Probability Criterion Based Location Tracking Approach for Mobility Management of Personal Communications Systems

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Abstract — **We consider a probability criterion based location tracking scheme for personal communications systems. This scheme seeks to minimize the average signaling cost due to both paging and registration for individual mobile users. A timer-based location update and a single-step dynamic paging procedure are used. Each user-network interaction resets the timer and modifies location record. The user registers when the timer expires. Users are paged over personally constructed *paging areas*. The size of these areas is an increasing function of the time since last user-network contact. The shape of the paging area depends on the particular probability distribution of user location.**

The problem formulation is applicable to arbitrary motion models assuming the associated user location distribution is available or can be estimated. As an illustrative example, we use a Brownian motion process for user motion and assume Poisson call arrivals. We then investigate the influence of user mobility and call parameters on the optimum timer value and on the optimum configuration of the paging area with P_c probability of paging success. We show that this scheme can reduce the cost by up to 50% as compared to the conventional timer-based scheme [7] and the cost lower-bounds that of the Adaptive Location Area Tracking Scheme [4] with a proper choice of the probability criterion P_c.

1 Introduction

Mobility management demands a significant portion of the network resources. Thus, future wireless networks will be confronted with the problem of how to keep track of user locations efficiently given limited network signaling resources. One of the strategies proposed is the time-based location tracking strategy which uses a timer to trigger location update (LU) and multiple-step procedure for paging [7].

Following previous work on optimal timer-based tracking method [1, 2], we formulate a timer-based location tracking problem but use different assumption and structure for LU rate and paging area (PA) dimensioning. We assume that only a fixed set of locations with a high probability denoted by P_c will be polled. If the user is not present in these locations, we stop and declare a paging failure rather than continuing until the user is found.

In cellular networks, every time a user gets a call or makes a call, the network knows the location (attached base station) of the user. Thus, we assume that each user-network interaction resets the timer and modifies location record. Because the *location uncertainty* of the user is growing with time since last user-network interaction, in order to maintain the P_c satisfactory probability of finding the user the network must page the user using a PA that changes dynamically according to the time duration since last user-network interaction.

The problem is formulated as an optimization problem that minimizes the combined paging and LU signaling cost with respect to a timer value T_r which is the maximum time for a user to wait before reporting its location to the system. The tradeoff between choosing a longer T_r to have lower LU rate but larger PA or vice versa is studied. Since location probability P_c comes into play, we call it the Probability Criterion Based Tracking Scheme.

The optimum PA to achieve P_c probability of paging success is investigated for arbitrary motion process and is proven to be simply defined by an isocline of the continuous probability distribution function of user location. As a particular example, we use a 2D Brownian motion mobility and Poisson call arrivals. Given P_c, the optimum timer value T_r^* is found as a function of the average *location uncertainty* and call arrival rate. The introduction of the probability criterion P_c brings another degree of freedom into this design problem. Performance analysis shows that this scheme can reduce the cost by up to 50% as compared to the conventional Fixed Time Based Scheme [7] and the minimum cost of this scheme lower-bounds that of the Adaptive Location Area Tracking Scheme [4] with proper choice of P_c.

2 Problem Formulation

Assume a model for user location can be described by a time-varying conditional pdf $p(x, y|x_0, y_0, t)$, conditioned on the location (x_0, y_0) where the user and network last contacted and t the time since this last contact. Given P_c, at time t, the location probability over a region $D(t)$ on the x-y plane is the integral:

$$I\{D(t)\} = P\{(x, y) \in D(t)\} = \int\int_{D(t)} p(x, y|x_0, y_0, t)\, dx\, dy \tag{1}$$

which is a function of the region $D(t)$.
The area of $D(t)$ is defined as:

$$\mathcal{A}\{D(t)\} = \int\int_{D(t)} dxdy \tag{2}$$

The problem of finding the optimum PA region $D_{PA}(t) = D^*(t)$ that has an area denoted by $A_{PA}(t)$ can be formulated as the following constrained optimization problem

$$\min_{\{D(t)\}} \quad \mathcal{A}\{D(t)\} = \min_{\{D(t)\}} \int\int_{D(t)} dx\, dy \tag{3}$$

$$\text{subject to} \quad \int\int_{D(t)} p(x, y|x_0, y_0, t)\, dx\, dy \geq P_c\,. \tag{4}$$

Knowing the incoming call arrival process and the call initiation process for the user, we can calculate the average length of the LU interval that is a function of the time-out parameter T_r. The longest LU interval is T_r. Correspondingly, the largest possible PA is the region $D_{PA}(t)|_{t=T_r} = D_{PA}(T_r)$. To have a lower LU cost, T_r should be larger to make the LU rate lower. While to have a lower paging cost, T_r should

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be smaller. These conflicting requirements form a T_r optimization problem.

2.1 Cost Structure

Assuming that incoming call arrivals are Poisson with mean λ_p (calls/unit time), the overall signaling cost for location tracking of a individual user has the following additive structure:

$$C = S_p \lambda_p \, N\left\{\frac{E[A_{PA}(t)]}{A_c}\right\} + S_r \frac{1}{E[\mathcal{T}]} \tag{5}$$

[signaling units/unit time/subscriber]

where S_p and S_r are cost coefficients for paging and the registration [signaling units/Event] respectively, $N\{\cdot\}$ is a counting function that counts every cell contained in the PA region D_{PA}, A_c is the cell area assuming all the cells in the system are identical, $E[A_{PA}(t)]$ is the average PA size and $E[\mathcal{T}]$ is the average LU interval. Notice that we assume each user-network contact point is a renewal point in the location update stochastic process. We approximate $N\{\cdot\}$ for the convenience of analysis as $N\left\{\frac{E[A_{PA}(t)]}{A_c}\right\} \approx \frac{E[A_{PA}(t)]}{A_c}$ assuming $E[A_{PA}(t)] \gg A_c$.

2.2 Average Location Update Interval

For simplicity, we ignore the LUs caused by call initiations and the call connection times. That is, we consider only the *roaming interval* [1,2] between last contact and either a paging or a LU event. We therefore seek to minimize the average cost over roaming intervals. As shown in Figure 1, the points on the time axis when the user's location records are updated are random renewal points which consist of both the call arrival points and the time-out registration points. LU_C denotes the LU caused by call arrivals and LU_T denotes the LU caused by time-out registrations. Every time interval in the figure is a LU interval.

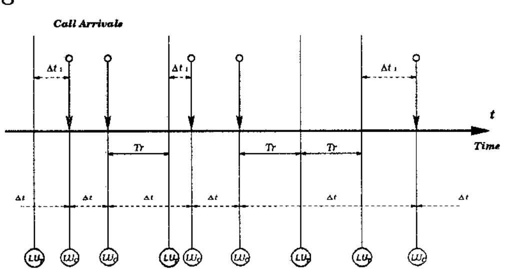

Figure 1: Call arrivals and renewal points for location updates.

According to the Poisson call arrival assumption, the inter-arrival time Δt is exponentially distributed, and the interval Δt_1 defined as the interval from a fixed point to next arrival point, is also exponentially distributed [8].

We define a random variable $\mathcal{T}$ as *location update interval.* With respect to the last location renewal point, all the call arrivals with inter-arrival intervals longer than the time-out parameter T_r will be terminated by location registration events. Thus the pdf of $\mathcal{T}$ is a truncated exponential distribution and can be calculated as

$$f_{\mathcal{T}}(t) = p_{\Delta t_1}(t)[u(t) - u(t-T_r)] + \left\{\int_{T_r}^{\infty} p_{\Delta t}(\tau)d\tau\right\}\delta(t - T_r) \tag{6}$$

as is shown in Figure 2. Then the average value of $\mathcal{T}$ is found to be:

$$E[\mathcal{T}] = \frac{1}{\lambda_p}[1 - e^{-\lambda_p T_r}] \tag{7}$$

2.3 Average Size of the Paging Area

The area of a PA $A_{PA}(t)$ depends on the time interval between the last location renewal and next call arrival, which is defined as the *paging arrival interval.* It is a random variable and has possible value as small as zero and as large as T_r^-. Because paging arrivals depend on call arrivals with interval Δt or Δt_1 less than T_r, the paging arrival interval has a conditional exponential pdf:

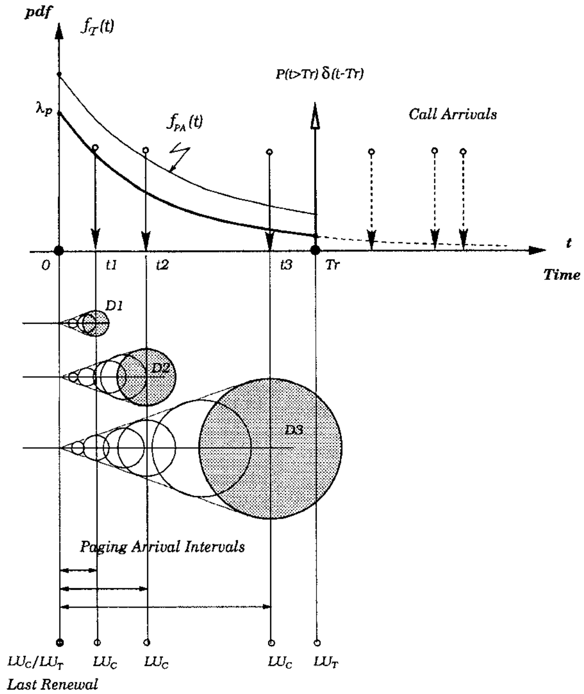

Figure 2: The pdf of LU interval $f_{\mathcal{T}}(t)$, the pdf of paging arrival interval $f_{PA}(t)$, and page arrivals with corresponding PAs. Circular PA shapes are due to an isotropic Brownian motion mobility assumption.

$$f_{PA}(t) = p_{\Delta t}(t \mid 0 \le t < T_r) = \frac{\lambda_p e^{-\lambda_p t}}{1 - e^{-\lambda_p T_r}}, \qquad 0 \le t < T_r \tag{8}$$

which is also shown in Figure 2. Hence the average PA size can be computed as:

$$E[A_{PA}] = \frac{\lambda_p}{1 - e^{-\lambda_p T_r}} \int_0^{T_r^-} A_{PA}(t)\, e^{-\lambda_p t} dt \tag{9}$$

3 Paging Area Optimization

We seek to use the least PA area to achieve the maximum probability of paging success specified by P_c, assuming that all the elements of the domain in the following discussions have the same size. As an aid to notion we define the size of a region D and the probability over a region D respectively to be

$$\begin{aligned} M(D) &= \int_D d\underline{x} \\ Q(D) &= \int_D p(\underline{x}|\underline{x}_0, t)d\underline{x}\,. \end{aligned}$$

Then the problem of optimum PA for arbitrary dimension can be formulated as

$$\min_{\{D\}} \quad M(D) \tag{10}$$

$$\text{subject to} \quad Q(D) \ge P_c\,. \tag{11}$$

This problem is solved by following theorem and corollary:

Theorem 1 (Optimum PA Theorem) *A smallest size region D^* which still meets a probability criterion $Q(D^*) \geq P_c$ can be constructed by choosing the elements $\underline{x}$ with largest $p(\underline{x}|\underline{x}_0, t)$ until their collective probability meets or exceeds P_c.*

Corollary 1 *For a density function $p(\underline{x}|\underline{x}_0, t)$ such that $Q(D_0) < P_c$, we can construct a solution set D^* for equations (10) and (11) by finding a constant ϕ^* such that*

$$\forall\, \underline{x} \in D^* \implies p(\underline{x}|\underline{x}_0, t) \geq \phi^*$$

and

$$p(\underline{z}|\underline{x}_0, t) < \phi^* \implies \underline{z} \notin D^*$$

and in addition $\forall \epsilon > 0$ with

$$S = \{\, \underline{s} \mid p(\underline{s}|\underline{x}_0, t) \geq \phi^* + \epsilon \,\}$$

we have $Q(S) < P_c$.

We omit all the proofs here which can be found in [4].

The analysis assumes no fundamental indivisible unit of area in the domain. Of course the fundamental elements are actually cells which cannot be further divided. The size of a PA region is then actually all the areas of the cells contained in the PA and thus the area of the PA is discrete. The location probability associated with the PA is a sum of the cell probabilities which is also discrete. The analysis is therefore only approximate and becomes more exact as the granulation of cell area relative to PA area becomes fine. For situations where cell sizes are variable and not of fine granularity, we have a difficult discrete packing problem which is only approximately solved by the analysis.

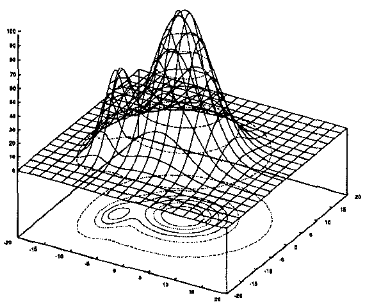

Figure 3: Different contour lines of a 2D location pdf function and their corresponding PA regions with boundaries defined by the projections of the contour lines on to x-y plane.

For our assumption with two-dimensional location pdf, Theorem 1 and Corollary 1 indicate that the optimum PA region on x-y plane under probability P_c is the region which has the minimum area and meanwhile attains the probability P_c. It consists of a set of locations with a boundary defined simply by the isocline of user location pdf formed by letting $p(x, y|x_0, y_0, t) = \phi^*$. Figure 3 serves to illustrate this result. The characteristics of a user's location pdf is used to determine the PA shape.

4 2D Mobility Assumption

Although the problem formulation is applicable to arbitrary mobility models and their location pdfs, it is useful to illustrate with an example. We use a two-dimensional isotropic Brownian motion process and assume that the diffusion constants $\mathcal{D}_x = \mathcal{D}_y = \mathcal{D}$, drift velocities $v_x = v_y = v = 0$ and the correlation coefficient $\rho_{xy} = 0$.

$$p_{xy}(x, y|x_0, y_0, t) = \frac{1}{2\pi\mathcal{D}t} \exp\left\{ \frac{-(x - x_0)^2}{2\mathcal{D}t} + \frac{-(y - y_0)^2}{2\mathcal{D}t} \right\} \quad (12)$$

where $\mathcal{D}$ represents the *location uncertainty* of the motion.

Assuming an initial point $x_0 = y_0 = 0$ and changing coordinates into polar, the expression can be further simplified to

$$p_{r,\theta}(r, \theta \mid r_0 = 0, \theta_0, t) = \frac{1}{2\pi\mathcal{D}t} \exp\left[-\frac{r^2}{2\mathcal{D}t} \right], \quad r \geq 0 \quad (13)$$

From this expression, the area of the PA that attains probability P_c can be found as

$$A_{PA}(t) = \pi \times R_c^2 = 2\pi\,\mathcal{D}\,t\, \ln\left(\frac{1}{1 - P_c}\right). \quad (14)$$

5 Cost Minimization and Choice of Probability Criterion

Using Equations 9 and 14, we can calculate the average PA size $E[A_{PA}]$. With Equation 7, the overall cost function becomes

$$C = S_p \frac{2\pi\mathcal{D}}{A_c} \left(1 - \frac{\lambda_p T_r\, e^{-\lambda_p T_r}}{1 - e^{-\lambda_p T_r}} \right) \ln\left(\frac{1}{1 - P_c}\right) + S_r \frac{\lambda_p}{1 - e^{-\lambda_p T_r}} \quad (15)$$

Differentiating C with respect to T_r and setting $\frac{\partial C}{\partial T_r} = 0$ gives an equation for the optimum value of T_r. There is no closed-form expression for the optimum time-out parameter T_r^*, but it can be calculated numerically.

Figures 4 and 5 are plots of the overall cost C with respect to the time-out parameter T_r for different values of the diffusion constant $\mathcal{D}$ or the mean call arrival rate λ_p. We observe from the the curves that there exists an optimum value for the time-out parameter T_r, which decreases when either D or λ_p increases.

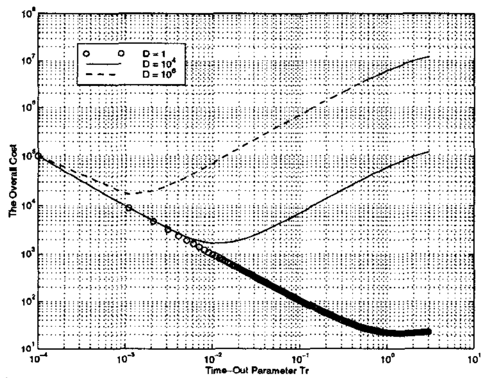

Figure 4: Effect of $\mathcal{D}$ on the cost function, with $\lambda_p = 1$, $P_c = 0.99$.

Figure 6 plots T_r^* as a function of *mobility index* $\frac{\mathcal{D}}{\lambda_p}$ under different P_c. We define *mobility index* [1,6] as the ratio $\mathcal{D}/\lambda_p$ and have found it used it to describe average location uncertainty in (length^2/call). Figure 7 is a plot of minimum cost C_{min} as a function of P_c for different values of mobility index. From these curves, we can see the effect of the mobility index on the optimum selection of the time-out parameter and the resulting cost. Under certain P_c, a higher mobility index requires a smaller time-out parameter and a higher tracking cost. For a user with a given mobility index, the

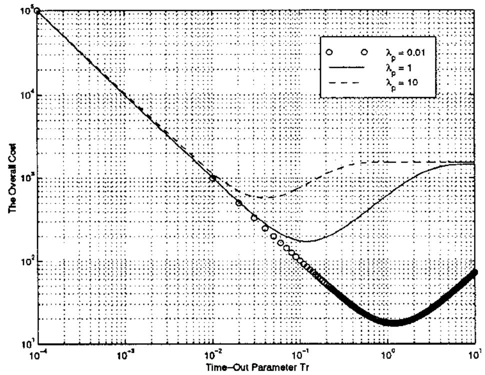

Figure 5: Effect of λ_p on cost function, with $\mathcal{D} = 100$, $P_c = 0.99$.

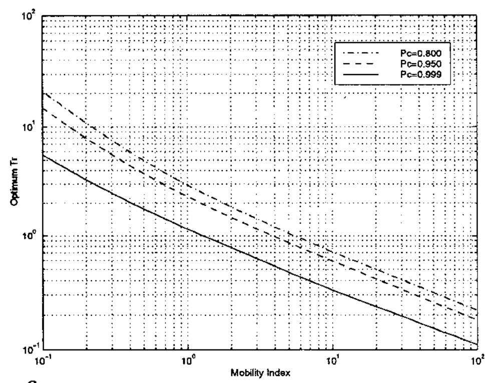

Figure 6: Optimum time-out parameter w.r.t. mobility index

higher the probability criterion selected, the smaller T_r^*, and the higher C_{min}.

The choice of the probability criterion P_c affects the quality of service since the remaining $(1-P_c)$ low probability area is left unsearched and $(1-P_c)$ represents the failure probability of the method. If a user happens to be in this area when an call arrives, the call is lost. P_c can be chosen large enough such that the probability of being outside the the paging coverage area is negligible. However, larger P_c means larger PA and thus higher cost. In fact, $P_c = 1$ implies infinite paging cost since all locations must be searched. This trade-off offers different location *performance/cost* choices from which users can select.

The group of curves in Figure 7 can be viewed as operating curves. Every point of a curve corresponds to a pair of values for performance and cost (P_c, C_{min}). According to the curves, users with different mobility indices may have different best choices of the operating points. When the mobility index is low, the system can track the user with little cost

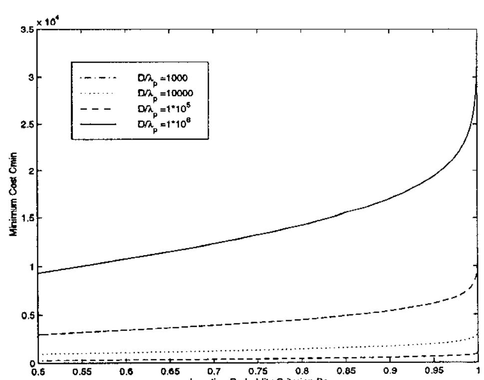

Figure 7: Minimum cost w.r.t. P_c

by using a high probability threshold. While when mobility index is high, using a lower probability threshold to track the user seems more appropriate. Using a high probability threshold to track a user with high mobility index will cause the tracking cost to increase drastically and this should be avoided except in special occasions. Therefore, we have to sacrifice certain amount of the service quality (probability of being located) to maintain a reasonable service cost. We assume, of course, that the user can always make an intelligent choice on how much he or she is willing to pay for what kind of location tracking service quality.

6 Performance Analysis

6.1 Comparison with the Fixed Periodic Time-Based Scheme

Let us assume that the probability criterion scheme proposed here and the conventional Fixed Periodic Time-Based Scheme [7] have the same LU cost and both schemes use a single-step blanket paging procedure. We can then compare the cost performance of both schemes. The main difference between the methods is that the Probability Criterion Based Scheme uses an optimized timer value and time-dependent PAs, while the Fixed Periodic Time-Based Scheme uses a fixed timer value and constant PAs for each call arrival. We choose the probability of successful paging $P_c = 0.99$ for both schemes.

We calculated the cost ratio of these schemes (Cost[prob. based]/Cost[fixed periodic]) and plot it in Figure 8 with respect to the registration interval of the Fixed Periodic Time Based Tracking Scheme.

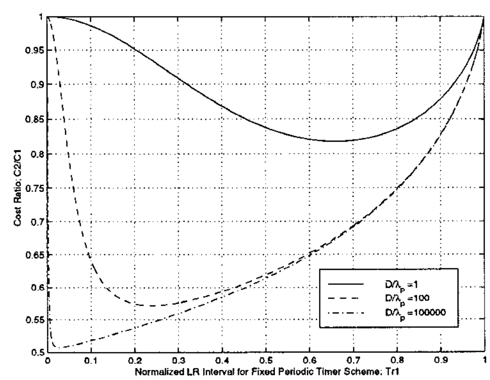

Figure 8: Cost ratio between Probability Criterion Based Scheme and Fixed Periodic Time Based Scheme w.r.t. registration interval T_{r1} of the latter scheme.

From these curves, we see that the Probability Criterion Based Tracking Scheme outperforms the Fixed Periodic Time Based Tracking Scheme. Performance gain increases with the mobility index $\mathcal{D}/\lambda_p$ and the upper limit for cost reduction is 50%, which occurs with high value of mobility index.

6.2 Comparison with the Adaptive Location Area Tracking Scheme

We compare the minimum cost C_{min} of the scheme proposed here with the Adaptive Location Area Tracking Scheme [4] under a one-dimensional cellular network assumption. Both C_{min} s must be calculated numerically. In Figure 9, we plot the C_{min} s of these two schemes as a function of mobility index $\mathcal{D}/\lambda_p$. For the Adaptive LA Tracking Scheme, we plot C_{min} for different values of velocity v. For the Probability Criterion Based Scheme, we plot C_{min} under different probability P_c. This figure shows that the Probability Criterion Based Tracking Scheme can outperform the Adaptive Location Area Tracking Scheme with proper choice of the probability criterion P_c. The curve with $P_c = 0.95$ lower bounds C_{min} of the Adaptive LA Tracking Scheme as shown in the figure. Notice that the performance of the Probability Criterion Based Scheme does not explicitly depend upon mobile velocity, but rather upon mobile *location uncertainty* as embodied in this instance by the mobility index $\mathcal{D}/\lambda_p$.

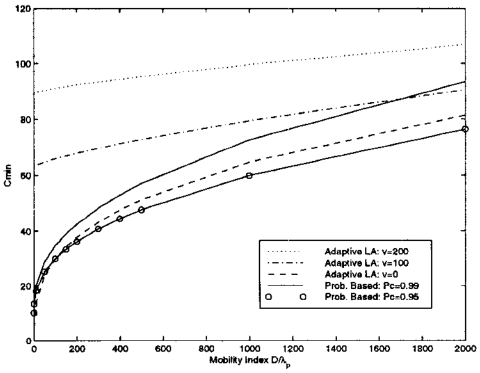

Figure 9: Comparison of the minimum cost C_{min} s w.r.t. mobility index $\mathcal{D}/\lambda_p$: Adaptive Location Area Tracking Scheme versus Probability Criterion Based Tracking Scheme.

7 Discussion of System Issues

Generally, we assume that identical cells cover the whole service area contiguously and without overlapping. Each cell can be identified by a geographic index (x, y). Base station broadcasts its beacon signal containing the index (x, y) periodically to all the users in the cell.

Both the network and user are equipped with timers which are reset to zero after each user-network interaction (paging, call initiation, registration, and hand-off, etc.). The network always modifies the location records of a user whenever there is a user-network interaction. Thus, it keeps the last known location of the user specified by (x_0, y_0). When no user-network interaction occurs before a time duration T_r, the user must register its current location with the network.

When a call arrives for a user, the network stops the network timer and computes the paging area (PA) for the user based on the current value of the timer, the location uncertainty $\mathcal{D}$ and the pre-specified location probability P_c. The computation gives a list of cells contained in the PA. Then network sends paging signals to all the cells in the list to page the user. The computation of the cell list which could also be done by the mobile, obviates the necessity of storing either at the mobile or in the network. The computation is a compression of the list information of sorts.

The network calculates the current best time-out parameter T_r^* after a LU event, according to the estimated mobility and call traffic parameters ($\mathcal{D}$ and λ_p) of the user in the previous period, and sends the timer value to the user. Thus, both network and user use the optimized timer value.

8 Conclusions

This study provides insight into the time-based mobility management problem, on how to set the timer value and configure the individual PAs to achieve the minimum cost. The study also gives a general formulation of the time-based location tracking problem which allows us to analyze time-based tracking schemes with variant paging procedures. The main conclusions are below.

(1) The study clearly shows the effects of average location uncertainty $\mathcal{D}$ and call arrival rate λ_p on the optimum *timer value* and the optimum *paging area* for location tracking.

(2) The optimum timer value T_r^* is a function of $\mathcal{D}$ and λ_p. A higher mobility index $\mathcal{D}/\lambda_p$ requires a shorter T_r and incurs a higher cost, which clearly indicates that if a user is location-volatile and receives calls less often, it is better to track the user with a smaller T_r. In contrast, if a user moves less and receives more calls, a longer T_r is more appropriate.

(3) With a continuous location probability distribution, the optimum PA to achieve the P_c probability of paging success consists of a set of locations with a boundary defined simply by the isocline of user location pdf within which the probability attains P_c.

(4) The scheme can offer different location *performance/cost* choices from which users can select.

(5) Performance analysis shows that this scheme can reduce cost by up to 50% as compared to the conventional Fixed Periodic Time-Based Scheme. The cost of this scheme lower-bounds that of the Adaptive LA Tracking Scheme with proper choice of the probability criterion P_c.

References

[1] C. Rose, "Minimizing the Average Cost of Paging and Registration: A Timer-Based Method", *ACM Journal of Wireless Networks*, vol.2, no.2, 1996, pp.109-116.

[2] C. Rose, "Minimization of Paging and Registration Cost Through Registration Deadlines", *Proc. IEEE ICC'95*, Seattle, WA, June 1995, pp.735-739.

[3] C. Rose, R. Yates, "Location Uncertainty in Mobile Networks: a theoretical framework", *IEEE Communications Magazine*, vol.35, no.2, February, 1997, pp.94-101.

[4] Z. Lei, C. Rose, "Wireless Subscriber Location Tracking for Adaptive Mobility Management", *WINLAB Tech. Report TR-131*, Rutgers University, September, 1996.

[5] D.J. Goodman, H. Xie, "Intelligent Mobility Management for Personal Communications",*IEE Colloquium on Mobility in Support of Personal Communications*, London, England, June 16, 1993.

[6] H. Xie, S. Tabbane, D.J. Goodman, "Dynamic Location Area Management and Performance Analysis" *Proc. 43rd IEEE VTC*, Secaucus, NJ, May 1993, pp.536-539.

[7] F.V. Baumann, I.G. Niemegeers, "An Evaluation of Location Management Procedures", *Conf. Record, 1994 3rd ICUPC* , San Diego, CA, Sept. 1994, pp.359-364.

[8] Athanasios Papoulis, *Probability, Random Variables and Stochastic Processes* , 2nd ed., McGraw-Hill 1984.