

MA215 Probability Theory

Exercise Sheet 9

Set: Friday 11th November; Hand in: Friday 18th November by 5pm.

1. If the joint probability density of X and Y is given by

$$f(x, y) = \begin{cases} x + y & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the joint cumulative distribution function (joint cdf) of these two random variables.

2. Find the joint probability density of the two random variables X and Y whose joint cumulative distribution function (joint cdf) is given by

$$F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}) & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Also use the joint probability density to determine $P(1 < X < 3, 1 < Y < 2)$.

3. Consider a circle of radius R and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point. (In other words, the point is uniformly distributed within the circle.) If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen, it follows, since (X, Y) is equally likely to be near each point in the circle, that the joint density function of X and Y is given by

$$f(x, y) = \begin{cases} c & \text{if } x^2 + y^2 \leq R^2 \\ 0 & \text{if } x^2 + y^2 > R^2 \end{cases}$$

for some value of c .

- (a) Determine the constant c .
 - (b) Find the marginal density functions of X and Y .
 - (c) Compute the probability that the distance from the origin of the point selected is not greater than a . ($0 \leq a \leq R$.)
 - (d) Are X and Y independent? Specify your reasons clearly.
4. A man and a woman decide to meet at a certain location. If each person independently arrives at a time uniformly distributed between 12 noon and 1 P.M., find the probability that the first person who arrives at the location has to wait longer than 10 minutes.