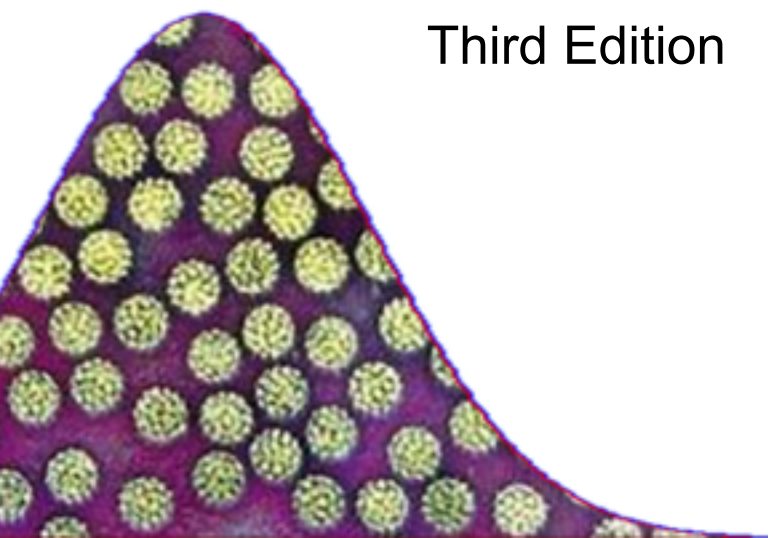
18. Two-sample problems for population means (σ unknown)

The Practice of Statistics in the Life Sciences

Third Edition



Objectives (PSLS Chapter 18)

Comparing two means (σ unknown)

- Two-sample situations
- *t*-distribution for two independent samples
- Two-sample *t* test
- Two-sample *t* confidence interval
- Robustness

Two samples situations

We may need to compare 2 treatments or 2 conditions. It is important to determine if the 2 samples are **independent** or not.

Independent samples: the individuals in both samples are chosen separately ("independently")

→ Chapter 18

Matched pairs samples: the individuals in both samples are related (for example, the same subjects assessed twice, or siblings)

→ Chapter 17

t distribution for 2 independent samples

We have **2 independent SRSs** coming from 2 populations with (μ_1,σ_1) and (μ_2,σ_2) unknown. We use $(\bar{x}_1, s_1)$ and $(\bar{x}_2, s_2)$ to estimate (μ_1,σ_1) and (μ_2,σ_2) respectively.

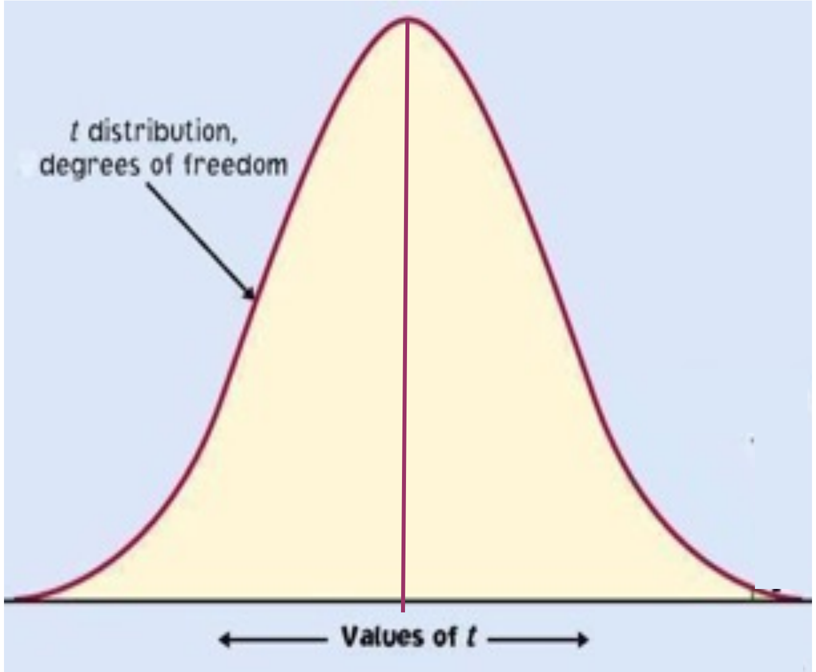
- Both populations *should* be Normally distributed.
- In practice, *it is enough that* both distributions have similar shapes and that the sample data contain no strong outliers.

The two-sample t statistic follows approximately a t distribution with a standard error SE (denominator) reflecting variation from both samples.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

The degrees of freedom of the t distribution are computed as follows:

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2}$$



Two-sample t test

If you have 2 independent random samples and want to test

H_0: $\mu_1 = \mu_2 \iff \mu_1 - \mu_2 = 0$

with either a one-sided or a two-sided alternative hypothesis:

Compute the ***t* statistic** and appropriate **df.**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \Rightarrow \boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{SE}}$$

Obtain and interpret the ***P*-value** (one- or two-sided, depending H_a).

Does smoking damage the lungs of children exposed to parental smoking?

Forced Vital Capacity (FVC) is the volume (in milliliters) of air that an individual can exhale in 6 seconds.

FVC was obtained for a sample of children not exposed to parental smoking and for a sample of children exposed to parental smoking.

Parental smoking	Mean FVC	*s*	*n*
Yes	75.5	9.3	30
No	88.2	15.1	30

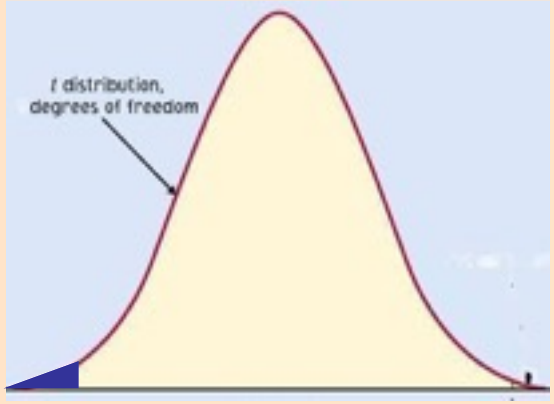
Is the mean FVC lower in the population of children exposed to parental smoking? We test:

$$H_0: \mu_{smoke} = \mu_{no} \iff (\mu_{smoke} - \mu_{no}) = 0$$

$$H_a: \mu_{smoke} < \mu_{no} \iff (\mu_{smoke} - \mu_{no}) < 0 \text{ (one-sided)}$$

Parental smoking	xbar	s	n
Yes	75.5	9.3	30
No	88.2	15.1	30

$$t = \frac{\bar{x}_{smoke} - \bar{x}_{no}}{\sqrt{\frac{s^2_{smoke}}{n_{smoke}} + \frac{s^2_{no}}{n_{no}}}} = \frac{75.5 - 88.2}{\sqrt{\frac{9.3^2}{30} + \frac{15.1^2}{30}}} = \frac{-12.7}{3.24} \approx -3.92$$



$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{(2.9 + 7.6)^2}{\frac{1}{29}(2.9)^2 + \frac{1}{29}(7.6)^2} \approx 48$$

In Table C, for df 40, we find: $|t| > 3.551$ ➔ $P < 0.0005$ (one-sided)

Software gives $P = 0.00014$, highly significant ➔ We reject H_0

Lung capacity is significantly impaired in children exposed to parental smoking, compared with children not exposed to parental smoking.

Geckos are lizards with specialized toe pads that enable them to easily climb even slick surfaces. Researchers want to know if male and female geckos differ significantly in the size of their toe pads. In a random sample of Tokay geckos, they find that the mean toe pad area is 6.0 cm^2 for the males and 5.3 cm^2 for the females. What is the appropriate null hypothesis here?

A. $H_0 : \bar{x}_{male} - \bar{x}_{female} = 0.7$

B. $H_0 : \bar{x}_{male} - \bar{x}_{female} = 0$

C. $H_0 : \mu_{male} - \mu_{female} = 0$

D. $H_0 : \mu_{differenceM-F} = 0$

E. $H_0 : \mu_{differenceM-F} = 0.7$

Should the alternative hypothesis be one-side or two-sided?

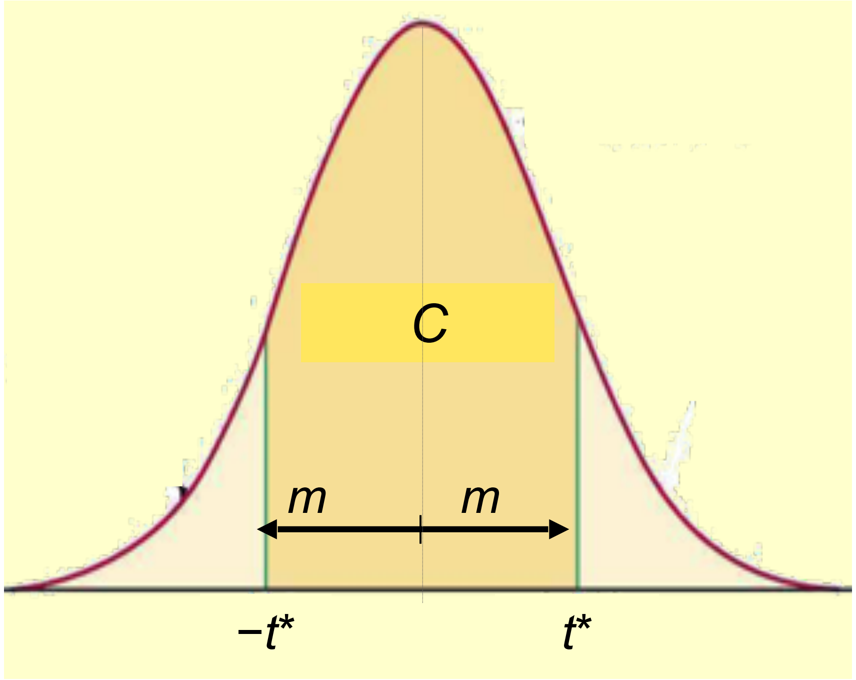
Two sample *t*-confidence interval

Because we have 2 independent samples we use the difference between both sample averages $(\bar{x}_1 - \bar{x}_2)$ to estimate $(\mu_1 - \mu_2)$.

- C is the area between $-t^*$ and t^*
- Find t^* in the line of Table C for the computed degrees of freedom

The margin of error m is:

$$\boxed{m = t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = t^* SE}$$



How does pesticide help seedling growth? Seeds are randomly assigned to be planted in pots with soil treated with pesticide or in pots with untreated soil. Seedling growth (in mm) is recorded after 2 weeks.

Treatment group				Control group			
24	61	59	46	42	33	46	37
43	44	52	43	43	41	10	42
58	67	62	57	55	19	17	55
71	49	54		26	54	60	28
43	53	57		62	20	53	48
49	56	33		37	85	42	

Group	n	$\bar{x}$	s
Treatment	21	51.48	11.01
Control	23	41.52	17.15

A 95% confidence interval for (μ_1 - μ_2) is:

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$\text{df} = \frac{\left(\frac{11.01^2}{21} + \frac{17.15^2}{23}\right)^2}{\frac{1}{20}\left(\frac{11.01^2}{21}\right)^2 + \frac{1}{22}\left(\frac{17.15^2}{23}\right)^2}$$

$$= \frac{344.486}{9.099} = 37.86$$

Using df = 30 from Table C, we get:

$$(51.48 - 41.52) \pm 2.042\sqrt{\frac{11.01^2}{21} + \frac{17.15^2}{23}}, \text{ or } \ 9.96 \pm 8.80 \text{ mm}$$

30	0.683	0.854	1.055	1.310	1.697	$\boxed{2.042}$	2.147	2.457
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423
	50%	60%	70%	80%	90%	95%	96%	98%
	Confidence level C							

Table C

t-Test: Two-Sample Assuming Unequal Variances	Treatment group	Control group
Mean	51.476	41.522
Variance	121.162	294.079
Observations	21	23
Hypothesized Mean Difference	-	
df	38	
t Stat	2.311	
P(T<=t) one-tail	0.013	
t Critical one-tail	1.686	
P(T<=t) two-tail	0.026	
t Critical two-tail	2.024	t^*

Excel

$$\text{for df} = 38,$$
$$m = 2.024 * 4.31 \approx 8.72$$

We are 95% confident that using pesticide yields seeds that are 1.2 to 18.7 mm longer on average after 2 weeks.

Independent Samples Test **SPSS**

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Growth	Equal variances assumed	2.362	.132	2.267	42	.029	9.95445	4.39189	1.09125	18.81765
	Equal variances not assumed			2.311	37.855	.026	9.95445	4.30763	1.23302	18.67588

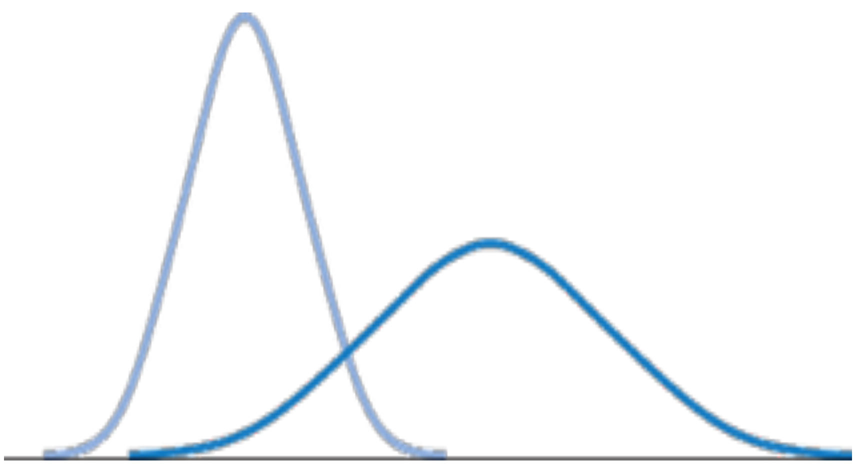
Robustness

The two-sample statistic is most **robust** when both sample sizes are equal and both sample distributions are similar. But even when we deviate from this, two-sample tests tend to remain quite robust.

As a guideline, a combined sample size (n_1 + n_2) of 40 or more will allow you to work even with the most skewed distributions.

Avoid the pooled two-sample *t* procedures

There are two versions of the two-sample *t* procedures: one **assuming equal variance ("pooled")** and one **not assuming equal variance** for the two populations. They have slightly different formulas and df.



Two Normally distributed populations with unequal variances

The pooled (equal variance) two-sample *t* test has degrees of freedom $n_1 + n_2 - 2$.

However, the assumption of equal variance is hard to check, and thus the **unequal variance test is safer**.

REVIEW: *t* procedures

One-sample *t* procedure One sample summarized by its mean $\bar{x}$ and standard deviation s. Population parameters μ and σ unknown.	Inference about μ
Matched pairs *t* procedure Two *paired* datasets (from a matched-pairs design). From the *n pairwise differences* we compute $(\bar{x}_{\text{diff}}, s_{\text{diff}})$. Population parameters μ_{diff} and σ_{diff} unknown.	Inference about μ_{diff}
Two-sample *t* procedure Two *independent* samples (unrelated individuals in the two samples). We summarize each sample separately with $(\bar{x}_1, s_1;\ \bar{x}_2, s_2)$. Population parameters μ_1 , σ_1 , μ_2 , σ_2 unknown.	Inference about $\mu_1 - \mu_2$

Which type of *t* inference procedure?
A: one sample, B: matched pairs, C: two samples ?

- Is blood pressure altered by use of an oral contraceptive? Comparing a sample of women not using an OC with a sample of women taking it.

- Average cholesterol level in general adult population is 175 mg/dl. Take a sample of adults with ‘high cholesterol’ parents. Is the mean cholesterol level higher in this population?

- Does bread lose vitamin with storage? Take a sample of bread loaves and compare vitamin content right after baking and again after 3 days later.

- Does bread lose vitamin with storage? Take a sample of bread loaves just baked and a sample of bread loaves stored for 3 days and compare vitamin content.

Table C

Degrees of freedom	Confidence level C											
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
One-sided P	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
Two-sided P	.50	.40	.30	.20	.10	.05	.04	.02	.01	.005	.002	.001