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①

1. (a). $\Omega = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

(b). $A = \{(1,4), (1,5), (1,6),$
$(2,3), (2,4), (2,5), (2,6),$
$(3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$B = \{(2,1),$
$(3,1), (3,2),$
$(4,1), (4,2), (4,3),$
$(5,1), (5,2), (5,3), (5,4),$
$(6,1), (6,2), (6,3), (6,4), (6,5)\}.$

$C = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}.$

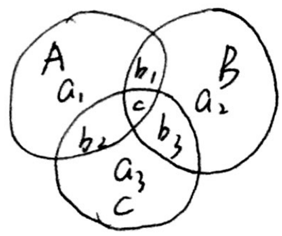
(c). $A\cap C=\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$

$$B\cup C=\{(2,1),\\ (3,1),(3,2),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (5,1),(5,2),(5,3),(5,4),\\ (6,1),(6,2),(6,3),(6,4),(6,5)\}$$

$$A\cap(B\cup C)=\{(3,2),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (5,1),(5,2),(5,3),(5,4),\\ (6,1),(6,2),(6,3),(6,4),(6,5).\}$$

2. $C=(A\cup B)\setminus(A\cap B)=(A\cup B)\cap(A\cap B)^c$

3. (a). ~~如~~图,




$$p(A)+p(B)+p(C)-p(A\cap B)-p(A\cap C)-p(B\cap C)\\ +p(A\cap B\cap C)$$

$$=(a_1+b_1+b_2+c)+(a_2+b_1+b_3+c)+(a_3+b_2+b_3+c)\\ -(b_1+c)-(b_2+c)-(b_3+c)+c$$

$$=a_1+a_2+a_3+b_1+b_2+b_3+c=p(A\cup B\cup C).$$

(b). $p(A\cup B)=p((A\setminus B)\cup(B\setminus A)\cup(A\cap B))$

$$=p(A\setminus B)+p(B\setminus A)+p(A\cap B)$$

$$=[p(A\setminus B)+p(A\cap B)]+[p(B\setminus A)+p(A\cap B)]-p(A\cap B)$$

$$=p(A)+p(B)-p(A\cap B).$$

$$p(A\cup B\cup C)=p[(A\cup B)\cup C]=p(A\cup B)+p(C)-p[(A\cup B)\cap C]$$

$$=p(A)+p(B)*-p(A\cap B)+p(C)-p[(A\cap C)\cup(B\cap C)]$$

$$=p(A)+p(B)+p(C)-p(A\cap B)-[p(A\cap C)+p(B\cap C)-p(A\cap B\cap C)]$$

$$=p(A)+p(B)+p(C)-p(A\cap B)-p(A\cap C)-p(B\cap C)+p(A\cap B\cap C).$$

4. $A\subset B \Rightarrow B = A\cup(B\setminus A)$, 且 $A\cap(B\setminus A)=\phi$

则 $P(B)=P(A)+P(B\setminus A)$, 即 $P(B\setminus A)=P(B)-P(A)$.

5. 令 $B_1=A_1$, $B_n = A_n\setminus\left(\bigcup_{i=1}^{n-1}A_i\right)$, $n\geq 2$,

则 $\{B_n; n\geq 1\}$ 不交, 且 $\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} B_n$

又 $P(A_n) = P\left(A_n\setminus \bigcup_{i=1}^{n-1}A_i\right) + P\left[A_n\cap\left(\bigcup_{i=1} A_i\right)\right] \geq P\left(A_n\setminus\left(\bigcup_{i=1}^{n-1} A_i\right)\right)$

则 $P\left(\bigcup_{n=1}^{\infty} A_n\right) = P\left(\bigcup_{n=1}^{\infty} B_n\right) = \sum_{n=1}^{\infty} P(B_n) \leq \sum_{n=1}^{\infty} P(A_n)$.

由于 $\forall n\geq 1$, $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$, 同理可得, $P\left(\bigcup_{n=1}^{k} A_n\right)\leq\sum_{n=1}^{k} P(A_n)$.

取 $k=2$, $A_1=A$, $A_2=B$, 得 $P(A\cup B)\leq P(A)+P(B)$.

6. (i). $n=2$时, $P(A_1\cup A_2) = P(A_1)+P(A_2)-P(A_1\cap A_2)$

设 $n=k$时, $P\left(\bigcup_{i=1}^{k} A_i\right) = \sum_{i=1}^{k} P(A_i) - \sum_{i<j\leq k} P(A_i\cap A_j)+\cdots+(-1)^{k-1}P(A_1\cap A_2\cap\cdots\cap A_k)$

$n=k+1$时, $P\left(\bigcup_{i=1}^{k+1}A_i\right) = P\left(\left(\bigcup_{i=1}^{k} A_i\right)\cup A_{k+1}\right) = P\left(\bigcup_{i=1}^{k} A_i\right)+P(A_{k+1}) - P\left(\left(\bigcup_{i=1}^{k} A_i\right)\cap A_{k+1}\right)$

$= \sum_{i=1}^{k+1}P(A_i) - \sum_{i<j\leq k} P(A_i\cap A_j)+\cdots+(-1)^{k-1}P(A_1\cap A_2\cap\cdots\cap A_k) - P\left(\bigcup_{i=1}^{k}(A_i\cap A_{k+1})\right)$

$=\sum_{i=1}^{k+1}P(A_i) - \sum_{i<j\leq k+1}P(A_i\cap A_j)+\cdots+(-1)^{k} P(A_1\cap A_2\cap\cdots\cap A_{k+1})$.

得证.

(ii). $n=2$, $P(A_1\cup A_2) = P(A_1)+P(A_2)-P(A_1\cap A_2)$

$n=3$, $P(A_1\cup A_2\cup A_3) = P(A_1)+P(A_2)+P(A_3)-P(A_1\cap A_2)-P(A_1\cap A_3)$
$-P(A_2\cap A_3)+P(A_1\cap A_2\cap A_3)$

$n=4$, $P\left(\bigcup_{i=1}^{4}A_i\right) = P(A_1)+P(A_2)+P(A_3)+P(A_4)-P(A_1\cap A_2)-P(A_1\cap A_3)$
$-P(A_1\cap A_4)-P(A_2\cap A_3)-P(A_2\cap A_4)-P(A_3\cap A_4)+P(A_1\cap A_2\cap A_3)$
$+P(A_1\cap A_2\cap A_4)+P(A_1\cap A_3\cap A_4)+P(A_2\cap A_3\cap A_4)-P(A_1\cap A_2\cap A_3\cap A_4)$

$n=5$, $P\left(\bigcup_{i=1}^{5}A_i\right) = \sum_{i=1}^{5}P(A_i) - \sum_{i<j\leq 5}P(A_i\cap A_j) + \sum_{i<j<k\leq 5}P(A_i\cap A_j\cap A_k)$
$-\sum_{i<j<k<l\leq 5}P(A_i\cap A_j\cap A_k\cap A_l)+P(A_1\cap A_2\cap A_3\cap A_4\cap A_5)$

7. (i). 记 $A_0=\Phi$, $B_n=A_n\setminus A_{n-1}$, $n\geq 1$, 则 $\bigcup_{n=1}^{\infty}A_n=\bigcup_{n=1}^{\infty}B_n$, $\{B_n, n\geq 1\}$ 不交

则 $P(\bigcup_{n=1}^{\infty}A_n)=P(\bigcup_{n=1}^{\infty}B_n)=\sum_{n=1}^{\infty}P(A_n\setminus A_{n-1})$

$=\sum_{n=1}^{\infty}[P(A_n)-P(A_{n-1})]=\lim_{n\to\infty}(P(A_n)-P(A_0))=\lim_{n\to\infty}P(A_n)$

(ii). 记 $B_n=(A_1\setminus A_n)$, $n\geq 1$, 则 $B_n\uparrow$

由(i)得, $P(\bigcup_{n=1}^{\infty}B_n)=\lim_{n\to\infty}P(B_n)\Rightarrow P(A_1\setminus\bigcap_{n=1}^{\infty}A_n)=P(A_1)-P(\bigcap_{n=1}^{\infty}A_n)$

$=\lim_{n\to\infty}[P(A_1)-P(A_n)]$

$=P(A_1)-\lim_{n\to\infty}P(A_n)$

$\Rightarrow \lim_{n\to\infty}P(A_n)=P(\bigcap_{n=1}^{\infty}A_n)$