Probability: Independent Events

Life is full of random events!

You need to get a "feel" for them to be a smart and successful person.

The toss of a coin, throwing dice and lottery draws are all examples of random events.

Sometimes an event can affect the next event.

Example: taking colored marbles from a bag: as you take each marble there are less marbles left in the bag, so the probabilities change.

We call those **Dependent Events**, because what happens **depends on** what happened before (learn more about this at Conditional probability).

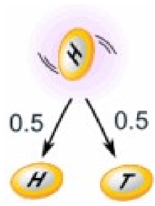
But otherwise they are **Independent Events** ...

Independent Events

Independent Events are **not affected** by previous events.

This is an important idea!

A coin does not "know" it came up heads before ...



.... each toss of a coin is a perfect isolated thing.

Example: You toss a coin and it comes up "Heads" three times ... what is the chance that **the next toss** will also be a "Head"?

The chance is simply ½ (or 0.5) just like ANY toss of the coin.

What it did in the past will not affect the current toss!

Some people think "it is overdue for a Tail", but *really truly* the next toss of the coin is totally independent of any previous tosses.

Saying "a Tail is due", or "just one more go, my luck is due" is called **The Gambler's Fallacy**

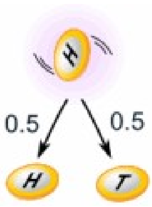
Of course your luck **may** change, because each toss of the coin has an equal chance.

Probability of Independent Events

"Probability" (or "Chance") is **how likely** something is to happen.

So how do we calculate probability?

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$



Example: what is the probability of getting a "Head" when tossing a coin?

Number of ways it can happen: 1 (Head)

Total number of outcomes: 2 (Head and Tail)

$$\text{So the probability} = \frac{1}{2} = 0.5$$

Example: what is the probability of getting a "4" or "6" when rolling a die?

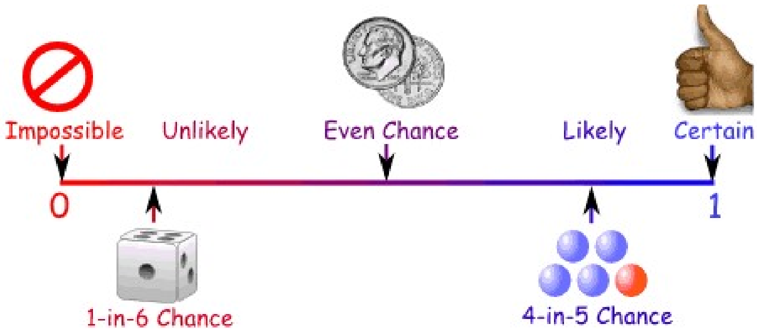
Number of ways it can happen: 2 ("4" and "6")

Total number of outcomes: 6 ("1", "2", "3", "4", "5" and "6")

$$\text{So the probability} = \frac{2}{6} = \frac{1}{3} = 0.333...$$

Ways of Showing Probability

Probability goes from **0** (imposssible) to **1** (certain):



It is often shown as a **decimal** or **fraction**.

Example: the probability of getting a "Head" when tossing a coin:

- As a decimal: **0.5**
- As a fraction: **1/2**
- As a percentage: **50%**
- Or sometimes like this: **1-in-2**

Two or More Events

We can calculate the chances of two or more **independent** events by **multiplying** the chances.

Example: Probability of 3 Heads in a Row

For each toss of a coin a "Head" has a probability of 0.5:

0.5

$0.5 \times 0.5 = 0.25$ (or $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$)

$0.5 \times 0.5 \times 0.5 = 0.125$ (or $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$)

And so the chance of getting 3 Heads in a row is **0.125**

So each toss of a coin has a ½ chance of being Heads, but **lots of Heads in a row** is unlikely.

Example: Why is it unlikely to get, say, 7 heads in a row, when *each* toss of a coin has a ½ chance of being Heads?

Because we are asking two different questions:

Question 1: What is the probability of 7 heads in a row?

Answer: ½×½×½×½×½×½×½ = 0.0078125 (less than 1%).

Question 2: Given that **we have just got 6 heads** in a row, what is the probability that **the next toss** is also a head?

Answer: ½, as the **previous** tosses don't affect the next toss.

You can have a play with the Quincunx to see how lots of independent effects can still have a pattern.

Notation

We use "P" to mean "Probability Of",

So, for Independent Events:

P(A and B) = P(A) × P(B)

Probability of A and B equals the probability of A times the probability of B

Example: your friend invites you to a movie, saying it starts some time on the weekend between 4 in the afternoon and midnight, but won't say more.

What are the chances it starts on Saturday between 6 and 8 at night?

Day: there are two days on the weekend, so **P(Saturday) = 0.5**

Time: between 4 and midnight is 8 hours, but you want between 6 and 8 which is only 2 hours:

P(Your Time) = 2/8 = 0.25

And:

P(Saturday and Your Time) = P(Saturday) × P(Your Time)
= 0.5 × 0.25
= **0.125**

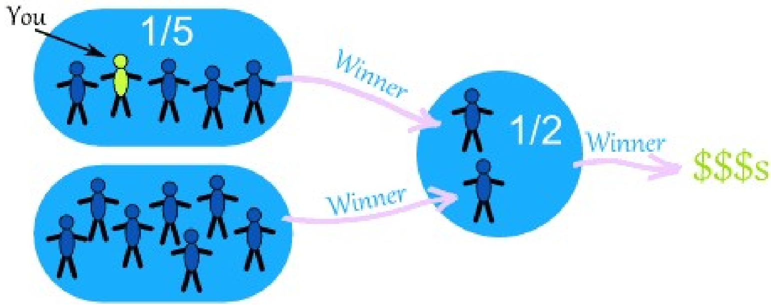
Or a 12.5% Chance

(Note: we could ALSO have worked out that you wanted 2 hours out of a total possible 16 hours, which is 2/16 = 0.125. Both methods work here.)

Another Example

Imagine there are two groups:

- A member of each group gets randomly chosen for the winners circle,
- **then** one of those gets randomly chosen to get the big money prize:



What is your chance of winnning the big prize?

- there is a **1/5 chance** of going to the winners circle
- and a **1/2 chance** of winning the big prize

So you have a 1/5 chance followed by a 1/2 chance ... which makes a 1/10 chance overall:

$$\frac{1}{5} \times \frac{1}{2} = \frac{1}{5 \times 2} = \frac{1}{10}$$

Or we can calculate using decimals (1/5 is 0.2, and 1/2 is 0.5):

$$0.2 \times 0.5 = \mathbf{0.1}$$

So your chance of winning the big money is **0.1** (which is the same as 1/10).

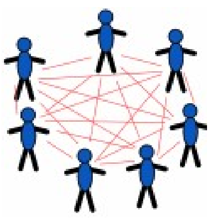
Coincidence!

Many "Coincidences" are, in fact, likely.

Example: you are in a room with 30 people, and find that Zach and Anna celebrate their birthday on the same day.

Would you say "wow, how strange", or "that seems reasonable, with so many people here".

In fact there is a **70% chance** that would happen ... so it is **likely**.



Why is the chance so high?

Because you are comparing everyone to everyone else (not just one to many).

And with 30 people that is **435 comparisons**

(Read Shared Birthdays to find out more.)

Example: Snap!

Did you ever say something **the same as someone else**, at the same time too?

Wow, how amazing!

But you were probably sharing an experience (movie, journey, whatever) and so your thoughts would be similar.

And there are only so many ways of saying something ...

... so it is like the card game "Snap!" ...

... if you speak enough words together, they will eventually match up.

So, maybe not so amazing, just simple chance at work.

Can you think of other cases where a "coincidence" was simply a likely thing?

Conclusion

- Probability is: (Number of ways it can happen) / (Total number of outcomes)
- Dependent Events (such as removing marbles from a bag) are affected by previous events
- Independent events (such as a coin toss) are **not** affected by previous events
- We can calculate the probability of 2 or more **Independent** events by **multiplying**
- Not all coincidences are really unlikely (when you think about them).

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