

THE SOUTH UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Tutorial 01

~~Set: Wednesday 7th September 2016; Hand in: Friday, 16th September 2016.~~

~~Note: Hand in your solutions no later than 4pm of Friday, 16th September.~~

1. Provide a strict proof for the following set relations.

(i) $B \setminus A = B \cap A^c$;

(ii) $(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$;

(iii) $(\bigcup_{k=1}^{\infty} A_k)^c = \bigcap_{k=1}^{\infty} A_k^c$;

~~(iv)~~ $(\bigcap_{k=1}^{\infty} A_k)^c = \bigcup_{k=1}^{\infty} A_k^c$;

~~(v)~~ $A \cup (\bigcap_{k=1}^{\infty} B_k) = \bigcap_{k=1}^{\infty} (A \cup B_k)$;

(vi) $A \cap (\bigcup_{k=1}^{\infty} B_k) = \bigcup_{k=1}^{\infty} (A \cap B_k)$.

As the generalizations of (iii) to (vi) we have the following general De Morgan's Laws and Distributive laws: For any index set I , we have

(vii) $(\bigcup_{i \in I} A_i)^c = \bigcap_{i \in I} (A_i)^c$;

~~(viii)~~ $(\bigcap_{i \in I} A_i)^c = \bigcup_{i \in I} (A_i)^c$;

~~(ix)~~ $A \cup (\bigcap_{i \in I} B_i) = \bigcap_{i \in I} (A \cup B_i)$;

~~(x)~~ $A \cap (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A \cap B_i)$.

2. A sequence of sets $\{A_1, A_2, \dots, A_n, \dots\}$ is called increasing if

$$A_1 \subset A_2 \subset A_3 \subset \dots \subset A_n \subset A_{n+1} \subset \dots.$$

Similarly, a sequence of sets $\{A_1, A_2, \dots, A_n, \dots\}$ is called decreasing if

$$A_1 \supset A_2 \supset A_3 \supset \dots \supset A_n \supset A_{n+1} \supset \dots.$$

Show that

(i) If $\{A_n; n \geq 1\}$ is an increasing set sequence, then for any $n \geq 1$, $\bigcup_{k=1}^n A_k = A_n$ and

$$\lim_{n \rightarrow \infty} A_n = \bigcup_{k=1}^{\infty} A_k = \bigcup_{n=1}^{\infty} A_n.$$

(ii) If $\{A_n; n \geq 1\}$ is a decreasing set sequence, then for any $n \geq 1$, $\bigcap_{k=1}^n A_k = A_n$ and

$$\lim_{n \rightarrow \infty} A_n = \bigcap_{k=1}^{\infty} A_k = \bigcap_{n=1}^{\infty} A_n.$$

3. Show that if A_1, A_2, \dots, A_n are all countable sets, then so is the n-tuple Cartesian product

$$A_1 \times A_2 \times \cdots \times A_n.$$

In particular, if A is a countable set, then so is A^n .

4. Suppose that the three sets A, B and C have the relationship $A \subset B \subset C$ and that $Card(A) = Card(C)$, then

$$Card(A) = Card(B) = Card(C),$$

where $Card(A)$ denotes the cardinal number of the set A etc.

5. Show that the set $[0, 1]$ is not countable.

6. Show that the Cardinal number of the real number \mathbb{R} is equal to the cardinal number of the open unit interval $(0, 1)$.

7. Suppose $\{A_n; n = 1, 2, \dots\}$ is an increasing sequence of sets.

Define $B_1 = A_1$, $B_2 = A_2 \setminus A_1$, and in general, $B_n = A_n \setminus A_{n-1} (n \geq 2)$. Show that

(i) $\{B_n; n \geq 1\}$ are disjoint.

(ii) For any $k \geq 1$, $\bigcup_{n=1}^k B_n = A_k$;

(iii) $\bigcup_{n=1}^{\infty} B_n = \bigcup_{n=1}^{\infty} A_n$.

8. Let S be the set of all the sequences with elements 0 and 1 only.

Is S countable or not? Prove your conclusion.