THE SOUTH UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Tutorial 02

Set: Monday 19th September 2016 Hand in: Monday 26th September.
Note: Hand in your solutions no later than 4pm of Monday, 26th September.

1. Two six-sided dice are thrown sequentially, and the face values that come up are recorded.
   (a) List the sample space.
   (b) List the elements that make up the following events:
      (1) $A =$ the sum of the two values is at least 5;
      (2) $B =$ the value for the first die is higher than the value of the second;
      (3) $C =$ the first value is 4.
   (c) List the elements of the following events:
      (1) $A \cap C$;
      (2) $B \cup C$;
      (3) $A \cap (B \cup C)$.

2. Let A and B be arbitrary events. Let C be the event that either A occurs or B occurs, but not both. Express C in terms of A and B using any of the basic operations of union, intersection, and complement.

3. Verify the following extension of the addition rule (a) by an appropriate Venn diagram and (b) by a formal argument using the axioms of probability and the propositions in the first chapter.

$$\begin{aligned} P(A \cup B \cup C) = {} & P(A) + P(B) + P(C) \\ & - P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & + P(A \cap B \cap C) \end{aligned}$$

4. Suppose A and B are two events such that $A \subset B$. show that

$$P(B \setminus A) = P(B) - P(A).$$

5. Suppose that $\{A_n; n \geq 1\}$ is a sequence of events which may not be disjoint. Show that the following sub-additive property is true:

$$P(\bigcup_{n=1}^{\infty} A_n) \leq \sum_{n=1}^{\infty} P(A_n).$$

Also, for any $k \geq 2$, we have

$$P(\bigcup_{n=1}^{k} A_n) \leq \sum_{n=1}^{k} P(A_n).$$

In particular, for any two events A and B, we have $P(A \cup B) \leq P(A) + P(B)$.

6. Suppose $\{A_i; 1 \leq i \leq n\}$ are events.

(i) Show that the following inclusion-exclusion formula is true.

$$\begin{aligned} P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j\leq n}\sum P(A_i \cap A_j) + \sum_{i<j<k\leq n}\sum\sum P(A_i \cap A_j \cap A_k) \\ - \ldots + (-1)^{n-1} P(A_1 \cap A_2 \ldots A_n). \end{aligned}$$

(ii) Write this formula for cases of $n = 2, n = 3, n = 4$ and $n = 5$ clearly.

7. (i) If $\{A_n; n \geq 1\}$ is an increasing sequence of events, i.e. for all $n \geq 1, A_n \subset A_{n+1}$, then $\lim_{n\to\infty} P(A_n) = P(\overset{\infty}{\underset{n=1}{\cup}} A_n)$.

(ii) If $\{A_n; n \geq 1\}$ is a decreasing sequence of events, i.e. for all $n \geq 1, A_n \supset A_{n+1}$, then $\lim_{n\to\infty} P(A_n) = P(\overset{\infty}{\underset{n=1}{\cap}} A_n)$.