Real-time Zero Phase Filtering for Heave Measurement

Hu Yongpan[1], Tao Limin[2]
Laboratory of Science and Technology on Integrated Logistics Support
College of Mechatronics and Automation
National University of Defense Technology, Changsha, China
Email: [1]*huyongpan1986@163.com*, [2]*tlm1964@sina.com*

Abstract *–Heave measurement is an important technology for many marine works. Inertial Measurement Unit (IMU), which is able to sense the accelerations of the ship motion and integrate them to compute heave velocity and displacement, is the most widely used device for heave measurement. High-pass filters are always used in heave measurement to separate the heave signals from integration drifts. However, linear filters inherently alter amplitude and phase characteristics of the desired signals and incur attenuation and phase-shift. A new method that performs real-time zero phase filtering to improve the accuracy of measurement is presented in this paper. The basic concept of the proposed zero phase filter is to design a pair of cascaded high-pass and all-pass IIR filters, such that the phase lead caused by the high-pass filter is balanced by the phase lag introduced by the all-pass filter. The weighted-frequency Fourier linear combiner (WFLC) algorithm is adopted as an adaptive frequency estimator to provide the instantaneous dominant frequency of the signal of interest to adjust the coefficients of all-pass filter to yield the required performance. The proposed filter is implemented in an experiment to measure the heave motion of a parallel motion simulator.*

Keywords *– Heave measurement, zero phase filter, WFLC*

I. INTRODUCTION

Many offshore works, such as underway replenishment, landing on and off of ship-carried airplanes and subsea exploring, are influenced by wave induced ship motion. Among the six degrees of freedom, the heave motion courses most troubles and must be compensated in conditions of certain sea state. As a necessary technology of heave compensation, heave measurement plays an important role in ocean engineering.

The most wildly used method of heave measurement is based on IMU, which contains accelerometers and gyroscopes. Heave acceleration of ship can be measured by IMU and heave displacement can be obtained through integration calculation. It is a well-known fact that the use of numerical integration of acceleration inherently causes errors to grow with time, which is commonly known as "integration drift". For that reason, measurement of displacement or position using inertial sensors is usually performed with the help of externally-referenced aided sensors or sensing systems. For instance, Godhavn combined GPS RTK with accelerometer to measure heave motion and got a higher accuracy in sea trial in comparison with using any device alone[1]. However, GPS RTK can't work without the assistant of reference station and getting a high-accuracy referencing signal on ocean is quite difficult. Due to the frequency difference between the desired signal and the integration drift, another more convenient and effective method is performing filtering or estimating to the integrated signals. In 1998, Godhavn proposed an adaptive high-pass filter which can change the filter settings according to the sea condition and applied it to the Seatex MRU[2]. In 2009, Yang et al designed an adaptive heave filter using a method similar with Godhavn's, and did more numerical simulation and testing analysis[3]. In 2011, Kuchler et al proposed a new observer based heave estimation method, in which the observer model is formulated by a sum of periodic components that approximate the heave motion of a vessel and used extended Kalman filter to estimate heave motion[4]. In 2012, Sun et al analyzed the influence of navigation solution on heave measurement and designed a FIR high-pass filter according to the analysis[5]. However, linear filters inherently alter amplitude and phase characteristics of the desired signals and incur attenuation and phase-shift, which become the main errors after filtering and no remarkable research has been taken to solve this problem so far. In order to improve the accuracy of heave measurement, this paper presents a new method that performs real-time zero phase filtering to the integrated signals, of which the basic concept is to design a pair of cascaded high-pass and all-pass IIR filters, such that the phase lead caused by the high-pass filter is balanced by the phase lag introduced by the all-pass filter. The weighted-frequency Fourier linear combiner (WFLC) algorithm is adopted as an adaptive frequency estimator to provide the instantaneous dominant frequency of the signal of interest to adjust the coefficients of all-pass filter to yield the required performance.

This paper will be structured as follows: In Section II we will introduce the principle of heave measurement based on IMU. In Section III we will describe the details of the proposed adaptive zero phase high-pass filter, including design and error analysis of traditional IIR high-pass filter, real-time phase compensation and adaptive frequency estimator. An experiment measuring the heave motion of a parallel motion simulator is given

978-1-4799-0759-5 /13/$31.00 ©2013 IEEE

using the proposed filter in section Ⅳ. Finally, section Ⅴ provides concluding remarks.

II. HEAVE MEASUREMENT BASED ON IMU

IMU usually contains tri-axes accelerometers and tri-axes gyros, which sense accelerations and angular rates respectively. IMU is fixed in the body coordinate frame of the ship (B-frame) while the reference of heave measurement is motion coordinate frame of the ship (R-frame). These two frames are defined as follows.

B-frame is fixed to the ship with its origin in the ship's center of gravity. The longitudinal axis OY_B points in the bow direction of the ship and the transverse axis OX_B points to the starboard of the ship. OZ_B is perpendicular with above two axes and points up.

R-frame is the same as B-frame when the ship is static. The origin locates in the balance point of the ship's center of gravity. OY_R and OX_R are always in the horizontal plane and OY_R points in the heading direction while OX_R points to the starboard position without roll. OZ_R is in the vertical direction and points up.

Generally, the fixed point F of IMU and the measurement point M are different. Because of the lever arm effect caused by the ship's roll and pitch, the heave motion of F and M are also different. The accelerations of the measurement point M can be calculated by:

$$\mathbf{a}_M^R = R_B^R(\mathbf{a}_F^B + \mathbf{b} + \mathbf{w}_a - (\boldsymbol{\omega}\times\boldsymbol{\omega} + \dot{\boldsymbol{\omega}})\times\mathbf{r}_{MF}^B) \tag{1}$$

where R_B^R denotes the transform matrix from B-frame to R-frame. $\mathbf{b}$ and $\mathbf{w}_a$ are the bias and measurement noise of the vertical accelerometer respectively. $\boldsymbol{\omega}$ stands for the angular rate output of IMU. $\mathbf{r}_{MF}^B$ is the vector from point M to point F represented in B-frame. The heave velocity and displacement of measurement point can be obtained by single and double integration of $\mathbf{a}_M^R$.

III. ADAPTIVE ZERO PHASE HIGHPASS FILTER

A. Design and error analysis of IIR high-pass filter

The sources of integration drift can be divided into two categories, which are measurement bias and misalignment respectively. The measured accelerations are usually mixed with high frequency noise and low frequency bias. Most measurement noise can be removed by the anti-aliasing filter and the integration process, but the bias and some other low frequency errors will accumulate. Moreover, when performing numerical integration, the initial velocity and displacement are usually set to 0, which are different from the actual values and will introduce misalignment errors.

For convenience, suppose the acceleration, velocity and displacement of heave motion are $a(t)$, $v(t)$, $p(t)$ and the initial time $t_0 = 0$. Let v_0 and p_0 denote the actual velocity and displacement at t_0. It is obvious that

$$v(t) = \int_0^t a(t)dt + v_0 \tag{2}$$

$$p(t) = \int_0^t v(t)dt + p_0 = \int_0^t\int_0^t a(t)d^2t + v_0 t + p_0 \tag{3}$$

If the initial velocity and displacement are set to 0 and the measured acceleration contains bias $b(t)$, the calculated velocity and displacement are

$$\hat{v}(t) = \int_0^t (a(t)+b(t))dt \tag{4}$$

$$\begin{aligned}\hat{p}(t) &= \int_0^t\int_0^t (a(t)+b(t))d^2t \\ &= p(t) - v_0 t - p_0 + \int_0^t\int_0^t b(t)d^2t\end{aligned} \tag{5}$$

Because the heave motion is generally periodic with a period less than 25s but integration drifts are low frequency components, a high-pass filter is the most direct solution to remove the drifts. After filtering, the calculated displacement equals to the true value theoretically, i.e.

$$\hat{p}(t) = p(t) \tag{6}$$

Digital filters are divided into two classes, IIR filters and FIR filters. FIR filters have strict linear phase response but need higher orders to achieve similar amplitude response with IIR filters. It has to be noticed that linear phase means the group delay $\tau(\omega) = -d\theta(\omega)/d\omega$ is constant but the phase error still exits. In this paper, an IIR filter is designed to process heave motion signals.

Designing an IIR filter usually needs two steps:

Step 1. design an analog prototype filter meeting the requests;

Step 2. change the transfer function of the analog filter $H_{hpa}(s)$ to a digital filter's $H_{hpd}(z)$.

We design a fourth order high-pass filter as the prototype filter:

$$H_{hpa}(s) = \frac{\hat{p}}{a_M}(s) = \frac{s^2}{(s^2 + 2\zeta\omega_c s + \omega_c^2)^2} \tag{7}$$

where s is the Laplace variable, ζ is the damping coefficient and ω_c is the cutoff frequency of the filter. For simplicity, in this paper $\zeta = 1/\sqrt{2}$, which is the critical damping coefficient.

In this paper, the bilinear transformation is adopted to perform the conversion between analog filter and digital filter, of which the basic relationship is:

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \tag{8}$$

where T represents the step size in the numerical integration.

The high-pass filter changes the amplitude and phase characteristic of the desired signals. Since the heave displacement is the double integration of the heave acceleration, we have

$$p(s) = \frac{a(s)}{s^2} \tag{9}$$

Combining with (7), we can get:

$$\hat{p}(s) = s^2 H_{hpa}(s) p(s) \tag{10}$$

Leave the phase-shift out of account, the amplitude error induced by the high-pass filter is

$$\left|1-\left|s^2 H_{hpa}(s)\right|\right| |_{s=j\omega} = \left|1-\frac{\omega^4}{\omega^4+\omega_c^4}\right| \to \left(\frac{\omega_c}{\omega}\right)^4 \text{ for } \omega \gg \omega_c \tag{11}$$

The total error of the high-pass filter is:

$$\left|1-s^2 H_{hpa}(s)\right| |_{s=j\omega} = \frac{\sqrt{\omega_c^8+8\omega^6\omega_c^2}}{\omega^4+\omega_c^4} \to 2\sqrt{2}\left(\frac{\omega_c}{\omega}\right) \text{ for } \omega \gg \omega_c \tag{12}$$

In comparison of (11) and (12), the error caused by phase-shift is much greater than that by amplitude attenuation. In order to improve the accuracy of heave measurement, finding a real-time phase compensation method is an important task.

B. Real-time phase compensation

A phase compensation method taking advantage of time reversing technology is given in literature[1], which is non-causal and can be applied off-line only. In this paper, real-time phase compensation is realized by designing an all-pass IIR filter cascaded after the high-pass filter, such that the phase lead caused by the high-pass filter is balanced by the phase lag introduced by the all-pass filter.

As is well known that a filter with zero phase at all frequencies must be non-causal, designing a stable all-pass filter to compensate the phase lead at all frequencies is impossible. It has been found the frequency spectrum of heave motion usually has only one peak, which is called the dominant frequency of the heave motion [2] and the accuracy of heave measurement will be improved remarkably if the phase lead at the dominant frequency is compensated.

Substituting $s = j\omega$ into (9), the relationship between the Fourier transforms of the displacement and the acceleration is obtained:

$$p(j\omega) = -\frac{1}{\omega^2} a(j\omega) \tag{13}$$

Let ω_0 denotes the dominant frequency of the heave motion, the phase response of the high pass filter at ω_0 is:

$$\theta_{hp}(\omega_0) = \arg H_{hpd}(e^{j\omega_0 T_s}) \tag{14}$$

where T_s represents the period of sample.

To balance the phase lead at ω_0, the all-pass filter must satisfy:

$$\theta_{ap}(\omega_0) = -\theta_{hp}(\omega_0) \tag{15}$$

Generally, the transfer function of an all-pass filter has the following form[6]:

$$H_{ap}(z) = \frac{N(z)}{D(z)} = \frac{\sum_{n=0}^{N} a_{N-n} z^{-n}}{\sum_{n=0}^{N} a_n z^{-n}} \tag{16}$$

It can be seen from (16) that the coefficients of the numerator and the denominator are reversely equal to each other. So the transfer function of the all pass filter can be totally obtained if the coefficients of the denominator are known.

The relationship between the phase response of the all-pass filter and its denominator follows:

$$\theta_{ap}(\omega) = -N\omega - 2\theta_D(\omega) \tag{17}$$

Let $N = 1$, the phase response of the denominator must satisfies:

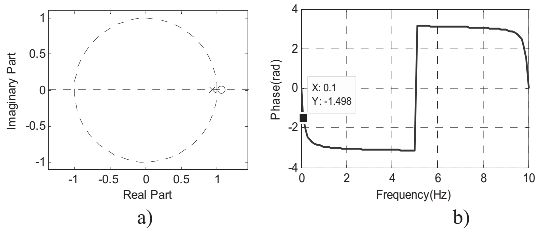
$$\theta_D(\omega_0) = \frac{\theta_{hp}(\omega_0) - \omega_0}{2} \tag{18}$$

The coefficients and the transfer function of the all-pass filter can be calculated then.

For example, given a displacement signal with a period of 10s, an fourth order high-pass filter with cut off frequency at 0.05Hz can be designed. Since $\omega_0 = 2\pi \times 0.1 = 0.628\ rad/s$, the phase response of high-pass filter is known $\theta_{hp}(\omega_0) = 1.5$. An all-pass filter is designed taking advantage of the above method, of which the transfer function is:

$$H_{ap}(z) = \frac{-1.07z^{-1}+1}{z^{-1}-1.07} \tag{19}$$

The distribution of zeros and poles of the all-pass filter is shown in Fig.1 a). All the poles are located in the unit circle, indicating the filter is stable. The phase response is shown in Fig.1 b), where the phase at 0.1Hz is -1.498 and is close to the objective of design.



a) b)

Fig.1. a)The distribution of zeros and poles of the all-pass filter;
b) The phase response of the all-pass filter

C. Adaptive Frequency Estimator

Since the design of all-pass filter is depend on the dominant frequency of heave motion, which is not constant throughout the process, the filtering system must be capable of detecting and tracking the dominant frequency of the signal and adjust the coefficients of the phase compensation all-pass filter accordingly.

We adopt the weighted-frequency Fourier linear combiner (WFLC) algorithm as an adaptive frequency estimator. The WFLC is an extension of the Fourier linear combiner (FLC), which is an adaptive algorithm that uses a dynamic truncated Fourier series model to estimate incoming quasi-periodic signals of known frequency by adapting the amplitude and phase of an artificially generated reference signal. The WFLC algorithm also adapt to the time-varying reference signal frequency, using a modification of the Least Mean Square (LMS) algorithm.

A block diagram of the WFLC algorithm is shown in Fig. 2.

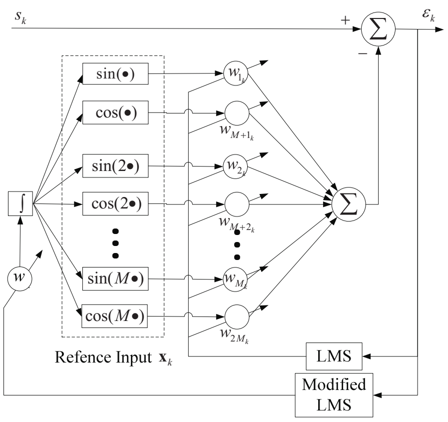


Fig. 2. The WFLC algorithm

The reference input vector to WFLC $\mathbf{x}_k = \left[x_{1_k} \cdots x_{2M_k}\right]^T$ is [7]:

$$x_{r_k} = \begin{cases} \sin\left(rT\sum_{t=0}^{k} w_{0_t}\right), & 1 \le r \le M \\ \cos\left((r-M)T\sum_{t=0}^{k} w_{0_t}\right), & M+1 \le r \le 2M \end{cases} \tag{20}$$

where M is the number of harmonics used, $k = 1, 2, \cdots$ represents time-index, T is a sampling period. The frequency can be estimated by updating every sample via:

$$\varepsilon_k = s_k - \mathbf{w}_k^T \mathbf{x}_k \tag{21}$$

$$w_{0_{k+1}} = w_{0_k} + 2\mu_0 \varepsilon_k \sum_{i=1}^{M} i\left(w_{i_k} x_{M+i_k} - w_{M+i_k} x_{i_k}\right) \tag{22}$$

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\mu \mathbf{x}_k \varepsilon_k \tag{23}$$

where $\mathbf{w}_k = \left[w_{1_k} \cdots w_{2M_k}\right]$ is the coefficient vector of the reference input, s_k is the desired signal to be modeled, i.e. the heave motion, and μ_0 and μ are the adaptive gain parameters.

In order to initialize the estimator, an FFT is applied to the acceleration data, and the amplitude spectrum of heave displacement can be obtained by $A(\omega) = \ddot{A}(\omega) / \omega^2$, then a peak detection algorithm is applied to find the dominant frequency ω_0. After several iteration, w_{0_k} will adapt to the desired signal's frequency.

If the desired signal is mixed with undesired components such as DC offset and low-frequency components, the accuracy of the estimation will descend. To make w_{0_k} adapt well to the desired signal's frequency, the adaptive frequency estimator must follows the high-pass filter. The block diagram of the complete adaptive zero phase high-pass filter is shown in Fig. 3.

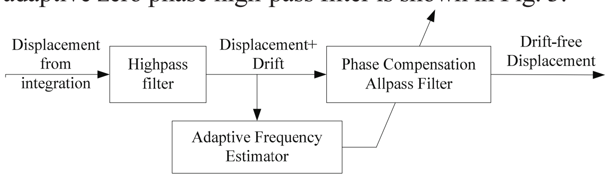


Fig. 3. Adaptive zero phase high-pass filter

IV. EXPERIMENT

An experiment is taken on a parallel motion simulator of six degrees of freedom. The experiment system includes the motion simulator, a single-axis accelerometer, a conditioning module, a low-pass anti-aliasing filter, a data acquisition card and a laptop computer. The structure diagram and the photo of the experiment system is shown in Fig. 4 and Fig. 5 a) respectively.

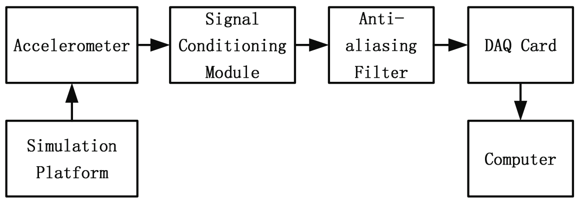
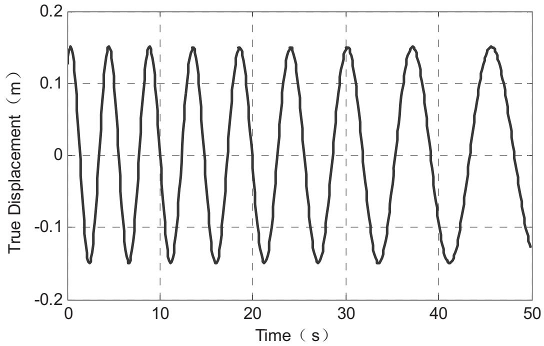


Fig. 4. Structure diagram of the experiment system

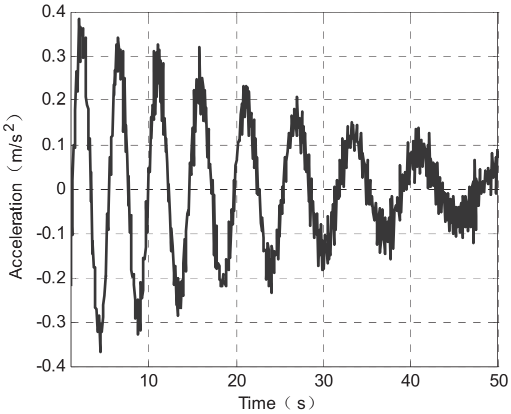
a)



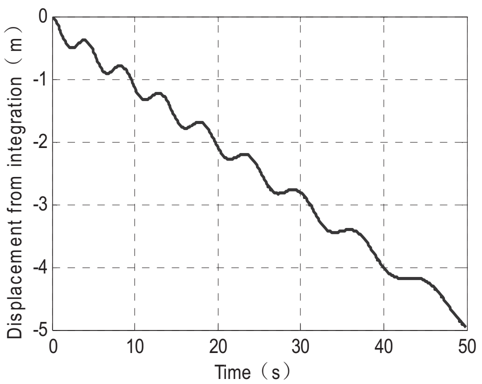
b)

Fig. 5. a) Photo of the experiment system, b) The displacement of the simulator

The simulator is set to generate a sinusoid-like heave motion with variable frequency, which is shown in Fig. 5 b). As it can be seen in Fig. 6 that the measured acceleration is mixed with noises and the displacement from integration drift fast.



a)



b)

Fig. 6. a) The measured acceleration, b) The displacement from integration

Fig.7 shows the plot of the true and measured heave displacement of the simulator, the solid line is the true displacement while the dotted line and the dashed line denote the measured values with and without phase compensation respectively. Due to the boundary effect, the data of the first 10 seconds are abandoned. From the figure, it can be seen that the uncompensated estimate has significant phase-shift, which is variable when the dominant frequency changes, but the proposed method can reduce most of them and provide accurate measurement. The effect of phase compensation is not influenced by the dominant frequency, which means the method is adaptive.

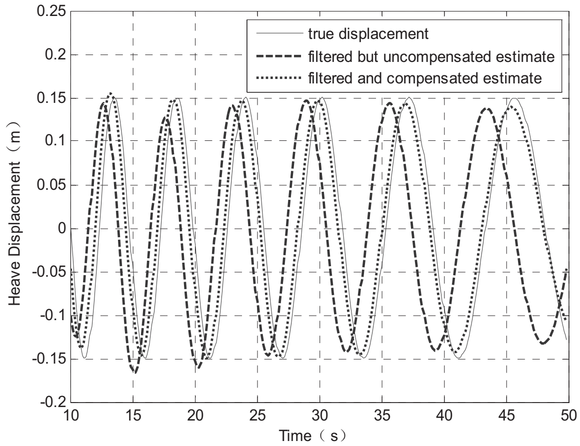


Fig. 7. Comparison of true heave displacement with compensated and uncompensated estimate

V. CONCLUSION

The paper presented a new signal processing method of heave measurement based on IMU. The basic principle of this method is taking advantage of all-pass filter to compensate the phase-shift reduced by high-pass filter. Knowledge on dominant frequency of the desired signal is obtained from the WFLC algorithm adaptively and is used to adjust the coefficient of all-pass filter. The proposed method performs as a real-time zero phase high-pass filter, which can improve the accuracy of the heave measurement and is also useful for other applications of displacement or position measurement.

REFERENCES

[1] GODHAVN J M. High quality heave measurements based on GPS RTk and accelerometer technology[C]. OCEANS 2000, 2000.
[2] GODHAVN J M. Adaptive tuning of heave filter in motion sensor[C]. OCEANS 1998, 1998.
[3] YANG W. Numerical simulation and testing analysis of adaptive heave motion measurement[C]. Zhangjiajie: 2009

International Conference on Measuring Technology and Mechatronics Automation, 2009.
[4] KUCHLER S. Heave motion estimation of a vessel using acceleration measurement[C]. MILANO:18th IFAC World Congress, 2011.
[5] SUN W, SUN F. Measurement technology of ship heave movement based on SINS resolving[J]. Chinese Journal of Scientific Instrument, 2012, 33（1）:167-172.
[6] RAJAMANI K, LAI Y S. A novel method for designing allpass digital filters[J]. IEEE SIGNAL PROCESSING LETTERS, 1999, 6(8)： 207-209.
[7] LATT W T. Real-time estimation and prediction of periodic signals from attenuated and phase-shifted sensed signals[C]. Singapore:2009 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, 2009.

AUTHOR BIOGRAPHY

Hu Yongpan was born in Shandong Province, China, in 1986. He received BS and MS from Hunan University and National University of Defense Technology, in 2007 and 2009, respectively. Now he is a PhD candidate in College of Mechatronics and Automation, National University of Defense Technology, China. His research interests include heave measurement and heave compensation.

Tao Limin was born in Hunan Province, China, in 1964. He received BS, MS and PhD from National University of Defense Technology, China, in 1984, 1987 and 2006, respectively. How he is a professor in College of Mechatronics and Automation, National University of Defense Technology, China. His research interests include state detection and fault diagnosis of machine.