COMP9020 - Review

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Topic 1: Set

1. Floor and ceiling

   - $\lfloor\rfloor$: floor
   - $\lceil\rceil$: ceiling
   - $\lfloor -X \rfloor = -\lceil X \rceil$
   - $\lfloor X + t \rfloor = \lfloor X \rfloor + t$ for t $\in$ Z

2. Divisibility, prime, gcd and lcm

   - $m \mid n$: m divides n (m is less)
   - $n \mid 0$ is true, and $0 \mid n$ is false, except n = 0
   - prime: $n > 1$ and $1 \mid n$ and $n \mid n$ only
   - relatively prime: $gcd(m, n) = 1$
   - gcd: greatest common divisor
   - lcm: least common multiple
   - gcd(m, n) * lcm(m, n) = $|m| * |n|$
   - Euclid's gcd algorithm: for m ¿ n, gcd(m, n) = gcd(m-n, n)

3. Set notation and construction

   - a set is a set of elements
   - Notation 1: S = {e1, e1, e1 ... }
   - Notation 2: S = {e: description of e}
   - symmetric difference 1: $A \oplus B = (A \cup B) \setminus (A \cap B)$
   - symmetric difference 2: $A \oplus B = (A \setminus B) \cup (B \setminus A)$
   - Subset: $\subseteq$, Proper subset: $\subsetneqq$
   - Power set: Pow(X) = $\{A : A \subseteq X\}$
   - Cardinality: $|X|$

- Always: $|Pow(X)| = 2^{|X|}$
- Set of Numbers: P $\subset$ N $\subset$ Z $\subset$ Q $\subset$ R

4. Laws of Sets Operations

- Commutativity
- Associativity
- Distribution
- Idempotence
- Identity
- Double Complementation
- De morgan Laws: $(A \cup B))^C = A^C \cap B^C$, $(A \cap B))^C = A^C \cup B^C$

5. Cartesian product

- (a, b): ordered pair
- $A \times B = \{(a,b)|a \in A, b \in B\}$

6. Formal language

- Σ: alphabet – a finite, none empty set
- λ: a empty word
- Σ^k: set of all words of length k
- Σ^*: set of all words
- Σ^+: set of all none empty words

Topic 2: Function Matrix and Relation0

1. Function Definition

- notation 1: $f : S \to T$
- notation 2: $f : x \mapsto y$
- notation 3: f(x) = y
- every input has an one and only one output
- Image: Im(f) = $\{f(x), x \in Dom(f)\}$
- $Im(f) \subset Codom(f)$
- Composition: $g \circ f = g(f(x))$ where Im(f) $\subset$ Dom(g)
- Identity: $f \circ Id = Id \circ f = f$

2. Function inverse

- surjective(onto): every output has a related input

$$Im(f) = Codom(f)$$

- injective(one-to-one): every input has an unique output

$$x \neq y \implies f(x) \neq f(y)$$
$$f(x) = f(y) \implies x = y$$

- bijective

$$surjective and injective$$

- inverse

$$f^{-1} : y \to x$$

- $f : D \to C, S_D \subseteq D, S_C \subseteq C$, then:
  $f(S_D) \subseteq C$ is the image, and $f^{\Leftarrow}(S_C) \subseteq D$ is the inverse image
  if $f^{-1}(S_C) = f^{\Leftarrow}(S_C)$

3. Matrix

- M_{mn} m is row and n is column

$$\begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \dots & & & \\ m_{m1} & m_{m2} & \dots & m_{mn} \end{bmatrix}$$

- Transpose M^T
  a matrix is called symmetric if $M^T = M$

- Sum

- product (first row second column)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} & a_{11} * b_{12} + a_{12} * b_{22} + a_{13} * b_{32} \\ a_{21} * b_{21} + a_{22} * b_{21} + a_{13} * b_{31} & a_{21} * b_{12} + a_{22} * b_{22} + a_{23} * b_{32} \end{bmatrix}$$

4. Relation Property

- a relation from S to T is a subset of $R \subseteq S \times T$
- x is related to y, denote xRy or R(x, y), or x,y $\in$ R, can be True or False
- Reflexive: $\forall x \implies xRx$
- Anti-reflexive: $\forall x \implies x \not{R} x$

- a relation can not be both reflexive and anti-reflexive
- Symmetric: $\forall x, y, xRy \implies yRx$
- Anti-Symmetric: $\forall x, y, xRy \land yRx \implies x = y$
- a relation can be both symmetric and anti-reflexive
- Transitive: $\forall x, y, z, xRy \land yRz \implies xRz$

5. Equivalence relation and Order relations

- Equivalence Relation: reflexive, symmetric and Transitive
- Partial Order$\preceq$: reflexive, antisymmetric, transitive
- lub: least upper bound, $x \in S \land x \succeq a, \forall a \in A$
- glb: greatest lower bound, $x \in S \land x \preceq a, \forall a \in A$
- Lattice: a poset where lub and glb exist for every pair of elements, then they exist for every finite subset
- Total Order$\leq$: Partial Order, Linearity: arrange every elements in a line $\forall a, b, a \leq b \lor b \leq a$
- Well order:Total Order, every subset has a least element
- Lexicographic order
- lenlex order

Topic 3: Graph theory

1. Definition: a collection of vertices and edges

- terminology:
- incident: edge is incident to vertices
- adjacent: vertex is adjacent to its neighbour vertices
- isolated
- types of graph:
- Undirected graph: edge = {v1, v2}
- directed graph: edge = (v1, v2)

2. Walk, trial and path

- walk:
- from a vertex to a vertex without any restriction
- close walk:
- only start point and end point is the same vertex
- trail:

- a walk without using repeated edges
- close trail:
- also called a circuit
- path:
- a walk with no repeated vertices
- close path:
- a close path is called a cycle

3. Connect graph

- connected graph:
- if there is an x-y path $\forall x, y \in V$, denote with k(G)
- simple:
- loop free, Undirected, no multiple edges

Topic 4: Logic

1.

2.

Topic 5: Induction

1.

2.

Topic 6: Recursion

1.

2.

Topic 7: Running time of programs

1.

2.

Topic 8: Counting

1.

2.

Topic 9: Probability and Expectation

1.

2.