COMP9020 - Assignment 3

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Question 1

On the left side:

$\binom{n}{k} = \frac{n!}{(n-k)!k!} = \frac{\frac{n!}{(n-k)}}{(n-k-1)!k!}$

$\binom{n}{k+1} = \frac{n!}{(n-k-1)!(k+1)!} = \frac{\frac{n!}{(k+1)}}{(n-k-1)!k!}$

$\binom{n}{k} + \binom{n}{k+1} = \frac{\frac{n!}{(n-k)} + \frac{n!}{(k+1)}}{(n-k-1)!k!}$

On the right side:

$= \frac{\frac{(n+1)n!}{(n-k)(k+1)}}{(n-k-1)!k!} = \frac{\frac{(n+1)!}{(n-k)(k+1)}}{(n-k-1)!(k)!}$

$\binom{n+1}{k+1} = \frac{(n+1)!}{(n-k)!(k+1)!} = \frac{\frac{(n+1)!}{(n-k)(k+1)}}{(n-k-1)!(k)!}$

Therefore, left side is equal to right side

Question 2

(a) let set X = $\{p, \neg p, q\}$
let set Y = $\{\wedge, \vee\}$
without using parenthesis, formulas can be constructed like this:
X1 Y1 X2 Y2 X3
the choices is:
$3! \times 2! = 12$
if we use 2 pairs of parenthesis, we can put them there:
((X1 Y1 X2) Y2 X3)
(X1 Y1 (X2 Y2 X3))
therefore, there are $12 \times 2 = 24$ different wff.

(b) there are 6 logical equivalence, as follows:
$X \vee p$
$X \vee q$
$X \vee \neg p$
$X \wedge p$
$X \wedge q$
$X \wedge \neg p$
X is the combination of the other 3 symbols, based on Commutativity, X is unique.

Question 3

(a) assume that the expected time is T, then:
$T = T_A * 1/4 + T_B * 1/4 + T_C * 1/4 + T_D * 1/4$
$T_A = 5$
$T_B = 3 + T$
$T_C = 8$
$T_D = 2 + T$
$T = 13/4 + (5 + 2T) * 1/4 = 9/2 + T/2$
$T = 9$ minutes

(b) assume that the expected time is T, then:
$T = 5 * 1/4 + T_B * 1/4 + 8/4 + T_D * 1/4$
$T_B = 3 + 5 * 1/3 + 8 * 1/3 + T_{BD} * 1/3$
$T_D = 2 + 5 * 1/3 + 8 * 1/3 + T_{DB} * 1/3$
$T_{BD} = 2 + 5 * 1/2 + 8 * 1/2 = 17/2 = 8.5$
$T_{DB} = 3 + 5 * 1/2 + 8 * 1/2 = 19/2 = 9.5$
Then:
$T_B = 10.167$
$T_D = 9.5$
$T = 8.167$ minutes

Question 4

(a) $p_1(n+1) = p_2(n) * 1/2$
$p_2(n+1) = p_1(n) + p_3(n) * 1/2$
$p_3(n+1) = p_2(n) * 1/2 + p_4(n) * 1/2$
$p_4(n+1) = p_3(n) + p_5(n) * 1/2$
$p_5(n+1) = p_4(n) * 1/2$

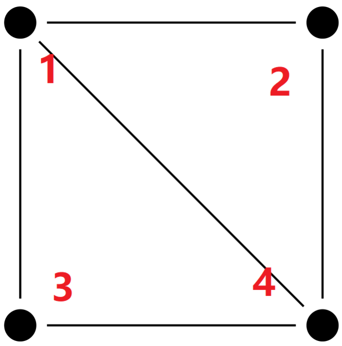
(b) $p_1(n) = p_2(n) * 1/2$
$p_2(n) = p_1(n) + p_3(n) * 1/2$
$p_3(n) = p_2(n) * 1/2 + p_4(n) * 1/2$
$p_4(n) = p_3(n) + p_5(n) * 1/2$

$p_5(n) = p_4(n) * 1/2$
$p_1(n) + p_2(n) + p_3(n) + p_4(n) + p_5(n) = 1$
by solving the equations:
$p_1(n) = 1/8$
$p_2(n) = 2 * p_1(n) = 1/4$
$p_3(n) = 2 * p_1(n) = 1/4$
$p_4(n) = 2 * p_1(n) = 1/4$
$p_5(n) = p_1(n) = 1/8$

(c) expected distance $= 0 * 1/8 + 1 * 1/4 + 2 * 1/4 + 3 * 1/4 + 4 * 1/8 = 2$

Question 5

(a) all possible colouring combination $= 3^4 = 81$



let calculate the number of all colouring combination which can lead to a 3 colouring graph:
first, let's color node 1, 2 and 4, since these 3 nodes all connect together, they must be colored differently.
There are $3! = 6$ different colouring combination of nodes 1, 2 and 4.
Then, let consider node 3, its color must be different from node 1 and 2, so its color is the same as node 4.
There are only 1 choice. Therefore, the probability is $6/81 = 7.4\%$

(b) There are 5 edges, namely (1, 2) (1, 3) (1, 4) (2, 4) (3, 4)
All possible colouring combination $= 6^5 = 7776$
First, let's color (1, 2):
We can choose from any "colour pairs (eg, (reg, green)),there are 6 choices.
Second, Let's color (1, 3):
If (1, 2) is colored by (red, green), then (1, 3) must be colored with (red, xx), because others will not be valid

Moreover, consider the conclusion we made in previous question
in order to get a 3-coloring graph, 2, and 3 must be in same color, which is (xx, green) for example.
Therefore, the "colour pairs" for (1, 3) is the same as (1, 2), there are 1 choice.
Third, let's color (1, 4):
color for 1 is fixed, and color for 4 must be different from 2(which is the same color as 3)
Therefore, there are 1 choices
In conclusion, the probability is $6/7776 = 1/1296$