MAT8010 Homeworks #7

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1(19H). Let $\mathcal{D}$ be a $3-(v,k,\lambda)$ design. Suppose that the derived design of $\mathcal{D}$ with respect to a point p is a symmetric design.

(1). Show that $\lambda(v-2)=(k-1)(k-2)$

Proof.

□

(2). Show that any two blocks of $\mathcal{D}$ meet in 0 or $\lambda+1$ points

(3). Show that the set of points not on a block B together with the blocks disjoint form B from a 2-design $\mathcal{D}^B$.

(4). Apply Fisher's inequality to the design $\mathcal{D}^B$ and deduce that $v=2k$ or otherwise $k=(\lambda+1)(\lambda+2)$ or $k=2(\lambda+1)(\lambda+2)$.

2. Let Γ be an srg(v, k, λ, μ) and let $-s$ be its smallest eigenvalue. If C is a coclique (independent set) of Γ, then $|C| \leqslant sv/(k+s)$, equality holds if and only if every vertex x of Γ not in C has exactly s neighbors in C.

3(21Q). Prove the so-called *Friendship Theorem:* At a party with n people ($n>3$) every two persons have exactly one mutual friend. Then there is a unique person at the party who is a friend of all the others. Use problem 1J.

Proof.

(1)This problem equal to following statement:

G is a finite graph in which any two vertices have precisely one common neighbor. Then there is a vertex which is adjacent to all the other vertices.

From Problem 1J, we know that

□

Let G be a simple graph on n vertices ($n>3$) with no vertex of degree $n-1$. Suppose that for any two vertices of G there is a unique vertex joined to both of them.