COMP9020 - Assignment 1

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Question 1

(a) we have

$$gcd(288, 120) = gcd(120, 48) = (48, 24) = (24, 24)$$

so

$$gcd(288, 120) = 24$$

(b) we know that

$$lcm(-91, 52) = lcm(91, 52)$$

first calculate

$$gcd(91, 52) = gcd(39, 52) = gcd(13, 39) = gcd(13, 13) = 13$$

according to [1]

$$gcd(91, 52) \times lcm(91, 52) = 91 \times 52 = 4732$$

we can know that

$$lcm(-91, 52) = \frac{4732}{gcd(91, 52)} = \frac{4732}{13} = 364$$

(c)

$$\because n + 1 > n \, for \, n \in N$$

according to Euclid's algorithm [2]

$$gcd(n + 1, n) = gcd(n + 1 - n, n) = gcd(n, 1)$$

$\therefore$ n and n+1 are relative prime, for $n \in N$

Question 2

(a)

$$\because\ Pow(\emptyset) = \emptyset$$
$$\therefore\ Pow(Pow(\emptyset)) = \emptyset$$
$$Card(\emptyset) = 0$$

(b)

$$A \cap (B \oplus C) = A \cap ((B \backslash C) \cup (C \backslash B))$$
$$= (A \cap (B \backslash C) \cup (A \cap (C \backslash B))$$

other the other hand

$$(A \cap B) \oplus (A \cap C) = ((A \cap B) \backslash (A \cap C)) \cup ((A \cap C) \backslash (A \cap B)$$

According to the definition of difference

$$= (A \cap (B \backslash C) \cup (A \cap (C \backslash B))$$

(c) if A = {1, 2} B = {2, 3} C = {3, 4}

$$A \oplus (B \cap C)$$
$$= (A \backslash (B \cap C)) \cup ((B \cap C) \backslash A)$$
$$= \{1, 2, 3\}$$

on the other hand

$$(A \oplus B) \cap (A \oplus C)$$
$$(A \backslash B) \cap (B \backslash A) \cap (A \backslash C) \cap (C \backslash A)$$
$$= (A \backslash (B \cup C)) \cap ((B \cap C) \backslash A)$$
$$= \emptyset$$

Question 3

(a)

$$\{\lambda, 1, 11, 111, 112, 12, 121, 122, 2, 21, 22, 211, 212, 221, 222\}$$

(b)

$$\{\lambda, 1, 2, 11, 12, 21, 22, 111, 112, 121, 122, 211, 212, 221, 222\}$$

Question 4

(a)
- $f(a) = f(b) = f(c) = 0$
- $f(a) = 0\ f(b) = 0\ f(c) = 1$
- $f(a) = 0\ f(b) = 1\ f(c) = 0$
- $f(a) = 0\ f(b) = 1\ f(c) = 1$
- $f(a) = 1\ f(b) = 0\ f(c) = 0$
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- $f(a) = 1\ f(b) = 1\ f(c) = 0$
- $f(a) = f(b) = f(c) = 1$

(b)

(i) for every $a \in A$, there are n choices
there for, the number of function is m^n

(ii) every relation can be seen as a function, so the number is:

$$m^n + n^m$$

(c) to list all function form {a, b, c} to {0,1}, we can do like this:
A is one subset of {a,b,c}
$A \to 0$
$\{a, b, c\}\backslash A \to 1$
therefore, Card(Pow{a,b,c}) is equal to the number in answer (a)

Question 5

(a) (i) Yes
because length(w) = length(v)
therefor, for any v ∈ L
ωv ∈ L if and only if ω' ∈ L

(ii) No
for v = (a) ωv = (aba) ∈ L
whereas $\omega' v$ = (ababa) ∉ L

(iii) No
for v = (a,a)
ωv = (aa) ∉ L
whereas $\omega' v$ = (baa) ∈ L

(iv) No
for v = (a)
ωv = (a) ∉ L
whereas $\omega' v$ = (bba) ∈ L

(v) Yes
if v $\in$ L, then $\omega v \in$ L and $\omega' v \in$ L
if v $\notin$ L, then $\omega v \notin$ L and $\omega' v \notin$ L

(b) wRw' is equal to length(w) mod 3 = length(w') mod 3
(R) for any v, it is obvious that wv $\in$ L if, and only if wv $\in$ L
(S) we know that wRw', for any v:
if w'v $\in$ L, then wv $\in$ L
if w'v $\notin$ L, then wv $\notin$ L
(T) we know that wRw' and w'Rw"
if w'v $\in$ L, then w"v $\in$ L
if w'v $\notin$ L, then w"v $\notin$ L
therefor, R is a equivalence relation

(c) three equivalence classes, as following:
[s1]= {(w,w' $\in$ R): length(w) mod 3 = length(w') mod 3 = 0}
[s2]= {(w,w' $\in$ R): length(w) mod 3 = length(w') mod 3 = 1}
[s3]= {(w,w' $\in$ R): length(w) mod 3 = length(w') mod 3 = 2}

References

[1] `https://en.wikipedia.org/wiki/Least_common_multiple#Fundamental_theorem_of_arithmetic`

[2] `https://en.wikipedia.org/wiki/Greatest_common_divisor#Using_Euclid.27s_algorithm`