Chapter 5 Hypothesis Testing

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Here are two important areas of **statistical inference**. The first one is the estimation of **parameters**, and the second is the **testing of hypotheses**. Based on observations from a random sample, statisticians follow a formal process to determine whether or not to reject a null hypothesis(H_0). This process is called **hypothesis testing**.

Here are four steps of hypothesis testing:

- *State the hypothese.* This involes stating the null and alternative by hypotheses.
- *formulate an analysis plan*. The analysis plan describle how to use sample data to evaluate the null hypothesis.
- *analyze sample data.*
- *Interpret results*. Apply the decision rule decribed in the anylysis plan. If the value of the test statistic is unlikely, based on the null hypothesis,reject the null hypothesis.

A *statistical hypothesis* is an assumption about a population parameter. This assumption may or may not be true. A research can conduct a statistical experiment to test the validity of this hypothesis. If the statistical hypothesis specifies the population distribution, it is called *simple hypothesis*. Otherwise, it is called a *composite hypothesis.*

The first, the hypothesis being testing, is called the *null hypothesis*, denoted by H_0. The second is called the *alternative hypothesis*, denoted by H_1 or H_a.

We

1 Type I error and Type II error

Being adjudged to be	The man is crimeless	The man is crime
Guilty(yes)	Type I Error(弃真)	
Guiltess(no)		Type II Error(取伪)

Table 1.

Rejeciton of the null hypothesis H_0 when it is ture is called *Type I error*. The probalibity of making a Type I error is denoted by

$$\begin{aligned}\alpha(\theta) = \Pr(\text{TypeI error}) &= \Pr(\text{rejecting } H_0 | H_o \text{ is true}|)\\ &= \Pr(\boldsymbol{x} \in C | \theta \in \Theta_0|)\end{aligned}$$

which is a function of θ defined in Θ_0. $\alpha(\theta)$ is called the *Type I error function.*

Typically, the Type I error is more serious thant the Tyoep] II error. But we can adjust to H_0 and H_1.

The values of the power function are the probilities of rejecting the null hypothesis H_0 fro variance values of the parameter θ. The power function plays the same role in hypothesis testing as that (MSE) played in estimation. The power function is goldgen standard in assessing the goodness of a test T or in comparing two competing tests T_1 and T_2.

$$\begin{aligned} p(\theta) &= \Pr \quad \theta \in \Theta_0 \\ &= \quad\quad \theta \in \Theta_1 \\ & \quad\quad\quad \theta \notin \Theta_0 \cup \Theta_1 \end{aligned}$$

Fix the probalility of Type I error at preassigned (small) level $a^*()$, then minimize the probability of Type II error. That is , consider the test with

$$\sup_{\theta \in \Theta_0} p(\theta) = \sup_{\theta \in \Theta_0} \alpha(\theta) \leqslant \alpha^*$$

and choose the one with the probability of Type II error $\beta(\theta)$ being minimized. So if $\alpha_{T_1}(\theta)$

2 The Neyman-Pearson Lemma

the most powerful test(MPT)

the uniformly most powerful test(UMPT) 一致最优检验