Statistical Mechanics II

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Lecture 7 Degenerated Fermi Gas

Outline

- General results for quantum gas
- Degenerated Fermi gas at zero temperature
- Degenerated Fermi gas at nonzero temperature
- Metal, insulator and semiconductor

The most probable distribution for Fermions

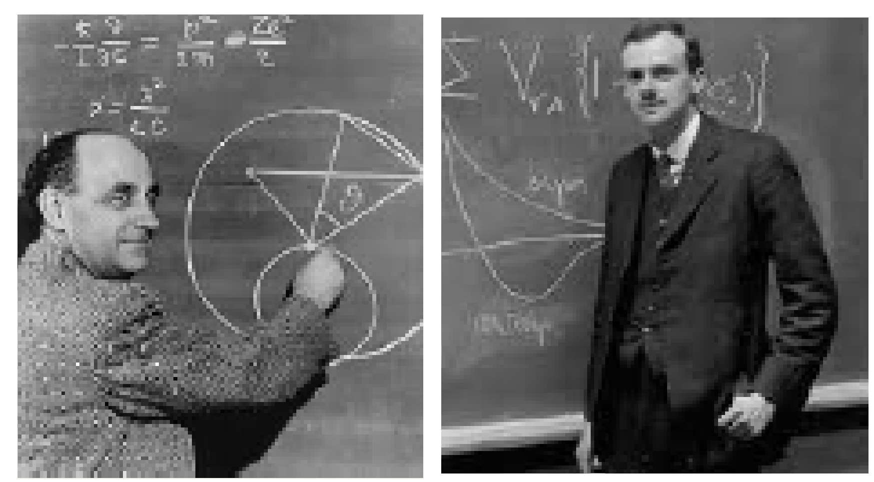
Total # of microstate for distribution $\{n_\lambda\}$ is

$$W_F(\{n_\lambda\}) = \prod_\lambda \begin{pmatrix} g_\lambda \\ n_\lambda \end{pmatrix} = \prod_\lambda \frac{g_\lambda!}{n_\lambda!\,(g_\lambda - n_\lambda)!}$$

$$\begin{aligned}
\mathcal{L} &= \ln W_F - \alpha\left(\sum_\lambda n_\lambda - N\right) - \beta\left(\sum_\lambda \varepsilon_\lambda n_\lambda - E\right) \\
&= \sum_\lambda \ln \frac{g_\lambda!}{n_\lambda!\,(g_\lambda - n_\lambda)!} - \alpha\left(\sum_\lambda n_\lambda - N\right) - \beta\left(\sum_\lambda \varepsilon_\lambda n_\lambda - E\right) \\
&= \sum_\lambda g_\lambda \ln g_\lambda - \sum_\lambda n_\lambda \ln n_\lambda - \sum_\lambda (g_\lambda - n_\lambda)\ln(g_\lambda - n_\lambda) \\
&\quad - \alpha\left(\sum_\lambda n_\lambda - N\right) - \beta\left(\sum_\lambda \varepsilon_\lambda n_\lambda - E\right)
\end{aligned}$$

$$0 = \frac{\partial \mathcal{L}}{\partial n_\lambda} = -\ln n_\lambda + 1 + \ln(g_\lambda - n_\lambda) + 1 - \alpha - \beta\varepsilon_\lambda$$

$$n_\lambda = \frac{g_\lambda}{e^{\alpha + \beta\varepsilon_\lambda} + 1}$$



Fermi-Dirac distribution

General results for degenerated quantum gas

Occupation number

$$z \equiv e^{\beta \mu}$$

$$n_{\vec{k}} = \frac{1}{e^{\beta\left(\varepsilon\left(\vec{k}\right)-\mu\right)} \mp 1}$$

$$\varepsilon\left(\vec{k}\right) = \frac{\hbar^2 k^2}{2m}$$

Occupation number

$$N = \sum_{\vec{k}} n_{\vec{k}} = \frac{V}{(2\pi)^3} \int n_{\vec{k}} d^3\vec{k} = \frac{V}{(2\pi)^3} \int d^3\vec{k} \frac{1}{e^{\beta\left(\varepsilon\left(\vec{k}\right)-\mu\right)} \mp 1}$$

$$n = \frac{N}{V} = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{e^{\beta(\varepsilon_k - \mu) \mp 1}}$$

$$E = \sum_{\vec{k}} \varepsilon_{\vec{k}} n_{\vec{k}}$$

$$\varepsilon = \frac{E}{V} = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\varepsilon\left(\vec{k}\right)}{e^{\beta(\varepsilon_k - \mu)\mp 1}} = \frac{V}{(2\pi)^3} \int d^3\vec{k} \frac{\varepsilon\left(\vec{k}\right)}{e^{\beta\left(\varepsilon\left(\vec{k}\right)-\mu\right)} \mp 1}$$

$$P = \frac{F}{A} = \frac{\overline{\Delta\left(mv_x\right)/\Delta t}}{A} = \frac{\overline{2mv_x}}{2LA/v_x} = \frac{\overline{mv_x^2}}{V} = \frac{1}{3}\overline{mv^2}\frac{1}{V} = \frac{2}{3}\overline{\frac{1}{2}mv^2}\frac{1}{V}$$

$$= \frac{1}{2}\int 2mv_x^2 f\left(\vec{q}, p, t\right) d^3p = \frac{2}{3}\int \frac{1}{2}mv^2 f\left(\vec{q}, p, t\right) d^3p$$

$$= \frac{2}{3}\int \frac{d^3\vec{k}}{(2\pi)^3} \varepsilon_{\vec{k}} f\left(\vec{p}, t\right) \frac{1}{V} = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\varepsilon$$

$$n = \frac{N}{V} = g \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{z \exp\left(\frac{\beta \hbar^2 k^2}{2m}\right) \mp 1}$$

$$\varepsilon = \frac{E}{V} = g \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{z^{-1} \exp\left(\frac{\beta \hbar^2 k^2}{2m}\right) \mp 1}$$

$$P = \frac{2}{3}\varepsilon$$

$$x \equiv \frac{\beta \hbar^2 k^2}{2m} \Rightarrow \frac{\hbar^2 k^2}{2m} = \frac{x}{\beta} \Rightarrow k = \sqrt{\frac{2mx}{\beta\hbar^2}} \Rightarrow dk = \frac{1}{2}\sqrt{\frac{2m}{\beta\hbar^2}}\frac{1}{\sqrt{x}}$$

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}} \qquad d^3k = 4\pi k^2 dk$$

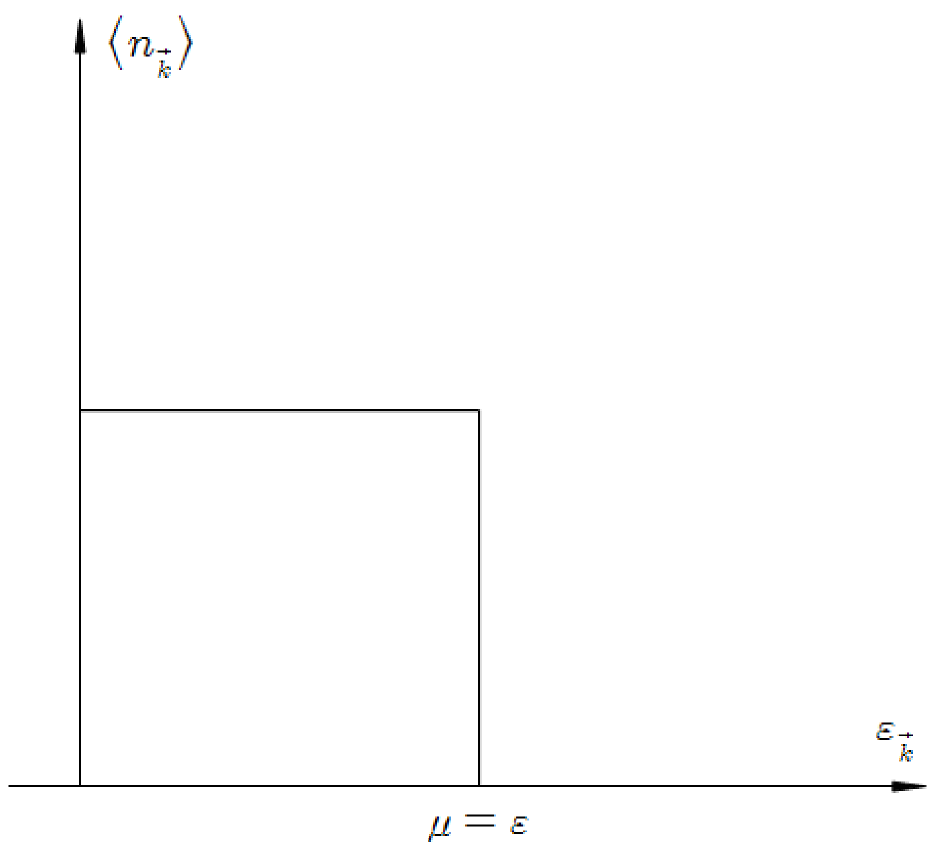
$$f_m^{\pm}(z) = \frac{1}{(m-1)!}\int_0^\infty \frac{x^{m-1}dx}{z^{-1}e^x \mp 1}$$

$$\begin{cases} n = \frac{g}{\lambda^3}\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{x^{\frac{1}{2}}\,dx}{z^{-1}e^x \mp 1} \\ \beta\varepsilon = \frac{g}{\lambda^3}\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{x^{\frac{3}{2}}\,dx}{z^{-1}e^x \mp 1} \end{cases} \longrightarrow \begin{cases} n = \frac{g}{\lambda^3} f^{\pm}_{\frac{3}{2}}(z) \\ \beta\varepsilon = \frac{3g}{2\lambda^3} f^{\pm}_{\frac{5}{2}}(z) \\ \beta P = \frac{2}{3}\varepsilon = \frac{g}{\lambda^3} f^{\pm}_{\frac{5}{2}}(z) \end{cases}$$

Occupation number at zero temperature

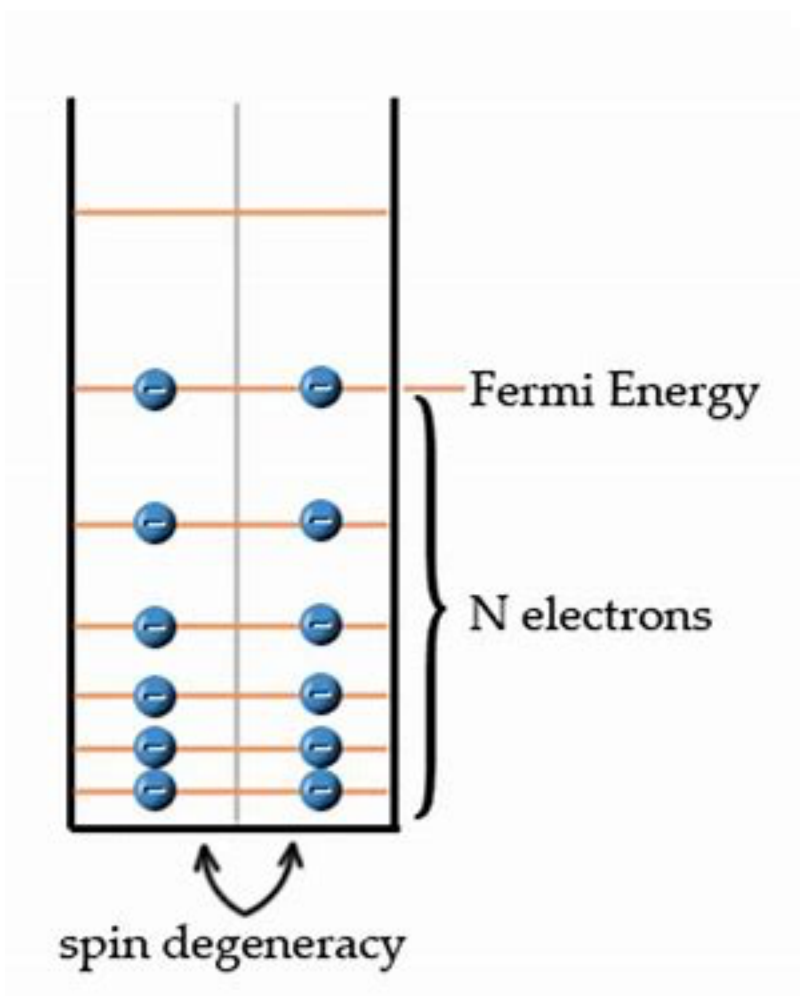
$$\langle n_{\vec{k}} \rangle = \frac{1}{e^{\beta\,\varepsilon\left(\vec{k}\right)-\mu}+1}$$

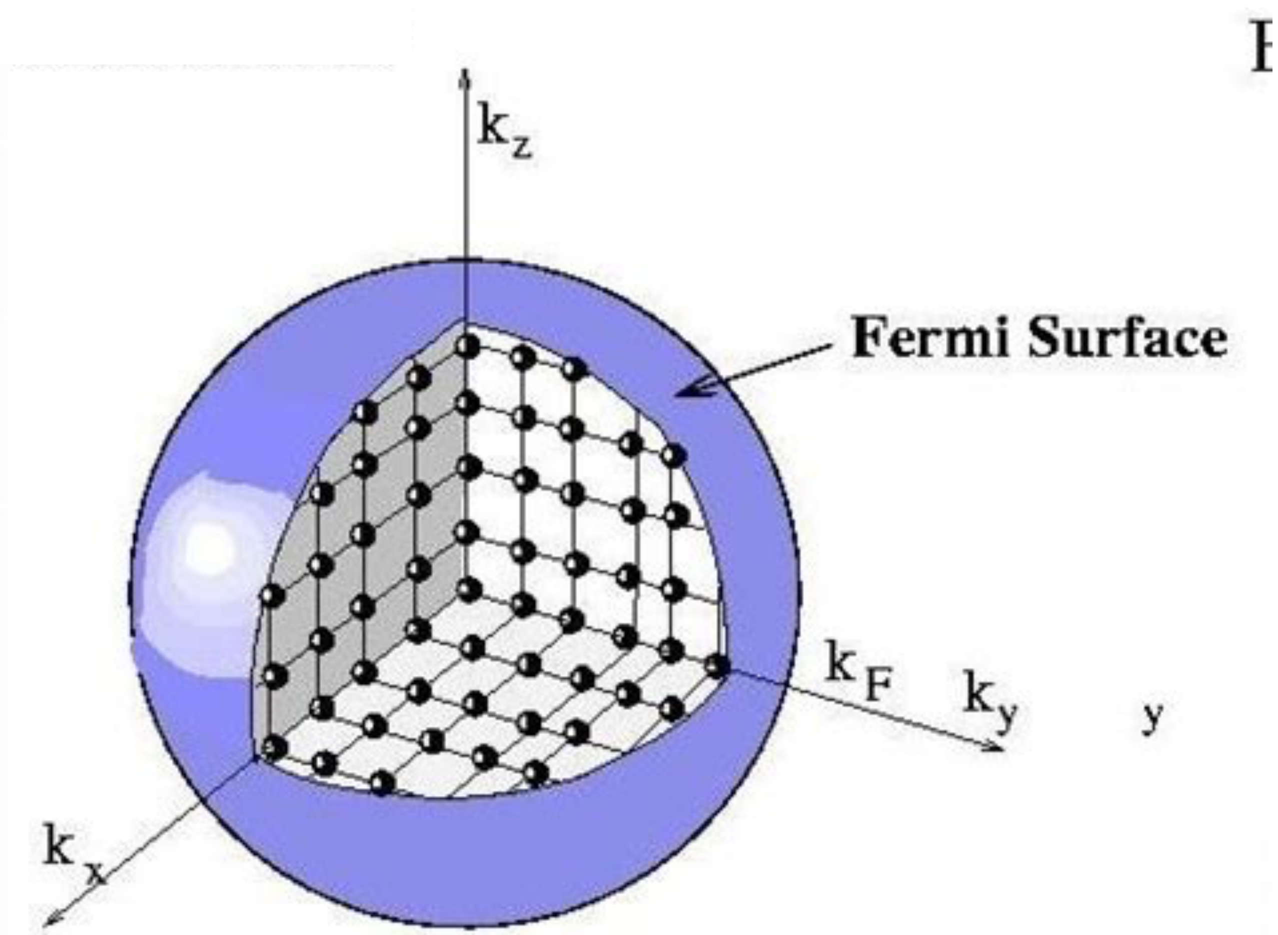
T = 0 Case



$$\begin{cases} \varepsilon\left(\vec{k}\right) < \varepsilon_F & n_{\vec{k}} = 1 \\ \varepsilon\left(\vec{k}\right) > \varepsilon_F & n_{\vec{k}} = 0 \end{cases}$$

Fermi surface





$$
\begin{aligned}
N &= \sum_{|\vec{k}|\leqslant k_F, \varepsilon_{\vec{k}}\leqslant \varepsilon_F} (2S+1) = gV\int^{|\vec{k}|\leqslant k_F} \frac{d^3\vec{k}}{(2\pi)^3} \\
&= gV\frac{4\pi k_F^3}{3}\frac{1}{8\pi^3} = g\frac{V}{6\pi^2}k_F^3 \qquad\qquad S=\frac{1}{2}, \qquad g=2
\end{aligned}
$$

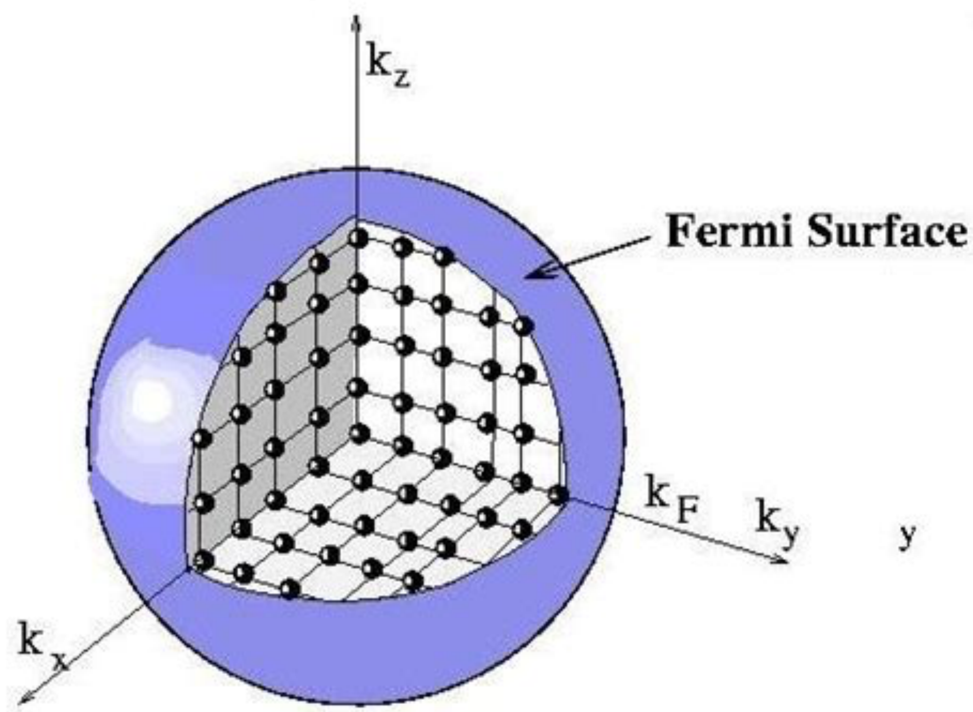
$$
n = \frac{N}{V} = \frac{g}{6\pi^2}k_F^3
$$

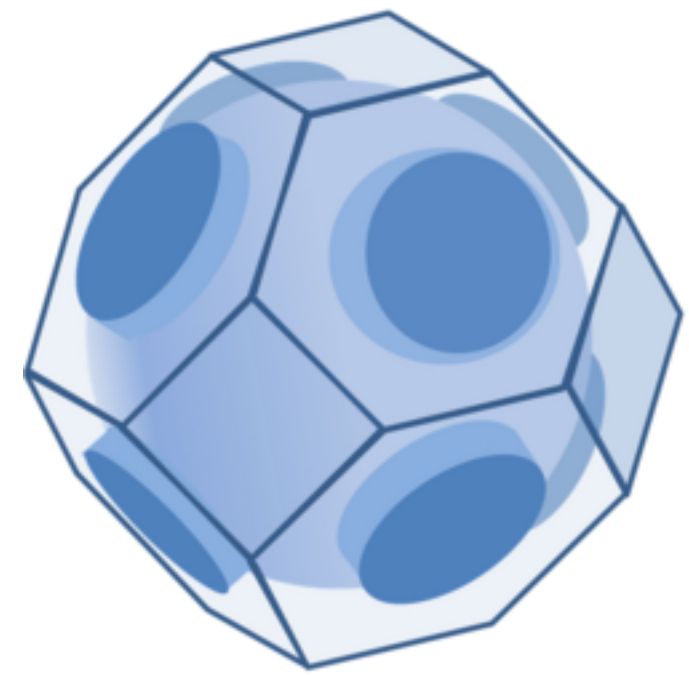
$$
k_F = \left(\frac{6\pi^2 n}{g}\right)^{\frac{1}{3}} \Longrightarrow \varepsilon_F(n) = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{6\pi^2 n}{g}\right)^{\frac{2}{3}}
$$

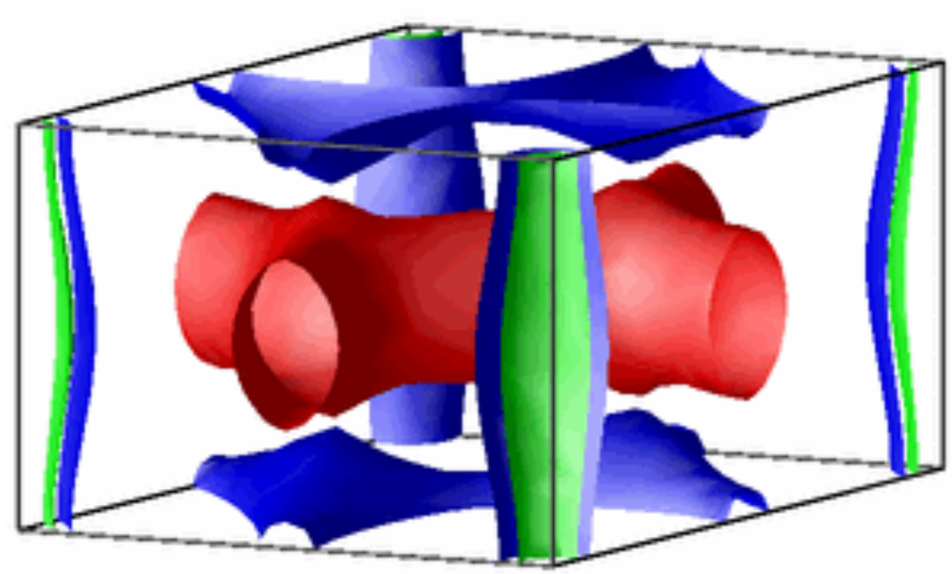
In QM, $T=0$, the system only has one ground state $\Omega = 1$.

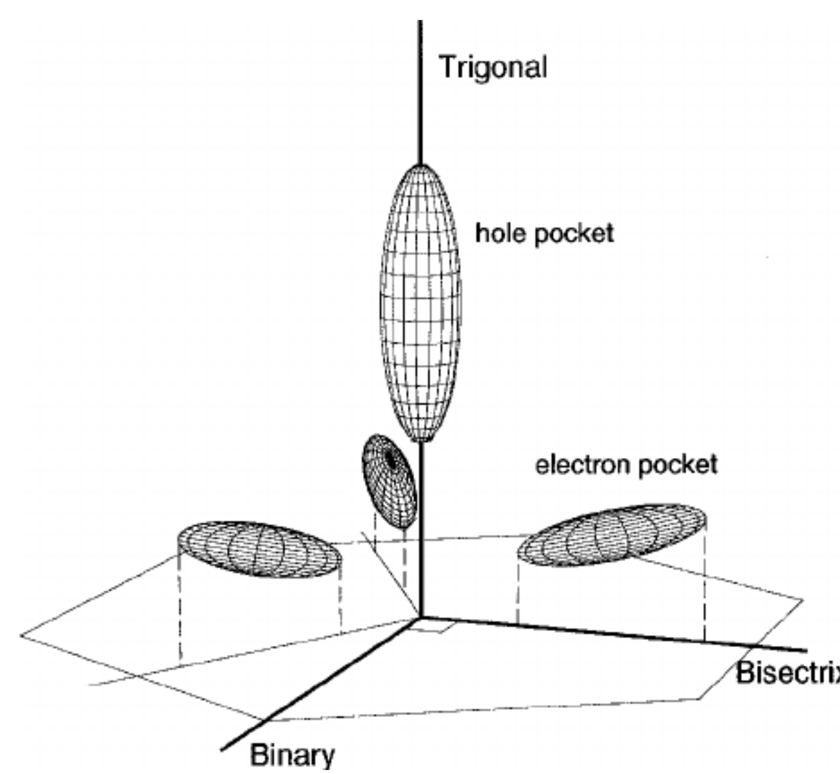
While in CM, $\Omega_{Classical} \propto \frac{V^N}{N!}$.

Fermi surface in solids

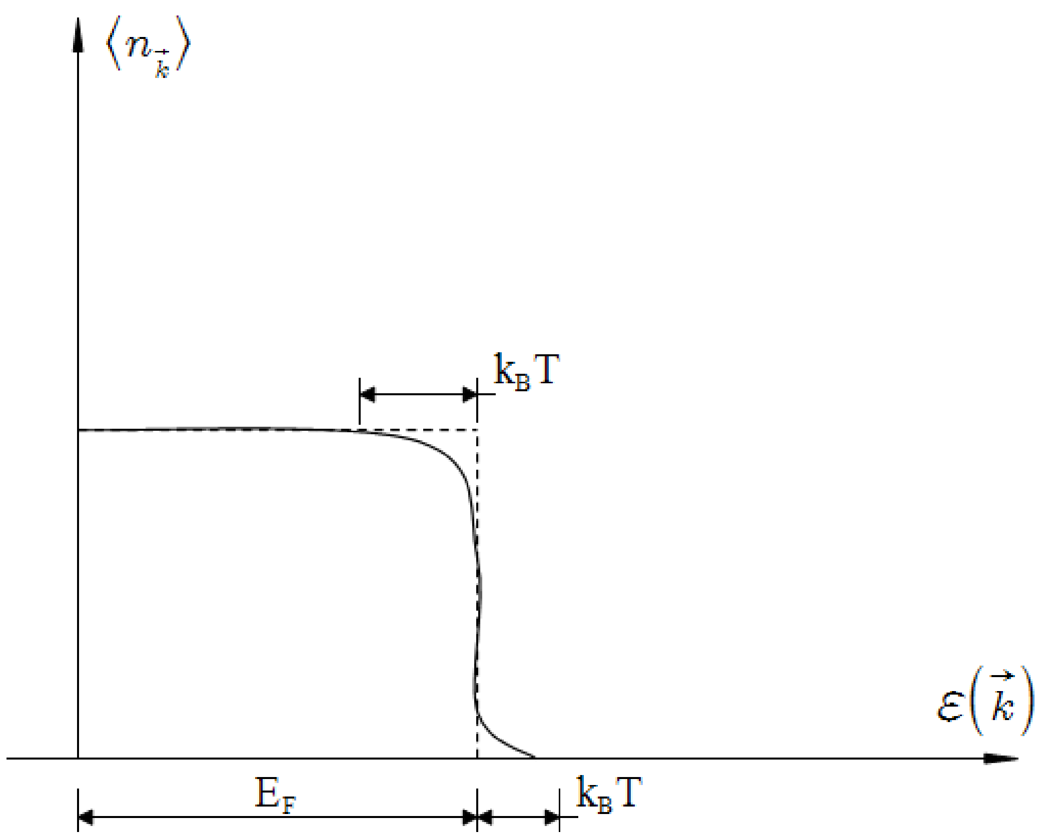








Occupation number at nonzero temperature



$$n \;= \frac{N}{V} = \frac{g}{\lambda^3} f^{-}_{\frac{2}{3}}(z) \qquad z = e^{\beta\mu}$$

$$f^{-}_{m}(z) = \frac{1}{(m-1)!}\int dx \frac{x^{m-1}}{z^{-1}e^{x}+1}$$

$$\begin{cases} n = \frac{g}{\lambda^3} f^{-}_{\frac{2}{3}}(z) \\ \beta P = \frac{g}{\lambda^3} f^{-}_{\frac{5}{2}}(z) = \frac{2}{3}\beta\,\varepsilon, \\ \varepsilon = \frac{3}{2} P = \frac{1}{\beta}\frac{3g}{2\lambda^3} f^{-}_{\frac{5}{2}}(z) \end{cases}$$

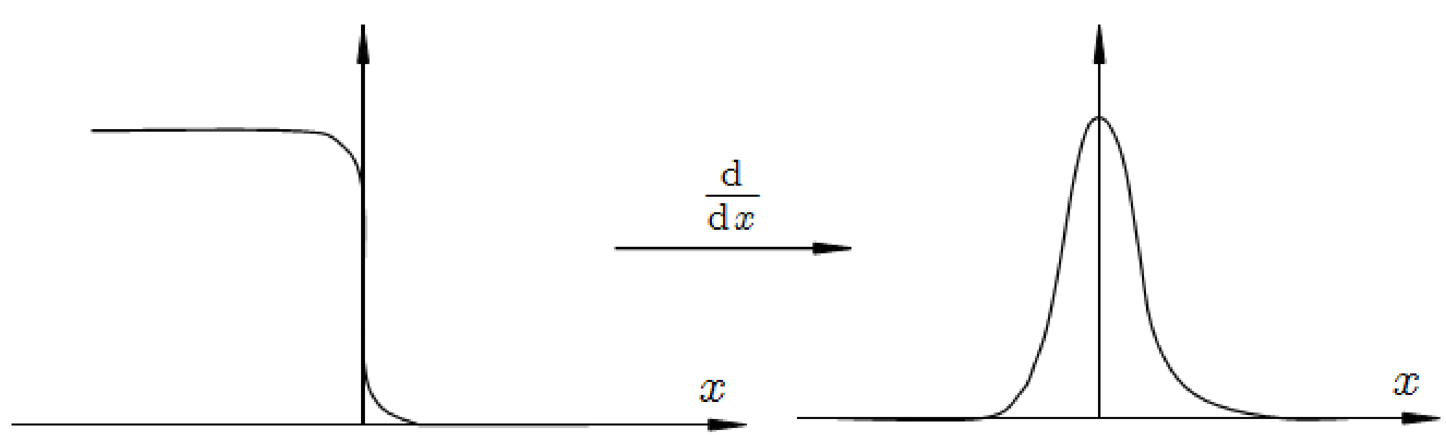
Properties of $f_m^-(\mathcal{Z})$

For $T \neq 0$ but is still very low, $\mathcal{Z} \to \infty$.

$$f_m^-(\mathcal{Z}) = \frac{1}{(m-1)!}\int_0^\infty dx \frac{x^{m-1}}{\mathcal{Z}^{-1}e^x + 1} = \frac{1}{m(m-1)!}\int_0^\infty \frac{dx^m}{\mathcal{Z}^{-1}e^x + 1}$$

$$= \frac{1}{m!}\left[\frac{x^m}{\mathcal{Z}^{-1}e^x + 1}\bigg|_{x=0}^{\infty} + \int_0^\infty \frac{x^m e^x \mathcal{Z}^{-1} dx}{(\mathcal{Z}^{-1}e^x + 1)^2}\right]$$

$$= \frac{1}{m!}\int_0^\infty x^m \frac{d}{dx}\left(\frac{-1}{\mathcal{Z}^{-1}e^x + 1}\right) dx$$



So we can expand, $x = \ln \mathcal{Z} + t, \quad t \in (-\infty, +\infty)$

$$\mathcal{Z}^{-1} e^{x} + 1 \quad = \quad \mathcal{Z}^{-1} e^{\ln \mathcal{Z} + t} + 1 \quad = \quad e^{t} + 1$$

$$f_{m}^{-}(\mathcal{Z}) \cong \frac{1}{m!} \int_{-\infty}^{+\infty} dt \, (\ln \mathcal{Z} + t)^{m} \frac{d}{dt} \left(\frac{-1}{e^{t} + 1} \right)$$

$$= \frac{1}{m!} \int_{-\infty}^{+\infty} dt \sum_{\alpha=0}^{\infty} \left(\begin{array}{c} m \\ \alpha \end{array} \right) t^{\alpha} (\ln \mathcal{Z})^{m-\alpha} \frac{d}{dt} \left(\frac{-1}{e^{t} + 1} \right)$$

$$= \frac{1}{m!} \sum_{\alpha=0}^{\infty} \frac{m!}{\alpha! \, (m - \alpha)!} (\ln \mathcal{Z})^{-\alpha} \int_{-\infty}^{+\infty} t^{\alpha} \frac{d}{dt} \left(\frac{-1}{e^{t} + 1} \right) dt$$

$\frac{e^{t}}{(e^{t}+1)^{2}}$ Symmetry $t \longrightarrow -t$

$$\frac{1}{\alpha!}\int_{-\infty}^{+\infty} t^{\alpha}\frac{d}{dt}\left(\frac{-1}{e^{t}+1}\right)=\begin{cases}0 & \text{for } \alpha \text{ odd}\\ \frac{2}{(\alpha-1)!}\int_0^{\infty} dt\,\frac{t^{\alpha-1}}{e^{t}+1}\equiv 2f_{\alpha}^{-}(1) & \text{for } \alpha \text{ even}\end{cases}$$

$$\lim_{\mathcal{Z}\to\infty} f_{m}^{-}(\mathcal{Z})=\frac{(\ln\mathcal{Z})^{m}}{m!}\sum_{\alpha=0}^{\text{even}} 2f_{\alpha}^{-}(1)\frac{m!}{(m-\alpha)!}(\ln\mathcal{Z})^{-\alpha}$$

$$=\frac{(\ln\mathcal{Z})^{m}}{m!}\left[1+\frac{\pi^2}{6}\frac{m(m-1)}{(\ln\mathcal{Z})^{2}}+\frac{7\pi^4}{360}\frac{m(m-1)(m-2)(m-3)}{(\ln\mathcal{Z})^{4}}\right]$$

In the degenerate limit $\mathcal{Z}\longrightarrow\infty$

$$\frac{n\lambda^3}{g}=f_{\frac{3}{2}}^{-}(\mathcal{Z})=\frac{(\ln\mathcal{Z})^{\frac{3}{2}}}{(3/2)!}\left[1+\frac{\pi^2}{6}\frac{3}{2}\frac{1}{2}(\ln\mathcal{Z})^{-2}+\cdots\right]$$

$$\frac{n\lambda^3}{g} = \frac{(\ln \mathcal{Z})^{\frac{3}{2}}}{(3/2)!} = \frac{(\ln \mathcal{Z})^{\frac{3}{2}}}{\frac{3}{4}\sqrt{\pi}}$$

$$\ln \mathcal{Z} = \left(\frac{4}{3\sqrt{\pi}}\frac{n\lambda^3}{g}\right)^{\frac{2}{3}} = \left(\frac{6\pi^2 n}{g}\right)^{\frac{2}{3}} \frac{\beta\hbar^2}{2m} = \frac{\beta\hbar^2 k_F^2}{2m} = \beta\,\varepsilon_F$$

$$(T \to 0)$$

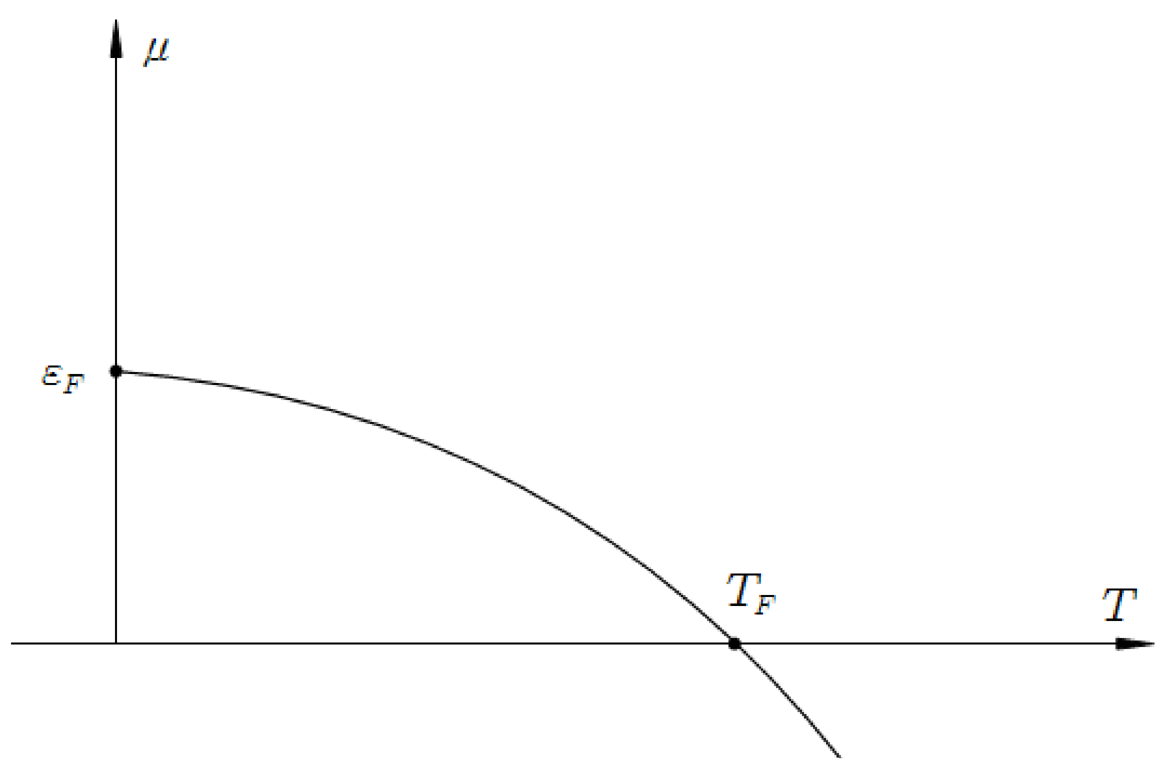
T small but not zero

$$\frac{n\lambda^3}{g} = \frac{(\ln \mathcal{Z})^{\frac{3}{2}}}{(3/2)!}\left[1 + \frac{\pi^2}{6}\frac{3}{2}\frac{1}{2}(\ln \mathcal{Z})^{-2}\right] = \frac{(\ln \mathcal{Z})^{\frac{3}{2}}}{(3/2)!}\left[1 + \frac{\pi^2}{8}(\ln \mathcal{Z})^{-2}\right]$$

$$\cong \frac{(\ln \mathcal{Z})^{\frac{3}{2}}}{(3/2)!}\left[1 + \frac{\pi^2}{8}\left(\frac{1}{\beta\,\varepsilon_F}\right)^2\right]$$

$$\ln \mathcal{Z} = \left(\frac{3}{2}\right)! \left(\frac{n\lambda^3}{g}\right)^{\frac{2}{3}} \left[1 + \frac{\pi^2}{8}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]^{-\frac{2}{3}} = \beta\,\varepsilon_F \left[1 + \frac{\pi^2}{8}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]^{-\frac{2}{3}}$$

$$= \beta\,\epsilon_F \left[1 + \frac{\pi^2}{8}\left(-\frac{2}{3}\right)\left(\frac{k_B T}{\varepsilon_F}\right)^2\right] = \beta\,\varepsilon_F \left[1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]$$

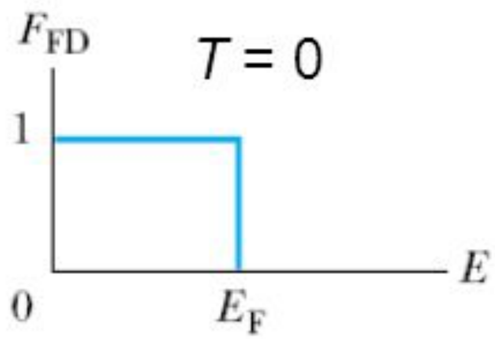


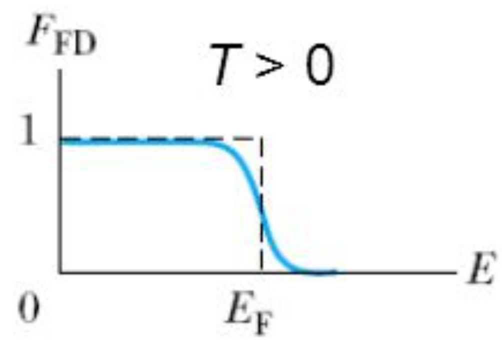
$\mu = k_B T \ln \mathcal{Z} > 0$ at low T,

$\mu = k_B T \ln \mathcal{Z} < 0$ at high T.

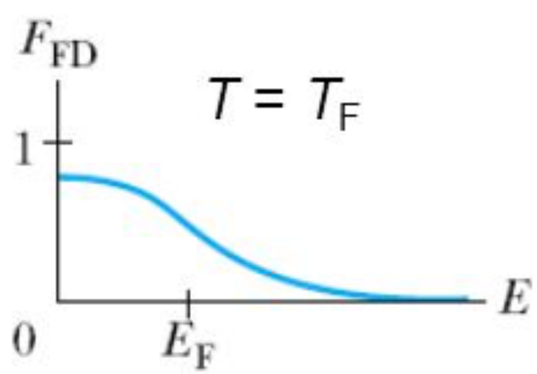
Change sign at $T \approx T_F = \frac{\varepsilon_F}{k_B}$.

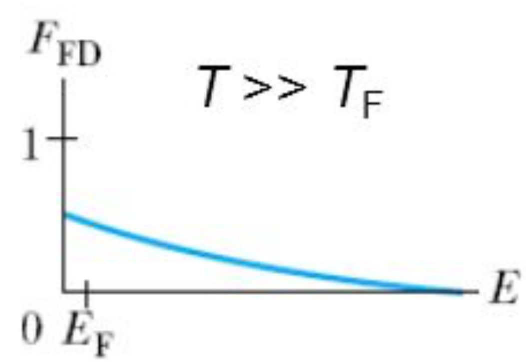
Fermi-Dirac Statistics





- As the temperature increases from $T = 0$, the Fermi-Dirac factor "smears out".
- **Fermi temperature**, defined as $T_F \equiv E_F / k$.   .





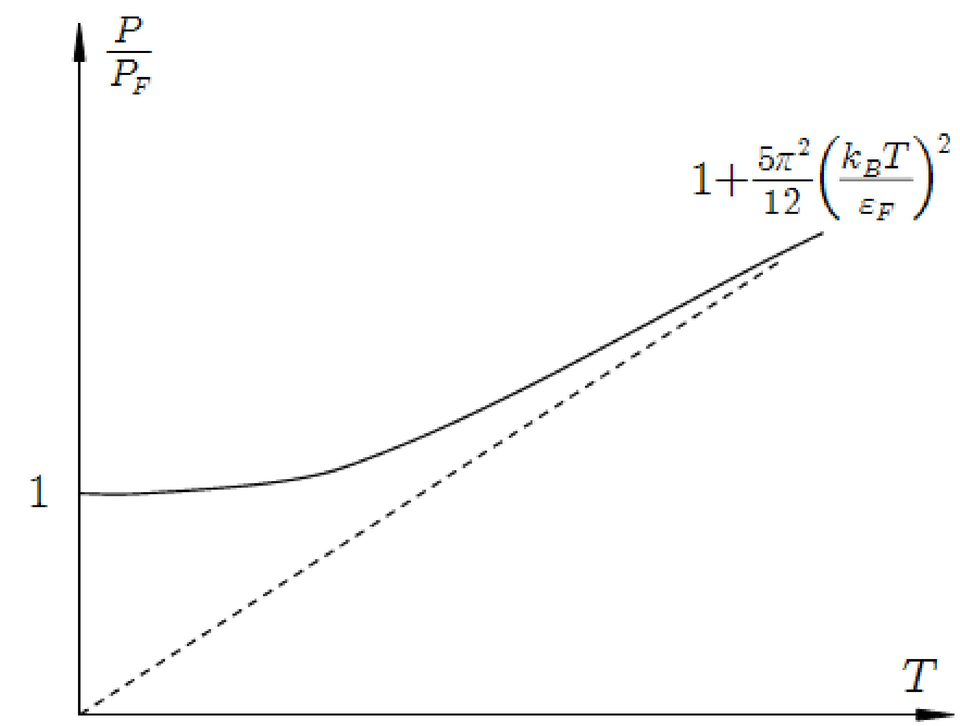
- When $T >> T_F$, F_{FD} approaches a decaying exponential.

Pressure

$$\beta P = \frac{g}{\lambda^3} f^{-}_{\frac{5}{2}}(z) = \frac{g}{\lambda^3}\frac{(\ln z)^{\frac{5}{2}}}{(5/2)!}\left[1 + \frac{\pi^2}{6}\frac{5}{2}\frac{3}{2}(\ln z)^{-2}\right]$$

$$= \frac{g}{\lambda^3}\frac{(\beta\varepsilon_F)^{\frac{5}{2}}}{(5/2)!}\left[1 + \frac{5\pi^2}{8}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]\left[1 - \frac{\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]^{\frac{5}{2}}$$

$$= \beta P_F\left[1 + \frac{5\pi^2}{12}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right]$$



Energy vs heat capacity

$$P_F = \frac{2}{3}\frac{E}{V} \xrightarrow{T=0} \frac{1}{V}\sum_{|\vec{k}|\leqslant k_F} g = g\frac{V}{V}\int_{k<k_F} \frac{d^3\vec{k}}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}$$

$$= \frac{g\hbar^2}{2\pi^3 2m}4\pi\int_0^{k_F} k^2 k^2 dk = \frac{g\hbar^2}{4\pi^2 m}\frac{1}{5}k_F^5$$

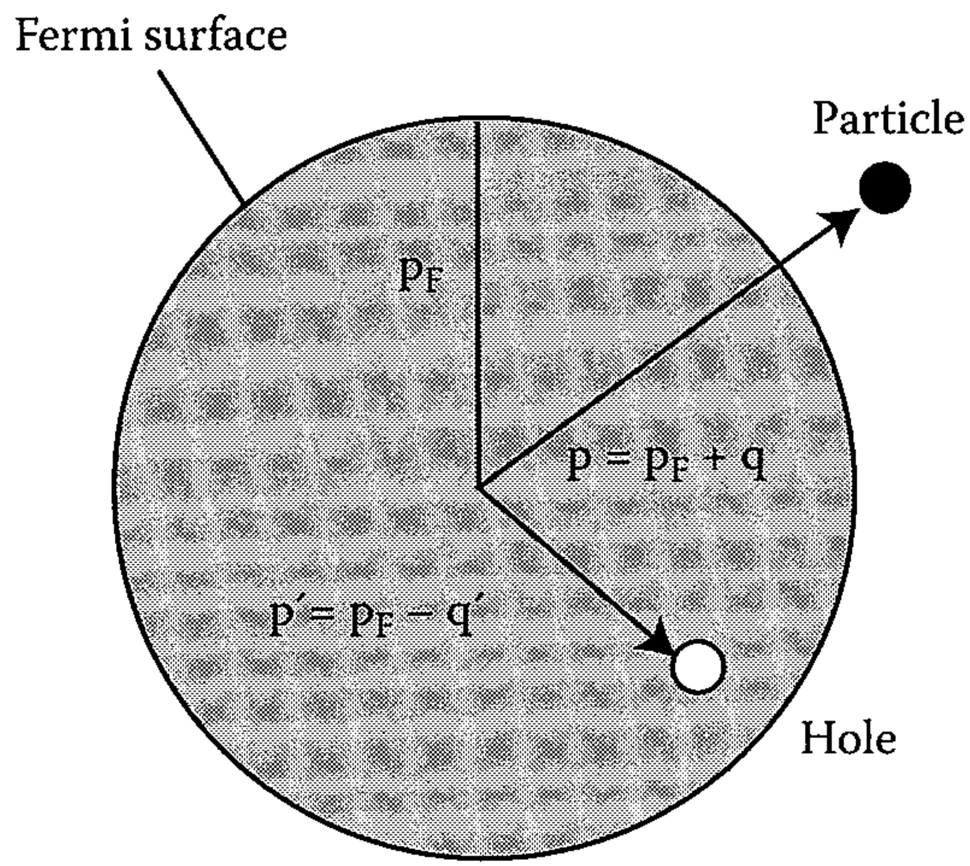
$$= \left(\frac{g k_F^3}{6\pi^2}\right)\frac{3\hbar^2}{2m\cdot 5}k_F^2 = n\frac{3}{5}\left(\frac{\hbar^2 k_F^2}{2m}\right) = \frac{3}{5}n\varepsilon_F$$

$$\frac{E}{V} = \frac{3}{2}P = \frac{3}{2}\frac{2}{5}n\varepsilon_F\left[1 + \frac{5\pi^2}{12}\left(\frac{T}{T_F}\right)^2 + \cdots\right]$$

$$C_V = \frac{dE}{dT} = \frac{d}{dT}\left[\frac{5}{12}\pi^2\left(\frac{T}{T_F}\right)^2\right]\frac{3}{5}n\varepsilon_F = \frac{\pi^2}{2}N k_B\left(\frac{T}{T_F}\right)$$

Why?

Particles and holes



The absence of a fermion of energy ϵ, momentum $\mathbf{p}$, charge e, corresponds to the presence of a hole with

$$\text{Energy} = -\epsilon$$

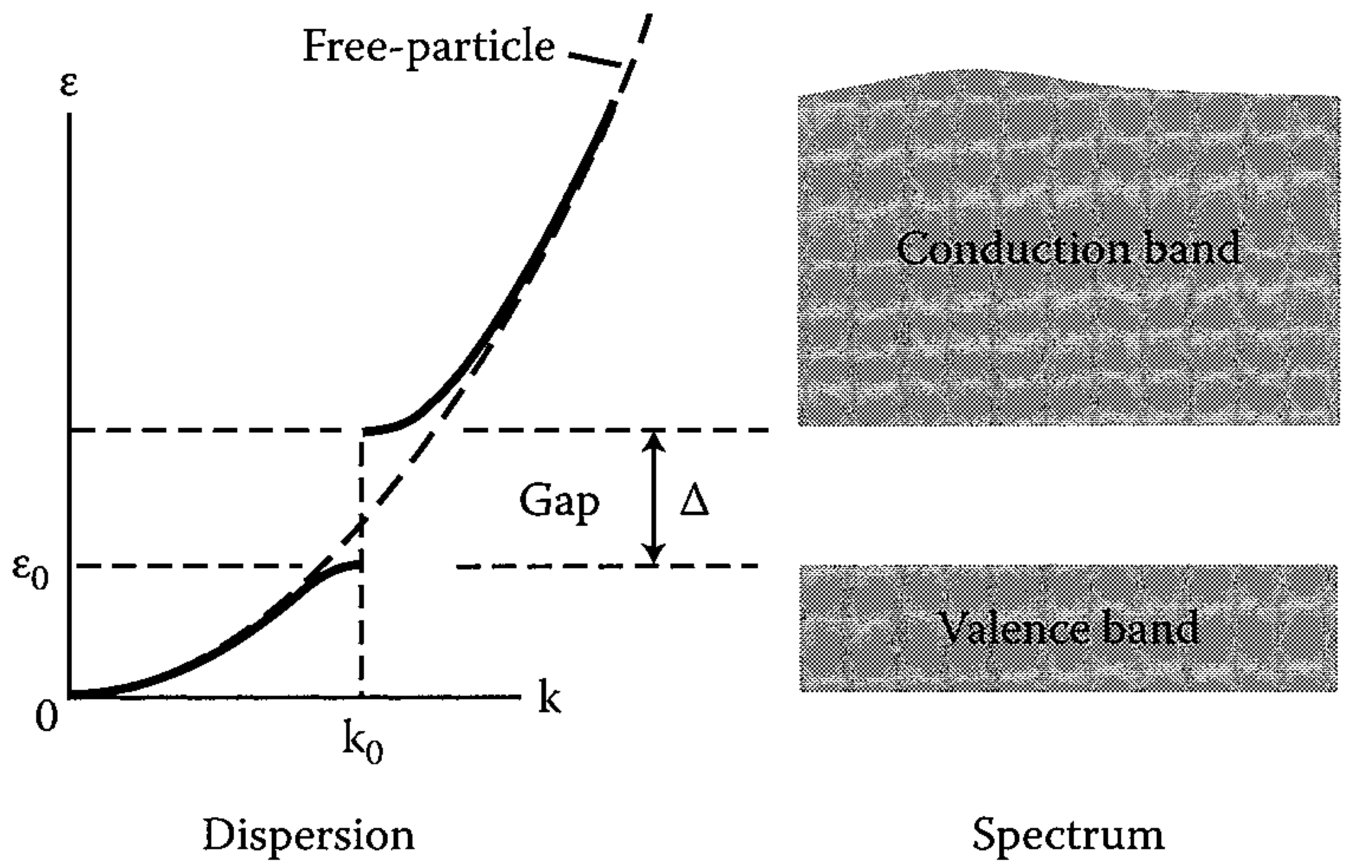
$$\text{Momentum} = -\mathbf{p}$$

$$\text{Charge} = -e$$

Particle: $$n_{\mathbf{p}} = \frac{1}{e^{\beta(\epsilon-\mu)}+1}$$

Hole: $$1 - n_{\mathbf{p}} = \frac{1}{e^{\beta(\mu-\epsilon)}+1}$$

Electrons in solid

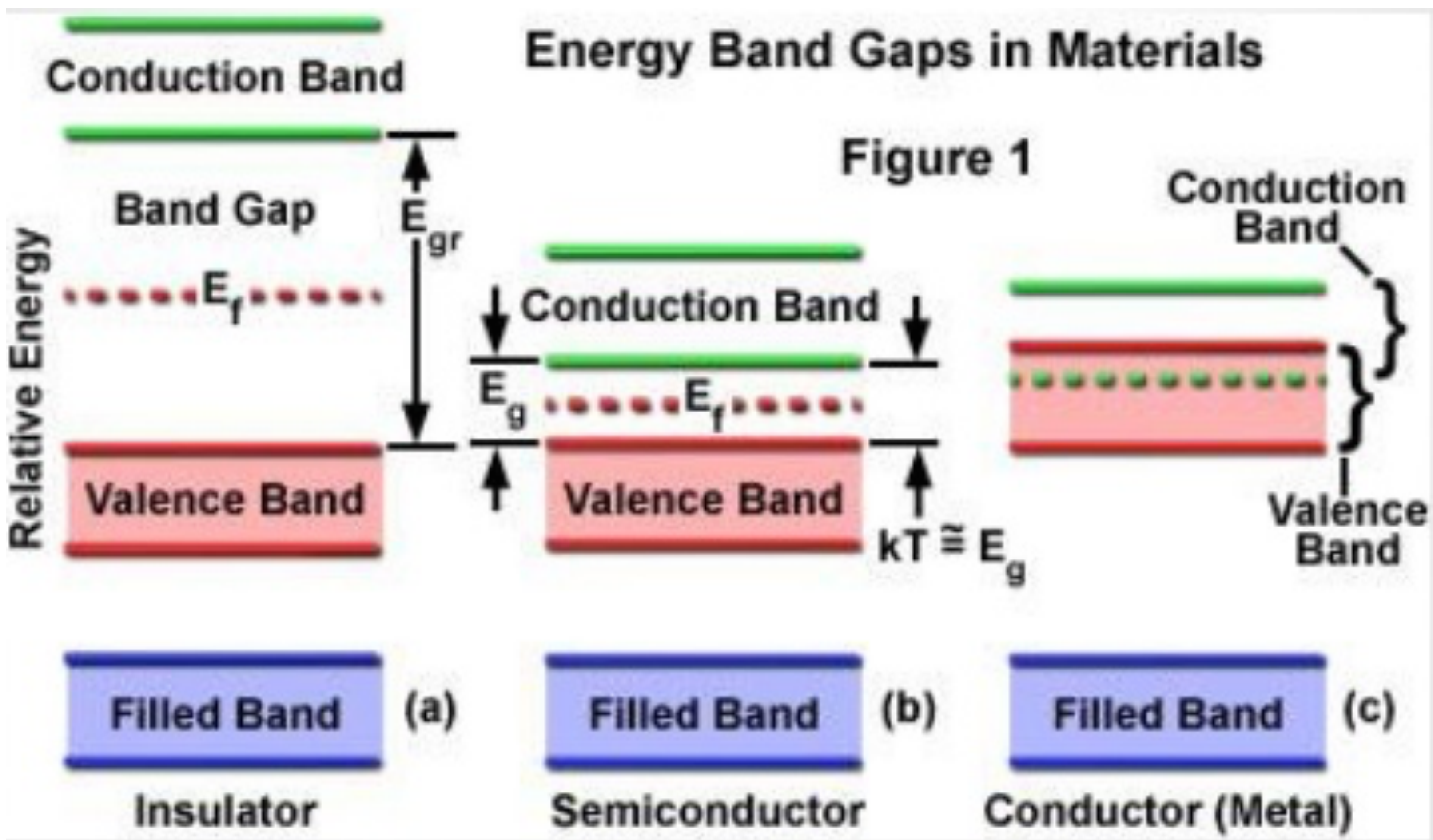


$$\epsilon_c = \epsilon_0 + \Delta + \frac{\mathbf{p}^2}{2m_c}$$

$$\epsilon_v = \epsilon_0 - \frac{\mathbf{p}^2}{2m_v}$$

Band structure in the energy of an electron
in a periodic potential

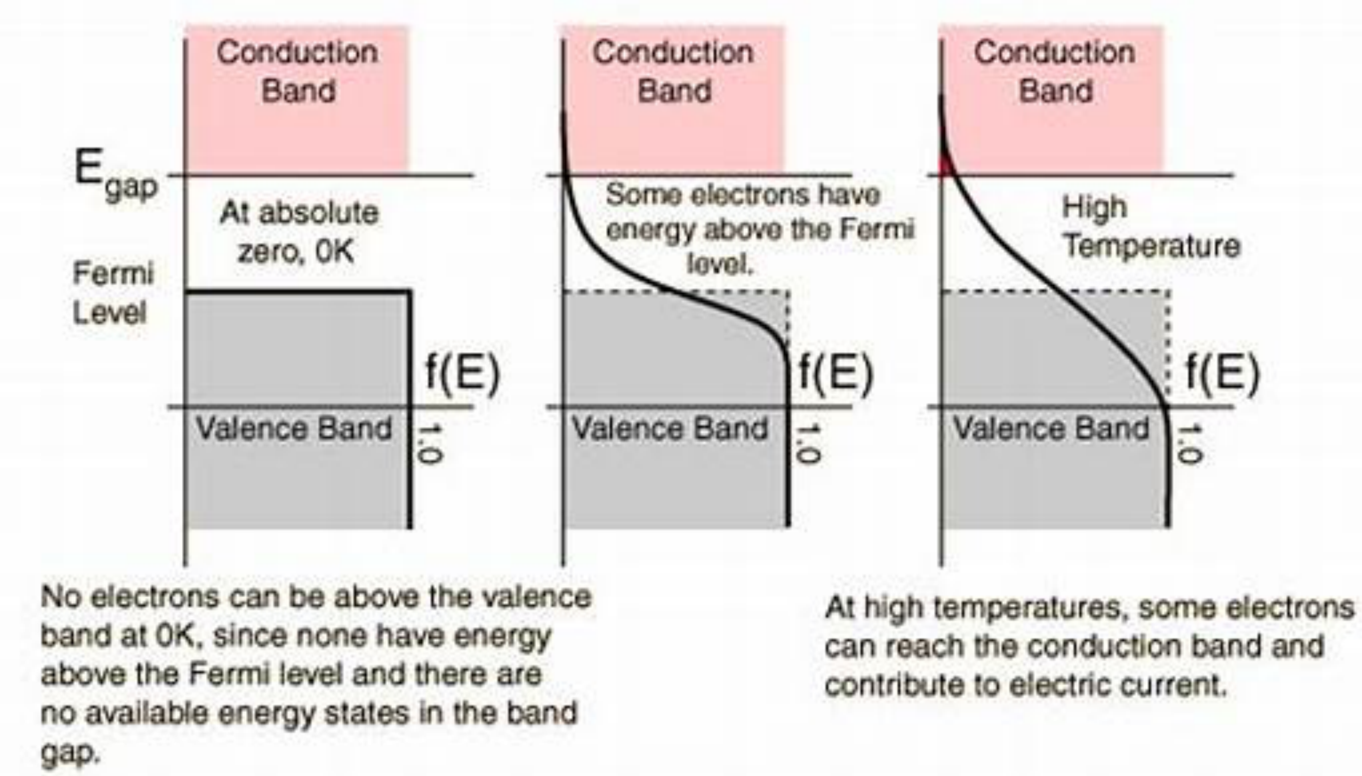
Metal,insulator and semicondutor



Nature semiconductor

Suppose at absolute zero the valence band is completely full, and the conduction band completely empty. Such a system is called a “natural semiconductor.” As the temperature increases, thermal excitation will cause some electrons to be excited across the gap into the conduction band, leaving holes in the valence band.

Calculating the particle and hole density



$$n_{\text{part}} = 2\int \frac{d^3p}{h^3}\frac{1}{e^{\beta(\epsilon_c-\mu)}+1}$$

$$n_{\text{hole}} = 2\int \frac{d^3p}{h^3}\frac{1}{e^{\beta(\mu-\epsilon_v)}+1}$$

$$n_{\text{part}} = n_{\text{hole}}$$

$$n_{\text{part}} \approx 2\int \frac{d^3p}{h^3}e^{-\beta(\epsilon_c-\mu)} = 2e^{-\beta(\epsilon_0+\Delta-\mu)}\left(\frac{m_c k_B T}{2\pi\hbar^2}\right)^{3/2}$$

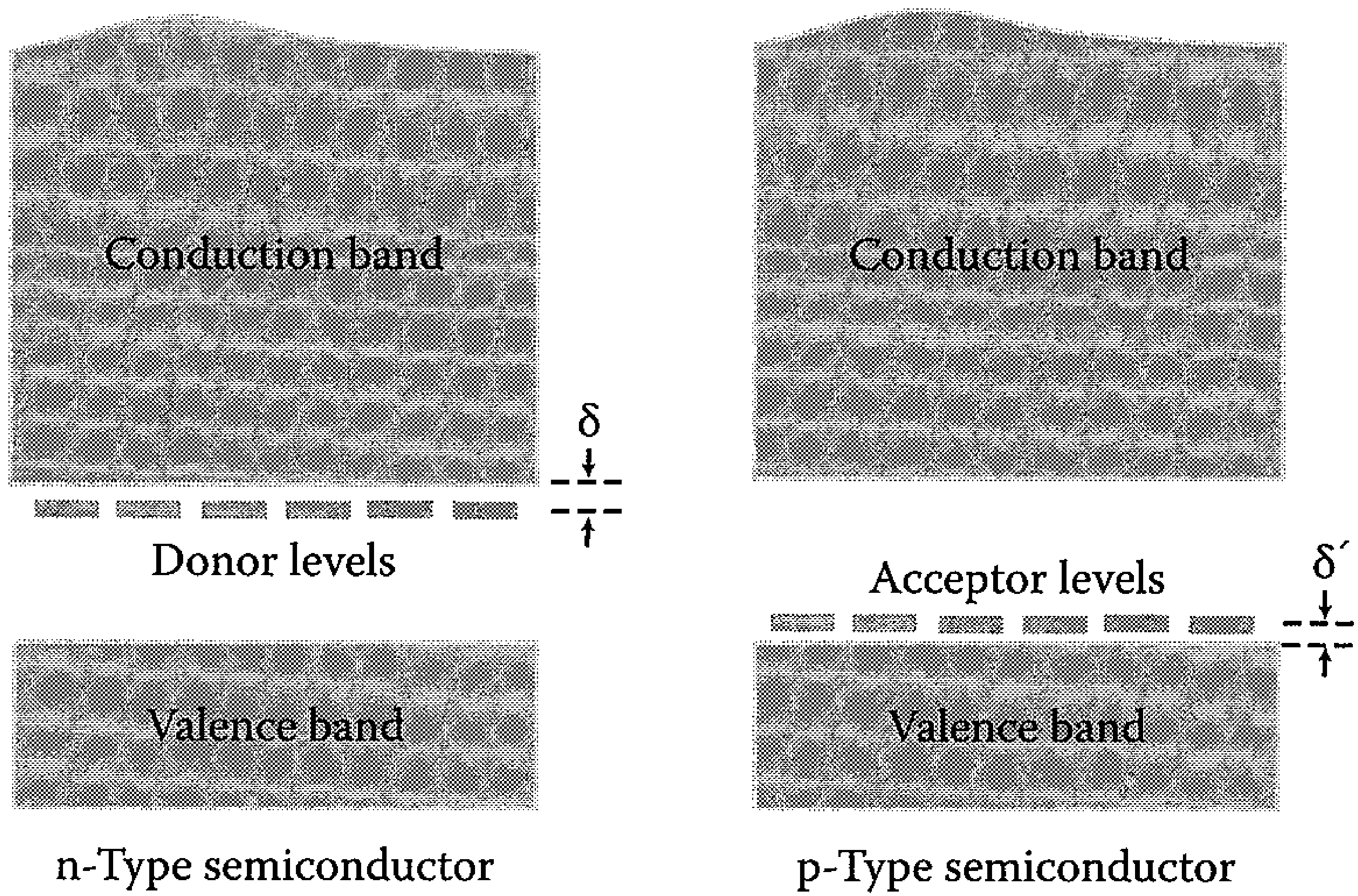
$$n_{\text{hole}} \approx 2\int \frac{d^3p}{h^3}e^{-\beta(\mu-\epsilon_v)} = 2e^{-\beta(\mu-\epsilon_0)}\left(\frac{m_v k_B T}{2\pi\hbar^2}\right)^{3/2}$$

$$\mu = \epsilon_0 + \frac{\Delta}{2} + \frac{3}{4} k_B T \ln \frac{m_c}{m_v}$$

$$n_{\text{part}} = n_{\text{hole}} = 2e^{-\beta\Delta/2} \left(\frac{\sqrt{m_c m_v} k_B T}{2\pi\hbar^2} \right)^{3/2}$$

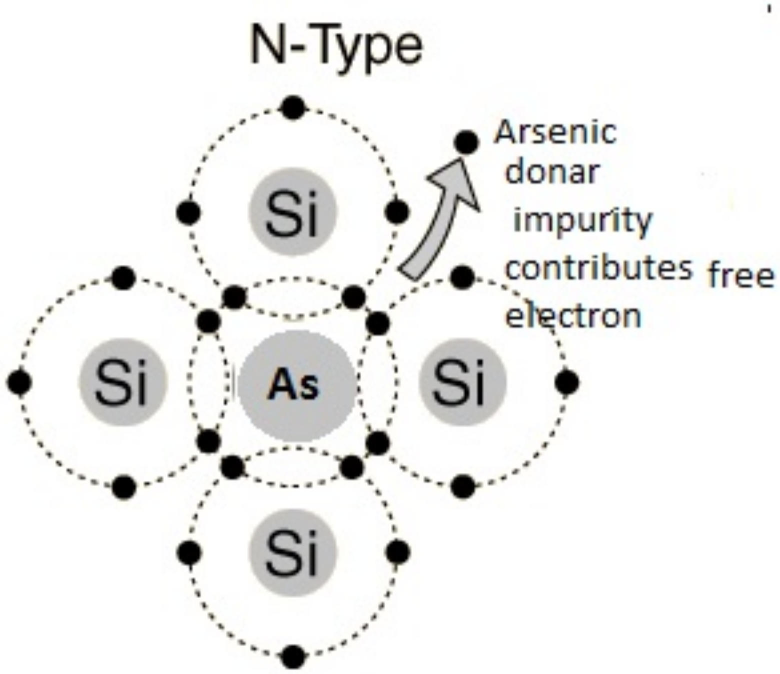
As an estimate, use the typical values $\Delta/k = 0.7\text{eV}$, and $m_c = m_v = m$, where m is the free electron mass. At room temperature $T = 300$ K, we find $n_{\text{part}} \approx 1.6 \times 10^{13}\ \text{cm}^{-3}$, which is to be compared with a charge carrier density of 10^{20} for a metal. The electrical conductivity of a natural semiconductor is therefore negligible.

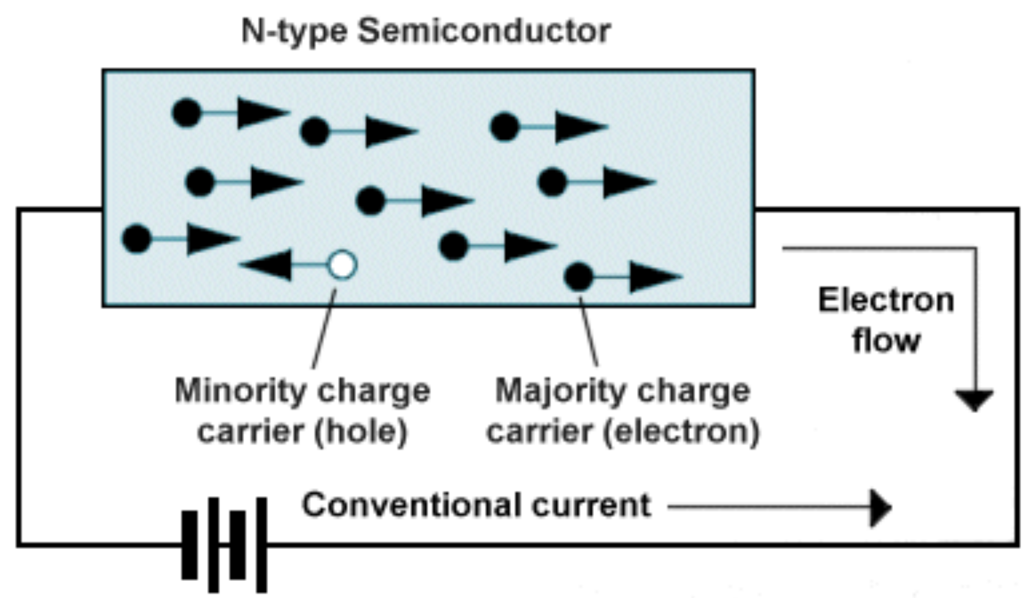
n/p-type semiconductor



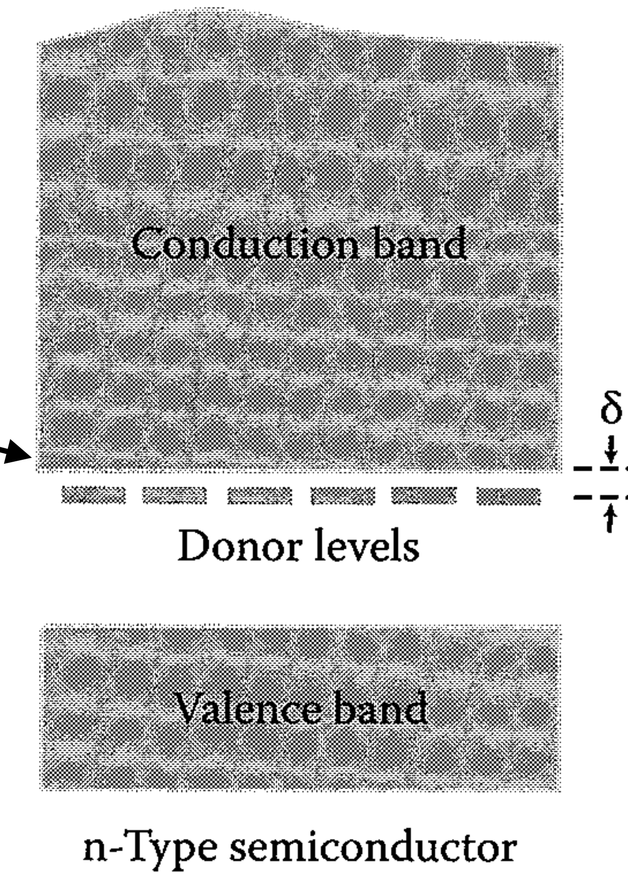
For the n-type semiconductor, let us measure energy with respect to the bottom of the conduction band. Suppose there are n_D bound states per unit volume, of energy $-\delta$.

Calculating the electron density in n-type semiconductor





$$n_{\text{donor}} = \frac{2n_{\text{D}}}{e^{-\beta(\delta+\mu)} + 1}$$



$$n_{\text{part}} = \frac{2z}{\lambda^3} \qquad \lambda = \sqrt{\frac{2\pi\hbar^2}{m_{\text{c}} k_B T}} \qquad z = e^{\beta\mu}$$

$$n_{\text{part}} \approx 2\int \frac{d^3p}{h^3} e^{-\beta(\epsilon_{\text{c}}-\mu)} = 2e^{-\beta(\epsilon_0+\Delta-\mu)} \left(\frac{m_{\text{c}} k_B T}{2\pi\hbar^2}\right)^{3/2}$$

$$\epsilon_0 + \Delta = 0$$

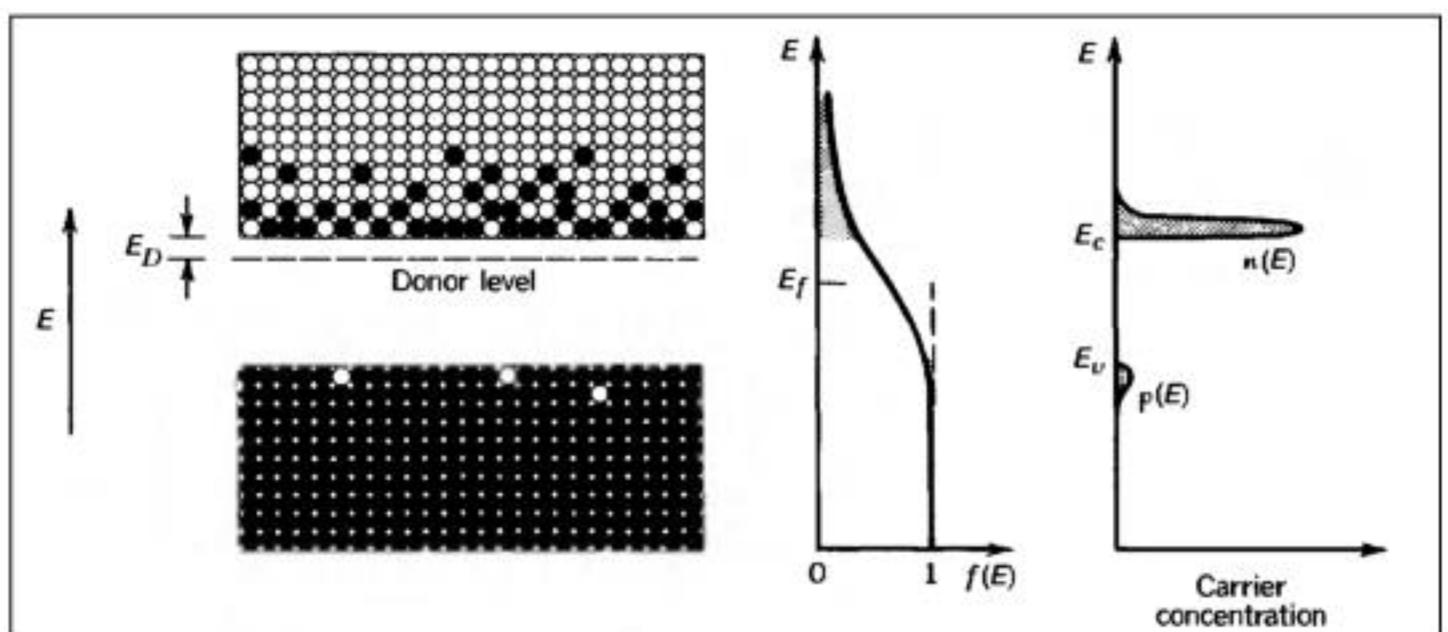
$$n_{\text{part}} + n_{\text{donor}} = 2n_{\text{D}}$$

$$\frac{2z}{\lambda^3} + \frac{2n_{\text{D}}}{z^{-1}e^{-\beta\delta} + 1} = 2n_{\text{D}}$$

$$e^{\beta\delta}z^2 + z - n_D\lambda^3 = 0$$

$$z = \frac{1}{2}e^{-\beta\delta}\left(\sqrt{4n_{\mathrm{D}}\lambda^3 + 1} - 1\right)$$

$$n_{\mathrm{part}} = \frac{e^{-\beta\delta}}{\lambda^3}\left(\sqrt{4n_{\mathrm{D}}\lambda^3 + 1} - 1\right)$$



Homework

PROBLEM 1: For a 2D many-Boson system with Hamiltonian,

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m}$$

we know we have the energy level and occupation as

$$\varepsilon(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m}$$

$$n_n = \frac{1}{e^{\beta\left[\hbar^2\mathbf{k}^2/2m-\mu\right]} - 1}$$

(a) If the particles number is N, what is the chemical potential of this system?

(b) At what the temperature, BEC will occurs? Or BEC won't occur ?

(c) Calculate the total energy as a function of temperature and the heat capacity of this system.

Due Nov. 26

PROBLEM 2: For a 2D many-Fermion system with Hamiltonian,

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m}$$

we know we have the energy level and occupation as

$$\varepsilon(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m}$$
$$n_n = \frac{1}{e^{\beta\left[\hbar^2\mathbf{k}^2/2m-\mu\right]} + 1}$$

(a) If the particles number is N, what is the chemical potential of this system at zero temperature?

(b) Calculate the total energy as a function of temperature and the heat capacity of this system.

PROBLEM 3: For a 1D many-Boson system with Hamiltonian,

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 x_i^2$$

we know we have the energy level and occupation as

$$\varepsilon_n = (n + \frac{1}{2}) h\omega$$
$$n_n = \frac{1}{e^{\beta[(n+1/2)h\omega - \mu]} - 1}$$

(a) If the particles number is N, what is the chemical potential of this system?

(b) At what the temperature, BEC will occurs?

(c) Calculate the total energy as a function of temperature and the heat capacity of this system.

PROBLEM 4: For a 1D many-Fermion system with Hamiltonian,

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 x_i^2$$

we know we have the energy level and occupation as

$$\varepsilon_n = (n + \frac{1}{2})h\omega$$
$$n_n = \frac{1}{e^{\beta[(n+1/2)h\omega-\mu]} + 1}$$

(a) If the particles number is N, what is the chemical potential of this system at zero temperature?

(b) Calculate the total energy as a function of temperature and the heat capacity of this system.

Summary

- Degenerated Fermi gas at zero temperature
- Degenerated Fermi gas at nonzero temperature
- Electrons and holes in solids
- Nature semiconductor and p-type (n-type) semiconductor