MA Probability Theory

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1. (ix) Prove:

① $A \cup (\bigcap_{i\in I} B_i) \Rightarrow x \in A$ or $x \in \bigcap_{i\in I} B_i$

If $\forall i \in I$, $x \in A \Rightarrow x \in A \cup B_i$ ($\forall i \in I$)

$\Rightarrow x \in \bigcap_{i\in I} (A \cup B_i)$

If $x \in \bigcap_{i\in I} B_i \Rightarrow$ For $\forall i \in I$, $x \in B_i$

$\Rightarrow x \in A \cup B_i$

$\Rightarrow x \in \bigcap_{i\in I} (A \cup B_i)$

So, $\bigcap_{i\in I}(A \cup B_i) \supset A \cup (\bigcap_{i\in I} B_i)$

② $\bigcap_{i\in I} (A \cup B_i) \Rightarrow x \in A$ or For $\forall i \in I$, $x \in B_i$

If $x \in A$, $\Rightarrow x \in A \cup (\bigcap_{i\in I} B_i)$

If for $\forall i \in I$, $x \in B_i \Rightarrow x \in \cancel{A \cup B_i} \bigcap_{i\in I} B_i$

$\Rightarrow x \in A \cup (\bigcap_{i\in I} B_i)$

So, $A \cup (\bigcap_{i\in I} B_i) \supset \bigcap_{i\in I} (A \cup B_i)$

At last, $A \cup (\bigcap_{i\in I} B_i) = \bigcap_{i\in I}(A \cup B_i)$

2. When $n=2$, Fixed $a_2 \in A_2$,

$\Rightarrow \{(a_1, a_2); a_1 \in A_1\}$ is countable

$\Rightarrow \bigcup_{a_2 \in A_2} \{(a_1, a_2); a_1 \in A_1\}$ is countable.

A

when $n=k$, assume

$\{(a_1, a_2, \cdots, a_k\}$ is countable

when $n=k+1$,

Fixed $a_{k+1} \in A_{k+1}$, $\{(a_1, a_2, \cdots, a_k, a_{k+1}\}$ is countable

$\Rightarrow \bigcup_{a_{k+1} \in A_{k+1}} \{(a_1, a_2, \cdots, a_k, a_{k+1}\}$ is countable.