Exercises: Empirical Approaches to Risk Metrics and Hedging

Fixed Income Securities - Tuckman and Serrat

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The following introduction applies to Questions 1 through 5. You are a market maker in the long-term EUR interest rate swaps. You typically have to hedge the interest rate risk of having received from, or paid to, a customer on a 20y interest rate swap. Given the transaction costs of hedhing with both 10s and 30s and the relatively short time you wind up having to hold any such hedge, you consider hedging these 20y swaps with either 10s or 30s, but not both. To that end, you run two single-variable regressions, both with changes in the 20y EUR swap rates as the dependent variable, but one regression with changes in the 10y swap rate as the independent variable and the other with changes in the 30y swap rate as the independent variable. The results over the period July 1st, 2009, to July 3rd, 2010, are given in the following table:

Independent Variable	Change in 10y	Change in 30y
R-squared	89.9%	96.3%
Standard Error	1.105	0.666

Table 1: Global statistics for the regression. Number of observations: 259

Regression coefficients	Value	Std. Error
Constant (α)	-0.017	0.069
Independent Variable (β)	1.001	0.021

Table 2: Regression statistics for the change in 10y.

Regression coefficients	Value	Std. Error
Constant (α)	-0.008	0.042
Independent Variable (β)	0.917	0.011

Table 3: Regression statistics for the change in 30y.

Problem 1. What are the 95% confidence intervals around the constant and slope coefficients of each regression?

Solution By the properties of a normally distributed random variable, values in an interval of 95% confidence fall within 2 standard deviations of the observed value. With the standard error represented as σ, this gives us for the 10y:

$$\begin{aligned}\alpha \pm 2\sigma &= (\alpha - 2\sigma, \alpha + 2\sigma) \\ &= (-0.017 - 2(0.069), -0.017 + 2(0.069)) \\ &= (-0.155, 0.121)\end{aligned} \tag{1}$$

and

$$\begin{aligned}\beta \pm 2\sigma &= (\beta - 2\sigma, \beta + 2\sigma) \\ &= (1.001 - 2(0.021), 1.001 + 2(0.021)) \\ &= (0.959, 1.043).\end{aligned} \tag{2}$$

With analogous computation for the 30y, we get

$$\alpha \pm 2\sigma = (-0.092, 0.076)$$

and

$$\beta \pm 2\sigma = (0.895, 0.939).$$

Problem 2. Use the confidence intervals just derived. Can you reject a) the hypothesis that the constant in the 10y regression equals 0? b) That the slope coefficient in the 30y regression equals 1?

Solution For the 10y, the constant, α, falls in the interval $(-0.155, 0.121)$ with 95% confidence. Since $0 \in (-0.155, 0.121)$, we cannot reject the hypothesis that $\alpha = 0$.

For the 30y, the slop coefficient, β, falls in the interval $(0.895, 0.939)$. Since $1 \notin (0.895, 0.939)$, we can reject this hypothesis with 95% confidence. However, note that at higher confidence levels the interval will be larger, and the hypothesis will not be so easily rejected.

Problem 3. As the swap market maker, you just paid fixed in \$100m notional of the 20y swaps. The DVO1s of the 10y, 20y, and 30y swaps are 0.0864, 0.1447, and 0.1911, respectively. Were you to hedge with 10y swaps, what would you trade to hedge? And with 30y swaps?

Solution In order to effectively hedge we want to create a DV01-neutral position. We must also consider the possible variation in 10y rate as dictated by $\beta^{(10)}$ provided in the question. The net position is

$$F^{(10)}\frac{DV01^{(10)}}{100} = F^{(20)}\frac{DV01^{(20)}}{100}\hat{\beta}^{(10)} \tag{3}$$

We want to compute the face value for the 10y hedge, so rearrange for $F^{(10)}$:

$$\begin{aligned} F^{(10)} &= F^{(20)}\frac{DV01^{(20)}}{DV01^{(10)}}\beta^{(10)} \\ &= \$100m \times \frac{0.1447}{0.0864} \times 1.001 \\ &= \$167.644\text{m.} \end{aligned} \tag{4}$$

Similarly for the 30y hedge:

$$\begin{aligned} F^{(30)} &= F^{(20)}\frac{DV01^{(20)}}{DV01^{(30)}}\beta^{(30)} \\ &= \$100m \times \frac{0.1447}{0.1911} \times 0.917 \\ &= \$69.435\text{m.} \end{aligned} \tag{5}$$

So whilst paying \$100m in the 20y swap, we could hedge by either receiving \$167.644m in the 10y swap, or receiving \$69.435m in the 30y swap.

Problem 4. Approximately what would be the standard deviation of the P&L of a hedged position of 20y swaps with 10y swaps? And if hedged with 30y swaps?

Solution The net P&L of the hedged position (when hedged with 10y swaps) is given by

$$P\&L = F^{(10)}\frac{DV01^{(10)}}{100}\Delta y^{(10)} - F^{(20)}\frac{DV01^{(20)}}{100}\Delta y^{(20)}.$$

Substituting Equation 4 (in Problem 3's solution) gives

$$\begin{aligned} P\&L &= \left(F^{(20)}\frac{DV01^{(20)}}{DV01^{(10)}}\beta^{(10)}\right)\frac{DV01^{(10)}}{100}\Delta y^{(10)} - F^{(20)}\frac{DV01^{(20)}}{100}\Delta y^{(20)} \\ &= F^{(20)}\tfrac{DV01^{(20)}}{100}\beta^{(10)}\Delta y^{(10)} - F^{(20)}\frac{DV01^{(20)}}{100}\Delta y^{(20)} \\ &= F^{(20)}\frac{DV01^{(20)}}{100}(\beta^{(10)}\Delta y^{(10)} - \Delta y^{(20)}). \end{aligned} \tag{6}$$

Using the linear regression formula, $\Delta y^{(20)} = \alpha + \beta^{(10)}\Delta y^{(10)} + \epsilon$, this becomes

$$P\&L = -F^{(20)}\frac{DV01^{(20)}}{100}(\alpha + \epsilon).$$

We know that ϵ is the error (variance) of the observed rates movements, and we also know that the constance *alpha* is largely unimportant. (Because most regression analyses of rates changes pass very close to the origin by construction.) This means that $\alpha + \epsilon$ approximates the standard error, and we can compute the standard deviation of the P&L as:

$$\begin{aligned} \text{sdev}(P\&L)_{10y} &\approx F^{(20)}\frac{DV01^{(20)}}{100}\sigma_1 0 \\ &= \$100m \times \frac{0.1447}{100} \times 1.105 \\ &= \$159,893.50. \end{aligned} \tag{7}$$

Similarly for the position hedged with 30y swaps:

$$\begin{aligned} \text{sdev}(P\&L)_{30y} &\approx F^{(20)}\frac{DV01^{(20)}}{100}\sigma_3 0 \\ &= \$100m \times \frac{0.1447}{100} \times 0.666 \\ &= \$96,370.20. \end{aligned} \tag{8}$$

> **Problem 5.** If you were to hedge with one of either the 10y or 30y swaps, which would it be and why?

Solution The standard deviation of the P&L calculated in Problem 4 gives the daily volatility of the hedged position. With a hedged position, it is good to minimise the volatility as much as possible. Clearly, the dollar volatility is lower or the 30y hedge, making it more desirable.

> **Problem 6.** Use the principal components in Table 6.5 (in the book) and the par swap data in Table 6.6 (in the book) to hedge 100 face amount of 10y swaps with 5y and 10y swaps with respect to the first two principal components.

Solution For a hedge, we want to neutralise the DV01. So we write the net P&L as

$$F^{(5)}\frac{DV01^{(5)}}{100}PC^{(5)} + F^{(30)}\frac{DV01^{(30)}}{100}PC^{(30)} = 100\frac{DV01^{(10)}}{100}PC^{(10)},$$

where $PC^{(t)}$ is the principal component for the swap of term t. Create two equations by substituting the level components into it, and then the slope components into it. Solving for $F^{(5)}$ and $F^{(30)}$ gives us

$$\begin{aligned} F^{(5)} &= 75.188, \\ F^{(30)} &= 31.53, \end{aligned} \tag{9}$$

with both values being relative to the 100 face amount of the 10y swaps.