MAT8010 Homework #2

Due Date: March 20, 2018

1. (3 points) Let $H_3(r)$ denote the number of 3×3 matrices with nonnegative integer entries such that each row and each column sum to r. Show that

$$H_3(r) = \binom{r+5}{5} - \binom{r+2}{5}.$$

2. (4 points) Let n and r be positive integers. A function $f : [n] \to [r]$ is called *monotone* if $x \leq y$ implies $f(x) \leq f(y)$, for all $x, y \in [n]$.

- Prove that the number of monotone surjections from $[n]$ to $[r]$ is $\binom{n-1}{r-1}$.
- Count monotone surjections from $[n]$ to $[r]$ by the PIE to prove that

$$\binom{n-1}{r-1} = \sum_{k=0}^{r-1} (-1)^k \binom{r}{k} \binom{r+n-k-1}{n}.$$

3. (4 points) 10F (Van Lint/Wilson, page 96) The second part "Can you prove this identity directly?" is mandatory.

4. (2 points) 10G (Van Lint/Wilson, page 96)

5. (2 points) Let k, n be fixed positive integers. Show that

$$\sum c_1 c_2 \cdots c_k = \binom{n+k-1}{2k-1},$$

where the sum ranges over all compositions $c_1 + c_2 + \cdots + c_k$ of n into k parts.