

I have ask Maggie Wu about Q6 and Nathan Pennerdt about Q8

$$\begin{aligned} Q_1: \quad \|\vec{u}\|_2 &= \sqrt{0^2 + 4^2 + (-1)^2} \\ &= \sqrt{17} = 4.12 \end{aligned}$$

$$\begin{aligned} Q_2: \quad \text{opposite direction: } &\begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \\ \text{unit vector } \hat{u} &= \left(\frac{1}{\|\vec{u}\|} \right) \vec{u} \\ &= \left(\frac{1}{\sqrt{0^2 + (-4)^2 + 1^2}} \right) \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \\ &= \frac{1}{\sqrt{17}} \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{pmatrix} = \begin{pmatrix} 0 \\ -16.49 \\ 4.12 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} Q_3: \quad \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} \\ &= \frac{0 \times (-2) + 4 \times 3 + (-1) \times 7}{\sqrt{0^2 + 4^2 + (-1)^2} \times \sqrt{(-2)^2 + 3^2 + 7^2}} \\ &= \frac{5}{\sqrt{17} \times \sqrt{62}} \\ &= \frac{5}{\sqrt{1054}} = 0.15 \end{aligned}$$

Q4: No, because the angle between \vec{u} and \vec{v} is not 0.
Or, $\cos \theta$ is not 0 in this case.

$$Q_5: \quad \|\vec{u}\|_1 = |0| + |4| + |-1| = 5$$

$$\begin{aligned}
 Q6: AB &= \begin{pmatrix} 2 \times 4 + 0 \times 0 + 1 \times 4 & 2 \times 2 + 0 \times 2 + 1 \times 0 & 2 \times 7 + 0 \times 1 + 1 \times 3 \\ 3 \times 4 + 1 \times 0 + 4 \times 4 & 3 \times 2 + 1 \times 2 + 4 \times 0 & 3 \times 7 + 1 \times 1 + 4 \times 3 \\ -2 \times 4 + 5 \times 0 + 0 \times 4 & -2 \times 2 + 5 \times 2 + 0 \times 0 & -2 \times 7 + 5 \times 1 + 0 \times 3 \end{pmatrix} \\
 &= \begin{pmatrix} 8+0+4 & -4+0+0 & 14+0+3 \\ 12+0+16 & -6-2+0 & 21+1+12 \\ -8+0+0 & 4+10+0 & -14-5+0 \end{pmatrix} \\
 &= \begin{pmatrix} 12 & -4 & 17 \\ 28 & -8 & 34 \\ -8 & 14 & -19 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 4 \times 2 + -2 \times 3 + 7 \times -2 & 4 \times 0 + -2 \times 1 + 7 \times 5 & 4 \times 1 + -2 \times 4 + 7 \times 0 \\ 0 \times 2 + 2 \times 3 + -1 \times -2 & 0 \times 0 + 2 \times 1 + -1 \times 5 & 0 \times 1 + 2 \times 4 + -1 \times 0 \\ 4 \times 2 + 0 \times 3 + 3 \times -2 & 4 \times 0 + 0 \times 1 + 3 \times 5 & 4 \times 1 + 0 \times 4 + 3 \times 0 \end{pmatrix} \\
 &= \begin{pmatrix} 8-6-14 & 0+2+35 & 4-8+0 \\ 0+6+2 & 0-2-5 & 0+8+0 \\ 8+0-6 & 0+0+15 & 4+0+0 \end{pmatrix} \\
 &= \begin{pmatrix} -12 & 37 & -4 \\ 8 & -7 & 8 \\ 2 & 15 & 4 \end{pmatrix}
 \end{aligned}$$

They are not equal

$$\begin{aligned}
 Q7: \text{tr}(AB) &= 12 - 8 - 19 = -15 \\
 \text{tr}(BA) &= -12 - 7 + 4 = -15
 \end{aligned}$$

$$\begin{aligned}
 Q8: &\begin{pmatrix} 8 & -4 & 14 \\ 12 & -6 & 21 \\ -8 & 4 & -14 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 10 & -5 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 3 \\ 16 & 0 & 12 \\ 0 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 12 & -4 & 17 \\ 28 & -8 & 34 \\ -8 & 14 & -19 \end{pmatrix}
 \end{aligned}$$

Q9: They are the same. They are different ways to calculate matrix multiplication.