# Analysis of Distance-Based Location Management in Wireless Communication Networks

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**Abstract**—The performance of dynamic distance-based location management schemes (DBLMS) in wireless communication networks is analyzed. A Markov chain is developed as a mobility model to describe the movement of a mobile terminal in 2D cellular structures. The paging area residence time is characterized for arbitrary cell residence time by using the Markov chain. The expected number of paging area boundary crossings and the cost of the distance-based location update method are analyzed by using the classical renewal theory for two different call handling models. For the call plus location update model, two cases are considered. In the first case, the intercall time has an arbitrary distribution and the cell residence time has an exponential distribution. In the second case, the intercall time has a hyper-Erlang distribution and the cell residence time has an arbitrary distribution. For the call without location update model, both intercall time and cell residence time can have arbitrary distributions. Our analysis makes it possible to find the optimal distance threshold that minimizes the total cost of location management in a DBLMS.

**Index Terms**—Cost analysis, distance-based location management scheme, renewal process, wireless communication network

## 1 Introduction

### 1.1 Motivation

In order to successfully and efficiently deliver incoming calls to a mobile terminal, a cellular wireless communication network that provides personal communication service (PCS) needs to constantly keep track of the location of a mobile terminal. Therefore, location and mobility management is an important and fundamental issue in wireless communication. A location management scheme is a key and critical component of any wireless communication network to effectively deliver network services to mobile users. There are two essential tasks in location management, namely, location update (location registration) and terminal paging (call delivery):

- *Location update* is the process for a mobile terminal to periodically notify its current location to a network so that the network can revise the mobile terminal's location profile in a location database.
- *Terminal paging* is the process for a network to search a mobile terminal by sending polling signals based on the information of its last reported location so that an incoming phone call can be routed to the mobile terminal.

The location database entry of a mobile terminal is updated when the mobile terminal performs a location update and/or when a network performs a terminal paging during the call delivery to the mobile terminal.

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Both location update and terminal paging consume significant communication bandwidth of a wireless network, battery power of mobile terminals, memory space in location registers and databases, and computing time at base stations. Therefore, both location update cost and terminal paging cost should be minimized. However, there is a tradeoff between the cost of location update and the cost of terminal paging. More location updates are likely to reduce the cost of terminal paging, but excessive location update wastes system resources while contributes little to call delivery. On the other hand, insufficient location update increases cost of terminal paging. A dynamic location management scheme has the capability to adjust its location update and terminal paging strategies based on the mobility pattern and incoming call characteristics of a mobile terminal, such that the combined cost of location update and terminal paging is minimized.

### 1.2 Existing Work

The design and analysis of any dynamic location management scheme depend on a mobility model of mobile terminals. Various mobility models have been proposed in the literature, including the shortest distance mobility model [2], [3], the fluid flow model [7], [12], the big move and the random walk models [13], the user mobility pattern scheme [15], the cell coordinates system [32], the isotropic diffusive motion model [34], 1D Markov chains [4], [11], [14], [31], [37], and 2D Markov chains [5], [7], [17], [24].

There are three location update methods, namely, the distance-based method, the movement-based method, and the time-based method [11]. Accordingly, there are three types of dynamic location management schemes, namely, *distance-based location management schemes* (DBLMS), *movement-based location management schemes* (MBLMS), and *time-based location management schemes* (TBLMS). A DBLMS (an

MBLMS and a TBLMS, respectively) employs the distance-based (movement-based and time-based, respectively) location update method. Furthermore, a DBLMS or an MBLMS or a TBLMS can use various terminal paging methods.

Dynamic location management has been studied extensively by many researchers. The performance of movement-based location management schemes has been investigated in [5], [11], [19], [27], [28], [29], [30], [33], [47], [50]. The performance of distance-based location management schemes has been studied in [3], [11], [14], [24], [31], [32]. The performance of time-based location management schemes has been considered in [4], [11], [12], [34]. Terminal paging methods with low cost and time delay have been studied by several researchers [2], [5], [10], [24], [36], [44], [46]. Other studied were reported in [7], [13], [15], [17], [18], [20], [35], [37], [45], [49]. Dynamic location management in a wireless communication network with a finite number of cells has been treated as an optimization problem which is solved by using bioinspired methods such as simulated annealing, neural networks, and genetic algorithms [8], [38], [40], [41], [42], [43]. The reader is also referred to the surveys in [6], [26], [39], [22, Chapter 15], and [23, Chapter 11].

For distance-based location management schemes, the studies in [11], [14], [31] assume that a wireless communication network has a linear cellular structure. Such modeling is certainly practically less interesting, since a wireless communication network has a 2D configuration. A discrete-time random walk model was developed in [24], where a wireless communication network has a 2D hexagonal cell structure. However, the Markov chain model of [24] implies that intercall times and cell residence times are geometrically distributed, and the model cannot be applied to intercall times and cell residence times that are continuous random variables with arbitrary distributions. A modified distance-based location update method is considered in [3] for the 2D mesh cell structure, where the objective is to find an optimal location registration area such that the location update cost is minimized. The study in [32] deviates more from other studies, where the mobility of a mobile terminal is not modeled using cell residence times but the speed and direction of movement in a cell coordinates system.

The investigation in [52] is the most related work to our study, in which, the expected number of location updates between two consecutive phone calls is derived, and the result is further used to determine the optimal distance threshold that minimizes the total location management cost. The main problem is that the transition probabilities of the Markov chain are not accurate (see Section 4 for detailed discussion). Another limitation is that it is assumed that the intercall time has an exponential distribution, although the cell residence time can have an arbitrary distribution. The approach in [52] is extended to find location distribution of a mobile terminal, which is very useful to reduce paging cost [51]. Analysis of a random walk of a mobile terminal in a hexagonal cell configuration is also attempted in [9]. In [48], the authors even considered irregular cell structures, where an arbitrary cell topology is represented by a random planar graph.

In summarizing the current research, we find that 1) there is no accurate description of the movement of a mobile terminal in 2D cell configurations; 2) there is no any characterization of the paging area (PA) residence time which is critical in studying the performance of a DBLMS; 3) and there is no analytical expression of the cost of the distance-based location update method. The objective of this paper is to solve the above problems.

### 1.3 Our Contributions

The present paper makes significant contributions to cost analysis and minimization of dynamic distance-based location management schemes in wireless communication networks. We solve three problems and the main contributions of the paper are threefold.

The first contribution of the paper is to develop a very accurate ring level Markov chain as a mobility model to describe the movement of a mobile terminal (Section 4). Results of the Markov chain are critical in studying the paging area residence time, which in turn, is critical in studying location update cost in a DBLMS. We would like to mention that there are many other important applications of the results from our Markov chain and paging area residence time. The Markov chain can be used to study location distribution of a mobile terminal in a paging area, which is critical in designing paging methods with reduced cost in a DBLMS or an MBLMS. Our results from the Markov chain and paging area residence time can also be employed to study reachability of a mobile terminal in a paging area, which is critical in analyzing the quality of service and in reducing paging cost in a TBLMS.

The second contribution of the paper is to characterize paging area residence time for arbitrary cell residence time by using the Markov chain (Theorems 8 and 9). The main obstacle in analyzing the performance of a DBLMS is to find the characterization of the paging area residence time based on known information of the cell residence time. It is clear that the paging area residence time is determined by the cell residence time, the movement pattern of a mobile terminal, and the size of a paging area. By using a Markov chain that characterizes the movement pattern of a mobile terminal in a wireless communication network, we successfully find the characterization of the paging area residence time using known information of the cell residence time.

The third contribution of the paper is to solve the main problem in analyzing the performance of a DBLMS, namely, to find the number of paging area boundary crossings within any time interval, which is directly related to the number of location updates in a DBLMS. By using the classical renewal theory, we are able to derive the expected number of paging area boundary crossings (Theorems 1-5). This leads to analytical results on location update cost in a DBLMS. The results from the Markov chain and paging area residence time are critical in the analysis of performance of a DBLMS for arbitrary intercall time and cell residence time. Our analysis makes it possible to find the optimal distance threshold that minimizes the total cost of location management in a DBLMS.

Our analysis of the distance-based location update method is conducted for two different call handling models. For the call plus location update (CPLU) model, we consider two cases. In the first case, the intercall time has an arbitrary distribution and the cell residence time has an exponential

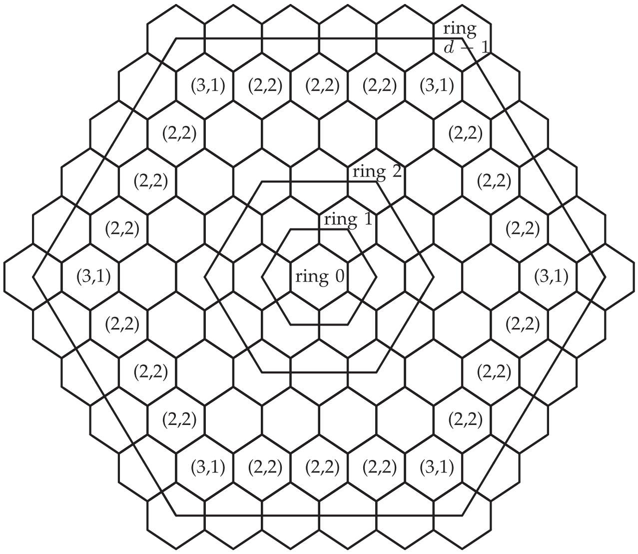


Fig. 1. The hexagonal cell configuration.

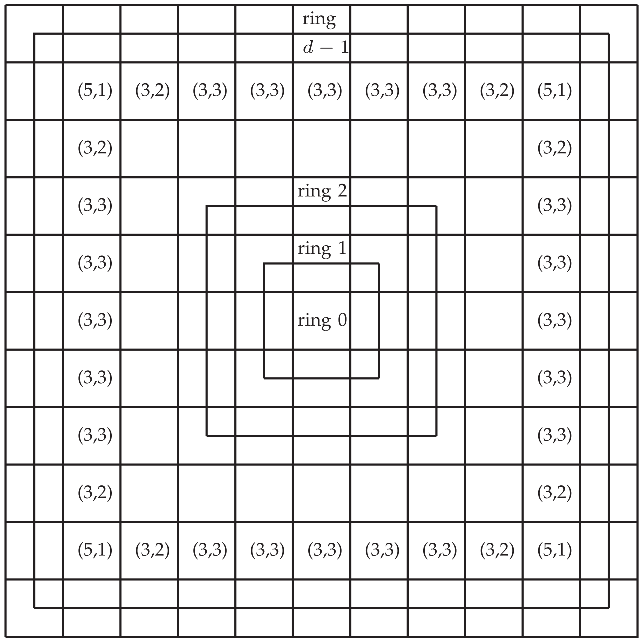


Fig. 2. The mesh cell configuration.

distribution (Theorem 10). In the second case, the intercall time has a hyper-Erlang distribution and the cell residence time has an arbitrary distribution (Theorem 11). For the call without location update (CWLU) model, both intercall time and cell residence time can have arbitrary distributions (Theorem 12). It is well known that the distance-based location update method has lower cost than the movement-based location update method. However, due to lack of analytical results of the cost of the distance-based location update method, it is not clear at all to what extent the distance-based location update method is better than the movement-based location update method. Our results in this paper give clear comparison of the two methods.

The rest of the paper is organized as follows. In Section 2, we describe the wireless communication network model used in this paper, location update methods, terminal paging methods, call handling models, cost analysis, and notations. In Section 3, we present the mathematical tools and results used in the paper, especially the probability distribution and expectation of the number of renewals in a random time interval in ordinary, equilibrium, and modified renewal processes. In Section 4, we develop a Markov chain as a mobility model to characterize the movement pattern of a mobile terminal in a wireless communication network. In Section 5, we derive an expression for the paging area residence time using an expression for the cell residence time based on the Markov chain. In Section 6, we study the number of paging area boundary crossings between two consecutive phone calls and the cost of location update in a DBLMS under two call handling models. In Section 7, we demonstrate simulation results to show the quality of our analysis and numerical data to show the impact of various parameters and performance optimization. We conclude the paper in Section 8.

## 2 Preliminaries

### 2.1 Dynamic Location Management Schemes

A dynamic location management scheme consists of a location update method, a terminal paging method, and a call handling model, all applied in a wireless communication network.

#### 2.1.1 Wireless Communication Networks

A wireless communication network has the common hexagonal cell configuration or mesh cell configuration. In the *hexagonal cell structure* (see Fig. 1), cells are hexagons of identical size and each cell has six neighbors. In the *mesh cell structure* (see Fig. 2), cells are squares of identical size and each cell has eight neighbors. Throughout the paper, we let $q$ be a constant such that $q = 3$ for the hexagonal cell configuration and $q = 4$ for the mesh cell configuration. By using the constant $q$, the hexagonal cell configuration and the mesh cell configuration can be treated in a unified way. For instance, we can say that each cell has $2q$ neighbors without mentioning the particular cell structure. The network is homogeneous in the sense that the behavior of a mobile terminal is statistically the same in all the cells.

Let $s$ be the cell registered by a mobile terminal in the last location update. The cells in a wireless networks can be divided into rings, where $s$ is the center of the network and called ring 0. The $2q$ neighbors of $s$ constitute ring 1. In general, the neighbors of all the cells in ring $r$, except those neighbors in rings $r - 1$ and $r$, constitute ring $r + 1$. For all $r \geq 0$, the cells in ring $r$ have distance $r$ to $s$. For all $r \geq 1$, the number of cells in ring $r$ is $2qr$. Notice that the rings are defined with respect to $s$. When a mobile terminal updates its location to another cell $s'$, $s'$ becomes the center of the network, and ring $r$ consists of the $2qr$ cells whose distance to $s'$ is $r$.

When a mobile terminal $u$ moves out of a cell $s$, it is normally assumed that $u$ moves into one of $s$s $2q$ neighbors with equal probability [19], [27], although this assumption is irrelevant in analyzing location update cost of movement-based schemes. However, how $u$ moves into the neighboring cells is very important in analyzing location update cost of distance-based schemes and quality of service of time-base schemes, and in reducing paging cost, since the way $u$ moves into the neighboring cells determines how fast or slow $u$ reaches the boundary of a paging area. When $u$ leaves ring 0 (i.e., $s_0$), $u$ definitely enters ring 1. When $u$ leaves a cell in ring $r \geq 1$, we are interested in $a$ defined as the probability that $u$ moves into ring $r+1$, and $b$ defined as the probability that $u$ moves into ring $r-1$. It is clear that the probability that $u$ remains in ring $r$ is $1-a-b$. For instance, if $u$ moves into one of $s$ s $2q$ neighboring cells with equal probability, then for a typical cell, we have $a = b = (q-1)/(2q)$.

#### 2.1.2 Location Update Methods

A mobile terminal $u$ constantly moves from cell to cell. Such movement also results in movement from ring to ring. Let the sequence of cells visited by $u$ before the next phone call be denoted as $s_0, s_1, s_2, \ldots, s_d, \ldots$, where $s_0 = s$ is $u$'s last registered cell (not the cell in which $u$ received the previous phone call) and considered as $u$'s current location. There are three location update methods proposed in the current literature, namely, the distance-based method, the movement-based method, and the time-based method. In the *distance-based location update method*, location update is performed as soon as $u$ moves into a cell $s_j$ in ring $d$, where $d$ is a distance threshold, i.e., the distance of $u$ from the last registered cell $s$ is $d$, such that $s_j$ is registered as $u$'s current location. It is clear that $j \geq d$, i.e., it takes at least $d$ steps for $u$ to reach ring $d$. In the *movement-based location update method*, location update is performed as soon as $u$ has crossed cell boundaries for $d$ times since the last location update, where $d$ is a movement threshold. It is clear that the sequence of registered cells for $u$ is $s_d, s_{2d}, s_{3d}, \ldots$ In the *time-based location update method*, location update is performed every $\tau$ units of time, where $\tau$ is a time threshold, regardless of the current location of $u$.

#### 2.1.3 Terminal Paging Methods

In all dynamic location management schemes, a current *paging area* consists of rings $0, 1, 2, \ldots, d-1$, where $d$ is some value appropriately chosen. We say that such a PA has radius $d$. Since the number of cells in ring $r$ is $2qr$, for all $r \geq 1$, the total number of cells in a PA is $qd^2 - qd + 1$. It should be noticed that a PA is defined with respect to the current location of a mobile terminal, and is changed whenever a mobile terminal updates its location. The radius $d$ of a PA can be adjusted in accordance with various cost and performance considerations. On the other hand, the location and size of a cell are fixed in a wireless network.

Two terminal paging methods have been proposed in the literature. In the *simple paging method*, the radius of a PA is fixed at $d$, where $d$ is the distance threshold used by a distance-based location update method, or the movement threshold used by a movement-based location update method, or appropriately chosen in accordance with the time threshold used by a time-based location update method. In the *selective paging method*, cells in a PA or the entire wireless communication network are divided into disjoint areas, such that these areas are paged one after another successively, until a mobile terminal is found. The advantage of the simple paging method is that a mobile terminal is guaranteed to find in one polling step. The disadvantage of the simple paging method is the high cost of polling $qd^2 - qd + 1$ cells. The advantage of the selective paging method is the low expected cost of paging. The disadvantage of the selective paging method is the long time delay in finding a mobile terminal. Since the main focus of this paper is to study location update cost in a DBLMS, we will only consider the simple paging method. We believe that analysis of the selective paging method as well as its impact on the performance of a DBLMS is a deep topic and deserves a separate paper to deal with.

#### 2.1.4 Call Handling Models and Renewal Processes

We will consider two different call handling models [29]. In the *call plus location update* model, the location of a mobile terminal is updated each time a phone call arrives. That is, in addition to distance-based or movement-based or time-based location updates, the arrival of a phone call also initiates location update and defines a new PA. This causes the original location update cycle of a mobile terminal being interrupted. In the *call without location update* model, the arrival of a phone call has nothing to do with location update, that is, a mobile terminal still keeps its original location update cycles.

### 2.2 Performance Analysis

#### 2.2.1 Notations

Throughout the paper, we use $\boldsymbol{P}[E]$ to denote the probability of an event $E$. For a random variable $T$, we use $\boldsymbol{E}(T)$ to represent the expectation of $T$ and $\lambda_T = \boldsymbol{E}(T)^{-1}$. The probability density function (pdf) of $T$ is $f_T(t)$, and the cumulative distribution function (cdf) of $T$ is $F_T(t)$. The Laplace transform of $f_T(t)$ and $F_T(t)$ for a nonnegative random variable $T$ are defined as

$$f_T^*(s) = \boldsymbol{E}(e^{-sT}) = \int_0^\infty e^{-st} f_T(t)dt,$$

and

$$F_T^*(s) = \int_0^\infty e^{-st} F_T(t)dt.$$

There are several important random variables in the study of dynamic location management. The *intercall time* $T_c$ is defined as the length of the time interval between two consecutive phone calls. The *cell residence time* $T_s$ is defined as the time a mobile terminal stays in a cell before it moves into a neighboring cell. The *paging area residence time* $T_m$ is defined as the time a mobile terminal stays in the current PA before it moves out of the PA. The quantity $\rho = \lambda_{T_c}/\lambda_{T_s}$ is the *call-to-mobility ratio*.

Throughout the paper, we use DBLMS $(d)$ to represent a dynamic location management scheme using the distance-based location update method with distance threshold $d$ and the simple paging method. We use DBLMS $(d)$-CPLU and DBLMS $(d)$-CWLU to represent a dynamic location management scheme using the distance-based location update method with distance threshold $d$ under the CPLU model and the CWLU model, respectively.

#### 2.2.2 Cost Analysis

The cost of dynamic location management contains two components, i.e., the cost of location update and the cost of terminal paging. The cost of location update is proportional to the number of location updates. If there are $X_u$ location update between two consecutive phone calls, the cost of location update is $\Delta_u X_u$, where $\Delta_u$ is a constant. Since $X_u$ is a random variable, the location update cost is actually calculated as $\Delta_u \boldsymbol{E}(X_u)$. The cost of terminal paging is proportional to the number of cells paged. If a PA has radius $d$, the cost of paging is $\Delta_p(qd^2 - qd + 1)$, where $\Delta_p$ is a constant.

In a DBLMS, location update is performed only when a mobile terminal moves out of the current PA. In an MBLMS, a mobile terminal may still resides in the current PA when a location update is performed. This implies that there might be unnecessary location updates. For the same value $d$ of distance threshold and movement threshold, an MBLMS requires more location update cost than a DBLMS. Using a DBLMS instead of an MBLMS can reduce location update cost. However, for large $d$, if the simple paging method is used, such reduction in location update cost does not significantly reduce the total location management cost because the paging cost grows dramatically as $d$ becomes large. Therefore, reducing both location update cost and paging cost is the only effective way to design high performance dynamic location management schemes.

Dynamic location management is per-terminal based. A mobile terminal is specified by $f_{T_c}(t)$ and $f_{T_s}(t)$, where $f_{T_c(t)}$ is the call pattern and $f_{T_s(t)}$ is the mobility pattern. Since a location update method determines the location update cost and a terminal paging method determines the terminal paging cost, for given a mobile terminal, we need to find a balanced combination of a location update method and a terminal paging method such that the total location management cost for the mobile terminal is minimized.

## 3 Renewal Processes

A *renewal process* is defined by a sequence of independent random variables $T_1, T_2, T_3, \ldots$, where $T_2, T_3, \ldots$ are a sequence of independent and identically distributed (i.i.d.) random variables with a common pdf, but $T_1$ may have a different pdf. (The reader is referred to [1] for a general introduction to the renewal theory.) A renewal process has many associated random variables and properties. The most interesting property related to our study is the number of renewals in a random period of time. We use $X(t)$ to denote the number of renewals in a time interval of length $t$. Let $S_j = T_1 + T_2 + \cdots + T_j$. If $S_j \le t < S_{j+1}$, we say that the number of renewals $X(t)$ in a time interval of length $t$ is $j$.

Let $X$ be the number of renewals in a random time interval of length $T_c$. Define

$$H(t) = \sum_{j=1}^{\infty} F_{S_j}(t).$$

The following theorem gives the probability distribution and the expectation of $X$for an arbitrary renewal process. The theorem will be used in Theorems 3 and 10. The proof is given in Appendix 1, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2012.123.

**Theorem 1.** *For an arbitrary renewal process, we have*

$$\boldsymbol{P}[X = j] = \int_0^\infty \big(F_{S_j}(t) - F_{S_{j+1}}(t)\big) f_{T_c}(t)dt,$$

*for all $j \ge 0$; furthermore, the expected number of renewals in a random time interval of length $T_c$ is*

$$\boldsymbol{E}(X) = \int_0^\infty H(t) f_{T_c}(t)dt,$$

*for arbitrary $T_1, T_2, T_3, \ldots$ and $f_{T_c}(t)$.*

### 3.1 Ordinary Renewal Processes

An *ordinary renewal process* is defined by a sequence of i.i.d. random variables $T_1$, $T_2$, $T_3, \ldots$, with a common pdf $f_T(t)$ [16]. In modeling dynamic location management schemes, the $T_i$s can be cell residence times, or paging area residence times, or location update times. When the $T_i$s are cell residence times, each renewal stands for a cell boundary crossing, and $X(t)$ is the number of cell boundary crossings in a time interval of length $t$. When the $T_i$s are paging area residence times, each renewal stands for a PA boundary crossing, and $X(t)$ is the number of PA boundary crossings in a time interval of length $t$. When the $T_i$s are location update times, each renewal stands for a location update, and $X(t)$ is the number of location updates in a time interval of length $t$. When $t$ is randomized according to intercall time distribution $f_{T_c}(t)$, we are essentially considering the number of cell boundary crossings, or the number of PA boundary crossings, or the number of location updates between two consecutive phone calls.

Based on $F^*_{S_j}(s)$, we can obtain $F_{S_j}(t)$ by performing the inversion integral

$$\begin{aligned} F_{S_j}(t) &= \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} F^*_{S_j}(s)e^{st}ds \\ &= \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{(f^*_T(s))^j}{s}\cdot e^{st}ds, \end{aligned}$$

for $t \ge 0$ and $\sigma > \sigma_a$, where the integration in the complex plane is taken to be a straight-line integration parallel to the imaginary axis and lying to the right of $\sigma_a$, the abscissa of absolute convergence for $F^*_{S_j}(s)$. The usual means for performing this integration is to carry out an integration around a closed contour, namely, a semicircle of infinite radius, so that the Cauchy residual theorem can be applied. This is most easily done if $F^*_{S_j}(s)$ is in rational form [25, p. 353]. This is indeed the case for a hyper-Erlang distribution of the $T_i$s, that is,

$$f_T(t) = \sum_{i=1}^{k} w_i \left(\frac{\lambda_i e^{-\lambda_i t}(\lambda_i t)^{\gamma_i - 1}}{(\gamma_i - 1)!}\right),$$

where $w_1 + w_2 + \cdots + w_k = 1$. Special forms of hyper-Erlang distributions include hyperexponential distributions ($\gamma_i = 1$ for all $1 \le i \le k$); exponential distributions ($k = 1$ and $\gamma_1 = 1$); chi-square distributions ($k = 1$ and $\lambda_1 = 1/2$); Erlang distributions ($k = 1$).

The following theorem will be used in Section 6.1. The proof is given in Appendix 1, available in the online supplemental material.

**Theorem 2.** *If the $T_i$s have a hyper-Erlang distribution, we have*

$$F_{S_j}(t) = \sum_{j_1+j_2+\cdots+j_k=j} \binom{j}{j_1, j_2, \ldots, j_k} \prod_{i=1}^{k} (w_i \lambda_i^{\gamma_i})^{j_i} \left( \prod_{i=1}^{k} \frac{1}{\lambda_i^{\gamma_i j_i}} + \sum_{i=1}^{k} \frac{1}{(\gamma_i j_i - 1)!} \left(\frac{\partial}{\partial s}\right)^{\gamma_i j_i - 1} \left[ \frac{e^{st}}{s} \prod_{i' \neq i} \frac{1}{(s+\lambda_{i'})^{\gamma_{i'} j_{i'}}} \right]_{s=-\lambda_i} \right),$$

*for all $j \geq 0$.*

### 3.2 Equilibrium Renewal Processes

Consider a mobile terminal $u$ moving through a sequence of cells $s_1, s_2, s_3, \ldots$, with a sequence of cell residence times $T_{s,1}, T_{s,2}, T_{s,3}, \ldots$. Assume that $u$ is in $s_1$ when a phone call arrives, i.e., $u$ has been in $s_1$ for a while. Clearly, $T_{s,1}$ is the *residual time* of $u$ in $s_1$ and does not have the same pdf as the other $T_{s,i}$s. Similarly, consider a mobile terminal $u$ moving through a sequence of paging areas $\mathrm{PA}_1, \mathrm{PA}_2, \mathrm{PA}_3, \ldots$, with a sequence of paging area residence times $T_{m,1}, T_{m,2}, T_{m,3}, \ldots$ Assume that $u$ is in $\mathrm{PA}_1$ when a phone call arrives, i.e., $u$ has been in $\mathrm{PA}_1$ for a while. Clearly, $T_{m,1}$ is the residual time of $u$ in $\mathrm{PA}_1$ and does not have the same pdf as the other $T_{m,i}$s.

An ordinary renewal process $T_1, T_2, T_3, \ldots$, where the pdf of $T_1$ is the residual time of an ordinary $T_i$, is called an *equilibrium renewal process*, which can be regarded as an ordinary renewal process that has been running for a long time before it is first observed [16]. Notice that if the $T_i$s have an exponential distribution, an equilibrium renewal process becomes an ordinary renewal process.

The expected number of renewals in a random time interval of an equilibrium renewal process is surprisingly easy to obtain. The following theorem is a simple and strong result in renewal theory [16]. The theorem will be used in Theorem 12. The proof is given in Appendix 1, available in the online supplemental material.

**Theorem 3.** *For any probability distributions of $T_c$ and $T$, the expected number of renewals in a random time interval of length $T_c$ is $\boldsymbol{E}(X) = \boldsymbol{E}(T_c)/\boldsymbol{E}(T) = \lambda_T/\lambda_{T_c}$ for an equilibrium renewal process.*

### 3.3 Modified Renewal Processes

A *modified renewal process* has all the properties of an ordinary renewal process $T_1$, $T_2$, $T_3, \ldots$, except that $T_1$ has a different pdf $f_{T_1}(t)$ from that of other $T_i$s which have the same pdf $f_T(t)$ [16]. Such a renewal process occurs in a DBLMS-CPLU, where each arriving phone call in the midst of cell residence time initiates a location update and creates a new PA. However, cell residence times cannot be altered. This results in a sequence of cell residence times forming an equilibrium renewal process, which results in a sequence of paging area residence times forming a modified renewal process.

The following theorem gives the probability distribution of $X$ in a modified renewal process when $T_c$ has a hyper-Erlang distribution with pdf:

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}(\lambda_{c,i} t)^{\gamma_{c,i}-1} e^{-\lambda_{c,i} t}}{(\gamma_{c,i}-1)!} \right),$$

where $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$. The proof is given in Appendix 1, available in the online supplemental material.

**Theorem 4.** *For a modified renewal process, if $T_c$ has a hyper-Erlang distribution, we have*

$$\boldsymbol{P}[X=0] = 1 - \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s}\right)^{\gamma_{c,i}-1} \left[ \frac{f_{T_1}^*(s)}{s} \right]_{s=\lambda_{c,i}},$$

*and*

$$\boldsymbol{P}[X=j] = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s}\right)^{\gamma_{c,i}-1} \left[ \left( \frac{f_{T_1}^*(s)(1-f_T^*(s))}{s} \right) (f_T^*(s))^{j-1} \right]_{s=\lambda_{c,i}},$$

*for all $j \geq 1$.*

The following theorem gives $\boldsymbol{E}(X)$ for a modified renewal process based on Theorem 4 by straightforward calculation. The theorem will be used in Theorem 11.

**Theorem 5.** *If $T_c$ has a hyper-Erlang distribution, the expected number of renewals in a random time interval of length $T_c$ is*

$$\boldsymbol{E}(X) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s}\right)^{\gamma_{c,i}-1} \left[ \frac{f_{T_1}^*(s)}{s(1-f_T^*(s))} \right]_{s=\lambda_{c,i}},$$

*for a modified renewal process with arbitrary $f_{T_1}(t)$ and $f_T(t)$.*

## 4 Mobility Modeling

### 4.1 A Cell Level Markov Chain

It is clear that the movement of a mobile terminal can be modeled by a continuous-time semi-Markov stochastic process with discrete state space. Such a process describes the location of a mobile terminal at any time. In such a process, the state space is a set of cells (discrete state space). State transition means moving into a neighboring cell. Since a mobile terminal can stay in a cell for arbitrary amount of time, state transitions can occur at any time (continuous time). Furthermore, since the cell residence time $T_s$ can have an arbitrary $f_{T_s}(t)$ (not necessarily an exponential distribution), such a process is not a Markov process. However, notice that at the instants of state transitions, the process behaves just like an ordinary Markov chain. Therefore, at those instants, we have an *embedded Markov chain* ([25, p. 23]).

Consider a mobile terminal $u$ in a cell $s$. After staying in $s$ for $T_s$ amount of time, $u$ moves out of $s$ and enters into one of the $2q$ neighbors of $s$. It is easily observed that the movement of a mobile terminal can be described by a 2D Markov chain. For each cell, we have a state in the Markov chain associated with the cell. The transition probability from a cell to a neighboring cell is $1/(2q)$. While the cell level Markov chain accurately describes the real cell structures, the number of states is exactly the same as the number of cells. As shown below, we will need to know the behavior of the Markov chain up to ring $d$, where $d$ is the radius of a PA, which implies that the number of states in the Markov chain is as large as $qd^2 + qd + 1$. Since we will perform multiplications of matrices of transition probabilities, the computational complexity will be $O((qd^2 + qd + 1)^3) = O(d^6)$, not mentioning the huge number of repetitions of matrix multiplications to achieve the desired numerical accuracy.

### 4.2 A Ring Level Markov Chain

To reduce the number of states, we can construct a ring level Markov chain which contains states $K_0, K_1, K_2, \ldots, K_d, \ldots$, where state $K_r$ means that a mobile terminal $u$ is in ring $r$, $r \geq 0$. Initially, $u$ is in state $K_0$. Instead of the probabilities of moving into neighboring cells, we are interested in the probabilities of moving into adjacent rings, i.e., the probability $a_r$ of moving into ring $r+1$ and the probability $b_r$ of moving into ring $r-1$. Let $p_{ij}$ denote the transition probability from $K_i$ to $K_j$, where $i, j \geq 0$. Then, we have $p_{01} = 1$, $p_{r,r+1} = a_r$, $p_{r,r-1} = b_r$, $p_{r,r} = 1 - a_r - b_r$, for all $r \geq 1$. All other $p_{ij}$s not specified above are zeros. Using the $a_r$s and the $b_r$s, the matrix of transition probabilities $P = [p_{ij}]$ is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ b_1 & 1-a_1-b_1 & a_1 & 0 & 0 & \cdots \\ 0 & b_2 & 1-a_2-b_2 & a_2 & 0 & \cdots \\ 0 & 0 & b_3 & 1-a_3-b_3 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

We use $p_{ij}^{(n)}$ to denote the $n$-step transition probability from $K_i$ to $K_j$, where $i, j \geq 0$. The following result is well known ([21, p. 383]), which will be used in Theorem 7.

**Theorem 6.** *If the nth power of $P$ is $P^n = [g_{ij}]$, we have $p_{ij}^{(n)} = g_{ij}$, for all $i, j \geq 0$.*

The $a_r$s and the $b_r$s can be obtained as follows. First, we make the assumption that all the cells in the same ring are visited with equal probability. Let the cells in ring $r$ be $s_1, s_2, \ldots, s_{2qr}$. Assume that cell $s_i$ has $x_i$ neighbors in ring $r+1$ and $y_i$ neighbors in ring $r-1$. We call such a cell as an $(x_i, y_i)$ cell. If $x$ is the average value of the $x_i$s and $y$ is the average value of the $y_i$s, we have $a_r = x/(2q)$ and $b_r = y/(2q)$. That is,

$$a_r = \frac{x_1 + x_2 + \cdots + x_{2qr}}{4q^2 r},$$

and

$$b_r = \frac{y_1 + y_2 + \cdots + y_{2qr}}{4q^2 r}.$$

Let us look at the hexagonal cell structure first (see Fig. 1). When $r = 1$, all the six cells in ring 1 are $(3,1)$ cells. Thus, we have $a_1 = 1/2$ and $b_1 = 1/6$. When $r > 1$, there are six $(3,1)$ cells at the six corners of a hexagonal ring, and all other cells are $(2,2)$ cells. Thus, we have

$$a_r = \frac{3 \cdot 6 + 2(6r-6)}{36r} = \frac{2r+1}{6r},$$

and

$$b_r = \frac{1 \cdot 6 + 2(6r-6)}{36r} = \frac{2r-1}{6r}.$$

It is clear that the above expressions for $a_r$ and $b_r$ are also valid for $a_1$ and $b_1$. Next, we examine the mesh cell structure (see Fig. 2). When $r = 1$, there are four $(3,1)$ cells and four $(5,1)$ cells. Thus, we have $a_1 = 1/2$ and $b_1 = 1/8$. When $r > 1$, there are four $(5,1)$ cells at the four corners of a square ring and eight $(3,2)$ cells that are adjacent to the corner cells, and all other cells are $(3,3)$ cells. Thus, we have

$$a_r = \frac{5 \cdot 4 + 3(8r-4)}{64r} = \frac{3r+1}{8r},$$

and

$$b_r = \frac{1 \cdot 4 + 2 \cdot 8 + 3(8r-12)}{64r} = \frac{3r-2}{8r}.$$

It is clear that the above expressions for $a_r$ and $b_r$ are also valid for $a_1$ and $b_1$. The $a_r$s and the $b_r$s in the hexagonal and mesh cell structures can be unified as follows:

$$a_r = \frac{(q-1)r+1}{2qr},$$

and

$$b_r = \frac{(q-1)r-q+2}{2qr},$$

for all $r \geq 1$. The above $a_r$ and $b_r$ for $q = 3$ were obtained in [5] and adopted in subsequent studies such as [52].

Next, we notice that the assumption that all the cells in the same ring are visited with equal probability does not reflect what happens in a real cell structure. It is easily observed that when $r > 1$, the six corner cells of a hexagon ring and the four corner cells in a square ring are visited less frequently than other cells in the same ring. This means that for a hexagon ring $r$, $r > 1$, the six $(3,1)$ cells should carry less weight in calculating the $a_r$s and the $b_r$s. Similarly, for a square ring $r$, $r > 1$, the four $(5,1)$ cells and the eight $(3,2)$ cells should carry less weight in calculating the $a_r$s and the $b_r$s. The consequence is that $a_r$ should be decreased and $b_r$ should be increased, i.e.,

$$a_r = \frac{(q-1)r+1-\delta_q}{2qr},$$

and

$$b_r = \frac{(q-1)r-q+2+\delta_q}{2qr},$$

for all $r > 1$, where $\delta_q$ is an appropriately chosen value to adjust $a_r$ and $b_r$. In our study, we set $\delta_3 = 0.15$ and $\delta_4 = 0.42$ empirically, which are very accurate, as verified in Tables 1, 3, and 4.

TABLE 1
Comparison of Numerical Data and Experimental Results of $N(d)$

| | Hexagonal Cell Structure | | | Mesh Cell Structure | | |
|---|---|---|---|---|---|---|
| $d$ | numerical | experiment. | relative diff. | numerical | experiment. | relative diff. |
| 1 | 1.00000 | 1.00000 | 0.00000% | 1.00000 | 1.00000 | 0.00000% |
| 2 | 3.33333 | 3.33610 | 0.00083% | 3.25000 | 3.31748 | 0.02076% |
| 3 | 7.32302 | 7.34127 | 0.00249% | 7.19301 | 7.24409 | 0.00710% |
| 4 | 12.95031 | 12.99110 | 0.00315% | 12.75221 | 12.73219 | − 0.00157% |
| 5 | 20.20851 | 20.28351 | 0.00371% | 19.90061 | 19.80443 | − 0.00483% |
| 6 | 29.09446 | 29.13525 | 0.00140% | 28.62535 | 28.49336 | − 0.00461% |
| 7 | 39.60639 | 39.60504 | − 0.00003% | 38.91923 | 38.70158 | − 0.00559% |
| 8 | 51.74322 | 51.88440 | 0.00273% | 50.77776 | 50.62325 | − 0.00304% |
| 9 | 65.50423 | 65.62642 | 0.00187% | 64.19796 | 64.05766 | − 0.00219% |
| 10 | 80.88891 | 81.01409 | 0.00155% | 79.17770 | 78.87586 | − 0.00381% |
| 11 | 97.89691 | 97.98460 | 0.00090% | 95.71544 | 95.58845 | − 0.00133% |
| 12 | 116.52794 | 116.70402 | 0.00151% | 113.81002 | 113.72454 | − 0.00075% |
| 13 | 136.78180 | 136.89059 | 0.00080% | 133.46053 | 133.29603 | − 0.00123% |
| 14 | 158.65832 | 158.83696 | 0.00113% | 154.66623 | 154.56775 | − 0.00064% |
| 15 | 182.15736 | 182.11244 | − 0.00025% | 177.42656 | 177.28756 | − 0.00078% |
| 16 | 207.27883 | 207.32104 | 0.00020% | 201.74102 | 201.63540 | − 0.00052% |
| 17 | 234.02262 | 233.81517 | − 0.00089% | 227.60923 | 227.77793 | 0.00074% |
| 18 | 262.38867 | 262.40102 | 0.00005% | 255.03084 | 255.37568 | 0.00135% |
| 19 | 292.37691 | 292.47607 | 0.00034% | 284.00557 | 284.46322 | 0.00161% |
| 20 | 323.98729 | 323.68674 | − 0.00093% | 314.53318 | 315.06902 | 0.00170% |

TABLE 2
The Probabilities $p_{0r}^{(d)}$

| $d$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=4$ | $r=5$ | $r=6$ | $r=7$ | $r=8$ | $r=9$ | $r=10$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $q=3$ | | | | | | | | | | | |
| 1 | 0.00000 | 1.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 2 | 0.16667 | 0.33333 | 0.50000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 3 | 0.05556 | 0.40903 | 0.33333 | 0.20208 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 4 | 0.06817 | 0.27940 | 0.37344 | 0.20208 | 0.07690 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 5 | 0.04657 | 0.25933 | 0.32200 | 0.24121 | 0.10254 | 0.02836 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 6 | 0.04322 | 0.21754 | 0.30601 | 0.24109 | 0.13462 | 0.04726 | 0.01026 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 7 | 0.03626 | 0.19606 | 0.27975 | 0.24415 | 0.15104 | 0.06857 | 0.02051 | 0.00366 | 0.00000 | 0.00000 | 0.00000 |
| 8 | 0.03268 | 0.17504 | 0.26113 | 0.23945 | 0.16417 | 0.08491 | 0.03278 | 0.00854 | 0.00129 | 0.00000 | 0.00000 |
| 9 | 0.02917 | 0.15957 | 0.24307 | 0.23427 | 0.17174 | 0.09899 | 0.04431 | 0.01496 | 0.00345 | 0.00045 | 0.00000 |
| 10 | 0.02660 | 0.14617 | 0.22784 | 0.22750 | 0.17659 | 0.11005 | 0.05526 | 0.02189 | 0.00658 | 0.00136 | 0.00016 |
| $q=4$ | | | | | | | | | | | |
| 1 | 0.00000 | 1.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 2 | 0.12500 | 0.37500 | 0.50000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 3 | 0.04688 | 0.40375 | 0.34375 | 0.20563 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 4 | 0.05047 | 0.29324 | 0.37287 | 0.20134 | 0.08208 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 5 | 0.03666 | 0.26344 | 0.32539 | 0.23879 | 0.10345 | 0.03227 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 6 | 0.03293 | 0.22533 | 0.30723 | 0.23715 | 0.13524 | 0.04954 | 0.01257 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 7 | 0.02817 | 0.20230 | 0.28200 | 0.23956 | 0.14932 | 0.07109 | 0.02270 | 0.00486 | 0.00000 | 0.00000 | 0.00000 |
| 8 | 0.02529 | 0.18193 | 0.26334 | 0.23446 | 0.16147 | 0.08602 | 0.03552 | 0.01009 | 0.00187 | 0.00000 | 0.00000 |
| 9 | 0.02274 | 0.16626 | 0.24575 | 0.22926 | 0.16786 | 0.09929 | 0.04662 | 0.01711 | 0.00439 | 0.00072 | 0.00000 |
| 10 | 0.02078 | 0.15298 | 0.23081 | 0.22259 | 0.17204 | 0.10924 | 0.05723 | 0.02417 | 0.00801 | 0.00187 | 0.00028 |

Let $\phi_{ij}^{(n)}$ be the probability that in a random walk starting from state $K_i$, the first entry to state $K_j$ occurs at the $n$th step, where $n \geq 1$. In particular, $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \ldots, \phi_{0d}^{(n)}, \ldots)$ is called the *first-passage distribution* for $K_d$. The following method is well known ([21, p. 388]), which will be used in Theorems 8 and 9.

**Theorem 7.** *The $\phi_{0d}^{(n)}$s can be calculated by using the following equation:*

$$p_{0d}^{(n)} = \sum_{v=1}^{n} \phi_{0d}^{(v)} p_{dd}^{(n-v)}, \quad n \geq 1,$$

*with* $p_{dd}^{(0)} = 1$.

Theorem 7 implies that $\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \ldots, \phi_{0d}^{(n)}, \ldots$ can be obtained successively as follows:

$$\phi_{0d}^{(1)} = p_{0d}^{(1)},$$
$$\phi_{0d}^{(n)} = p_{0d}^{(n)} - \sum_{v=1}^{n-1} \phi_{0d}^{(v)} p_{dd}^{(n-v)}, \quad n \geq 2.$$

The quantity $\phi_{0d}^{(n)}$ is the probability that starting from ring 0, a mobile terminal $u$ takes exactly $n$ steps to reach ring $d$ for the first time and then immediately performs location update. We notice that $\phi_{0d}^{(1)} = \phi_{0d}^{(2)} = \cdots = \phi_{0d}^{(d-1)} = 0$, i.e., $u$ takes at least $d$ steps to reach ring $d$.

The $\phi_{0d}^{(n)}$s, where $n \geq 1$, can be computed based on $P_d$, the $(d+1) \times (d+1)$ submatrix of $P$ consisting of the first $d+1$ rows and the first $d+1$ columns, because the $\phi_{0d}^{(n)}$s only depend on the behavior of the Markov chain in states $K_0, K_1, K_2, \ldots, K_d$. (In fact, the last row of $P_d$, i.e., the $p_{dj}$s, are immaterial, because this finite Markov chain with $d+1$ states has the same first-passage distribution $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \ldots, \phi_{0d}^{(n)}, \ldots)$ for $K_d$ as the original infinite Markov chain, no matter what the $p_{dj}$s are.) Such computation involves the matrix powers $P_d, P_d^2, P_d^3, \ldots, P_d^n, \ldots$, to provide $p_{0d}^{(n)}$ and $p_{dd}^{(n)}$ for all $n \geq 1$.

The expected number of steps for a random walk starting from $K_0$ to reach $K_d$ is

$$N(d) = \sum_{n=1}^{\infty} n\phi_{0d}^{(n)} = \sum_{n=d}^{\infty} n\phi_{0d}^{(n)}.$$

In Fig. 1 of Appendix 2, available in the online supplemental material, we display numerical values of $N(d)$. We observe that $N(d)$ grows very fast as $d$ increases. To be more specific, we compute $N'(d)$ when

$$a_r = b_r = \frac{q-1}{2q}.$$

We find that

$$N'(d) = \frac{d(qd-1)}{q-1},$$

and

$$N(d) > \frac{1}{2} N'(d),$$

for all $d \geq 1$. This implies that $N(d) = O(d^2)$. The quadratic growth rate of $N(d)$ implies significant reduction in location update cost in DBLMSs compared with MBLMSs (see the comment after Theorem 12).

In Table 1, we compare the numerical data of $N(d)$ obtained from our Markov chain and the experimental data of $N(d)$ obtained from computer simulations. For each $d$, we simulate a random walk starting from $K_0$ until $K_d$ is reached and record the length of the random walk. The average length of 500,000 random walks is reported with 99 percent confidence interval $\pm 0.25713$ percent. We observe that the numerical data of $N(d)$ obtained from our Markov chain and the experimental results of $N(d)$ obtained from computer simulations are extremely close, with relative difference no more than $\pm 0.02$ percent. Although the exact values of the $a_r$s and the $b_r$s are still mysteries to us, our values of the $a_r$s and the $b_r$s are able to provide very reliable performance data.

In Table 2, we show $p_{0r}^{(d)}$, that is, the probability that a mobile terminal is in ring $r$ after crossing cell boundaries for $d$ times, where $1 \leq d \leq 10$. It is noticed that when $d > 3$, the probability that a mobile terminal is in ring $d$ after crossing cell boundaries for $d$ times is very small. This implies that an MBLMS makes more location updates than necessary, since a mobile terminal is still inside a PA with high probability after crossing cell boundaries for $d$ times.

In Tables 1 and 2 of Appendix 3, available in the online supplemental material, we show numerical values of the first-passage distribution $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \ldots, \phi_{0d}^{(n)}, \ldots)$ for $K_d$.

We would like to mention that our approach developed in this section is also applicable to an irregular cell structure [45], which can easily be divided into rings, as long as the transition probabilities, i.e., the $a_r$s and the $b_r$s, are available [52].

## 5 Paging Area Residence Time

### 5.1 Ordinary Paging Area Residence Time

A paging area residence time is ordinary if the last location update for the current paging area is ordinary. An *ordinary paging area residence time* $T_m$ is a sum of random number $n$ of i.i.d. random variables $T_{s,1}, T_{s,2}, \ldots, T_{s,n}$, which are cell residence times with $f^*_{T_{s,i}}(s) = f^*_{T_s}(s)$, for all $1 \le i \le n$. By paging area residence time, we mean ordinary paging area residence time, unless otherwise indicated (e.g., see Section 6.1).

The following theorem is the most important result of our study, which will be used in Theorems 10, 11, and 12.

**Theorem 8.** *The paging area residence time $T_m$ for a PA with radius $d$ is characterized by*

$$f^*_{T_m}(s) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} \left( f^*_{T_s}(s) \right)^n,$$

*and*

$$\boldsymbol{E}(T_m) = N(d)\boldsymbol{E}(T_s),$$

*for arbitrary cell residence time $T_s$ characterized by $f^*_{T_s}(s)$.*

**Proof.** For a fixed $n$, we use

$$T_m(n) = T_{s,1} + T_{s,2} + \cdots + T_{s,n}$$

to represent the paging area residence time under the condition that it takes $n$ steps for a mobile terminal to reach ring $d$, where $d$ is the radius of a PA. Hence, we obtain

$$f^*_{T_m(n)}(s) = \prod_{i=1}^{n} f^*_{T_{s,i}}(s) = \left( f^*_{T_s}(s) \right)^n.$$

Since the random variable $n$ has distribution $(\phi^{(d)}_{0d}, \phi^{(d+1)}_{0d}, \phi^{(d+2)}_{0d}, \ldots)$, we get

$$\boldsymbol{P}[T_m \le t] = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} \boldsymbol{P}[T_m(n) \le t],$$

that is,

$$F_{T_m}(t) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} F_{T_m(n)}(t),$$

and

$$f_{T_m}(t) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} f_{T_m(n)}(t).$$

By the property that the Laplace transform of $w_1 f_1(t) + w_1 f_2(t)$ is $w_1 f^*_1(s) + w_1 f^*_2(s)$, we have

$$f^*_{T_m}(s) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} f^*_{T_m(n)}(s).$$

Since $\boldsymbol{E}(T_m(n)) = n\boldsymbol{E}(T_s)$, we get

$$\boldsymbol{E}(T_m) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} \boldsymbol{E}(T_m(n)) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} n\boldsymbol{E}(T_s)$$

$$= \boldsymbol{E}(T_s) \sum_{n=d}^{\infty} n\phi^{(n)}_{0d} = N(d)\boldsymbol{E}(T_s).$$

The theorem is proven. □

When $T_s$ has an exponential distribution with

$$f_{T_s}(t) = \lambda_s e^{-\lambda_s t},$$

and

$$f^*_{T_s}(s) = \frac{\lambda_s}{s + \lambda_s},$$

$T_m(n)$ has an Erlang distribution with

$$f_{T_m(n)}(t) = \frac{\lambda_s e^{-\lambda_s t} (\lambda_s t)^{n-1}}{(n-1)!},$$

and

$$f^*_{T_m(n)}(s) = \left( \frac{\lambda_s}{s + \lambda_s} \right)^n,$$

and $T_m$ has a hyper-Erlang distribution with

$$f_{T_m}(t) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} \left( \frac{\lambda_s e^{-\lambda_s t} (\lambda_s t)^{n-1}}{(n-1)!} \right),$$

and

$$f^*_{T_m}(s) = \sum_{n=d}^{\infty} \phi^{(n)}_{0d} \left( \frac{\lambda_s}{s + \lambda_s} \right)^n.$$

We emphasize that even when $T_s$ has an exponential distribution, $T_m$ has a hyper-Erlang distribution. It is not reasonable to assume that $T_m$ has a simple probability distribution such as an exponential distribution or a hyperexponential distribution.

### 5.2 Modified Paging Area Residence Time

An *ordinary location update* is performed at the boundary of a cell residence time. In a DBLMS-CWLU and an MBLMS-CWLU, all location updates are ordinary. A *modified location update* is performed in the midst of a cell residence time. In a DBLMS-CPLU and an MBLMS-CPLU, the arrival of a phone call initiates a modified location update. However, all subsequent location updates before the next phone call are ordinary.

A paging area residence time is modified if the last location update for the current paging area is modified. A *modified paging area residence time* $T'_m$ is a sum of random number $n$ of random variables $T_{s,1}, T_{s,2}, \ldots, T_{s,n}$, where $T_{s,2}, T_{s,3}, \ldots, T_{s,n}$ are cell residence times with $f^*_{T_{s,i}}(s) = f^*_{T_s}(s)$, for all $2 \le i \le n$, but $T_{s,1}$ is the residual cell residence time with $f^*_{T_{s,1}}(s) = \lambda_{T_s}(1 - f^*_{T_s}(s))/s$.

Similar to Theorem 8, we have the following result, which will be used in Theorems 10 and 11.

**Theorem 9.** *The modified paging area residence time $T'_m$ for a PA with radius $d$ is characterized by*

$$f^*_{T'_m}(s) = \sum_{n=d}^{\infty} \phi_{0d}^{(n)} \left( \frac{\lambda_{T_s}(1 - f^*_{T_s}(s))(f^*_{T_s}(s))^{n-1}}{s} \right),$$

*and*

$$E(T'_m) = \frac{E(T_s^2)}{2E(T_s)} + (N(d) - 1)E(T_s),$$

*for arbitrary cell residence time $T_s$ characterized by $f^*_{T_s}(s)$.*

**Proof.** Let $T'_m(n) = T_{s,1} + T_{s,2} + \cdots + T_{s,n}$. It is clear that

$$\begin{aligned} f^*_{T'_m(n)}(s) &= f^*_{T_{s,1}}(s) \prod_{i=2}^{n} f^*_{T_{s,i}}(s) \\ &= \frac{\lambda_{T_s}(1 - f^*_{T_s}(s))(f^*_{T_s}(s))^{n-1}}{s}. \end{aligned}$$

Thus, we get

$$\begin{aligned} f^*_{T'_m}(s) &= \sum_{n=d}^{\infty} \phi_{0d}^{(n)} f^*_{T'_m(n)}(s) \\ &= \sum_{n=d}^{\infty} \phi_{0d}^{(n)} \left( \frac{\lambda_{T_s}(1 - f^*_{T_s}(s))(f^*_{T_s}(s))^{n-1}}{s} \right). \end{aligned}$$

Since

$$E(T_{s,1}) = \frac{E(T_s^2)}{2E(T_s)},$$

(see [25, (5.15), p. 173]), we have

$$E(T'_m(n)) = \frac{E(T_s^2)}{2E(T_s)} + (n - 1)E(T_s),$$

which gives rise to

$$E(T'_m) = \sum_{n=d}^{\infty} \phi_{0d}^{(n)} E(T'_m(n)) = \frac{E(T_s^2)}{2E(T_s)} + (N(d) - 1)E(T_s).$$

This proves the theorem. □

If $T_s$ is an exponential random variable, $T'_m$ is identical to $T_m$.

## 6 Cost of Location Management

In a distance-based location management scheme DBLMS $(d)$, where $d$ is a distance threshold and also the radius of a PA, location update is performed as soon as a mobile terminal moves out of the current PA. Therefore, the number of location update between two consecutive phone calls is related to the number of paging area boundary crossings between two consecutive phone calls.

Let us consider two consecutive phone calls, i.e., the previous call $C_1$ and the next call $C_2$. Assume that a mobile terminal $u$ resides in $\mathrm{PA}_1$ when it receives $C_1$. Before $u$ receives the next phone call $C_2$, $u$ moves through $\mathrm{PA}_1$, $\mathrm{PA}_2, \ldots, \mathrm{PA}_{X_m+1}$, and receives $C_2$ in $\mathrm{PA}_{X_m+1}$, where $X_m$ denotes the number of PA boundary crossings. Let $T_{m,i}$ denote the residence time of $u$ in $\mathrm{PA}_i$, where $i \geq 1$. The $T_{m,i}$s (except possibly $T_{m,1}$) are i.i.d. random variables with the same pdf $f_{T_m}(t)$. The sequence of random variables $T_{m,1}, T_{m,2}, T_{m,3}, \ldots$ is a renewal process.

### 6.1 The CPLU Model

In a DBLMS $(d)$ under the CPLU model, when the previous call $C_1$ arrives, location update is performed immediately and $\mathrm{PA}_1$ is treated as a new PA and the paging area residence time $T_{m,1}$ is set to zero. However, when $C_1$ arrives, a mobile terminal is still in the midst of a cell residence time, which implies that $T_{m,1}$ is a modified paging area residence time. Consequently, the sequence of random variables $T_{m,1}, T_{m,2}, T_{m,3}, \ldots$ is a modified renewal process. Let $X_m$ denote the number of PA boundary crossings (i.e., the number of renewals) between two consecutive phone calls (i.e., within a time interval of random length $T_c$). Then, the number of location update is $X_u = X_m + 1$, where we include the location update performed right after $C_1$ arrives.

The expected number of PA boundary crossings $E(X_m)$ can be obtained by using Theorem 1, where we need to calculate

$$F^*_{S_j}(s) = \frac{f^*_{T'_m}(s)(f^*_{T_m}(s))^{j-1}}{s}$$

and

$$H^*(s) = \sum_{j=1}^{\infty} F^*_{S_j}(s) = \frac{f^*_{T'_m}(s)}{s(1 - f^*_{T_m}(s))},$$

and

$$H(t) = \sum_{j=1}^{\infty} F_{S_j}(t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{f^*_{T'_m}(s) e^{st}}{s(1 - f^*_{T_m}(s))} ds,$$

where $f^*_{T_m}(s)$ and $f^*_{T'_m}(s)$ are given by Theorems 8 and 9. We have the following result.

**Theorem 10.** *The total cost of location management in a DBLMS $(d)$ under the CPLU model is*

$$D_{\mathrm{CPLU}}(d) = \Delta_u(E(X_m) + 1) + \Delta_p(qd^2 - qd + 1),$$

*where*

$$E(X_m) = \int_0^{\infty} H(t) f_{T_c}(t) dt,$$

*for any $f_{T_c}(t)$ and $f_{T_s}(t)$.*

If the cell residence time $T_s$ has an exponential distribution, by Theorem 8, the paging area residence time $T_m$ has a hyper-Erlang distribution. Furthermore, since the residual cell residence time of an exponential $T_s$ has the same distribution as $T_s$, the modified paging area residence time $T'_m$ has an identical hyper-Erlang distribution as the ordinary paging area residence time $T_m$. Hence, the sequence of random variables $T_{m,1}, T_{m,2}, T_{m,3}, \ldots$ is an ordinary renewal process, and Theorem 2 can be applied to find $F_{S_j}(t)$.

If $T_c$ has a hyper-Erlang distribution, the expected number of PA boundary crossings $E(X_m)$ can be obtained using Theorem 5.

**Theorem 11.** *If $T_c$ has a hyper-Erlang distribution, the total cost of location management in a DBLMS $(d)$ under the CPLU model is*

$$D_{\mathrm{CPLU}}(d) = \Delta_u(\boldsymbol{E}(X_m)+1) + \Delta_p(qd^2 - qd + 1),$$

*where*

$$\boldsymbol{E}(X_m) = \sum_{i=1}^{k_c} w_{c,i}\left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!}\right) \left(-\frac{\partial}{\partial s}\right)^{\gamma_{c,i}-1}\left[\frac{f^*_{T'_m}(s)}{s(1-f^*_{T_m}(s))}\right]_{s=\lambda_{c,i}},$$

*for any mobile terminal, where $f^*_{T_m}(s)$ and $f^*_{T'_m}(s)$ are given by Theorems 8 and 9.*

### 6.2 The CWLU Model

In a DBLMS $(d)$ under the CWLU model, when the previous call $C_1$ arrives, a mobile terminal $u$ has been in $\mathrm{PA}_1$ for a while. Hence, $T_{m,1}$ is the residual time of $u$ in $\mathrm{PA}_1$ and does not have the same pdf as the other $T_{m,i}$s. The sequence of random variables $T_{m,1}, T_{m,2}, T_{m,3}, \ldots$ is an equilibrium renewal process. According to Theorems 3 and 8, we have

$$\mathbf{E}(X_m) = \frac{\mathbf{E}(T_c)}{\mathbf{E}(T_m)} = \frac{\mathbf{E}(T_c)}{N(d)\mathbf{E}(T_s)} = \frac{1}{N(d)}\left(\frac{\lambda_{T_s}}{\lambda_{T_c}}\right) = \frac{1}{\rho N(d)},$$

where $\rho = \lambda_{T_c}/\lambda_{T_s}$ is the *call-to-mobility ratio*. The number of location update is $X_u = X_m$ and $\boldsymbol{E}(X_m)$ is the expected number of location update between two consecutive phone calls.

**Theorem 12.** *For any probability distributions of $T_c$ and $T_s$, the total cost of location management in a DBLMS $(d)$ under the CWLU model is*

$$D_{\mathrm{CWLU}}(d) = \frac{\Delta_u}{\rho N(d)} + \Delta_p(qd^2 - qd + 1),$$

*for any mobile terminal.*

It is proven in [29] that for any probability distributions of $T_c$ and $T_s$, the location update cost in an MBLMS $(d)$ under the CWLU model is

$$\frac{\Delta_u}{\rho d}.$$

We have already known from Section 4 that

$$N(d) = \alpha \cdot \frac{d(qd-1)}{q-1},$$

where $\alpha$ depends on $d$ and $q$ but is in a small range of $0.5 < \alpha < 0.7$ for all $d \geq 2$. Since $N(d)$ is significantly greater than $d$ even for small to moderate $d$, compared to an MBLMS $(d)$-CWLU, the location update cost in a DBLMS $(d)$-CWLU is significantly reduced.

If we set $\alpha$ to be a constant in $(0.5, 0.7)$, e.g., $\alpha = 0.6$, then we get a closed-form expression of $D_{\mathrm{CWLU}}(d)$:

$$D_{\mathrm{CWLU}}(d) = \frac{\beta}{d(qd-1)} + \Delta_p(qd^2 - qd + 1),$$

where

$$\beta = \frac{\Delta_u(q-1)}{\alpha\rho}.$$

The performance of a DBLMS $(d)$-CWLU is determined by $d$. We can find $d^*$ such that the total location update and terminal paging cost is minimized. To minimize $D_{\mathrm{CWLU}}(d)$, let us consider the derivative of $D_{\mathrm{CWLU}}(d)$,

$$\frac{\partial D_{\mathrm{CWLU}}(d)}{\partial d} = -\beta\left(\frac{2qd-1}{d^2(qd-1)^2}\right) + \Delta_p q(2d-1).$$

The optimal value of $d^*$ which minimizes $D_{\mathrm{CWLU}}(d)$ in a DBLMS $(d)$-CWLU is either $\lfloor d \rfloor$ or $\lceil d \rceil$, where $d$ satisfies the following equation:

$$\beta(2qd-1) = \Delta_p qd^2(2d-1)(qd-1)^2.$$

## 7 Simulation Results and Numerical Data

In this section, we present simulation results and numerical data. We consider a hyperexponential distribution of $T_c$ with

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i}\lambda_{c,i}e^{-\lambda_{c,i}t},$$

where $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$, and a hypergamma distribution of $T_s$ with

$$f_{T_s}(t) = \sum_{j=1}^{k_s} w_{s,j}\left(\frac{\lambda_{s,j}e^{-\lambda_{s,j}t}(\lambda_{s,j}t)^{\gamma_{s,j}-1}}{\Gamma(\gamma_{s,j})}\right),$$

and

$$f^*_{T_s}(s) = \sum_{j=1}^{k_s} w_{s,j}\left(\frac{\lambda_{s,j}}{s+\lambda_{s,j}}\right)^{\gamma_{s,j}},$$

where $w_{s,1} + w_{s,2} + \cdots + w_{s,k_s} = 1$. It is clear that

$$\rho = \left(\sum_{i=1}^{k_c}\frac{w_{c,i}}{\lambda_{c,i}}\right)^{-1}\left(\sum_{j=1}^{k_s} w_{s,j}\left(\frac{\gamma_{s,j}}{\lambda_{s,j}}\right)\right).$$

By Theorem 11, the total cost of location management is

$$D_{\mathrm{CPLU}}(d) = \Delta_u\left(1 + \sum_{i=1}^{k_c} w_{c,i}\left(\frac{f^*_{T'_m}(\lambda_{c,i})}{1-f^*_{T_m}(\lambda_{c,i})}\right)\right) + \Delta_p(qd^2 - qd + 1),$$

for DBLMS $(d)$-CPLU, where

$$f^*_{T_m}(\lambda_{c,i}) = \sum_{n=d}^{\infty}\phi_{0d}^{(n)}\left(f^*_{T_s}(\lambda_{c,i})\right)^n,$$

and

$$f^*_{T'_m}(\lambda_{c,i}) = \sum_{n=d}^{\infty}\phi_{0d}^{(n)}\left(\frac{\lambda_{T_s}(1-f^*_{T_s}(\lambda_{c,i}))(f^*_{T_s}(\lambda_{c,i}))^{n-1}}{\lambda_{c,i}}\right),$$

TABLE 3
Comparison of Analytical Data and Simulation Results of $E(X_u)$ $(q = 3)$

| | CPLU | | | CWLU | | |
|---|---|---|---|---|---|---|
| $d$ | analytical | simulation | relative differ. | analytical | simulation | relative differ. |
| 1 | 21.00000 | 20.89574 | – 0.49646% | 20.00000 | 19.93129 | – 0.34356% |
| 2 | 6.82490 | 6.81898 | – 0.08664% | 6.00000 | 5.98488 | – 0.25193% |
| 3 | 3.52852 | 3.50588 | – 0.64165% | 2.73111 | 2.72296 | – 0.29849% |
| 4 | 2.34065 | 2.33121 | – 0.40326% | 1.54436 | 1.54152 | – 0.18419% |
| 5 | 1.79277 | 1.78085 | – 0.66473% | 0.98968 | 0.98767 | – 0.20370% |
| 6 | 1.50071 | 1.49678 | – 0.26148% | 0.68742 | 0.68675 | – 0.09660% |
| 7 | 1.33007 | 1.32093 | – 0.68730% | 0.50497 | 0.50317 | – 0.35625% |
| 8 | 1.22408 | 1.21809 | – 0.48963% | 0.38652 | 0.38515 | – 0.35497% |
| 9 | 1.15536 | 1.15016 | – 0.45066% | 0.30532 | 0.30354 | – 0.58488% |
| 10 | 1.10942 | 1.10508 | – 0.39146% | 0.24725 | 0.24631 | – 0.38126% |
| 11 | 1.07800 | 1.07450 | – 0.32520% | 0.20430 | 0.20373 | – 0.27536% |
| 12 | 1.05613 | 1.05337 | – 0.26100% | 0.17163 | 0.17067 | – 0.55855% |
| 13 | 1.04070 | 1.03865 | – 0.19717% | 0.14622 | 0.14721 | 0.67551% |
| 14 | 1.02970 | 1.02826 | – 0.13919% | 0.12606 | 0.12610 | 0.03248% |
| 15 | 1.02178 | 1.02053 | – 0.12193% | 0.10980 | 0.10962 | – 0.15591% |
| 16 | 1.01604 | 1.01517 | – 0.08608% | 0.09649 | 0.09621 | – 0.29266% |
| 17 | 1.01186 | 1.01098 | – 0.08698% | 0.08546 | 0.08578 | 0.36996% |
| 18 | 1.00879 | 1.00828 | – 0.05051% | 0.07622 | 0.07663 | 0.52897% |
| 19 | 1.00654 | 1.00605 | – 0.04812% | 0.06840 | 0.06804 | – 0.53337% |
| 20 | 1.00487 | 1.00456 | – 0.03152% | 0.06173 | 0.06208 | 0.55917% |

TABLE 4
Comparison of Analytical Data and Simulation Results of $E(X_u)$ $(q = 4)$

| | CPLU | | | CWLU | | |
|---|---|---|---|---|---|---|
| $d$ | analytical | simulation | relative differ. | analytical | simulation | relative differ. |
| 1 | 21.00000 | 20.93631 | – 0.30329% | 20.00000 | 19.98818 | – 0.05910% |
| 2 | 6.96982 | 6.84804 | – 1.74717% | 6.15385 | 6.02806 | – 2.04403% |
| 3 | 3.57576 | 3.55838 | – 0.48615% | 2.78048 | 2.76407 | – 0.58995% |
| 4 | 2.36612 | 2.35431 | – 0.49922% | 1.56836 | 1.57024 | 0.11988% |
| 5 | 1.81093 | 1.80285 | – 0.44627% | 1.00499 | 1.00961 | 0.45906% |
| 6 | 1.51512 | 1.50847 | – 0.43901% | 0.69868 | 0.70717 | 1.21466% |
| 7 | 1.34193 | 1.33298 | – 0.66658% | 0.51388 | 0.51995 | 1.18027% |
| 8 | 1.23392 | 1.22689 | – 0.56943% | 0.39387 | 0.39515 | 0.32417% |
| 9 | 1.16354 | 1.15805 | – 0.47177% | 0.31154 | 0.31272 | 0.38057% |
| 10 | 1.11620 | 1.11043 | – 0.51678% | 0.25260 | 0.25403 | 0.56755% |
| 11 | 1.08359 | 1.07917 | – 0.40855% | 0.20895 | 0.20904 | 0.03987% |
| 12 | 1.06073 | 1.05653 | – 0.39634% | 0.17573 | 0.17592 | 0.10502% |
| 13 | 1.04447 | 1.04090 | – 0.34182% | 0.14986 | 0.15006 | 0.13410% |
| 14 | 1.03277 | 1.03017 | – 0.25213% | 0.12931 | 0.12925 | – 0.04385% |
| 15 | 1.02428 | 1.02227 | – 0.19646% | 0.11272 | 0.11287 | 0.12890% |
| 16 | 1.01807 | 1.01610 | – 0.19424% | 0.09914 | 0.09911 | – 0.03127% |
| 17 | 1.01350 | 1.01203 | – 0.14566% | 0.08787 | 0.08763 | – 0.27302% |
| 18 | 1.01012 | 1.00862 | – 0.14905% | 0.07842 | 0.07863 | 0.26283% |
| 19 | 1.00761 | 1.00657 | – 0.10290% | 0.07042 | 0.06986 | – 0.79969% |
| 20 | 1.00574 | 1.00492 | – 0.08092% | 0.06359 | 0.06353 | – 0.08539% |

and

$$f_{T_s}^*(\lambda_{c,i}) = \sum_{j=1}^{k_s} w_{s,j} \left( \frac{\lambda_{s,j}}{\lambda_{c,i} + \lambda_{s,j}} \right)^{\gamma_{s,j}}.$$

By Theorem 12, the total cost of location management is

$$D_{\text{CWLU}}(d) = \frac{\Delta_u}{\rho N(d)} + \Delta_p(qd^2 - qd + 1),$$

for DBLMS $(d)$-CWLU.

We assume the following parameter settings. For $T_c$, let $k_c = 3$, $w_{c,1} = 0.25$, $w_{c,2} = 0.50$, $w_{c,3} = 0.25$, $\lambda_{c,1} = \lambda_c/2$, $\lambda_{c,2} = \lambda_c$, $\lambda_{c,3} = 2\lambda_c$, where $\lambda_c$ is a variable. For $T_s$, let $k_s = 3$, $w_{s,1} = 0.25$, $w_{s,2} = 0.50$, $w_{s,3} = 0.25$, $\lambda_{s,1} = \lambda_{s,2} = \lambda_{s,3} = \lambda_s$, $\gamma_{s,1} = \gamma_s/2$, $\gamma_{s,2} = \gamma_s$, $\gamma_{s,3} = 2\gamma_s$, where $\lambda_s$ and $\gamma_s$ are variables. It is easy to verify that $E(T_c) = 9/(8\lambda_c)$, and $E(T_s) = 9\gamma_s/(8\lambda_s)$, which give $\rho = E(T_s)/E(T_c) = \lambda_c\gamma_s/\lambda_s$. Furthermore, we have $\text{Var}(T_s) = (19\gamma_s^2 + 72\gamma_s)/(64\lambda_s^2)$, and the coefficient of variation of $T_s$,

$$C_{T_s} = \left( \sqrt{19\gamma_s^2 + 72\gamma_s} \right) / (9\gamma_s).$$

The total cost of location management is now determined by seven variables, namely, $q$, $d$, $\lambda_c$, $\lambda_s$, $\gamma_s$, $\Delta_p$, and $\Delta_u$. By Theorems 11 and 12, the total cost of location management is determined by five variables, namely, $q$, $d$, $\rho$, $\Delta_p$, and $\Delta_u$, where $\rho$ is given by $\lambda_c$, $\lambda_s$, and $\gamma_s$. We will fix $\Delta_p$ and examine the impact of $\Delta_u$, and fix $\lambda_c$ and see the impact of $\lambda_s$ and $\gamma_s$.

### 7.1 Simulation Results

Extensive simulations have been conducted to validate our analytical results on the expected number $E(X_u)$ of location updates between two consecutive phone calls. In Tables 3 and 4, we display and compare our analytical data and simulations results for the hexagonal cell structure with $q = 3$ and the mesh cell structure with $q = 4$, respectively, where $\lambda_c = 1$, $\lambda_s = 40$, and $\gamma_s = 2$. The analytical data of $E(X_u)$ for the CPLU model are calculated by using Theorem 11, and the analytical data of $E(X_u)$ for the CWLU model are calculated by using Theorem 12. The simulation results of $E(X_u)$ are obtained by simulating a mobile terminal with random intercall time and random cell residence time specified by the above probability density functions $f_{T_c}(t)$ and $f_{T_s}(t)$. For every pair of consecutive phone calls, we record the number of location updates between the two consecutive phone calls. We then report the average number of location updates between all consecutive phone calls. The number of phone calls is as large as 500,000, such that the maximum 99 percent confidence interval of our simulation results is about $\pm 1.42622$ percent. The relative difference between a simulation result and its corresponding analytical datum is also given. We observe that for the hexagonal cell structure, the relative differences of all our simulation results are no more than $\pm 0.69$ percent. For the mesh cell structure, the relative differences of all our simulation results are no more than $\pm 1.22$ percent, except for the case $d = 2$. These simulation results demonstrate the high quality and accuracy of our analytical data.

### 7.2 Numerical Data

In Appendix 4, available in the online supplemental material, with Figures 2-15, we present numerical data to show the impact of various parameters and performance optimization.

## 8 Concluding Remarks

The paper has made several contributions to cost analysis and performance optimization of dynamic distance-based location management schemes in wireless communication networks. We have developed a Markov chain as a mobility model to describe the movement of a mobile terminal in 2D cellular structures. We characterized the paging area residence time for arbitrary cell residence time by using the Markov chain. By using the classical renewal theory, we have analyzed the expected number of paging area boundary crossings and the cost of the distance-based location update method for two different call handling models. Our analytical results enable clear comparison of the distance-based location update method and the movement-based location update method.

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