MAT8010 Homework #3

Due Date: April 3, 2018

1. (3 points) Find a closed form expression for each of the following generating functions:

(a) $\sum_{n=0}^{\infty}(n+2)^2 x^n$,

(b) $\sum_{n=0}^{\infty}(n+2)^2 \frac{x^n}{n!}$,

(c) $\sum_{n=0}^{\infty}(n+2)^2 \binom{2n}{n} x^n$.

2. (2 points) Problem 14K, page 149 (Van Lint/Wilson).

3. (2 points) Let n be a positive integer, q a prime power, and let $f_2(n,q)$=the number of co-prime pairs of monic polynomials of degree n over GF(q). Find a simple formula for $f_2(n,q)$.

4. (3 points) An L-tile is a 2×2 square with the upper right 1×1 square removed; no rotations are allowed. Let a_n be the number of tilings of a $4 \times n$ rectangle using tiles that are either 1×1 squares or L-tiles. Find a closed form for the generating function $1 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$ (i.e., write the generating function as a rational function).

5. For positive integers m and n, let $f(m,n)$ denote the number of n-tuples $(x_1, x_2, \ldots, x_n)$ of integers such that

$$|x_1| + |x_2| + \cdots + |x_n| \leq m.$$

(a). (2 points) Find a recurrence formula for $f(m,n)$.

(b). (1 point) Use the recurrence formula to prove that $f(m,n) = f(n,m)$.

(c). (1 point) Do part (b) by using the generating function method.

(d). (2 points) Find a closed formula for $\sum_{n \geq 0} f(n,n) x^n$.