# MAT8010 Homework #6

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1. Show that if an $\mathcal{S}(3,6,v)$ exists, then $v \equiv 2$ or $6 (\mathrm{mod}\ 20)$.

   Prove:

   Calculate $b_i$, $i = 3, 2, 1, 0$; $b_1 \notin \mathbb{Z}$

2. 

3. Let $\mathcal{S} = (\mathcal{P}, \mathcal{B})$ be a $t - (v, k, \lambda)$ design, and let $x_1, x_2, ..., x_{t+1}$ be points of $\mathcal{P}$. Suppose that $\mu$ blocks of $\mathcal{S}$ contain all these points. Use the PIE to show that the number of blocks containing none of points $x_1, x_2, ..., x_{t+1}$ is $N + (-1)^{t+1}\mu$, where $N$ depends on $t, v, k, \lambda$ only. Deduce that if $t$ is even and $v = 2k+1$, then $(\mathcal{P} \cup \{y\}, \{\mathcal{B} \cup \{y\}, \mathcal{P}\langle\backslash\rangle\mathcal{B} | B \in \mathcal{B})$ is a $(t+1) - (v+1, k+1, \lambda)$-design.

   Prove:

4. Let $X$ be a $v$-set, and let $0 \leq t \leqslant k \leqslant v$ be integers. The subset inclusion matrix $W_{tk}(v)$ is a (0, 1)-matrix whose rows are indexed by the $t$-subsets $T$ of $X$ and whose columns are indexed by the $k$-subsets $K$ of $X$, with the $(T, K)$-entry being 1 if $T \subseteq K$, and 0 otherwise. Prove that $W_{tk}(v)$ has full rand over $\boldsymbol{Q}$; that is $\mathrm{rank}_{\boldsymbol{Q}}(W_{tk}(v)) = \min\{\binom{v}{t}, \binom{v}{k}\}$.

   Prove:

5. Let $O$ be a subset of the points of a projective plane of order $n$ such that no three points of $O$ are on one line. Show that $|O| \leqslant n+1$ if $n$ is odd and that $|O| \leq n+2$ if $n$ is even. A set of $n+1$ points, no three on a line, is called an oval; a set of $n+2$ points, no three on a line, is a hyperoval. Two constructions of $PG_2(4)$ were given in Example 19.7 and Problem 19F. In each case, construct a hyperoval.

   Prove:

6. Let $G$ be simple graph and let $A$ be the adjacency matrix of $G$. The eigenvalues of $A$ are called the eigenvalues of $G$. Also we denote the largest vertex degree of $G$ by $\Delta(G)$. Prove that (i) the eigenvalue of $G$ with the largest absolute value is $\Delta(G)$ if and only if some connected component of $G$ is $\Delta(G)$-regular; (ii) The multipicity of $\Delta(G)$ as an eigenvalue of $G$ is the number of $\Delta(G)$-regular components of $G$.

   Prove:

   (i)

   (ii)