

## MAT8010 Homework #4

Due Date: April 17, 2018

1. (2 points) Problem 15C, page 154 (Van Lint/Wilson).
2. (2 points) Find a bijection proof of the fact that  $u(n) = o(n)$ , where  $u(n)$  is the number of partitions of  $n$  with unequal parts, and  $o(n)$  is the number of partitions of  $n$  with odd parts.
3. (2 points) Problem 1F, page 9 (Van Lint/Wilson).
4. (2 points) Problem 1H, page 9. (Supplement to Problem 1H: Assume that  $G$  is a simple graph on the vertex set  $\{1, 2, \dots, n\}$ . Let  $A$  be an adjacency matrix of  $G$  as defined in Problem 1H. Show that  $\text{trace}(A^2)$  equals twice the number of edges of  $G$ , and  $\text{trace}(A^3)$  is six times the number of triangles in  $G$ .)
5. (2 points) Let  $A_1, A_2, \dots, A_{1066}$  be subsets of a finite set  $X \neq \emptyset$  such that  $|A_i| > |X|/2$  for all  $i$ ,  $1 \leq i \leq 1066$ . Prove that there exist ten elements  $x_1, x_2, \dots, x_{10}$  (not necessarily distinct) of  $X$  such that every  $A_i$  contains at least one of  $x_1, x_2, \dots, x_{10}$ .
6. (2 points) Problem 3C, page 29, (Van Lint/Wilson).