Ex. 4

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1. Prove: If the conditional probabilities exist,

$$P(A_1)\cdot P(A_2|A_1)\cdot P(A_3|(A_1\cap A_2))\cdots P(A_n|A_1\cap A_2\cap\cdots A_{n-1})$$

$$= P(A_1)\cdot\frac{P(A_2\cap A_1)}{P(A_1)}\cdot\frac{P(A_1\cap A_2\cap A_3)}{P(A_1\cap A_2)}\cdots\frac{P(A_1\cap A_2\cap\cdots A_{n-1}\cap A_n)}{P(A_1\cap A_2\cap A_3\cap\cdots A_{n-1})}$$

$$= P(A_1\cap A_2\cap\cdots A_n) \quad \square$$

2. (a) P(red ball is drawn) = P((red ball is drawn ∩ head) ∪ (red ball is drawn ∩ tail))

$$= \frac{1}{2}\times\frac{2}{4}+\frac{1}{2}\times\frac{2}{7}=\frac{11}{28}$$

$$\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{2}{7}=\frac{31}{70}$$

(b) P(head up | red ball)

$$= \frac{P(\text{head up and red ball})}{P(\text{red ball})} = \frac{\frac{1}{2}\times\frac{2}{4}}{\frac{11}{28}} = \frac{7}{11}$$

$$\frac{\frac{1}{2}\times\frac{3}{5}}{\frac{31}{70}} = \frac{3}{10}\times\frac{70}{31} = \frac{21}{31}$$

3. (a) The probability: 4 : 45

$$P(A|RED) = \frac{P(RED|A)\cdot P(A)}{P(RED)}$$

$$\frac{4}{9}\times\frac{3}{10}+\frac{5}{9}\times\frac{2}{10}=\frac{11}{45}$$

(b) The probability:

$$\frac{\frac{3}{9}}{\frac{11}{45}} = \frac{\frac{1}{15}}{\frac{11}{45}} = \frac{9}{22}$$

$$\frac{4}{9}\times\frac{45}{11}$$

$$\frac{\frac{3}{10}\times\frac{4}{9}}{\frac{11}{45}} = \frac{6}{11}$$

4. $\frac{1}{2}$

A: 另一个抽屉是 silver

S: 已知这是一个抽屉是 silver

$$P(A|S) = \frac{P(S|A)\cdot P(A)}{P(S)} = \frac{P(A\cap S)}{P(S)} = \frac{\frac{1}{3}}{\frac{3}{6}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

5. Prove:

(1) $Q(\Omega) = P(\Omega \mid B) = \dfrac{P(\Omega \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$

(2) For any event A $Q(A) = \dfrac{P(A \cap B)}{P(B)} \geq 0$

~~(3) $Q(\emptyset) = P(\emptyset \mid B) = \frac{P(\emptyset \cap B)}{P(B)} = \frac{P(\emptyset)}{P(B)} = 0$~~

(3) If A and C are disjoint, then

$Q(A \cup C) = P(A \cup C \mid B) = \dfrac{P[(A \cup C) \cap B]}{P(B)} = \dfrac{P[(A \cap B) \cup (C \cap B)]}{P(B)} = \dfrac{P(A \cap B) + P(C \cap B)}{P(B)}$

$= P(A \mid B) + P(C \mid B) = Q(A) + Q(C)$

Also true for finitely many of disjoint events.

So, $Q(A) = P(A \mid B)$ is a probality measure.

Thus, $Q(A \cup C) = Q(A) + Q(C) - P(A \cap C)$

$Q(A^c) = 1 - Q(A)$

6. Prove: $\because$ A, B, C is mutually independent

$\therefore$ A, B, C also is pairwise independent

(1) $\because P[(A \cap B) \cap C] = P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = P(A \cap B) \cdot P(C)$

$\therefore$ $A \cap B$ and C is independent

(2) P(a current flow betwee A & B)

$= P[(A_1 \cap A_4) \cup (A_1 \cap A_3 \cap A_5) \cup (A_2 \cap A_5) \cup (A_2 \cap A_3 \cap A_4)]$

$= \cancel{P_1 + P_4 P_1 P_4 + P_1 + P}$

$= p_1p_4 + p_1p_3p_5 + p_2p_5 + p_2p_3p_4 - p_1p_3p_4p_5 - p_1p_2p_4p_5 - p_1p_2p_3p_4$
$- p_1p_2p_3p_5 - p_1p_2p_3p_4p_5 - p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5$
$- p_1p_2p_3p_4p_5 = p_1p_4 + p_2p_5 + p_1p_3p_5 + p_2p_3p_4 - p_1p_3p_4p_5 - p_1p_2p_4p_5 - p_1p_2p_3p_4 - p_1p_2p_3p_5$
$- p_2p_3p_4p_5 + p_1p_2p_3p_4p_5$

7. (a) P(a current flow between A & B) $= P[(A_1 \cap A_2 \cap A_5) \cup (A_3 \cap A_4 \cap A_5)]$

$= p_5\big(+p_1p_2(1-p_3)(1-p_4) + p_3p_4(1-p_1)(1-p_2) + p_1p_2(1-p_3)p_4 + p_3p_4(1-p_1)p_2 + p_1p_2p_3(1-p_4) + p_3p_4p_1(1-p_2) + p_3p_4p_1p_2\big)$

$= p_1p_2p_5 + p_3p_4p_5 - p_1p_2p_3p_4p_5$

(b) $P[(A \cup B) \cap C]$

$= P[(A \cap C) \cup (B \cap C)]$

$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$

$\because$ $P(A \cup B) \cdot P(C)$

$= [P(A) + P(B) - P(A \cap B)] \cdot P(C)$

$= P(A) \cdot P(C) + P(B) \cdot P(C) - P(A \cap B) \cdot P(C)$

$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) = P[(A \cup B) \cap C]$ □

6. (2)

$C = P(C \mid 3) \cdot P(3) + P(C \mid 3^c) \cdot P(3^c)$

$= P(1 \cup 2) \cdot P(4 \cup 5) \cdot P(3)$
$+ P(3^c)[P(1 \cap 4) + P(2 \cap 5)]$

$= (P_1 + P_2)(P_4 + P_5)P_3$
$+ (1-P_3)[P(1) \cdot P(4) + P(2) \cdot P(5)]$

$=$

A