Math Stat Practice I

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March 27, 2018

1 Part I

1.1

a) If $X \sim \text{Bionomial}(n,p)$, then pmf is $\binom{n}{x}p^x(1-p)^{n-x}$

b) The mgf of X is $(\text{pe}^t+q)^n$, then

$E(Z^2) = \frac{d^2(\text{pe}^t+1-p)^n}{\text{dt}^2}|_{t=0} = n^2P^2 - np^2 + np$

$\text{Var}(Z) = E(Z^2) - E(Z)^2 = n^2p^2 - np^2 + np - n^2p^2 = np(1-p)$

c) The m.g.f of Y is $\exp\{\lambda(e^t-1)\}$,then

$M_t(X+Y) = M_t(X)\cdot M_t(Y) = \lambda(pe^t+1-q)^n(e^t-1)$

1.2

a) The marginal distribution of X is

$$Pr(X=1) = Pr(1,1)+Pr(1,2)+Pr(1,3)+Pr(1,4) = \frac{1}{4}$$
$$Pr(X=2) = Pr(2,2)+Pr(2,3)+Pr(2,4) = \frac{1}{4}$$
$$Pr(X=3) = Pr(3,3)+Pr(3,4) = \frac{1}{4}$$
$$Pr(X=4) = Pr(4,4) = \frac{1}{4}$$

b) The pmf of $X+Y$ is

X+Y	2	3	4	5	6	7	8
Probability	1/16	1/16	3/16	2/16	4/16	1/16	4/16

Table 1.

1.3

a) Using the point-wise formula, hhe marginal distribution of X is

$f_X(x) = \left\{\int_{S_Y}\frac{f_{(Y|X|)}(y|x|)}{f_{(X|Y|)}(x|y|)}d\,y\right\}^{-1} = \left\{\int_0^b \frac{x}{y}\exp(y\,x - x\,y)\frac{1-e^{-b\,y}}{1-e^{-b\,x}}d\,y\right\}^{-1} = \frac{1-e^{-b\,x}}{x}\left\{\int_0^b\frac{1-e^{-b\,y}}{y}dy\right\}^{-1} =$

b) When $b=+\infty$, $f_X(X)$ don't exsit.

1.4

a) The marginal distributions of X is

$p_1 = \left\{\frac{b_{11}}{a_{11}}+\frac{b_{12}}{a_{12}}+\frac{b_{13}}{a_{13}}+\frac{b_{14}}{a_{14}}\right\}^{-1} = \{\frac{7}{6}+\frac{4}{6}+\frac{7}{6}+\frac{7}{6}\}^{-1} = \frac{6}{25}$

$p_2 = \left\{\frac{b21}{a_{21}} + \frac{b_{22}}{a_{22}} + \frac{b23}{a_{23}} + \frac{b_{24}}{a_{24}}\right\}^{-1} = \left\{\frac{7}{7} + \frac{4}{7} + \frac{7}{7} + \frac{7}{7}\right\}^{-1} = \frac{7}{25}$

$p_3 = \left\{\frac{b_{31}}{a_{31}} + \frac{b_{32}}{a_{32}} + \frac{b_{33}}{a_{33}} + \frac{b_{34}}{a_{34}}\right\}^{-1} = \left\{\frac{7}{12} + \frac{4}{131112} + \frac{7}{12} + \frac{7}{12}\right\}^{-1} = \frac{11121112}{25}$

$q_1 \propto \frac{b_{11}}{a_{11}} = \frac{7}{6}, q_2 \propto \frac{b_{12}}{a_{12}} = \frac{4}{6}, q_3 \propto \frac{b_{13}}{a_{13}} = \frac{7}{6}, q_4 = \frac{b_{34}}{a_{34}} = \frac{7}{6}$

$\Rightarrow q_1 = \frac{7}{25}, q_2 = \frac{4}{25}, q_3 = \frac{7}{25}, q_4 = \frac{7}{25}$

b) The joint distribution of (X, Y) is

(X,Y)	1	2	3
1	$42/25^2$	$49/25^2$	$84/25^2$
2	$24/25^2$	$28/25^2$	$48/25^2$
3	$42/25^2$	$49/25^2$	$84/25^2$
4	$42/25^2$	$49/25^2$	$84/25^2$

Table 2.

1.6

a) $\Pr(1/4 < X < 5/8) = \Pr(X < 5/8) - \Pr(X < 1/4) = 1 - 2(1 - 5/8)^2 - 2\left(\frac{1}{4}\right)^2 = \frac{19}{32}$

b) $f(x) = F'(x) = \begin{cases} 0 & x < 0 \\ 4x & 0 \leqslant x < 1/2 \\ -4(x-1) & 1/2 \leqslant x < 3/4 \\ 0 & 3/4 \leqslant x \end{cases}$

$\mathrm{Var}(X) = E(X^2) - E(X)^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - (\int_{-\infty}^{\infty} x f(x)dx)^2 = \frac{11}{16} - \frac{11^2}{48^2} \cong 0.635$

1.10

2 Part II