

MA215 Probability Theory

Exercise Sheet 11

Set: Monday 5th December; Hand in: Monday 12th December by 4pm.

1. Suppose $Y = e^X$ where X is normally distributed with parameters μ and σ^2 . Use the following two methods to obtain $E(Y)$.
 - (a) First obtain the pdf of Y , denoted by $f_Y(y)$ and then find $E(Y)$ by using $f_Y(y)$.
 - (b) Find $E[Y]$ directly by viewing Y as a function of X and then using the formula of getting the expected value of a function of the random variable X .

2. Suppose the random variable X obeys the uniformly distribution over interval $[a, b]$. Find $E(X^2)$ and then obtain the value of $E(X^2) - (E(X))^2$.

3. Suppose X is normally distributed random variable with parameters μ and σ^2 . Find the value of $E(X^2) - (E(X))^2$

4. (a) If the probability density function of an absolutely continuous random variable X is given by

$$f(x) = \begin{cases} \frac{1}{x(\ln 3)} & \text{for } 1 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

find $E(X)$, $E(X^2)$, and $E(X^3)$.

- (b) Use the results of part (a) to determine $E(X^3 + 2X^2 - 3X + 1)$.

5. If the probability density function of an absolutely continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{2} & \text{for } 0 < x \leq 1 \\ \frac{1}{2} & \text{for } 1 < x \leq 2 \\ \frac{3-x}{2} & \text{for } 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

find the expected value of $g(X) = X^2 - 5X + 3$.

6. The two continuous random variables X and Y have joint pdf

$$f(x, y) = x + y \quad (0 \leq x \leq 1, 0 \leq y \leq 1)$$

Find $E[(X + Y)^2]$.