Math Statistics Assignment 4

by Yuejian Mo

11510511

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4.1.

Show that the distrubution of T_{Welch} defined in (4.13) can be approximated by a $t-$distribution with v degrees of freedom, where

$$v=\left\{\frac{c^2}{n_1-1}+\frac{(1-c)^2}{n_2-1}\right\}^{-1} \text{and}\, c=\frac{S_1^2/n_1}{S_1^2/n_1+S_2^2/n_2}$$

Proof:

4.2.

a) From (4.21), we obtain

$$\frac{\sqrt{n}\left(\bar{X}_n-\mu\right)}{\sqrt{\mu(1-\mu)}}\sim N(0,1)$$

Thus

$$1-\alpha \doteq \Pr\left\{-z_{\alpha/2}\leqslant\frac{\sqrt{n}\left(\overline{X_n}-\lambda'\right)}{\sigma(\lambda')}\leqslant z_{\alpha/2}\right\}$$

$$1-0.05=\Pr\left\{-1.96=-z_{0.025}\leqslant\frac{\sqrt{100}(6.25-\lambda')}{\lambda'}\leqslant z_{0.025}=1.96\right\}$$

So an approximate equal-tail 95% CI for λ is [5.2258, 7.7736]

b) From (4.22), we obtain

$$1-0.05\doteq\Pr\left\{-z_{\alpha_2}\leqslant\frac{10(6.25-\lambda')}{\lambda'}\leqslant z_{1-0.05+\alpha_2}\right\}$$

$$\frac{6.25}{1+z_{0.95+\alpha_2}}\leqslant\lambda'\leqslant\frac{6.25}{1-z_{\alpha_2}}$$

Thus, the length of CI is

$$L(\alpha_2)=\frac{6.25}{1-z_{\alpha 2}}-\frac{6.25}{1+z_{0.95+\alpha_2}}$$

Then the short CI with $\hat{\alpha}_2$ is

$$\hat{\alpha}_2=\arg\min_{0\leqslant\alpha_2\leqslant\alpha}L(\alpha_2)$$

4.3.

a) $\overline{X}=\frac{3.3-0.3-0.6-0.9}{4}=0.375$

$\overline{X} \pm z_{\frac{1-a}{a}} \frac{\sigma}{\sqrt{n}} = 0.375 \pm 1.645 \frac{3}{\sqrt{4}}$

The 90% CI for μ is $[-2.0925, 2.8425]$

b) $S^2 = \frac{(3.3-0.375)^2 + (-.03-0.375)^2 + (-0.6-0.375)^2 + (-0.9-0.375)^2}{4-1} = 15.863$

$S = 3.9828,\ t(0.05, 3) = 1.638,\ \overline{X} \pm t(0.05, 3)\frac{S}{\sqrt{n}} = 0.375 \pm 1.638 \frac{3.9828}{\sqrt{4}}$

The 90% CI for μ is $[-2.8869, 3.6369]$

4.4.

4.5.

$\bar{X}_A = 81.625,\ \bar{X}_B = 75.875,\ S_A = 12.070, S_B = 10.106$

4.6.

a) Here is a pivor

$$2n\overline{X} \sim \text{Gamma}(2n/2, 1/2) = \chi^2(2n)$$

b)

4.7.

a) $\overline{X} = \frac{\sum_i^{10} x_i}{n} = 55.087$

$\left[\frac{\chi^2(1-\alpha/2, 2n)}{2n\,\overline{X}}, \frac{\chi^2(\alpha/2, 2n)}{2n\overline{X}}\right] = \left[\frac{9.591}{2 \times 10 \times 55.087}, \frac{34.170}{2 \times 10 \times 55.087}\right]$

The exact 95% equal-tail CI for λ is [0.0087053, 0.031015]

b) So, we can say an exact 95% equal-tail CI for $1/\lambda$ is $[1/0.031015, 1/0.0087053] =$ [32, 114.87]