MAT8010 Homework #6

Due Date: June 7, 2018

1. (2 points) Problem 19B, page 219 (Van Lint/Wilson).

2. (4 points) Problem 19E, page 224 (Van Lint/Wilson).

3. (4 points) Let $\mathbf{S} = (\mathcal{P}, \mathcal{B})$ be a $t-(v,k,\lambda)$ design, and let $x_1, x_2, \ldots, x_{t+1}$ be points of $\mathcal{P}$. Suppose that μ blocks of $\mathbf{S}$ contain all these points. Use the PIE to show that the number of blocks containing none of the points $x_1, x_2, \ldots, x_{t+1}$ is $N + (-1)^{t+1}\mu$, where N depends on t, v, k, λ only. Deduce that if t is even and $v = 2k+1$, then $(\mathcal{P} \cup \{y\}, \{B \cup \{y\}, \mathcal{P} \setminus B \mid B \in \mathcal{B})$ is a $(t+1) - (v+1, k+1, \lambda)$-design.

4. (4 points) Let X be a v-set, and let $0 \leq t \leq k \leq v$ be integers. The subset inclusion matrix $W_{tk}(v)$ is a $(0,1)$-matrix whose rows are indexed by the t-subsets T of X and whose columns are indexed by the k-subsets K of X, with the (T,K)-entry being 1 if $T \subseteq K$, and 0 otherwise. Prove that $W_{tk}(v)$ has full rank over $\mathbf{Q}$; that is $\text{rank}_{\mathbf{Q}}(W_{tk}(v)) = \min\{\binom{v}{t}, \binom{v}{k}\}$.

5. (2 points) Problem 19I, page 226/227 (Van Lint/Wilson).

6. (4 points) Let G be a simple graph and let A be the adjacency matrix of G. The eigenvalues of A are called the eigenvalues of G. Also we denote the largest vertex degree of G by $\Delta(G)$. Prove that (i) the eigenvalue of G with the largest absolute value is $\Delta(G)$ if and only if some connected component of G is $\Delta(G)$-regular; (ii) The multiplicity of $\Delta(G)$ as an eigenvalue of G is the number of $\Delta(G)$-regular components of G.