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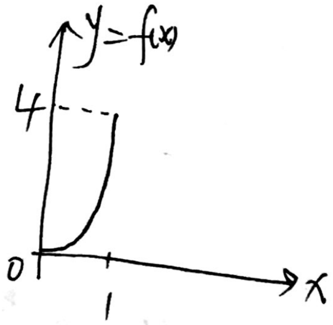
Tuto 6.

1. (a) $1 = \int_{\mathbb{R}} f(x)dx = \int_0^1 cx^3 dx = \frac{c}{4}x^4\Big|_0^1 = \frac{c}{4} \Rightarrow c = 4.$

(b)



(c). $x \le 0,\quad F(x) = \int_{-\infty}^{x} f(x)dx = 0,$

$0 < x < 1 \quad F(x) = \int_{-\infty}^{x} f(x)dx = \int_0^x 4x^3 dx = x^4.$

$x \geq 1 \quad F(x) = \int_{-\infty}^{x} f(x)dx = \int_0^1 4x^3 dx = 1.$

即

$$F(x) = \begin{cases} 0, & x \le 0 \\ x^4, & 0 < x < 1 \\ 1, & x \geq 1. \end{cases}$$

2. 若 $X \sim$ exponential (λ). 则 $F_X(x) = 1 - e^{-\lambda x}$. $x \geq 0$.

$$P(X > s+t \mid X > t) = \frac{P(X > s+t)}{P(X > t)}$$

$$= \frac{1 - F_X(s+t)}{1 - F_X(t)}$$

$$= \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}}$$

即X有无记忆性.

$$= e^{-\lambda s} = P(X > s).$$

3. $X \sim exponential(0.5)$ $\quad F(x) = 1 - e^{-0.5x}$, $\quad x \geq 0$.

(a) $P(X > 1) = 1 - F(1) = e^{-0.5}$

(b) $P(X > 2 \mid X > 1) = P(X > 1) = e^{-0.5}$.

4. 记 X 为 t 小时内的电话次数.

则 $X \sim Poisson(\mu t)$. $\quad P(X = k) = e^{-\mu t} \cdot \frac{(\mu t)^k}{k!}$, $\quad k = 0, 1, 2, \cdots$

$$P(X = 0) = e^{-\mu t}.$$

$$P(T \leq t) = P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-\mu t}.$$

$$F_T(t) = P(T \leq t) = 1 - e^{-\mu t}.$$

即 $T \sim exponential(\mu)$.

5. (a). $F(m) = 1 - \exp\left[-\left(\frac{m}{\alpha}\right)^\beta\right] = 0.5$

$\Rightarrow$ $\quad \cancel{\exp} - \left(\frac{m}{\alpha}\right)^\beta = \ln 0.5$

$\Rightarrow$ $\quad m = \alpha (\ln 2)^{\frac{1}{\beta}}$

(b). $f(x) = F'(x) = -\exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right] \cdot \left[-\beta \left(\frac{x}{\alpha}\right)^{\beta-1} \cdot \frac{1}{\alpha}\right]$

$\quad = \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} \cdot \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right]$ $\quad x > 0$.

$f(x) = 0$, $\quad x \leq 0$.

Tuto 7

2. (a) $X \sim N(58.4, 2.9^2)$. 则 $Y = \frac{X-58.4}{2.9} \sim N(0, 1)$

则 $P(57 \le X \le 61) = P\left(\frac{57-58.4}{2.9} \le Y \le \frac{61-58.4}{2.9}\right)$

$= \Phi\left(\frac{2.6}{2.9}\right) - \Phi\left(-\frac{1.4}{2.9}\right)$

$= \Phi\left(\frac{2.6}{2.9}\right) + \Phi\left(\frac{1.4}{2.9}\right) - 1$

$=$

(b) $0.9 = P(X \le c) = P\left(Y \le \frac{c-58.4}{2.9}\right) = \Phi\left(\frac{c-58.4}{2.9}\right)$

$\Rightarrow c = \Phi^{-1}(0.9) \cdot 2.9 + 58.4 =$

3. (a). 由于 $X \in R$, 则 $Y = \exp(X) > 0$, 即 $Y \in (0, +\infty)$.

(b). 记 $g(x) = e^x$. 则 $g^{-1}(x) = \ln x$, $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$

则 $f_Y(x) = f_X(g^{-1}(x)) \cdot \left|\frac{d}{dx} g^{-1}(x)\right|$

$= \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right]$. $x > 0$,

4. (a). $X \in R$, $Y = aX + b \in R$.

(b). 记 $g(x)=ax+b$. 则 $g^{-1}(x)=\frac{x-b}{a}$. $f_X(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$

$$f_Y(x)=f_X(g^{-1}(x))\cdot\left|\frac{d}{dx}g^{-1}(x)\right|$$

$$=\frac{1}{\sqrt{2\pi}\sigma|a|}\exp\left[-\frac{(\frac{x-b}{a}-\mu)^2}{2\sigma^2}\right]$$

$$=\frac{1}{\sqrt{2\pi}\sigma|a|}\exp\left\{-\frac{[x-(a\mu+b)]^2}{2a^2\sigma^2}\right\}$$

(c). 由 Y 的 p.d.f. $f_Y(x)$ 可知, $Y\sim N(a\mu+b,\ a^2\sigma^2)$.

5. (a). $X\sim N(0,1)$. $Y=X^2\geq 0$, 即 $Y\in[0,+\infty)$.

(b). $x\geq 0$, $F_Y(x)=P(Y\leq x)=P(X^2\leq x)$

$$=P(-\sqrt{x}\leq X\leq\sqrt{x})$$

$$=F_X(\sqrt{x})-F_X(-\sqrt{x})$$

$F_X(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$,

则 $f_Y(x)=F_Y'(x)=f_X(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}-f_X(-\sqrt{x})\cdot\left(-\frac{1}{2\sqrt{x}}\right)$

$$=\frac{1}{\sqrt{x}}\cdot f_X(\sqrt{x})$$

$$=\frac{1}{\sqrt{2\pi x}}e^{-\frac{x}{2}},\qquad x\geq 0.$$

$x<0$ 时, $F_Y(x)=0$. $\Rightarrow f_Y(x)=0$.