THE SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Probability Theory

Exercise Sheet 5

Set: Tuesday 11th October; Hand in: Wednesday 19th October.

1. Random variable X is said to follow the logarithmic distribution with parameter p, where $0 < p < 1$, if X has probability mass function

$$\Pr(X = k) \quad = \quad A\left(\frac{p^k}{k}\right) \text{ for } k = 1, 2, 3, \ldots$$

for some appropriate constant A.

Find the value of the constant A in terms of the parameter p.

2. Suppose that independent trials, each having a probability p, $0 < p < 1$, of being a success, are performed until a success occurs. Let X equal the number of trials required. What are the possible values of X ? Write down the probability mass function of X .

3. Suppose that independent trials, each having a probability p, $0 < p < 1$, of being a success, are performed until a total of r successes is accumulated, where $r \geq 1$ is a positive integer. Let X equal the number of trials required. What are the possible values of X ? Write down the probability mass function of X .

4. Assuming that each dart has probability 0.2 of hitting its target, give the (cumulated) probability distribution of the number of darts one should throw at the target to get the first successful hit. What is the probability distribution of the number of throws required to get two hits? Finally what is the probability of at least one hit in n throws, and what is the smallest value of n for which this is greater than 0.9?

5. The number of phone calls received at a certain residence in any period of c hours is a Poisson random variable with parameter $\lambda = 0.5c$.

   (a) What is the probability that the phone rings during a given 15 minute period?

   (b) How long a period must one wait for the probability of at least one phone call during that period to be at least 0.5?