No: Homework 2 109-1 機器學習 - SVM 推導.

Date: /

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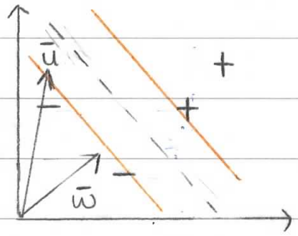
SVM ⇒ 監督式學習，以統計風險最小化的原則來估測一個分類的超平面。




$\bar{w}\cdot\bar{u} \geq C$ ← 常數

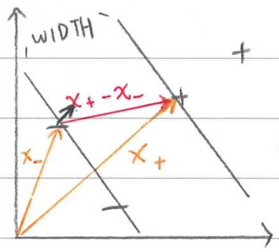
① $\bar{w}\cdot\bar{u}+b \geq 0$ + 樣本

$C=-b$

⇒ "+" : $\bar{w}+\bar{x}_+ + b \geq 1$ ➜ $y_i(x_i\cdot\bar{w}+b)\geq 1$ ➜ $y_i(x_i\bar{w}+b)-1\geq 0$

"−" : $\bar{w}+\bar{x}_- + b \leq -1$ ➜ $y_i(x_i\cdot\bar{w}+b)\leq 1$ ⇒ ② $y_i(x_i\bar{w}+b)-1=0$

↳ for $x_i$ in gutter




$\text{WIDTH} = (\bar{x}_+ - \bar{x}_-)\cdot\frac{\bar{w}}{\|w\|} = \frac{2}{\|w\|}$

↳ $\bar{x}_+ = 1-b$ , $\bar{x}_- = 1+b$

$\text{MAX}\ \frac{2}{\|w\|} \Rightarrow \text{MAX}\ \frac{1}{\|w\|} \Rightarrow \min\|w\| \Rightarrow \min \frac{1}{2}\|w\|^2$

$L=\frac{1}{2}\|\bar{w}\|^2-\sum\alpha_i[y_i(\bar{w}\cdot\bar{x}_i+b)-1]$

$\frac{\partial L}{\partial w}=\bar{w}-\sum\alpha_i y_i x_i=0$ ➜ $\bar{w}=\sum_i\alpha_i y_i x_i$ ③

$\frac{\partial L}{\partial b}=-\sum\alpha_i y_i=0$ ➜ $\boxed{\sum\alpha_i y_i=0}$

$L=\frac{1}{2}(\sum\alpha_i y_i\bar{x}_i)(\sum\alpha_j y_j\bar{x}_j)-\sum\alpha_i y_i x_i(\sum\alpha_j y_j x_j)-\underbrace{\boxed{\sum\alpha_i y_i}}_{=0}b+\sum\alpha_i$

$=\sum\alpha_i-\frac{1}{2}\sum_i\sum_j\alpha_i\alpha_j y_i y_j x_i x_j$ ④ 求最大值

$\sum\alpha_i y_i\bar{x}_i\cdot\bar{u}+b\geq 0$ ➜ "+"

不可線性分離: $\phi(\bar{x}_i)\cdot\phi(\bar{x}_j)$ to MAX $K(x_i,x_j)=\phi(\bar{x}_i)\cdot\phi(\bar{x}_j)$

$\phi(x_i)\cdot\phi(x_u)$

⇒ ① $(\bar{u}\cdot\bar{v}+1)^n$ ② $e^{-\frac{\|x_i-y_i\|}{\sigma}}$