Probability Theory 115105-11 HW3

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1\. (a) sample space: ~~All the elements following mean~~.

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), 8
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

~~All the~~ The set which contain all the elements above is ~~space~~ sample space.

(b)

(1) A contain all the ~~ele~~ following element.
(1,4), (1,5), (1,6)
(2,3), (2,4), (2,5), (2,6), (3,2)
(3,3), (3,4), (3,5), (3,6), (4,1)
(4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

(2) B contain all following element
(2,1)
(3,1), (3,2)
(4,1), (4,2), (4,3)
(5,1), (5,2), (5,3), (5,4)
(6,1), (6,2), (6,3), (6,4), (6,5)

(3) C contain all following element
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(c) (1) $A \cap C = C = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

(2) ~~$B \cup C = \{$~~ $B \cup C$ contains all the following elements:
(2,1)
(3,1), (3,2)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), ~~(5,5), (5,6)~~
(6,1), (6,2), (6,3), (6,4), (6,5)

(3)

(3,2)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4)

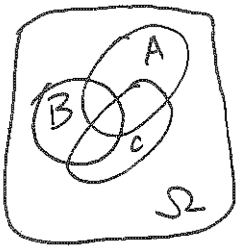
(6,1), (6,2), (6,3), (6,4), (6,5)

2. $C = (A \cup B) \cap (A \cap B)^c$

How set link with probability?

可以用字母表示

3. (a) let



~~P(A∪B∪C)~~

~~Because~~ $P(A\cap B)$, $P(A\cap C)$, $P(B\cap C)$, ~~$P(A\cap B\cap C)$~~ are added ~~more~~ more one times; $P(A\cap B\cap C)$ is deleted more one time.

~~P(A∪B∪C) should delete one~~ time.

(b) prove:

According to (a), and it can be proved that $A\cap(B\cup C)^c$, $B\cap(A\cup C)^c$, $C\cap(A\cup B)^c$, $(A\cap B)\cap(A\cap B\cap C)^c$, $(B\cap C)\cap(A\cap B\cap C)^c$, $(A\cap C)\cap(A\cap B\cap C)^c$, $A\cap B\cap C$ are disjoint.

$$P(A\cup B\cup C) = P[A\cap(B\cup C)^c] + P[B\cap(A\cup C)^c] + P[C\cap(B\cup A)^c] + P[(A\cap B)\cap(A\cap B\cap C)^c] + P[(B\cap C)\cap(A\cap B\cap C)^c] + P[(A\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C) \quad \cdots\cdots (A)$$

Then

$$P(A) = P[A\cap(B\cup C)^c] + P[(A\cap B)\cap(A\cap B\cap C)^c] + P[(A\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C) \quad \cdots\cdots (B_1)$$

$$P(B) = P[B\cap(A\cup C)^c] + P[(A\cap B)\cap(A\cap B\cap C)^c] + P[(B\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C) \quad \cdots\cdots (B_2)$$

$$P(C) = P[A\cap(B\cup C)^c] + P[(A\cap C)\cap(A\cap B\cap C)^c] + P[(B\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C) \quad \cdots\cdots (B_3)$$

Take B_1, B_2, B_3 to A.

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P[(A\cap B)\cap(A\cap B\cap C)^c] - P[(A\cap C)\cap(A\cap B\cap C)^c] - P[(B\cap C)\cap(A\cap B\cap C)^c] - P(A\cap B\cap C) - P(A\cap B\cap C)$$

$$= P(A) + P(B) + P(C) - \{P[(A\cap B)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C)\} - \{P[(A\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C)\}$$

$- \{P[(B\cap C)\cap(A\cap B\cap C)^c] + P(A\cap B\cap C)\} + P(A\cap B\cap C)$

$= P(A)+P(B)+P(C)$
$\quad - P(A\cap B) - P(A\cap C) - P(B\cap C)$
$\quad + P(A\cap B\cap C)$ $\blacksquare$

4. prove:

$\because \forall x \in B\cap A^c \Rightarrow x\in B$ and $x \notin A$

$\therefore$ A and $B\cap A^c$ is disjoint

Then $P(B) = P(A) + P(B\cap A^c)$

$P(B\cap A^c) = P(A) - P(B)$

$P(B\setminus A) = P(A) - P(B)$ $\blacksquare$

5. prove:

(1) $\bigcup_{k=1}^{n} A_n \setminus \bigcup_{k=1}^{n-1} A_n = A_n \setminus \bigcup_{k=1}^{n-1} A_k = B_n$

$\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty}\left(\bigcup_{k=1}^{n} A_n \setminus \bigcup_{k=1}^{n-1} A_n\right) = \bigcup_{n=1}^{\infty} B_n$

$P\left(\bigcup_{n=1}^{\infty} A_n\right) = P\left(\bigcup_{n=1}^{\infty} B_n\right) = \sum_{i=1}^{\infty} P(B_i)$

$\because$ B_n is subset of A_n, $\therefore$ $P(B_i) \le P(A_i)$

$\therefore$ $P\left(\bigcup_{n=1}^{\infty} A_n\right) \le \sum_{i=1}^{\infty} P(A_i)$

(2) let when $k \geq 2$, $i > k$, $A_i = \emptyset$

$P\left(\bigcup_{n=1}^{\infty} A_n\right) = P\left(\bigcup_{n=1}^{k} A_n\right) \le \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{k} P(A_i)$

(3) $P(A_1 \cup A_2) \le P(A_1) + P(A_2)$

6. (1) $n=2$时，等式成立

~~假设~~ 假设$n=k$时成立，当$n=k+1$时

$$P\left(\bigcup_{i=1}^{k+1} A_i\right) = P\left(\bigcup_{i=1}^{k} A_i \cup A_{k+1}\right) = P\left(\bigcup_{i=1}^{k} A_i\right) + P(A_{k+1}) - P\left[A_{k+1} \cap \bigcup_{i=1}^{k} A_i\right]$$

$$= \sum_{i=1}^{k+1} P(A_i) - \sum\sum_{i<j\le k} P(A_j \cap A_i) + \sum\sum\sum_{i<j<q\le k} P(A_i \cap A_j \cap A_q) \cdots (-1)^{n+1} P(A_1 \cap A_2 \cap \cdots \cap A_n)$$

$$- P\left(\bigcup_{i=1}^{k} (A_{k+1} \cap A_i)\right)$$

$$= \sum_{i=1}^{k+1} P(A_i) - \sum\sum_{i<j\le k} P(A_j \cap A_i) + \sum\sum\sum_{i<j<q\le k} P(A_i \cap A_j \cap A_q) \cdots (-1)^{k+1} P(A_1 \cap A_2 \cap \cdots \cap A_k)$$

$$- \left(\sum_{i=1}^{k} P(A_{k+1} \cap A_i) - \sum_{i<j\le k} P(A_{k+1} \cap A_i \cap A_j) + \cdots + (-1)^{k+1} P(A_1 \cap A_2 \cap \cdots \cap A_{k+1})\right)$$

(2) $$= \sum_{i=1}^{k+1} P(A_i) - \sum\sum_{i<j\le k+1} (A_i \cap A_j) + \cdots + (-1)^{k+2} (A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_{k+1})$$

$n=2$: $$P\left(\bigcup_{i=1}^{2} A_i\right) = \sum_{i=1}^{2} P(A_i) - \sum_{i<j\le 2} P(A_i \cap A_j)$$

$$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$n=3$: $$P\left(\bigcup_{i=1}^{3} A_i\right) = \sum_{i=1}^{3} P(A_i) - \sum_{i<j\le 3} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k\le 3} (A_i \cap A_j \cap A_k)$$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3)$$
$$+ P(A_1 \cap A_2 \cap A_3)$$

$n=4$: $$P\left(\bigcup_{i=1}^{4} A_i\right) = \sum_{i=1}^{4} P(A_i) - \sum_{i<j\le 4} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k\le 4} (A_i \cap A_j \cap A_k)$$

$$- \sum\sum\sum\sum_{i<j<k<f\le 4} (A_i \cap A_j \cap A_k \cap A_f)$$

$$= P(A_1) + P(A_2) + P(A_3) + P(A_4) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_1 \cap A_4)$$
$$- P(A_2 \cap A_3) - P(A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_4)$$
$$+ P(A_1 \cap A_3 \cap A_4) + P(A_2 \cap A_3 \cap A_4)$$
$$- P(A_1 \cap A_2 \cap A_3 \cap A_4)$$

$n=5$: $$P\left(\bigcup_{i=1}^{5} A_i\right) = P(A_1) + P(A_2) + P(A_3) + P(A_4) + P(A_5)$$
$$+ P(A_1 \cap A_2) + P(A_1 \cap A_3) + P(A_1 \cap A_4) + P(A_1 \cap A_5)$$
$$+ P(A_2 \cap A_3) + P(A_2 \cap A_4) + P(A_2 \cap A_5)$$
$$+ P(A_3 \cap A_4) + P(A_3 \cap A_5)$$
$$+ P(A_4 \cap A_5)$$
$$+ P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_4) + P(A_1 \cap A_2 \cap A_5)$$
$$+ P(A_1 \cap A_4 \cap A_3) + P(A_1 \cap A_4 \cap A_5) + P(A_1 \cap A_3 \cap A_5)$$
$$+ P(A_2 \cap A_4 \cap A_3) + P(A_2 \cap A_4 \cap A_5) + P(A_2 \cap A_3 \cap A_5) + P(A_3 \cap A_4 \cdots$$

$+ P(A_1 \cap A_2 \cap A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_3 \cap A_5) + P(A_1 \cap A_2 \cap A_4 \cap A_5)$
$+ P(A_1 \cap A_3 \cap A_4 \cap A_5) + P(A_2 \cap A_3 \cap A_4 \cap A_5)$
$+ P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5)$

7. Prove:

(1) $A_n \setminus A_{n-1} = B_n$ 取 $A_0 = \phi$

$$P\left(\bigcup_{i=1}^{\infty} A_n\right) = P\left(\bigcup_{i=1}^{\infty} B_n\right) = \sum_{i=1}^{\infty} P(B_i) = \sum_{i=1}^{\infty} [P(A_n) - P(A_{n-1})]$$

$$= \lim_{k \to \infty} (P(A_k) - P(A_0))$$

$$= \lim_{k \to \infty} P(A_k) - P(A_0)$$

$$= \lim_{n \to \infty} P(A_n)$$

(2) Let $B_n = A_1 \setminus A_n$, so B_n is increase set.

$\bigcup_{i=1}^{\infty} B_n = \bigcup_{i=1}^{\infty} (A_1 \setminus A_n) = A_1 \setminus \bigcap_{i=1}^{\infty} A_n$

Then $\lim_{n \to \infty} P(B_n) = P\left(\bigcup_{i=1}^{\infty} B_n\right)$

$$\lim_{n \to \infty} P(A_1 \setminus A_n) = P\left(A_1 \setminus \bigcap_{i=1}^{\infty} A_n\right)$$

$$\lim_{n \to \infty} P(A_1) - \lim_{n \to \infty} P(A_n) = P(A_1) - P\left(\bigcap_{i=1}^{\infty} A_n\right)$$

$$\lim_{n \to \infty} P(A_n) = P\left(\bigcap_{i=1}^{\infty} A_n\right)$$

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