Exercises: Returns, Spreads and Yields

Fixed Income Securities - Tuckman and Serrat

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Problem 1. The price of the $\frac{3}{4}$s of May 31, 2012 was 99.961 as of May 31, 2010. Calculate the price using the discount factors in Table 2.3. Is the bond trading cheap or rich to those discount factors? Then, using trial-and-error, express the price difference as a spread to the spot rate curve implied by those discount factors.

Solution The table is a typo in the book, it should be Table 1.3. We have the following discount factors:

Date	Discount factor
11/2010	0.99925
05/2011	0.99648
11/2011	0.99135
05/2012	0.98532

Given the dates, we have 4 cash flows of 0.75%, and the price is

$$\begin{aligned} P &= \frac{c}{2}\sum_{t=1}^{2T} d\Big(\frac{t}{2}\Big) + \Big(100 + \frac{c}{2}\Big)d(T) \\ &= \frac{0.75}{2}(0.99925 + 0.99648 + 0.99135 + 0.98532) + \Big(100 + \frac{0.75}{2}\Big)(0.98532) \\ &= 100.391 \end{aligned} \tag{1}$$

We have a market price of 99.961 and a present value of 100.391. Since the market price is lower than the PV, the bond is trading cheap.

In order to express the price difference as a spread to the spot rate curve, we need to calculate the spot rates. We know that spot rates $\hat{r}$ are expressed as

$$\begin{aligned} 1 &= \Big(1 + \frac{\hat{r}(t)}{2}\Big)d(t) \\ \implies \hat{r}(t) &= 2\Big[\Big(\frac{1}{d(t)}\Big)^{\frac{1}{2t}} - 1\Big] \\ \hat{r}(0.5) &= 0.150\% \\ \hat{r}(1) &= 0.353\% \\ \hat{r}(1.5) &= 0.580\% \\ \hat{r}(2) &= 0.741\% \end{aligned} \tag{2}$$

We can express the market price of a bond in terms of these spot rates and spreads:

$$\begin{aligned}P &= \frac{c/2}{1+\frac{r(0.5)}{2}+\frac{s}{2}} + \frac{c/2}{\left(1+\frac{r(1)}{2}+\frac{s}{2}\right)^2} + \frac{c/2}{\left(1+\frac{r(1.5)}{2}+\frac{s}{2}\right)^3} + \frac{100+c/2}{\left(1+\frac{r(2)}{2}+\frac{s}{2}\right)^2} \\ \implies 99.961 &= \frac{0.375}{1+\frac{0.150}{2}+\frac{s}{2}} + \frac{0.375}{\left(1+\frac{0.353}{2}+\frac{s}{2}\right)^2} + \frac{0.375}{\left(1+\frac{0.580}{2}+\frac{s}{2}\right)^3} + \frac{100+0.375}{\left(1+\frac{0.741}{2}+\frac{s}{2}\right)^2} \\ \implies s &= 0.0307\%\end{aligned} \tag{3}$$

Problem 2. The yield of the $\frac{3}{4}$s of May 31, 2012, was 0.7697% as of May 31, 2010. Verify that this is consistent wiht the price in Question 1.

Solution We need to price the bond using the yield given, and for it to equal 99.961. There are 4 cash flows, so we have

$$\begin{aligned}P &= \frac{c/2}{1+\frac{y}{2}} + \frac{c/2}{\left(1+\frac{y}{2}\right)^2} + \frac{c/2}{\left(1+\frac{y}{2}\right)^3} + \frac{100+c/2}{\left(1+\frac{y}{2}\right)^4} \\ &= \frac{0.375}{1+\frac{0.007697}{2}} + \frac{0.375}{\left(1+\frac{0.007697}{2}\right)^2} + \frac{0.375}{\left(1+\frac{0.007697}{2}\right)^3} + \frac{100.375}{\left(1+\frac{0.007697}{2}\right)^4} \\ &= 99.961,\end{aligned} \tag{4}$$

which shows that the yield is consistent with the price from the previous question.

Problem 3. The price of the $4\frac{3}{4}$s of May 31, 2012, was 107.9531 as of May 31, 2010. What was the yielf of the bond?

Solution using the equation relating price with yield from the previous question, we have

$$107.9531 = \frac{2.371}{1+\frac{y}{2}} + \frac{2.371}{\left(1+\frac{y}{2}\right)^2} + \frac{2.371}{\left(1+\frac{y}{2}\right)^3} + \frac{102.371}{\left(1+\frac{y}{2}\right)^4} \tag{5}$$

Using trial-and-error or something like Wolfram Alpha gives

$$y = 0.736\%$$

Problem 4. Did you get a higher yield for the $4\frac{3}{4}$s from Question 3 than the yield of the $\frac{3}{4}$s given in Question 3? Is that what you expected? Why or why not?

Solution The yield on the $4\frac{3}{4}$s is 0.736%, and the yield on the $\frac{3}{4}$s is 0.7697%, so the yield is higher for the $\frac{3}{4}$s.

In Question 1, we computed the simple rates across the curve (Equation 2), and they demonstrated a rising term structure. With rising term structures, for any given maturity higher-coupon bonds have a lower yield due to more discounting across the curve. Hence, our higher-coupon bond, the $4\frac{3}{4}$s, has a lower yield.

Problem 5. An investor purchases the $4\frac{3}{4}$s of May 31, 2012 on May 31, 2010, at the yield given in Question 3. Exactly 6 months later the investor sells the bond at the same yield. What is the price of the bond on the sale date and what is the investor's total return from the bond over those 6 months?

Solution The investor buys the bond at a price of 107.9531 and a yield of 0.736%. We know that it is sold at this yield 6 months later, so we can compute the new price, remembering that there is now one less cash flow to discount because it is 6 months later:

$$\begin{aligned} P_{\text{sell}} &= \frac{c/2}{1+\frac{y}{2}} + \frac{c/2}{\left(1+\frac{y}{2}\right)^2} + \frac{100+c/2}{\left(1+\frac{y}{2}\right)^3} \\ &= \frac{2.375}{1+\frac{0.00736}{2}} + \frac{2.375}{\left(1+\frac{0.00736}{2}\right)^2} + \frac{102.375}{\left(1+\frac{0.00736}{2}\right)^3} \\ &= 105.9770 \end{aligned} \tag{6}$$

This gives the following total return:

$$\begin{aligned} R &= \frac{P_1(\mathbb{R}) + c - P_0(\mathbb{R})}{P_0(\mathbb{R})} \\ &= \frac{105.9770 + 2.375 - 107.9531}{107.9531} \\ &= 0.3695\% \end{aligned} \tag{7}$$

Problem 6. Interpret your answer to Question 5. In what way is the return significant or interesting? Explain why an investor would buy a premium bond when it is only going to be worth par at maturity. How does this relate to your work in Question 5?

Solution The return is interesting because:

$$\begin{aligned} R = 0.3695\% &\approx \frac{0.736\%}{2} = \frac{y}{2} \\ R &\approx \frac{y}{2}. \end{aligned} \tag{8}$$

We know that under unchanged yield assumptions in carry-rolldown scenarios, the one-period return is equal to the yield. Here we have a half-period return and an unchanged yield, so the above result of $R \approx y/2$ makes sense.

When an investor holds a premium bond, as it pulls towards par they will be earning cash flow payments above the market rate, so are happy to own one despite it maturing at par. This effect can be seen from the calculation in Question 5, where we showed that, despite dropping in price, the investor earned a positive return.

Start **End**	**05/2010** **11/2010**	**11/2010** **05/2011**	**05/2011** **11/2011**	**Price**
Initial forwards	0.193%	0.600%	1.080%	100.190
Term structure	0.149%	0.556%	1.036%	
Spreads	0.044%	0.044%	0.044%	

Table 1: Initial term structure.

Start **End**	**05/2010** **11/2010**	**11/2010** **05/2011**	**05/2011** **11/2011**	**Price**
Carry-rolldown forwards		0.193%	0.600%	100.353
Term structure		0.149%	0.556%	
Spreads		0.044%	0.044%	

Table 2: Rates after considering carry-rolldown.

Problem 7. Re-compute the sample return decomposition of Tables 3.2 and 3.3 of the text, replacing the assumption of realised forwards with the assumption of an unchanged term structure.

Solution We have two pricing dates: May 2010 and November 2010. The initial term structure of the bond can be seen in Table 1, pricing in May 2010. This is nothing other than replicating the initial term structures from the text.

Now the second pricing date is November 2010, 6 months later. We know that the term structure is unchanged (the assumption required by the question), so the rates in Table 1 can all just shift to the right (i.e. the term structure is the same when rolled forward 6 months). So this gives Table 2 with the carry-rolldown forward rates. The price of 100.353 comes from discounting the $\frac{3}{4}$s coupons with forward rates (remember that in text the price decomposition is done with the $\frac{3}{4}$s bond):

$$
\begin{aligned}
P &= \frac{c/2}{1+\frac{f(0.5)}{2}} + \frac{100+c/2}{\left(1+\frac{f(0.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)} \\
&= \frac{0.375}{1+\frac{0.00193}{2}} + \frac{100.375}{\left(1+\frac{0.00193}{2}\right)\left(1+\frac{0.00600}{2}\right)} \\
&= 100.353.
\end{aligned}
\tag{9}
$$

Now we need to consider the rate-change forwards. From the text, we know that the market sees a parallel shift in the curve of 10bps lower over the 6 months, so we have a net rate change across the term stucture of -10bps. Subtracting this from the carry-rolldown forward rates gives us Table 3. The new bond price comes from implementing the same calculation as that done for the carry-rolldown forwards:

Start	**05/2010**	**11/2010**	**05/2011**	
End	**11/2010**	**05/2011**	**11/2011**	**Price**
Rate-change forwards		0.093%	0.500%	100.453
Term structure		0.049%	0.456%	
Spreads		0.044%	0.044%	

Table 3: Rates after considering the market rate changes.

Start	**05/2010**	**11/2010**	**05/2011**	
End	**11/2010**	**05/2011**	**11/2011**	**Price**
Spread-change forwards		0.049%	0.456%	100.497
Term structure		0.049%	0.456%	
Spreads		0.000%	0.000%	

Table 4: Rates are considering the market spread changes.

$$\begin{aligned} P &= \frac{0.375}{1+\frac{0.00093}{2}} + \frac{100.375}{\left(1+\frac{0.00093}{2}\right)\left(1+\frac{0.00500}{2}\right)} \\ &= 100.453. \end{aligned} \tag{10}$$

Finally, we need to consider the spread-change forwards. From the text, we know that over the 6 month period the spreads converge to nothing, so we simply reduce the spreads to 0, and leave the term structure the same. The forwards are calculated by adding the term structure and the spread, so these become $0.049 + 0 = 0.049$, and $0.456 + 0 = 0.456$. This can all be seen in Table 4, with the price calculated as above.

We can summarise everything in Table 5. From this we can give a P&L decomposition in Table 6. The total P&L can easily be calculated by adding the carry-rolldown P&L, rates-change P&L, spread-change P&L, and the coupon payment:

$$\begin{aligned} \text{Carry-roll P\&L} &= 100.353 - 100.190 = 0.163 \\ \text{Rates-change P\&L} &= 100.453 - 100.353 = 0.100 \\ \text{Spread-change P\&L} &= 100.497 - 100.453 = 0.044 \\ \text{Coupon payment} &= 0.375 \\ \text{Total P\&L} &= 0.163 + 0.100 + 0.044 + 0.375 = 0.682 \end{aligned} \tag{11}$$

Start **End**	**05/2010** **11/2010**	**11/2010** **05/2011**	**05/2011** **11/2011**	**Price**
Initial forwards	0.193%	0.600%	1.080%	100.190
Term structure	0.149%	0.556%	1.036%	
Spreads	0.044%	0.044%	0.044%	
Carry-rolldown forwards		0.193%	0.600%	100.353
Term structure		0.149%	0.556%	
Spreads		0.044%	0.044%	
Rate-change forwards		0.093%	0.500%	100.453
Term structure		0.049%	0.456%	
Spreads		0.044%	0.044%	
Spread-change forwards		0.049%	0.456%	100.497
Term structure		0.049%	0.456%	
Spreads		0.000%	0.000%	

Table 5: Summary of all rates changes and prices.

Initial price	100.190
Price appreciation	+0.307
Carry-rolldown	+0.163
Rates-change	+0.100
Spread-change	+0.044
Cash-carry	+0.375
Coupon	+0.375
Financing	0.000
Total P&L	**0.682**

Table 6: Summary of P&Ls.

Problem 8. Start with any upward-sloping term structure. Then replicate the 0-coupon, par, and 9% coupon curves. Add a curve for a security that makes equal fixed payments to various maturities, i.e. a mortgage.

Solution We have seen that for upward sloping term structures, the 0-coupon yield curve lies above the par yield curve, which lies above the 9% coupon yield curve. The question is asking us to place the curve for our new bond relative to these three.

A bond that makes equal fixed payments to various maturities, such as a mortgage, is an annuity. Annuities pay a fixed dollar amount each period (like rolling over many zero-coupon bonds), and the annuity's yield is the average of the zero rates associated with each of its cash flows. This means that it definitely lies below the zero-coupon curve, due to simple arithmetic.

Now we need to consider its position relative to the coupon bond. A coupon bond is a stream of coupon payments of equal size plus a large payment of par at maturity. Since an annuity is a stream of fixed dollar amounts, a coupon bond is simply an annuity (the stream of fixed payments) plus a zero bond of the same maturity (final principal payment). Hence the coupon bond yield is an average of an annuity yield and a 0-coupon yield. Since the 0-coupon yield lies above the coupon bond yield, the annuity must lie below the coupon bond yield.

Hence, with rising term structures, the curve of a security making equal fixed payments to various maturities (an annuity) lies below the 9% coupon bond yield curve, which lies below the par yield curve, which lies below the 0-coupon bond yield curve.

Problem 9. In the subsection "News Excerpt: Sale of Greek Government Bonds in March, 2010", approximately what is the yield on seven-year Spanish debt?

Solution We are told that the Greek 7 year yield of 6% lies 3.34% above the bund yield. This gives a bund yield of 2.66%. We are also told that Spanish debt lies 0.61% above the bund yield. So Spanish debt has yield

$$y = 3.27\%$$