

1.1

a) If $X \sim \text{Binomial}(n, p)$, then pmf is $\binom{n}{x} p^x (1-p)^{n-x}$

b) The mgf of X is $(pe^t + q)^n$, then

$$E(Z^2) = \frac{d^2(t+1-p)^n}{dt^2} \Big|_{t=0} = n^2 p^2 - np^2 + np$$

$$\text{Var}(Z) = E(Z^2) - E(Z)^2 = n^2 p^2 - np^2 + np - n^2 p^2 = np(1-p)$$

c) The m.g.f of Y is $\exp\{\lambda(e^t - 1)\}$, then

$$M_t(X+Y) = M_t(X) \cdot M_t(Y) = \lambda(pe^t + 1 - q)^n (e^t - 1)$$

1.2

a) The marginal distribution of X is

$$Pr(X=1) = Pr(1,1) + Pr(1,2) + Pr(1,3) + Pr(1,4) = \frac{1}{4}$$

$$Pr(X=2) = Pr(2,2) + Pr(2,3) + Pr(2,4) = \frac{1}{4}$$

$$Pr(X=3) = Pr(3,3) + Pr(3,4) = \frac{1}{4}$$

$$Pr(X=4) = Pr(4,4) = \frac{1}{4}$$

b) The pmf of $X+Y$ is

X+Y	2	3	4	5	6	7	8
Probability	1/16	1/16	3/16	2/16	4/16	1/16	4/16

1.3

a) Using the point-wise formula, the marginal distribution of X is

$$f_X(x) = \left\{ \int_{S_Y} \frac{f_{(Y|X)}(y|x)}{f_{(X|Y)}(x|y)} dy \right\}^{-1} = \left\{ \int_0^b \frac{x}{y} \exp(yx - xy) \frac{1-e^{-by}}{1-e^{-bx}} dy \right\}^{-1} =$$

$$\frac{1-e^{-bx}}{x} \left\{ \int_0^b \frac{1-e^{-by}}{y} dy \right\}^{-1} =$$

b) When $b = +\infty$, $f_X(X)$ doesn't exist.

1.4

a) The marginal distributions of X is

$$p_1 = \left\{ \frac{b_{11}}{a_{11}} + \frac{b_{12}}{a_{12}} + \frac{b_{13}}{a_{13}} + \frac{b_{14}}{a_{14}} \right\}^{-1} = \left\{ \frac{7}{6} + \frac{4}{6} + \frac{7}{6} + \frac{7}{6} \right\}^{-1} = \frac{6}{25}$$

$$p_2 = \left\{ \frac{b_{21}}{a_{21}} + \frac{b_{22}}{a_{22}} + \frac{b_{23}}{a_{23}} + \frac{b_{24}}{a_{24}} \right\}^{-1} = \left\{ \frac{7}{7} + \frac{4}{7} + \frac{7}{7} + \frac{7}{7} \right\}^{-1} = \frac{7}{25}$$

$$p_3 = \left\{ \frac{b_{31}}{a_{31}} + \frac{b_{32}}{a_{32}} + \frac{b_{33}}{a_{33}} + \frac{b_{34}}{a_{34}} \right\}^{-1} = \left\{ \frac{7}{12} + \frac{4}{12} + \frac{7}{12} + \frac{7}{12} \right\}^{-1} = \frac{12}{25}$$

$$q_1 \propto \frac{b_{11}}{a_{11}} = \frac{7}{6}, q_2 \propto \frac{b_{12}}{a_{12}} = \frac{4}{6}, q_3 \propto \frac{b_{13}}{a_{13}} = \frac{7}{6}, q_4 = \frac{b_{34}}{a_{34}} = \frac{7}{6}$$

$$\Rightarrow q_1 = \frac{7}{25}, q_2 = \frac{4}{25}, q_3 = \frac{7}{25}, q_4 = \frac{7}{25}$$

b) The joint distribution of (X, Y) is

(X, Y)	1	2	3
1	$42/25^2$	$49/25^2$	$84/25^2$
2	$24/25^2$	$28/25^2$	$48/25^2$
3	$42/25^2$	$49/25^2$	$84/25^2$
4	$42/25^2$	$49/25^2$	$84/25^2$

1.6

1.10