MAT8010 Homework #5

Due Date: May 24, 2018

1. (2 points) Problem 4A, page 38 (Van Lint/Wilson).

2. (2 points) Problem 4H, page 41 (Van Lint/Wilson).

3. (2 points) Let $H_3(r)$ denote the number of 3×3 matrices with nonnegative integer entries such that each row and each column sum to r. Show that

$$H_3(r) = \binom{r+5}{5} - \binom{r+2}{5}.$$

Do this problem by using Theorem 5.5. Do not use the PIE.

4. (2 points) For $n \geq 6$, let $\mathcal{H} \subseteq 2^{[n]}$ be such that for any two distinct $A, B \in \mathcal{H}$, $|A \cap B|$ is even. Prove that $|\mathcal{H}| \leq 2^{n/2}$ if n is even, and $|\mathcal{H}| \leq 1 + 2^{(n-1)/2}$ if n is odd. (Here note that we do not require $|A|$ to be even for $A \in \mathcal{H}$.)

5. (2 points) Suppose that in a town of n residents, every club has an even number of members, and any two distinct clubs have an odd number of members in common. Then the maximum number of clubs is n if n is odd, and $n-1$ if n is even.

6. (2 points) Problem 6C, page 58, (Van Lint/Wilson).