5. Prove:

(1) $Q(\Omega) = P(\Omega \mid B) = \dfrac{P(\Omega \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$

(2) For any event A, $Q(A) = \dfrac{P(A\cap B)}{P(B)} \geq 0$

(3) If A and C are disjoint, then

$$Q(A\cup C) = P(A\cup C \mid B) = \frac{P[(A\cup C)\cap B]}{P(B)} = \frac{P[(A\cap B)\cup (C\cap B)]}{P(B)} = \frac{P(A\cap B)+P(C\cap B)}{P(B)}$$

$$= P(A\mid B)+P(C\mid B) = Q(A)+Q(C)$$

Also true for finitely many of disjoint events.

So, $Q(A) = P(A\mid B)$ is a probolity measure.

Thus, $Q(A\cup C) = Q(A)+Q(C) - P(A\cap C)$

$$Q(A^c) = 1 - Q(A)$$

6. Prove: $\because$ A, B, C is mutually independent

$\therefore$ A, B, C also is pairwise independent

(1) $\because P[(A\cap B)\cap C] = P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C) = P(A\cap B)\cdot P(C)$

$\therefore A\cap B$ and C is independent

(2)

P(a current flow betwee A & B)

$= P[(A_1\cap A_4)\cup(A_1\cap A_3\cap A_5)\cup(A_2\cap A_5)\cup(A_2\cap A_3\cap A_4)]$

$= \cancel{P_3 P_1 + P_4 P_1 P_4 + P_1 + P}$

$= P_1P_4 + P_1P_3P_5 + P_2P_5 + P_2P_3P_4 - P_1P_3P_4P_5 - P_1P_2P_4P_5 - P_1P_2P_3P_4$
$- P_1P_2P_3P_5 - P_1P_2P_3P_4P_5 - P_2P_3P_4P_5 + P_1P_2P_3P_4P_5 + P_1P_2P_3P_4P_5 + P_1P_2P_3P_4P_5$
$- P_1P_2P_3P_4P_5 = P_1P_4 + P_2P_5 + P_1P_3P_5 + P_2P_3P_4 - P_1P_3P_4P_5 - P_1P_2P_4P_5 - P_1P_2P_3P_4 - P_1P_2P_3P_5$
$- P_2P_3P_4P_5 + P_1P_2P_3P_4P_5$

7. (a) P(a current flow between A & B) $= P_5\big(P_1P_2(1-P_3)(1-P_4) + P_3P_4(1-P_1)(1-P_2)$
$+ P_1P_2(1-P_3)P_4 + P_3P_4(1-P_1)P_2$
$+ P_1P_2P_3(1-P_4) + P_3P_4P_1(1-P_2)$
$+ P_3P_4P_1P_2\big)$

$= P[(A_1\cap A_2\cap A_5)\cup(A_3\cap A_4\cap A_5)]$

$= P_1P_2P_5 + P_3P_4P_5 - P_1P_2P_3P_4P_5$

(b) $P[(A\cup B)\cap C]$

$= P[(A\cap C)\cup(B\cap C)]$

$= P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)$

又: $P(A\cup B)\cdot P(C)$

$= [P(A)+P(B)-P(A\cap B)]\cdot P(C)$

$= P(A)\cdot P(C) + P(B)\cdot P(C) - P(A\cap B)\cdot P(C)$

$= P(A\cap C) + P(B\cap C) - P(A\cap B\cap C) = P[(A\cup B)\cap C]$ □