Mathematical Stastics Assignment 2

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1 Part I

2.1 Calculate the expectation and variance of the $T \sim t(n)$ via the stochastic representation (SR):

$$T \overset{d}{=} \frac{Z}{\sqrt{Y/n}},$$

Where $Z \sim N(0,1), Y \sim \chi^2(n)$ and Z and Y are indepented.**2.7** Let x_1, x_2 be a random sample form the $N(o, \sigma^2)$ population.

Solve:

2.2 Let $X_1, ..., X_n$ are iid obey Beta(3,2). Find the sampling distributions of $X_{(1)} = \min\{X_1, ..., X_n\}$ and $X_d = \max\{X_1, ..., X_n\}$.

2.3 Let

2.4 Let $X_i \sim \text{Gamma}(a_i, 1), i = 1, ...n,$ and $X_1, ..., X_n$ are mutually independent. Difine

$$Y_i = \frac{X_i}{m}$$

(a) Derive the distribution of the statistic

$\underline{X}$

(b) Find the constant k, such that

$$P_r\left\{\frac{(X_1+X_2)^2}{(X_1+X_2)^2+(X_1-X_2)^2} < k\right\} = 00.01$$

Solve:

2.8 Show that if X and Y are independent exponential random variables with $\lambda = 1$, then X/Y follows an F distribution. Also, identify the degrees of freedom.

Prove: