

MA 215 Probability Theory

Exercise Sheet 8

Set: Friday 4th November; Hand in: Friday 11th November by 5pm.

1. Suppose that the continuous random variable X has pdf

$$f_X(x) = \begin{cases} kx(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate the constant k .

Find the non-zero range of possible Y values and the pdf $f_Y(y)$ of Y when (a) $Y = -3X + 3$,
(b) $Y = 1/X$.

2. Suppose that the random variable X has cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ (1 - \cos(x))/2 & 0 \leq x \leq \pi \\ 1 & x > \pi \end{cases}$$

and that $Y = \sqrt{X}$.

What is the non-zero range of Y ?

Find the cumulative distribution function $F_Y(y)$ of y , and hence find the pdf of Y .

3. Suppose that the two random variables X and Y have joint probability cumulative function $F(x, y)$. Show that $F(x, y)$ possesses the following properties:

- (a) For any fixed x , $F(x, y)$ is a non-decreasing function of y and, similarly, for any fixed y , $F(x, y)$ is a non-decreasing function of x .
- (b) $F(x, y) \rightarrow 1$ when both $x \rightarrow +\infty$ and $y \rightarrow +\infty$.
- (c) $F(x, y) \rightarrow 0$ when either $x \rightarrow -\infty$ or $y \rightarrow -\infty$.
- (d) If $x_1 < x_2$ and $y_1 < y_2$, then

$$\Pr(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) + F(x_1, y_1)$$

4. Suppose that the two discrete random variables X and Y have joint probability mass function given by

	$Y = 1$	$Y = 2$	$Y = 3$	$Y = 4$
$X = 1$	2/32	3/32	4/32	5/32
$X = 2$	3/32	4/32	5/32	6/32

Obtain the marginal probability mass function of X .

5. Continuous random variables X and Y have joint pdf

$$f(x, y) = x + y \quad (0 \leq x \leq 1, 0 \leq y \leq 1)$$

- (a) Find the marginal pdfs of X and Y .
- (b) Find $\Pr(X > Y)$,
- (c) Find $\Pr(X \leq 0.5)$.