# Math Stat

## 1 Part I

1.1

a) If $X \sim \text{Bionomial}(n,p)$, then pmf is $\binom{n}{x} p^x (1-p)^{n-x}$

b) The mgf of $X$ is $(\mathrm{pe}^t + q)^n$, then

$E(Z^2) = \frac{d^2(\mathrm{pe}^t + 1 - p)^n}{\mathrm{dt}^2}|_{t=0} = n^2 P^2 - n p^2 + n p$

$\mathrm{Var}(Z) = E(Z^2) - E(Z)^2 = n^2 p^2 - n p^2 + n p - n^2 p^2 = n p (1-p)$

c) The m.g.f of $Y$ is $\exp\{\lambda(e^t - 1)\}$,then

$M_t(X+Y) = M_t(X) \cdot M_t(Y) = \lambda(p e^t + 1 - q)^n (e^t - 1)$

1.2

a) The marginal distribution of $X$ is

$$\begin{aligned} Pr(X=1) &= Pr(1,1) + Pr(1,2) + Pr(1,3) + Pr(1,4) = \frac{1}{4} \\ Pr(X=2) &= Pr(2,2) + Pr(2,3) + Pr(2,4) = \frac{1}{4} \\ Pr(X=3) &= Pr(3,3) + Pr(3,4) = \frac{1}{4} \\ Pr(X=4) &= Pr(4,4) = \frac{1}{4} \end{aligned}$$

b) The pmf of $X+Y$ is

| X+Y | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Probability | 1/16 | 1/16 | 3/16 | 2/16 | 4/16 | 1/16 | 4/16 |

**Table 1.**

1.3

a) Using the point-wise formula, hhe marginal distribution of $X$ is

$f_X(x) = \left\{\int_{S_Y} \frac{f_{(Y|X|)}(y|x|)}{f_{(X|Y|)}(x|y|)} d y\right\}^{-1} = \left\{\int_0^b \frac{x}{y} \exp(y x - x y) \frac{1 - e^{-b y}}{1 - e^{-b x}} d y\right\}^{-1} = \frac{1 - e^{-b x}}{x} \left\{\int_0^b \frac{1 - e^{-b y}}{y} dy\right\}^{-1} =$

b) When $b = +\infty$, $f_X(X)$ don't exsit.

1.4

a) The marginal distributions of $X$ is

$p_1 = \left\{\frac{b_{11}}{a_{11}} + \frac{b_{12}}{a_{12}} + \frac{b_{13}}{a_{13}} + \frac{b_{14}}{a_{14}}\right\}^{-1} = \left\{\frac{7}{6} + \frac{4}{6} + \frac{7}{6} + \frac{7}{6}\right\}^{-1} = \frac{6}{25}$

$p_2 = \left\{\frac{b21}{a_{21}} + \frac{b_{22}}{a_{22}} + \frac{b23}{a_{23}} + \frac{b_{24}}{a_{24}}\right\}^{-1} = \left\{\frac{7}{7} + \frac{4}{7} + \frac{7}{7} + \frac{7}{7}\right\}^{-1} = \frac{7}{25}$

$p_3 = \left\{\frac{b_{31}}{a_{31}} + \frac{b_{32}}{a_{32}} + \frac{b_{33}}{a_{33}} + \frac{b_{34}}{a_{34}}\right\}^{-1} = \left\{\frac{7}{12} + \frac{4}{131112} + \frac{7}{12} + \frac{7}{12}\right\}^{-1} = \frac{1112112}{25}$

$q_1 \propto \frac{b_{11}}{a_{11}} = \frac{7}{6}$, $q_2 \propto \frac{b_{12}}{a_{12}} = \frac{4}{6}$, $q_3 \propto \frac{b_{13}}{a_{13}} = \frac{7}{6}$, $q_4 = \frac{b_{34}}{a_{34}} = \frac{7}{6}$

$\Rightarrow q_{1=}\frac{7}{25}$, $q_2 = \frac{4}{25}$, $q_3 = \frac{7}{25}$, $q_4 = \frac{7}{25}$

b) The joint distribution of $(X, Y)$ is

| $(X,Y)$ | 1 | 2 | 3 |
|---|---|---|---|
| 1 | $42/25^2$ | $49/25^2$ | $84/25^2$ |
| 2 | $24/25^2$ | $28/25^2$ | $48/25^2$ |
| 3 | $42/25^2$ | $49/25^2$ | $84/25^2$ |
| 4 | $42/25^2$ | $49/25^2$ | $84/25^2$ |

**Table 2.**

1.6

a) $\Pr(1/4 < X < 5/8) = \Pr(X < 5/8) - \Pr(X < 1/4) = 1 - 2(1 - 5/8)^2 - 2(\frac{1}{4})^2 = \frac{19}{32}$

b) $f(x) = F'(x) = \begin{cases} 0 & x < 0 \\ 4x & 0 \leqslant x < 1/2 \\ -4(x-1) & 1/2 \leqslant x < 3/4 \\ 0 & 3/4 \leqslant x \end{cases}$

$\mathrm{Var}(X) = E(X^2) - E(X)^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - (\int_{-\infty}^{\infty} x f(x) dx)^2 = \frac{11}{16} - \frac{11^2}{48^2} \cong 0.635$

1.10

# 2 Part II