THE SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

MA215 Introduction to Probability Theory

Exercise Sheet 3

Set: Wednesday 21st September Hand in: Friday 30th September

Hand in your solutions no later than 4pm of Friday, 30th September.

1. Show that if the conditional probabilities exist, then

$$\begin{aligned} &P(A_1 \cap A_2 \cap \cdots \cap A_n) \\ &= P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)\cdots P(A_n|A_1 \cap A_2 \cap \cdots A_{n-1}) \end{aligned}$$

2. Urn A has three red balls and two white balls, and urn B has two red balls and five white balls. A fair coin is tossed; if it lands heads up, a ball is drawn from urn A, and otherwise a ball is drawn from urn B.

   (a) What is the probability that a red ball is drawn?

   *(b) If a red ball is drawn, what is the probability that the coin landed heads up?

3. Urn A has four red, three blue, and two green balls. Urn B has two red, three blue, and four green balls. A ball is drawn from urn A and put into urn B, and then a ball is drawn from urn B.

   (a) What is the probability that a red ball is drawn from urn B?

   (b) If a red ball is drawn from urn B, what is the probability that a red ball was drawn from urn A?

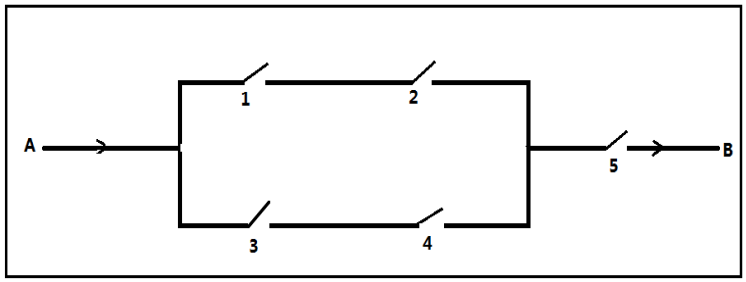
4. There are three cabinets, A, B, and C, each of which has two drawers. Each drawer contains one coin; A has two gold coins, B has two silver coins, and C has one gold and one silver coin. A cabinet is chosen at random, one drawer is opened, and a silver coin is found. What is the probability that the other drawer in that cabinet contains a silver coin?

5. If B is an event, with $P(B) > 0$, show that the set function $Q(A) = P(A|B)$ is a probability measure. Thus, we can use the following formulas in lectures:

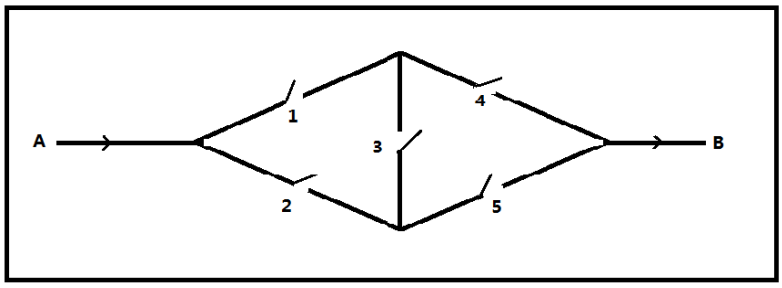
$$\begin{aligned} P(A \cup C|B) &= P(A|B) + P(C|B) - P(A \cap C|B) \\ \text{and} \quad P(A^c|B) &= 1 - P(A|B) \end{aligned}$$

6. Show that if A, B, and C are mutually independent, then $A \cap B$ and C are independent and $A \cup B$ and C are independent.

7. The probability of the closing of the ith relay in the circuits shown is given by $p_i, i = 1,2,3,4,5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits?



(a)



(b)