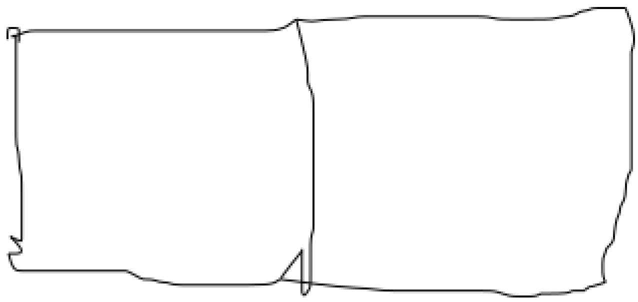
COMP9020 - Assignment 2

Jack Jiang (z5129432)

24 September 2017

Question 1

(a) For graph G = (E, V) as follows:



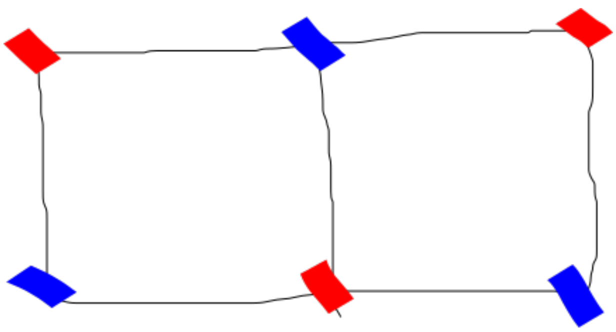
For every $e = (v, w) \in E$

$$c(v) \neq c(w)$$

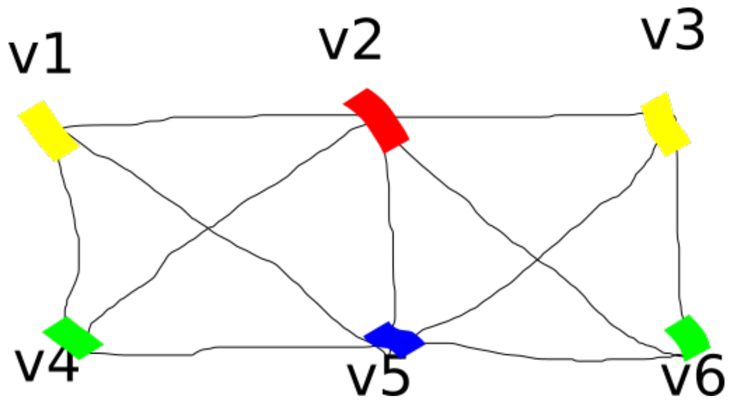
The mininum number of colors to sufficient effect such a mapping, denoted:

$$\chi(G)$$

(b) The minimum number of colors is 2, as follows:



(c) The connection of the graph changes, as follows:



Because v2 connect to v3, so they must be different colours, $c(v2) \neq c(v3)$ v2 and v3 connect to all other vertex, so other vertex must use different colours other than c(v2) and c(v3)

$$c(v1) \neq c(v2) \neq c(v3)$$

also, v1 connect to v4, so $c(v1) \neq c(v4)$, at lease we should use 4 different colors Because I can use 4 different colors as shows in graph, therefore:

$$\chi(G) = 4$$

Question 2

(a) $\{\mathrm{A}_v : \sum_{c \in C} P_{v,c}\}$

(b) $\{\mathrm{B}_v : \forall c, d \in C, \neg(P_{v,c} \wedge P_{v,d})\}$

(c) $\{\mathrm{C}_{u,v} : \forall c \in C, \neg(P_{v,c} \wedge P_{u,c})\}$

(d) $\{\varphi_G : \forall (u,v) \in E,\ \neg(P_{v,c} \wedge P_{u,c})$ and $\forall c, d, e \in C,\ \sum_{v \in V}(P_{v,c} \vee P_{v,d} \vee P_{v,e})\ \}$

Question 3

(a) (R):

$$x \vee x = x \implies x \sqsubseteq x \, for \, all \, x \in S$$

(AS):

$$x \sqsubseteq y \implies x \vee y = y$$

$$y \sqsubseteq x \implies y \vee x = x$$

$$\because x \vee y = y \vee x$$

$$\therefore x = y$$

(T):

$$x \sqsubseteq y \implies x \vee y = y$$

$$y \sqsubseteq z \implies y \vee z = z$$

$$x \vee z = x \vee (y \vee z) = (x\ lory) \vee z = y \vee z = z$$

$$\therefore x \sqsubseteq z$$

(L): not satisfied
for x, y are not empty, if x $\vee y = \emptyset$, then:
neither $x \sqsubseteq y$, nor $y \sqsubseteq x$

(b) $\sqsubseteq$ is corresponse to $\subseteq$

$$x \subseteq y \iff x \cup y = y$$

(c) Draw a trueth table:

x	y	$x \vee y$	$(x \vee y) \rightleftarrows y$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

Therefore, the logic equivilence is:

$$xy + \bar{x}y + \overline{xy} = y + \overline{xy}$$

Question 4

(a) assume that add(0, n) = n
for n=0, add(0, 0) = 0
for n>0, add(0, n+1) = add(0, n) + 1 = n + 1
therefore,for n $\in$ N,
add(0, n) = n
because add(n, 0) = n
therefore, P(n) holds for all n $\in$ N

(b) a + b = n $\implies$ b = n - a
assume that add(a, n-a) = n
for n=a, add(a, n-a) = add(a, 0) + 0 = a = n
for n a, add(a, n-a+1) = add(a, n-a) + 1 = n + 1
therefore, add(a, n-a) = n
follow similar steps, we can prove that:
add(b, n-b) = n
Therefore add(a, b) = add(b, a) = a + b = n

Question 5

(a)

(b)

(c)