRT (10), goodness of fit test (15).
- There is 1 question in Appendix B with a total of 10 Marks.

## **6). Key Points for Chapters 1–4**

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**Please see “Lecture 28: Review for Midterm Test”**

## 7). Key Points for Chapter 5



## 31. Type I error function

$$\begin{aligned}\alpha(\theta) &= \Pr(\text{Type I error}) && \text{弃真} \\ &= \Pr(\text{rejecting } H_0 | H_0 \text{ is true}) \\ &= \Pr(x \in C | \theta \in \Theta_0),\end{aligned}\tag{5.3}$$

where  $C$  is the critical region.

## 32. Type II error function

$$\begin{aligned}\beta(\theta) &= \Pr(\text{Type II error}) \\ &= \Pr(\text{accepting } H_0 | H_0 \text{ is false}) \\ &= \Pr(x \in C' | \theta \in \Theta_1),\end{aligned}\tag{5.4}$$

where  $C'$  is the acceptance region.

### **33. Power function and the relationship with $\alpha(\theta)$ and $\beta(\theta)$**

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$$p(\theta) = \Pr(\text{rejecting } H_0 | \theta) = \Pr(\mathbf{x} \in \mathcal{C} | \theta) \quad (5.6)$$

— If  $\theta \in \Theta_0$ ,

$$p(\theta) = \Pr(\mathbf{x} \in \mathcal{C} | \theta \in \Theta_0) = \alpha(\theta). \quad (5.7)$$

— If  $\theta \in \Theta_1$ ,

$$p(\theta) = \Pr(\mathbf{x} \in \mathcal{C} | \theta \in \Theta_1) = 1 - \beta(\theta). \quad (5.8)$$

## 34. Neyman–Pearson Lemma

Let  $X_1, \dots, X_n$  be a random sample from a population with the pdf (or pmf)  $f(x; \theta)$ . Let the likelihood function be  $L(\theta)$ . Then a test  $\varphi$  with critical region

$$C = \left\{ \mathbf{x} = (x_1, \dots, x_n)^\top : \frac{L(\theta_0)}{L(\theta_1)} \leq k \right\} \quad (5.12)$$

and size  $\alpha$  is the **most powerful test** of size  $\alpha$  for testing  $H_0: \theta = \theta_0$  against  $H_1: \theta = \theta_1$ , where  $k$  is a value determined by the size  $\alpha$ .

## 35. Uniformly Most Powerful Test

- Definition 5.3 on page 197
- Examples 5.7 and 5.8
- Questions 5.1–5.9 in Assignment 5.

## 36. How to find a UMPT

- (a) For the given two composite hypotheses  $H_0: \theta \in \Theta_0$  and  $H_1: \theta \in \Theta_1$ , first consider two simple hypotheses  $H_{0s}: \theta = \theta_0 \in \Theta_0$  and  $H_{1s}: \theta = \theta_1 \in \Theta_1$ .
- (b) By using the Neyman–Pearson lemma, to find a most powerful test  $\varphi$  of size  $\alpha$  with critical region  $\mathbb{C}$ .

- (c) If  $\mathbb{C}$  is free from  $\theta_1$ , then  $\varphi$  is the UMPT of size  $\alpha$  for testing  $H_{0s}: \theta = \theta_0 \in \Theta_0$  against  $H_1: \theta \in \Theta_1$ .
- (d) If  $\sup_{\theta \in \Theta_0} p_\varphi(\theta) = \alpha = p_\varphi(\theta_0)$ , then  $\varphi$  is the UMPT of size  $\alpha$  for testing  $H_0: \theta \in \Theta_0$  against  $H_1: \theta \in \Theta_1$ .

## 37. Likelihood Ratio (LR) Test

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- Step 1: Find  $\hat{\theta}^R$ , the restricted MLE of  $\theta$  in  $\Theta_0$ , and  $\hat{\theta}$ , the (un)restricted MLE of  $\theta$  in  $\Theta$ .
- Step 2: Calculate the LR statistic

$$\lambda(\mathbf{x}) = L(\hat{\theta}^R)/L(\hat{\theta}). \quad (5.23)$$

- Step 3: Determine  $\lambda_\alpha$  or  $c$  in

$$\mathbb{C} = \{\mathbf{x}: \lambda(\mathbf{x}) \leq \lambda_\alpha\} = \{\mathbf{x}: \log \lambda(\mathbf{x}) \leq c\}$$

based on

$$\alpha = \Pr\{\log \lambda(\mathbf{x}) \leq c | H_0 \text{ is true}\}$$

## 38. How to determine the $c$ in LR Test

- Step I. Express  $\lambda(x)$  or  $\log \lambda(x)$  as a function of a statistic  $Q$ , e.g.,  $\lambda(x) = h(Q)$ , such that under  $H_0$ ,  $Q$  follows a given distribution (e.g., Normal, Gamma, Chi-squares, t, F).
- Step II. Prove that  $h(Q)$  is **concave** with a maximum or **convex** with a minimum.

- Step III. If  $h(Q)$  is concave, then

$$h(Q) \leq c$$

is equivalent to

$$Q \leq c_1 \quad \text{or} \quad Q \geq c_2.$$

- Step IV. If  $h(Q)$  is convex, then  $h(Q) \geq c$

is equivalent to

$$Q \leq c_1 \quad \text{or} \quad Q \geq c_2.$$

- Step V. Use the equal-tail approach, let

$$\alpha/2 = \Pr(Q \leq c_1 | H_0)$$

and

$$\alpha/2 = \Pr(Q \geq c_2 | H_0)$$

to determine the  $c_1$  and  $c_2$ .

## 39. Tests on Normal Means

- One-sample normal test when variance is known (§5.4.1)
- One-sample  $t$  test (§5.4.2)
- Two-sample  $t$  test (§5.4.3)

## **40. The critical region approach and the *p*-value approach**

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- If you are asked to use the critical region approach, you must obey.
- If you are asked to use the *p*-value approach, you must obey.
- You could use either approach if the approach is not be specified in questions.

## 41. Goodness of Fit Test

Please review §5.5.1 – §5.5.3

- 42. All questions in Assignments 1–5.**
- 43. All questions in Tutorials.**

# End of the Review



GOD Bless You!