I asked Qi Ma for Q8 and Joshua for Q1.

Q1: In general, if the number of equation is smaller than the number of unknown variables, this system is under-determined system, which means this system has more than 1 solution.

Q2:
$$\left(\begin{array}{cccc|c} 1 & 3 & 5 & -1 & 0 \\ 2 & 0 & 4 & 1 & 0 \\ 2 & -1 & 3 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 5 & -1 & 0 \\ 0 & -6 & -6 & 3 & 0 \\ 0 & -7 & -7 & 2 & 0 \end{array}\right) \begin{array}{l} \\ (2^{nd}\text{ row}) - 2\times(1^{st}\text{ row}) \\ (3^{rd}\text{ row}) - 2\times(1^{st}\text{ row}) \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 5 & -1 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & 0 \\ 0 & -7 & -7 & 2 & 0 \end{array}\right) \quad (2^{nd}\text{ row}) \times -\frac{1}{6}$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 5 & -1 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \end{array}\right) \quad (3^{rd}\text{ row}) + 7\times(2^{nd}\text{ row})$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 3 & 5 & -1 & 0 \\ 0 & 1 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \quad (3^{rd}\text{ row}) \times -\frac{2}{3}$$

Q3: There are 3 pivots
First column, Second column, Fourth column as showed above.

Q4: $x_1 + 3x_2 + 5x_3 - x_4 = 0$

$$x_2 + x_3 - \tfrac{1}{2}x_4 = 0$$

$$x_4 = 0$$

$$\begin{cases} x_4 = 0 \\ x_2 = -x_3 \\ x_1 = -2x_3 \end{cases}$$

- Solution set in parametric form:

$$\left\{ \begin{pmatrix} -2x_3 \\ -x_3 \\ x_3 \\ 0 \end{pmatrix} : x_3 \in R \right\}$$

$$= \left\{ \begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} x_3 : x_3 \in R \right\}$$

Q5: The null space has a basis formed by the set

$$\begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix}.$$

Dimension $= 1$, it equals the number of columns without leading entries.

Q6: The column space of A is

$$\left\{ \begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}3\\0\\-1\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix} \right\}$$

$$\text{rank}(A) = \text{span}\left(\begin{pmatrix}1\\2\\2\end{pmatrix}, \begin{pmatrix}3\\0\\-1\end{pmatrix}, \begin{pmatrix}-1\\1\\0\end{pmatrix} \right) = 3$$

Q7: The rank-nullity theorem states that the rank and nullity (dimension of the kernel) sum to the number of columns in a given matrix.

In this case:

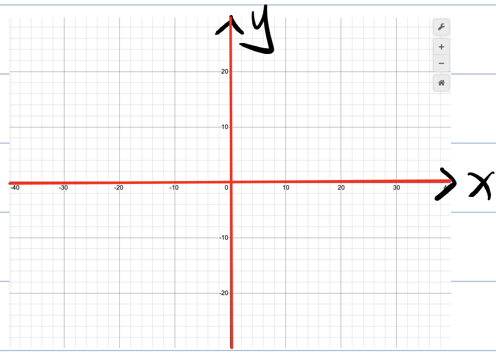
d (# of variables) $= 4$

$\text{rank}(A) = 3$

$\text{nullity}(A) = 1$

$\therefore d = \text{ran}(A) + \text{nullity}(A)$

Q8:



As showed above, $xy=0$ has two lines (red) along with axis x and y.

S is not a subspace in $\mathbb{R}^2$.

if we choose $\vec{v}=(1,0)$, $\vec{w}=(0,1)$ we can see those 2 vectors are both in S

However, $\vec{v}+\vec{w}=(1,1)$ is not in S.

$\therefore$ it conflict with the second condition of definition

$\therefore$ S is not a subspace in R^2.