COMP9020 - Assignment 2

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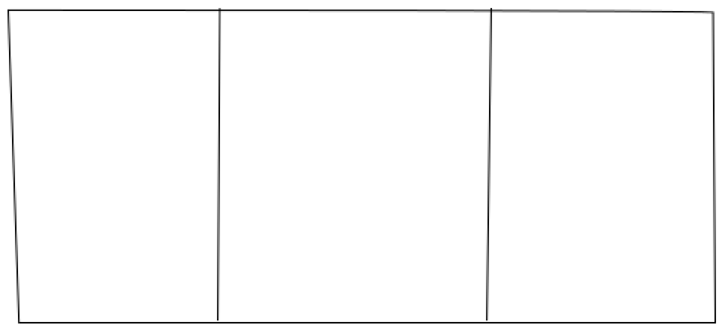
Question 1

(a) For graph G = (E, V) as follows:



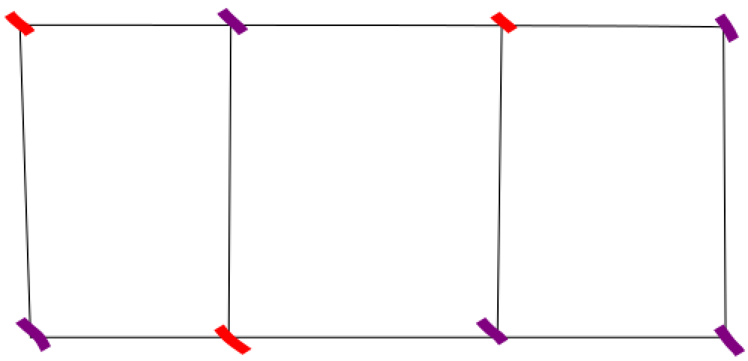
For every $e = (v, w) \in E$

$$c(v) \neq c(w)$$

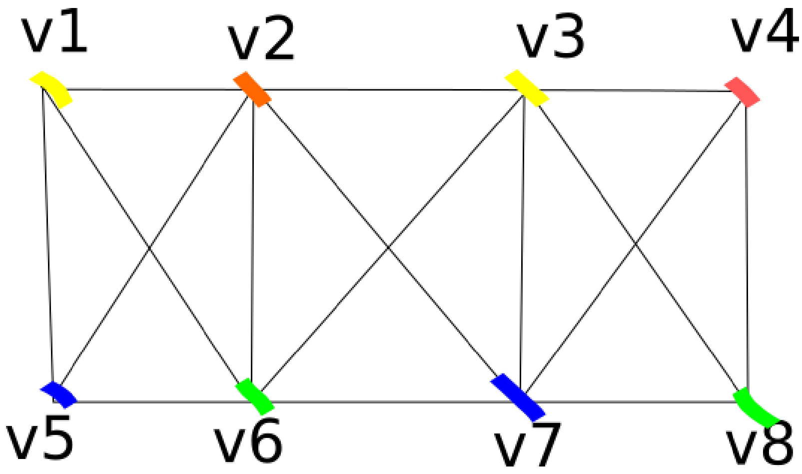
The mininum number of colors to sufficient effect such a mapping, denoted:

$$\chi(G)$$

(b) The minimum number of colors is 2, as follows:



(c) The connection of the graph changes, as follows:



Look at vertex v2, v3, v6, v7, they are connected to each other.
Therefor, at least we should use 4 different colors.

$$c(v2) \neq c(v3) \neq c(v6) \neq c(v7)$$

Because I can use 4 different colors as shows in graph, therefore:

$$\chi(G) = 4$$

Question 2

(a) $\{\mathrm{A}_v : \sum_{c \in C} P_{v,c}\}$

(b) $\{\mathrm{B}_v : \forall c, d \in C, \neg(P_{v,c} \wedge P_{v,d})\}$

(c) $\{\mathrm{C}_{u,v} : \forall c \in C, \neg(P_{v,c} \wedge P_{u,c})\}$

(d) $\{\varphi_G : \forall (u, v) \in E, \neg(P_{v,c} \wedge P_{u,c})$ and $\forall c, d, e \in C, \sum_{v \in V}(P_{v,c} \vee P_{v,d} \vee P_{v,e})\ \}$

Question 3

(a) (R):

$$x \vee x = x \implies x \sqsubseteq x \, for \, all \, x \in S$$

(AS):

$$x \sqsubseteq y \implies x \vee y = y$$

$$y \sqsubseteq x \implies y \vee x = x$$

$$\because x \vee y = y \vee x$$

$$\therefore x = y$$

(T):

$$x \sqsubseteq y \implies x \vee y = y$$

$$y \sqsubseteq z \implies y \vee z = z$$

$$x \vee z = x \vee (y \vee z) = (x\ lory) \vee z = y \vee z = z$$

$$\therefore x \sqsubseteq z$$

(L): not satisfied
for x, y are not empty, if x $\vee y = \emptyset$, then:
neither $x \sqsubseteq y$, nor $y \sqsubseteq x$

(b) $\sqsubseteq$ is coresponse to $\subseteq$

$$x \subseteq y \iff x \cup y = y$$

(c) Draw a truth table:

x	y	$x \vee y$	$(x \vee y) \rightleftarrows y$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

Therefore, the logic equivilence is:

$$xy + \bar{x}y + \overline{xy} = y + \overline{xy}$$

Question 4

(a) assume that add(0, n) = n
for n=0, add(0, 0) = 0
for n>0, add(0, n+1) = add(0, n) + 1 = n + 1
therefore,for n $\in$ N,
add(0, n) = n
because add(n, 0) = n
therefore, P(n) holds for all n $\in$ N

(b) a + b = n $\implies$ b = n - a
assume that add(a, n-a) = n
for n=a, add(a, n-a) = add(a, 0) + 0 = a = n
for n a, add(a, n-a+1) = add(a, n-a) + 1 = n + 1
therefore, add(a, n-a) = n
follow similar steps, we can prove that:
add(b, n-b) = n
Therefore add(a, b) = add(b, a) = a + b = n

Question 5

(a) $rec\_a(n):$
$if n < 2 : return\, n$
$else:$
$x := rec_a(n-1) -- > T(n-1)$
$y := rec_a(n-2) -- > T(n-2)$
$return 5x - 6y -- > 1$
The total cost is $T(n-1) + T(n-2) + 1$
$T(1) = 0$
$T(2) = 0$
$T(n) = T(n-1) + T(n-2) + 1$
Where $T(n+1)$ is a fibonacci number
according to Binet's Formula [1]
$T(n+1) = \frac{\phi^n - (-\phi^{-n})}{\sqrt{5}}$
where ϕ is golden ratio
therefore the upper bound for $rec\_a(n)$ is ϕ^n where $\phi = 1.618$
$iter\_a(n):$
$if n < 2 : return\, n$
$else:$
$x := 1$
$y := 0$
$i := 1 -- > 3$
$while i < n:$
$i := x$
$x := 5x - 6y$
$y := t$
$i := i + 1 -- > 4n$
$return\, x$
The total cost is $4n + 4$
The upper bound for $iter\_a(n)$ is n

(b) draw a table

n	a_n	$a_n + 2^n$
0	0	1
1	1	3
2	5	9
3	19	27
4	65	81
5	211	243
6	665	729
7	2059	2187

Guess $a_n = 3^n - 2^n$

for n = 0, $a_n = 0$ holds
for n = 1, $a_n = 3 - 2 = 1$ holds
assume $a_n = 3^n - 2^n$ holds for some n > 1
$a_n = 5 * a_{n-1} - 6a_{n-2}$
$= 5 * (3^{n-1} - 2^{n-1}) - 6 * (3^{n-2} - 2^{n-2})$
$= 5 * (3 * 3^{n-2} - 2 * 2^{n-2}) - 6 * (3^{n-2} - 2^{n-2})$
$= 15 * 3^{n-2} - 10 * 2^{n-2} - 6^{n-2} + 6 * 2^{n-2}$
$= 9 * 3^{n-2} - 4 * 2^{n-2}$
$= 3^n - 2^n$ holds for all n $\in$ N

(c) $calc\_a(n)$:
return 3**n-2**n
the upper bound of $calc_a(n)$ is 1, so it is more efficient

References

[1] `http://mathworld.wolfram.com/BinetsFibonacciNumberFormula.html`