Modeling of Tracking Area List-Based Location Update Scheme in Long Term Evolution

Xian Wang*, Xianfu Lei†, Rose Qingyang Hu†, and Yi Qian‡
*School of Information Science and Technology, Southwest Jiaotong University, Chengdu, Sichuan 610031, China
†Department of Electrical and Computer Engineering, Utah State University, Logan, UT 84322, USA
‡Department of Computer and Electronics Engineering, University of Nebraska-Lincoln, Omaha, NE 68588, USA
E-mail: drwangxian@gmail.com, xflei81@yahoo.com.cn, rose.hu@usu.edu, yqian@ieee.org

***Abstract*—Long Term Evolution uses a new location update (LU) scheme, called a tracking area list (TAL)-based scheme, to overcome the defects of the LU scheme used in 2G and 3G cellular networks. Under the TAL-based LU scheme, each time a user equipment (UE) performs an LU, it is allocated a group of tracking areas, referred to as a TAL, within which the UE can move freely without any LU. The UE performs an LU when moving out of the TAL. The performance of the TAL-based LU scheme depends on the allocated TAL. In this paper we develop a mathematical model to analyze the signaling overhead of the TAL-based LU scheme for local UEs whose mobility exhibits strong regularity. We derive formulas for the LU cost and the paging cost of a TAL allocation strategy. With these formulas we can find an optimal TAL allocation strategy to minimize the signaling cost of the TAL-based LU scheme.**

I. Introduction

In a cellular network, the location of a user equipment (UE) must be determined before the successful delivery of an incoming call to the UE. The network uses a mechanism called location management to keep track of the UE location. Location management consists of two essential operations, namely, location update (LU) and paging. LU is the process through which the UE dynamically updates its location registration in network databases. Paging is the process through which when an incoming call arrives, the network pinpoints the area where the UE currently resides so as to deliver the call by broadcasting a paging message in cells close to the cell where the last LU before the call arrival occurs. To facilitate location management, the coverage of the network is partitioned into registration areas, each of which comprises an aggregation of cells. In Global System for Mobile Communication (GSM) and Universal Mobile Telecommunications System (UMTS), a registration area is called a location area (LA) in the circuit-switched service domain, and called a routing area (RA) in the packet-switched service domain. In Long Term Evolution (LTE) a registration area is called a tracking area (TA).

In GSM and UMTS, an UE performs an LU whenever moving into a new LA/RA. Define a paging area as an area within which the UE can be located upon the arrival of an incoming call. The above LU scheme used in GSM and UMTS is intrinsically a static LU scheme, owing to the fact that all the UEs in the network have paging areas of the same size, i.e., the size of an LA/RA, irrespective of the diverse mobility and traffic characteristics of individual UEs. Intuitively, for UEs with dense call arrivals, frequent LUs are necessary to reduce the paging area size and accordingly the paging cost; whereas for UEs with sparse call arrivals, infrequent LUs are necessary. Imagine that for a UE that never has an incoming call, LU is not needed at all. Hence, the static LU scheme is cost-ineffective. A cost-effective LU scheme is such a scheme that is dynamic enough to adapt to diverse mobility and traffic characteristics. In addition to the problem of cost-ineffectiveness, the static LU scheme is widely criticized for the following problems, namely, ping-pong LU effect and uneven distribution of LU signaling. The ping-pong LU effect arises when a UE that moves back-and-forth among multiple LAs/RAs performs LUs in the same way as in the case of straight line movement. Under the static LU scheme, all the LUs occur in the boundary cells of an LA/RA, causing the uneven distribution of LU signaling. In an extreme case that a large number of UEs simultaneously move into a boundary cell, e.g., all the UEs in a train moving into a new LA/RA, there is a risk of an LU signaling storm in the boundary cell due to a large number of LUs launched in a short period. This is an undesirable case, because it could degrade the quality-of-service in the boundary cell and even lead to signaling congestion.

To tackle the problems associated with the static LU scheme used in GSM and UMTS, LTE adopts a tracking area list (TAL)-based LU scheme. Under this scheme, each time the UE performs an LU in a TA, it is allocated a group of TAs, called a TAL, by the Mobility Management Entity (MME), the entity responsible for mobility management in LTE. The UE can move within the TAs contained in the TAL without any LU, referred to as TA update (TAU) in LTE. Only after the UE moves into a new TA that does not belong to the TAL, does it need to perform a TAU toward the MME, which will in turn allocate the UE a new TAL that includes the new TA. In contrast with the static LU scheme, the TAL-based LU scheme has the following advantages. (a) It is dynamic enough to adapt to diverse mobility and traffic characteristics. (b) It can avoid the ping-pong LU effect through allocating the UE a TAL that contains all the TAs among which the UE moves back-and-forth. (c) It can evenly distribute the LU signaling among TAs by allocating different UEs with different TALs.

The allocation of TALs determines the performance of the TAL-based LU scheme. References [1]–[5] devote to this issue. All these references assume that a TA contains only one cell.

This assumption makes the concept of TA in LTE insignificant. In [1] Chung treated the TAL-based LU as a movement-based LU (MBLU) scheme, under which an LU occurs when the number of cells crossed by the UE reaches a threshold called a movement threshold. In [2] Ferragut and Mangues-Bafalluy treated the TAL-based LU scheme as a distance-based LU (DBLU) scheme, under which an LU occurs when the distance in terms of cells traveled by the UE reaches a threshold called a distance threshold. Both [1] and [2] proposed a method for adjusting the movement threshold or the distance threshold so as to reduce the signaling cost. When the TAL-based LU scheme is treated as an MBLU or a DBLU scheme, the TAL allocated to the UE consists of TAs arranged in concentric cycles around a central TA where the last TAU occurs. In this case the flexible range within which the movement threshold and the distance threshold can be freely adjusted is severely limited, because 3GPP TS 23.401 ([6]) prescribes that the number of TAs contained in a TAL cannot exceed sixteen, so that the optimal movement threshold or distance threshold may not meet this prescription. In [3]–[5] Razavi *et al.* developed three algorithms for allocating TAL, namely, a local search algorithm, an intuitive rule of thumb algorithm, and an algorithm based on UE traces. These algorithms have the following shortcomings. (a) Each cell forms a tailored TAL by including the neighboring cells that bring the least total LU and paging cost into the TAL without considering the impact on neighboring cells, so that it is impossible to achieve global optimization within the whole network. (b) A cell gives only one common TAL to all the UEs that perform TAU in the cell without considering the diverse mobility and traffic characteristics of individual UEs, so that this common TAL may be cost-ineffective for some UEs. Moreover, this approach of allocating TAL brings the problem of ping-pong LU effect and the problem of uneven distribution of the LU signaling encountered by the static LU scheme used in GSM and UMTS to LTE.

The UEs in the network can be classified into two broad groups, namely, local UEs and global UEs, with respect to their activity scope. Local UEs refer to the UEs that are local residents of the current region; whereas global UEs refer to the UEs that are not local residents of the current region. The mobility of global UEs has weak regularity, because as new comers to a place, these UEs often want to visit more sites in a short time. On the contrary, local UEs often follow an ordered pattern to haunt several fixed sites, so that their mobility exhibits strong regularity [7], [8]. Thus, it is feasible to allocate a local UE an optimal TAL that can minimize the signaling cost.

In this paper, we develop an embedded Markov chain model to analyze the signaling cost of the TAL-based LU scheme for local UEs. We derive formulas for the TAU cost and the paging cost of an arbitrary TAL allocation strategy. We also give a method to preclude ineligible strategies. With these formulas, we can search for an optimal strategy among all the eligible TAL allocation strategies. The model developed and the formulas derived in this paper can be exploited to optimize the performance of the TAL based-LU scheme.

The remaining paper is organized as follows. Section II introduces a system model. Section III develops an embedded Markov chain model to calculate the signaling cost of a TAL allocation strategy. Section IV validates the formulas derived in this paper by simulation, and relies on these formulas to search for the optimal strategy. Section V concludes this paper.

II. System Model

A. Mobility Model

In this paper we consider the location management for a local UE who follows a relatively fixed pattern to visit a number of fixed sites that are adjacent to each other. Assume that the UE's activity scope consists of n TAs, identified as TA_1, TA_2, $\ldots$, TA_n, $n = 1, 2, \ldots$. Denote by TAL_x, $x = 1, 2, \ldots, n$, the TAL allocated to the UE when the UE attaches or performs a TAU in TA_x. Note that TA_x must be included in TAL_x. There are a total of $2^{(n-1)\times n}$ possible TAL allocation strategies, each of which can be represented by a square matrix of size n, which is denoted by **TAL**,

$$\mathbf{TAL} = (\mathrm{TAL}(x, y))_{n\times n}, \quad \mathrm{TAL}(x, y) \in \{0, 1\}. \tag{1}$$

$\mathrm{TAL}(x, y) = 1$ means that TA_y is included in TAL_x, $x, y = 1, 2, \ldots, n$; and $\mathrm{TAL}(x, y) = 0$ bears the opposite meaning. The movements of the UE among the n TAs can be represented by a square matrix of size n, denoted by $\mathbf{p}$,

$$\mathbf{p} = (p(x, y))_{n\times n}, \quad 0 \le p(x, y) \le 1, \tag{2}$$

where $p(x, y)$ is the probability that after leaving TA_x the UE moves to TA_y, and satisfies

$$\sum_{y=1}^{n} p(x, y) = 1,$$
$$p(x, x) = 0,$$

for $x = 1, 2, \ldots, n$. Denote by t_x, $x = 1, 2, \ldots, n$, the UE's residence time in TA_x. Assume that t_x's for $x = 1, 2, \ldots, n$ are independent and that t_x follows an exponential distribution of rate μ_x.

B. Traffic Model

Assume that the call arrivals to the UE follow a Poisson process of rate λ. That is, the time between the arrivals of two consecutive calls, which is referred to as the call inter-arrival time and denoted by t_c, follows an exponential distribution of rate λ.

C. Parameter Setting for an Example Used for Case Study

For the purpose of case study, in this paper we assume that $n = 5$. In this case there are a total of 2^{20} possible TAL allocation strategies. We exemplify the modeling of a TAL allocation strategy using an example. Matrix $\mathbf{p}$ of the example is set as

$$\mathbf{p} = \begin{bmatrix} 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0.6 & 0 & 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0 & 0.1 & 0.7 \\ 0 & 0.3 & 0.5 & 0 & 0.2 \\ 0.1 & 0.3 & 0.2 & 0.4 & 0 \end{bmatrix}. \tag{3}$$

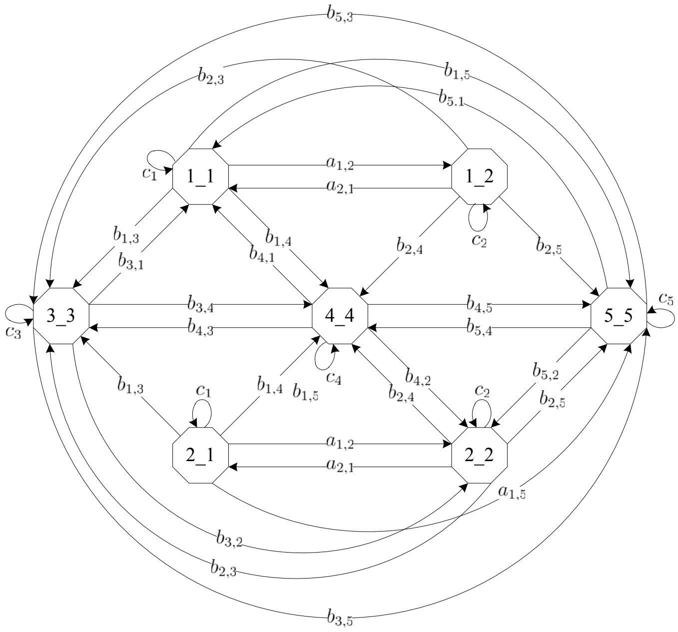


Fig. 1. Embedded Markov chain at the TA level for the TAL allocation strategy given in (4).

Matrix **TAL** of the example is set as

$$\mathbf{TAL} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

which is equivalent to the following TAL allocation strategy

$$\begin{cases} \mathrm{TAL}_1 = \mathrm{TAL}_2 = \{\mathrm{TA}_1, \mathrm{TA}_2\}, \\ \mathrm{TAL}_x = \{\mathrm{TA}_x\}, \quad x \in \{3, 4, 5\}. \end{cases}$$

Parameters μ_x's of the example are set as

$$[\mu_1, \mu_2, \ldots, \mu_5] = [1, 0.1, 10, 0.02, 50]\lambda. \tag{5}$$

Note that in practice the above parameters can be obtained easily by the network. Parameter λ can be obtained by analyzing a UE's call record. Parameters $\mathbf{p}$ and μ_x's can be obtained by exercising for a long enough period a special TAL allocation strategy where TAL_x contains only TA_x, $x = 1, 2, \ldots, n$.

III. Signaling Cost of the TAL-Based LU Scheme

A. Embedded Markov Chain

Given a TAL allocation strategy, the TAL-based LU scheme can be modeled by an embedded Markov chain (EMC). Fig. 1 shows an EMC for the TAL allocation strategy given in (4). In the EMC state x_y, $x, y = 1, 2, \ldots, n$, represents the state that the TAL allocated to the UE is TAL_x and the UE is currently residing in TA_y of TAL_x. Denote by $\mathbf{S}$ the state space of the EMC. It follows that for the example $\mathbf{S} = \{1\_1, 1\_2, 2\_1, 2\_2, 3\_3, 4\_4, 5\_5\}$. When the EMC is in state x_y, there are three events that can cause the EMC to leave state x_y. The first event is that the UE leaves TA_y to another TA, say TA_z, $z = 1, 2, \ldots, n$ and $z \neq y$, which is included in the current TAL TAL_x. The second event is that the UE leaves TA_y to TA_z that is not included in TAL_x; and after entering TA_z the UE will initiate an TAU toward the MME and afterward will be allocated a new TAL by the latter, i.e., TAL_z. The third event is that an incoming call arrives before the UE moves out of its current TA TA_y; and upon the call arrival the network will page the UE in all the TAs of TAL_x and afterward a call connection will be established. In this paper we assume that the duration time of the call is negligible compared with the residence time in TA_y, so that after the completion of the call, the EMC remains in state x_y. Denote, respectively, by $a_{y,z}$, $b_{y,z}$ and c_y the probabilities with which the above three events take place. It follows that for state x_y, $x\_y \in \mathbf{S}$,

$$\begin{cases} a_{y,z} = \Pr(t_c > t_y)p(y,z) = \dfrac{\mu_y}{\lambda + \mu_y} p(y,z), \\ \qquad z = 1, 2, \ldots, n, z \neq y, \text{ and } \mathrm{TA}_z \in \mathrm{TAL}_x, \\ b_{y,z} = \Pr(t_c > t_y)p(y,z) = \dfrac{\mu_y}{\lambda + \mu_y} p(y,z), \\ \qquad z = 1, 2, \ldots, n, z \neq y, \text{ and } \mathrm{TA}_z \notin \mathrm{TAL}_x, \\ c_y = \Pr(t_c < t_y) = \dfrac{\lambda}{\lambda + \mu_y}. \end{cases}$$

Denote by π_{x_y}, $x\_y \in \mathbf{S}$, the steady-state probability of state x_y. Note that π_{x_y} is directly proportional to the number of times rather than the duration time state x_y has been visited by the EMC. Referring to Fig. 1, we can write out the set of balance equations for the example

$$\begin{cases} \pi_{1\_1}(1-c_1) = \pi_{1\_2}a_{2,1} + \pi_{3\_3}b_{3,1} + \pi_{5\_5}b_{5,1}, \\ \pi_{1\_2}(1-c_2) = \pi_{1\_1}a_{1,2}, \\ \pi_{2\_1}(1-c_1) = \pi_{2\_2}a_{2,1}, \\ \pi_{2\_2}(1-c_2) = \pi_{2\_1}a_{1,2} + \pi_{3\_3}b_{3,2} + \pi_{4\_4}b_{4,2} + \pi_{5\_5}b_{5,2}, \\ \pi_{3\_3}(1-c_3) = (\pi_{1\_1} + \pi_{2\_1})\, b_{1,3} + (\pi_{1\_2} + \pi_{2\_2})\, b_{2,3} \\ \qquad + \pi_{4\_4}b_{4,3} + \pi_{5\_5}b_{5,3}, \\ \pi_{4\_4}(1-c_4) = (\pi_{1\_1} + \pi_{2\_1})\, b_{1,4} + (\pi_{1\_2} + \pi_{2\_2})\, b_{2,4} \\ \qquad + \pi_{3\_3}b_{3,4} + \pi_{5\_5}b_{5,4}, \\ \pi_{5\_5}(1-c_5) = (\pi_{1\_1} + \pi_{2\_1})\, b_{1,5} + (\pi_{1\_2} + \pi_{2\_2})\, b_{2,5} \\ \qquad + \pi_{3\_3}b_{3,5} + \pi_{4\_4}b_{4,5}. \end{cases} \tag{6}$$

The above equations combined with the normalization condition that

$$\sum\nolimits_{x\_y \in \mathbf{S}} \pi_{x\_y} = 1 \tag{7}$$

can yield a solution for π_{x_y}'s, $x\_y \in \mathbf{S}$.

Denote by r_{x_y}, $x\_y \in \mathbf{S}$, the sojourn time in state x_y. When the leaving from state x_y is due to the first or the second event, we have $t_c > t_y$ and $r_{x\_y} = t_y$. When the leaving is due to the third event, we have $t_c < t_y$ and $r_{x\_y} = t_c$. Therefore, it follows that for state $x\_y \in \mathbf{S}$,

$$\begin{cases} r_{x\_y} = \min(t_c, t_y), \\ \overline{r}_{x\_y} \triangleq E(r_{x\_y}) = \dfrac{1}{\lambda + \mu_y}. \end{cases}$$

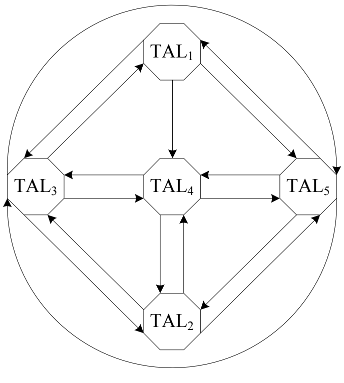


Fig. 2. Embedded Markov chain at the TAL level for the TAL allocation strategy given in (4).

Denote by r the time between two consecutive state transitions. It follows that

$$\begin{cases} r = \sum_{x\_y \in \mathbf{S}} \pi_{x\_y} r_{x\_y}, \\ \overline{r} \triangleq E(r) = \sum_{x\_y \in \mathbf{S}} \pi_{x\_y} \overline{r}_{x\_y}. \end{cases}$$

B. Precluding Ineligible TAL Allocation Strategies

Let $\mathbf{\Pi} = \{\pi_{x\_y}, x\_y \in \mathbf{S}\}$ be the steady-state distribution of the EMC for a TAL allocation strategy. Denote by l the length of $\mathbf{\Pi}$, i.e., the number of entries in $\mathbf{\Pi}$. $\mathbf{\Pi}$ can be expressed in a matrix form as

$$\begin{cases} \mathbf{\Pi P} = \mathbf{\Pi}, \\ \mathbf{\Pi}(\mathbf{P} - \mathbf{I}) = \mathbf{0}, \end{cases}$$

where $\mathbf{P}$ represents the transition matrix of the EMC, and $\mathbf{I}$ is the $l \times l$ identity matrix. Combing the normalization condition given in (7) with the last equation yields

$$\mathbf{\Pi A} = [1, 0, \ldots, 0]_{1 \times l}$$

where

$$\mathbf{A} = \left(a_{i,j}\right)_{l \times l},$$
$$a_{i,j} = \begin{cases} (\mathbf{P} - \mathbf{I})_{i,j}, & i = 1, \ldots, l, j = 2, \ldots, l, \\ 1, & i = 1, \ldots, l, j = 1. \end{cases}$$

It follows from the last equation that when $\mathbf{A}$ is non-singular,

$$\mathbf{\Pi} = [1, 0, \ldots, 0]_{1 \times l} \mathbf{A}^{-1}.$$

When $\mathbf{A}$ is singular, there is no unique solution for $\mathbf{\Pi}$. Thus, it is necessary to find a way to determine whether a TAL allocation strategy has a unique steady-state distribution $\mathbf{\Pi}$ or not.

Given a TAL allocation strategy, if we focus on the transitions among TALs, we can construct another EMC composed of n states, namely, TAL_1, …, TAL_n. In this EMC state TAL_x, $x = 1, 2, \ldots, n$, is the aggregation of states x_y's, for $y = 1, 2, \ldots, n$ and $\text{TA}_y \in \text{TAL}_x$, in the EMC shown in Fig. 1 that focuses on the transitions among TAs. To avoid confusion, when necessary we call the EMC for a TAL allocation strategy focusing on the transitions among TAs *the EMC at the TA level*, and call the one focusing on the transitions among TALs *the EMC at the TAL level*. Fig. 2 shows an EMC at the TAL level for the TAL allocation strategy given in (4). In Fig. 2 a directed arc from TAL_i to TAL_j is included if after leaving TAL_i the UE can transit to TAL_j, $i, j = 1, 2, \ldots, n$ and $i \neq j$. Here the transition from TAL_i to TAL_j means that (a) $\text{TA}_j \notin \text{TAL}_i$, and that (b) there exists at least one integer k, $k = 1, 2, \ldots, n$, such that $\text{TA}_k \in \text{TAL}_i$ and $p(k, j) > 0$. Therefore, the EMC at the TAL level for a TAL allocation strategy has relation with not only the strategy itself but also matrix $\mathbf{p}$ given in (2) that describes the UE movements among TAs.

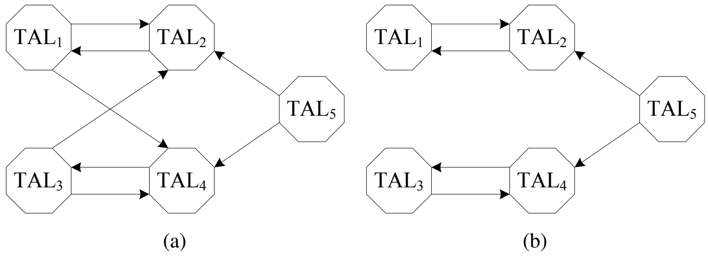


Fig. 3. Embedded Markov chains at the TAL level. (a) For the TAL allocation strategy given in (8). (b) For the TAL allocation strategy given in (9).

As a contrast to the TAL allocation strategy given in (4), we give another two strategies whose $\mathbf{TAL}$ matrices are as follows

$$\mathbf{TAL} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}, \tag{8}$$

$$\mathbf{TAL} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}. \tag{9}$$

The EMCs at the TAL level for the two strategies given in (8) and (9) are shown, respectively, in Figs. 3(a) and 3(b).

Define *a unichain* as a finite-state Markov chain that contains a single recurrent class plus, perhaps, some transient states [9]. It is pointed out in [9] that a finite-state Markov chain has a unique steady-state distribution if and only if the chain is a unichain. Since the EMC at the TAL level for a TAL allocation strategy is an aggregation representation of the EMC at the TA level, it follows that if the EMC at the TAL level is a unichain, then so does the EMC at the TA level; and vice versa. Thus, we can rely on the EMC at the TAL level to judge whether a TAL allocation strategy has a unique solution for $\mathbf{\Pi}$. Since the space size of the EMC at the TAL level equals n and is usually smaller than that of the EMC at the TA level, it is easier to use the EMC at the TAL level than to use the EMC at the TA level to do the judging task. The EMC shown in Fig. 2 is a unichain with no transient state; the one shown in Fig. 3(a) is a unichain with one transient state, i.e., TAL_5; the one shown in Fig. 3(b) has two recurrent classes, i.e., $\{\text{TAL}_1, \text{TAL}_2\}$ and $\{\text{TAL}_3, \text{TAL}_4\}$, so that it is not a unichain. The EMC shown in Fig. 3(b) has no unique steady-state distribution. For example, when the UE's initial state is TAL_1 the steady-state distribution is totally different from the one when the initial state is TAL_3. When the initial state is TAL_1, the EMC oscillates forever between states TAL_1 and TAL_2, and can never reaches states TAL_3 and TAL_4; and vice versa when the initial state is TAL_3. Thus, we consider TAL allocation strategies whose EMCs at the TAL level are not unichains as ineligible strategies. Ineligible strategies should be precluded from the candidate strategies among which the optimal strategy is searched for. When $n = 5$ and matrix $\mathbf{p}$ is given in (3), 56950 (i.e., 5.43%) of the 2^{20} possible TAL allocation strategies are ineligible.

C. Paging Scheme

Denote by $p(x\_y)$, $x\_y \in \mathbf{S}$, the probability that when an incoming call arrives, the UE resides in TA_y of TAL_x, given that the UE's current TAL is TAL_x. It follows that for $x\_y \in \mathbf{S}$,

$$p(x\_y) = \pi_{x\_y}\bar{r}_{x\_y} \Bigg/ \sum_{\substack{z=1 \\ \text{TA}_z \in \text{TAL}_x}}^{n} \pi_{x\_z}\bar{r}_{x\_z} .$$

Given the distribution of $p(x\_y)$, following the theorems developed by Zhu and Leung in [10] we can design an optimal sequential paging scheme for TAL_x that can meet a bound on the paging delay and at the same time minimize the paging cost. This issue is out of the scope of this paper. We consider only the parallel paging scheme. Define a paging area as an area within which the UE can be located when an incoming call arrives. Under the parallel paging scheme, all the TAs in the paging area are paged simultaneously. In the TAL-based LU scheme, when the UE's current TAL is TAL_x, $x = 1, 2, \ldots, n$, the paging area consists of all the TAs in TAL_x.

D. Signaling Cost

Denote by $N_{\text{TA}}(x)$ the number of TAs in TAL_x, $x = 1, 2, \ldots, n$. Denote, respectively, by $N_{\text{TAU,ST}}$ and $N_{\text{paging,ST}}$ the expected number of TAUs performed and the expected number of TAs paged due to a state transition in the EMC at the TA level. It follows that

$$\begin{cases} N_{\text{TAU,ST}} = \displaystyle\sum_{x\_y \in \mathbf{S}} \pi_{x\_y} \sum_{\substack{z=1 \\ \text{TA}_z \notin \text{TAL}_x}}^{n} b_{y,z}, \\ N_{\text{paging,ST}} = \sum_{x\_y \in \mathbf{S}} \pi_{x\_y} c_y N_{\text{TA}}(x). \end{cases}$$

Specifically, for the example we have

$$\begin{cases} N_{\text{TAU,ST}} = (\pi_{1\_1} + \pi_{2\_1})(1 - c_1 - a_{1,2}) \\ \qquad + (\pi_{1\_2} + \pi_{2\_2})(1 - c_2 - a_{2,1}) \\ \qquad + \pi_{3\_3}(1 - c_3) + \pi_{4\_4}(1 - c_4) + \pi_{5\_5}(1 - c_5), \\ N_{\text{paging,ST}} = 2(\pi_{1\_1}c_1 + \pi_{2\_1}c_1 + \pi_{1\_2}c_2 + \pi_{2\_2}c_2) \\ \qquad + \pi_{3\_3}c_3 + \pi_{4\_4}c_4 + \pi_{5\_5}c_5. \end{cases}$$

Denote, respectively, by $N_{\text{TAU,UT}}$ and $N_{\text{paging,UT}}$ the expected number of TAUs performed and the expected number of TAs paged per unit time. Note that both the $N_{\text{TAU,ST}}$ TAUs and the paging of $N_{\text{paging,ST}}$ TAs occur during time r. It follows that

$$N_{\text{sub,UT}} = N_{\text{sub,ST}} / \bar{r}$$

where sub $\in$ {TAU, paging}. Denote, respectively, by N_{TAU} and N_{paging} the expected number of TAUs performed and the expected number of TAs paged during the call inter-arrival time. It follows that

$$N_{\text{sub}} = N_{\text{sub,UT}} \frac{1}{\lambda} = N_{\text{sub,ST}} \frac{1}{\bar{r}} \frac{1}{\lambda}.$$

Denote by $p_{\text{TAL}}(x)$ the probability that when an incoming call arrives, the UE resides in any TA of TAL_x, $x = 1, 2, \ldots, n$. It follows that

$$p_{\text{TAL}}(x) = \left[\sum_{\substack{y=1 \\ \text{TA}_y \in \text{TAL}_x}}^{n} \pi_{x\_y}\bar{r}_{x\_y} \right] \frac{1}{\bar{r}}, \quad x = 1, 2, \ldots, n.$$

N_{paging} can be calculated using a different approach as

$$N_{\text{paging}} = \sum_{x=1}^{n} p_{\text{TAL}}(x) N_{\text{TA}}(x).$$

Denote, respectively, by δ_{TAU} and δ_{paging} the unit cost of performing a TAU and the unit cost of performing a paging in a TA. Denote by C the signaling cost of the TAL-based LU scheme due to the TAU and paging operations during the call inter-arrival time. It follows that

$$C = N_{\text{TAU}}\delta_{\text{TAU}} + N_{\text{paging}}\delta_{\text{paging}}.$$

IV. Performance Evaluation

A. Preparations

In this section we will conduct simulations to validate the accuracy of some numerical results calculated as per the analytical formulas derived in this paper. As a convention, only after a simulation has entered a steady state can we begin to gather simulation data required to calculate a performance metric. The conventional way to realize the steady-state is to run the simulation for a sufficiently long time before gathering the simulation data. This is a timing-consuming process. Moreover, it is difficult to judge whether the simulation has entered the steady state. Note that in the steady state, the UE follows the steady-state distribution to reside in states x_y's, $x\_y \in \mathbf{S}$. Therefore, from a statistical perspective, the steady state can be realized in the following approach without the warming-up process of the conventional way. Let $\text{PM}(x\_y)$ be a performance metric obtained by simulating a number of incoming calls when at the beginning of the simulation the UE resides in state x_y, $x\_y \in \mathbf{S}$. Denote by N_{call} the number of incoming calls simulated to obtain $\text{PM}(x\_y)$. The performance metric, denoted by PM, can be expressed

$$\text{PM} = \sum_{x\_y \in \mathbf{S}} \pi_{x\_y}\bar{r}_{x\_y} \tfrac{1}{\bar{r}} \text{PM}(x\_y)$$

where term $\pi_{x\_y}\bar{r}_{x\_y} / \bar{r}$ is the steady-state probability that the UE resides in state x_y.

To calculate or simulate the signaling cost, we must first set the cost of a TAU (i.e., δ_{TAU}) and the cost of a paging in

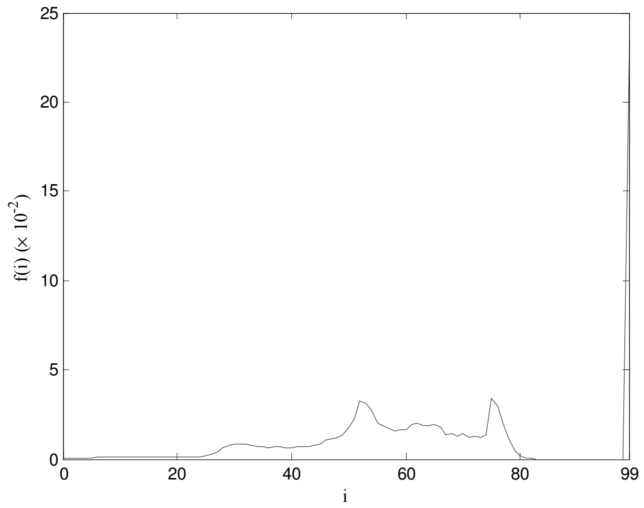


Fig. 4. Probability mass function of the signaling cost when the all-in-one strategies are not excluded from the eligible TAL allocation strategies. Parameters $\mathbf{p}$ and μ_x's are given, respectively, in (3) and (5); and $\delta_{\text{paging}} = 1$. $C_{\min} = 1.5226$ and $C_{\max} = 5$.

a TA (i.e., δ_{paging}). The cost of a TAU depends on the TAU procedure, so that the cost is fixed. The cost of a paging in a TA is determined by the cost of a paging in a cell and the TA size, i.e., the number of cells in a TA. Since the cost of a paging in a cell is fixed, the cost of a paging in a TA is directly proportional to the TA size. Therefore, we let $\delta_{\text{TAU}} = 9$, and consider a small TA size and a large TA size by letting $\delta_{\text{paging}} = 1$ and $\delta_{\text{paging}} = 9$. In this section we focus on the probability mass function (pmf) of the signaling cost. Denote by $C_{\min}$/ $C_{\max}$ the signaling cost of the optimal/worst strategy chosen from all the eligible TAL allocation strategies. Denote by $\mathbf{TAL}_{\min}$ the $\mathbf{TAL}$ matrix of the optimal strategy. Denote by N_{div} the number of equal-length sub-intervals interval $[C_{\min}, C_{\max}]$ is divided into. Thus, the length of a sub-interval is

$$\delta \triangleq [C_{\max} - C_{\min}] \,/\, N_{\text{div}}.$$

For an integer i, $i = 0, 1, \ldots, N_{\text{div}} - 1$, the pmf of i, denoted by $f(i)$, is defined as the percentage of eligible TAL allocation strategies whose signaling costs fall in $[C_{\min} + i\delta, C_{\min} + (i + 1)\delta)$. In other words, if we randomly select a strategy from all the eligible strategies, then

$$f(i) \triangleq \Pr[i\delta \le C - C_{\min} < (i + 1)\delta], \quad i = 0, 1, \ldots, N_{\text{div}} - 1.$$

We set $n = 5$ and $N_{\text{div}} = 100$ in this section.

B. Performance Evaluation

Adopt the $\mathbf{p}$ and μ_x's given, respectively, in (3) and (5); and assume that $\delta_{\text{paging}} = 1$. Fig. 4 shows the pmf of the signaling cost. Define a special type of TAL allocation strategies, referred to as *all-in-one strategies*, to be the strategies that have one and only one TAL containing all the n TAs. In the current case, 23.46% (i.e., 232656) of the eligible strategies are all-in-one strategies. For all-in-one strategies, there is no TAU and the paging area comprises all the n TAs, i.e., $N_{\text{TAU}} = 0$ and $N_{\text{paging}} = n$. In Fig. 4, the all-in-one strategies are the worst strategies. Since it is meaningless to pay much attention to the all-in-one strategies, in the following we will exclude these strategies from the eligible strategies.

Adopt the same parameters except δ_{paging} as those used in Fig. 4. Fig. 5 shows the pmf of the signaling cost when the all-in-one strategies are excluded from the eligible TAL allocation strategies. In Fig. 5, when $\delta_{\text{paging}} = 1$, matrix $\mathbf{TAL}_{\min}$ is as follows

$$\mathbf{TAL}_{\min} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}. \tag{10}$$

In Fig. 5, when $\delta_{\text{paging}} = 9$, matrix $\mathbf{TAL}_{\min}$ is as follows

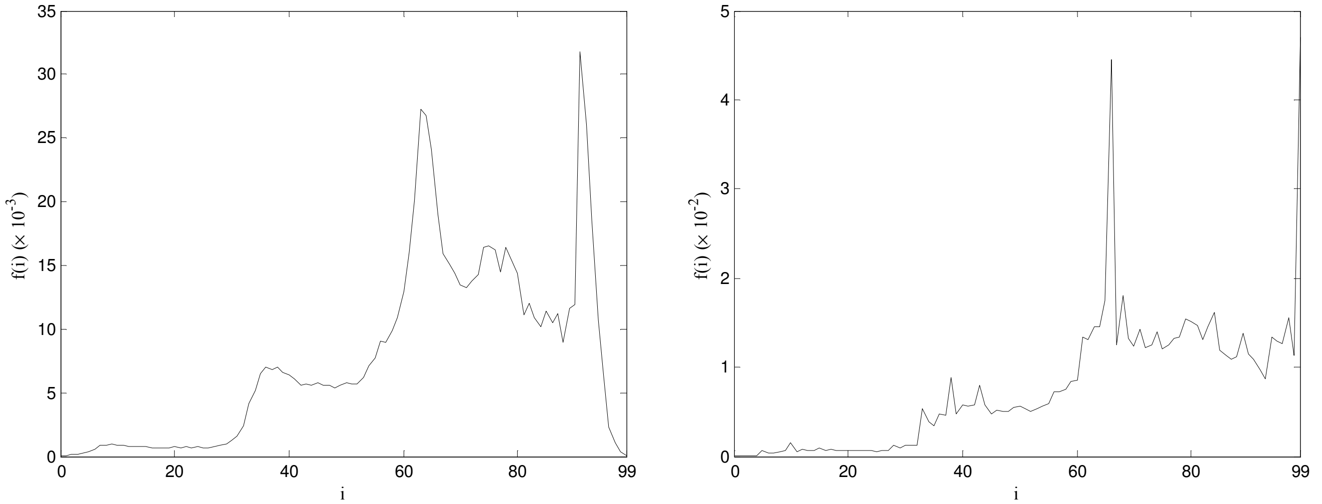
$$\mathbf{TAL}_{\min} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}. \tag{11}$$

Under the optimal strategy for $\delta_{\text{paging}} = 1$, the three TAs with the smallest TA residence times, i.e., TA_5, TA_3 and TA_1, are grouped into a TAL; and the two TAs with the largest TA residence times, i.e., TA_4 and TA_2, individually form two TALs. The rationale behind the above TAL allocation strategy is as follows. (a) The residence times in TA_5, TA_3 and TA_1 are very small and thus the UE will frequently move out from them, so that separating them into different TALs will incur high TAU cost. On the other hand, grouping them into a single TAL will cause the increase in the paging cost. However, this increase in the paging cost is worth for exchanging for the decrease in the TAU cost. (b) The residence times in TA_4 and TA_2 are very large and thus the UE will not frequently move out from them, so that separating them into different TALs can decrease the paging cost. The change when it comes to the optimal strategy for $\delta_{\text{paging}} = 9$ is that, instead of three TAs in the optimal strategy for $\delta_{\text{paging}} = 1$, only the two TAs with the smallest TA residence times, i.e., TA_5 and TA_3, are grouped into a TAL; and TA_1 forms a separate TAL. This reduction in TAL size is because, compared with $\delta_{\text{paging}} = 1$, when $\delta_{\text{paging}} = 9$, the paging cost will increase a lot, so that the TAL size should be reduced to decrease the paging cost and increase the TAU cost. Through this way, the difference between the TAU cost and the paging cost will not enlarge too much.

Table I compares the expected number of TAUs, the expected number of paged cells, and the signaling cost calculated as per the formulas derived in this paper with those obtained through simulation. In Table I the entry *rel. diff.* represents the relative difference of the analytical result to the simulation result. Table I suggests that the analytical and simulation results match closely.

V. Conclusions

The performance of the TAL-based LU scheme used in LTE is solely determined by the TAL allocated to a UE. The



(a) (b)

Fig. 5. Probability mass function of the signaling cost when the all-in-one strategies are excluded from the eligible TAL allocation strategies. Parameters $\mathbf{p}$ and μ_x's are given, respectively, in (3) and (5). (a) $\delta_{\text{paging}} = 1$. $C_{\min} = 1.5226$ and $C_{\max} = 4.4013$. (b) $\delta_{\text{paging}} = 9$. $C_{\min} = 9.5885$ and $C_{\max} = 36.3425$.

TABLE I
COMPARISON BETWEEN ANALYTICAL AND SIMULATION RESULTS WHEN PARAMETERS $\mathbf{p}$ AND μ_x'S ARE GIVEN, RESPECTIVELY, IN (3) AND (5). $N_{\text{CALL}} = 10^5$

	$\delta_{\text{paging}} = 1$				$\delta_{\text{paging}} = 9$		
the strategy given in (10)	N_{TAU}	analytical	0.0549	the strategy given in (11)	N_{TAU}	analytical	0.0633
		simulation	0.0545			simulation	0.0634
		rel. diff. (%)	0.5951			rel. diff. (%)	0.1919
	N_{paging}	analytical	1.0288		N_{paging}	analytical	1.0021
		simulation	1.0280			simulation	1.0020
		rel. diff. (%)	0.0707			rel. diff. (%)	0.0036
	C	analytical	1.5226		C	analytical	9.5885
		simulation	1.5189			simulation	9.5893
		rel. diff. (%)	0.2402			rel. diff. (%)	0.0080

existing studies on TAL allocation either simply treated the TAL-based LU scheme as an MBLU or a DBLU scheme, or ignored the mobility and traffic characteristics of individual UEs and thus cannot exploit the full potential of the TAL-based LU scheme. In this paper, we developed an embedded Markov chain model to analyze the signaling cost of the TAL-based LU scheme for local UEs whose mobility exhibits strong regularity. We derived formulas for the TAU cost and the paging cost of a TAL allocation strategy, whose accuracy was tested through simulation. Some formulas can be further used to design optimal sequential paging schemes. We also proposed an approach to preclude ineligible TAL allocation strategies. With the formulas derived in this paper, we can find an optimal strategy among all the eligible strategies to optimize the performance of the TAL-based LU scheme.

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