COMP9020 - Review

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Topic 1: Numbers, Sets, and Alphabets

1. Floor and ceiling
   - $\lfloor\rfloor$: floor
   - $\lceil\rceil$: ceiling
   - $\lfloor -X \rfloor = -\lceil X \rceil$
   - $\lfloor X + t \rfloor = \lfloor X \rfloor + t$ for t $\in$ Z
2. Divisibility, prime, gcd and lcm
   - $m \mid n$: m divides n (m is less)
   - $n \mid 0$ is true, and $0 \mid n$ is false, except n = 0
   - prime: $n > 1$ and $1 \mid n$ and $n \mid n$ only
   - relatively prime: $gcd(m, n) = 1$
   - gcd: greatest common divisor
   - lcm: least common multiple
   - gcd(m, n) * lcm(m, n) = $|m| * |n|$
   - Euclid's gcd algorithm: for m ¿ n, gcd(m, n) = gcd(m-n, n)
3. Set notation and construction
   - a set is a set of elements
   - Notation 1: S = {e1, e1, e1 ... }
   - Notation 2: S = {e: description of e}
   - symmetric difference 1: $A \oplus B = (A \cup B) \setminus (A \cap B)$
   - symmetric difference 2: $A \oplus B = (A \setminus B) \cup (B \setminus A)$
   - Subset: $\subseteq$, Proper subset: $\Subset$
   - Power set: Pow(X) = $\{A : A \subseteq X\}$
   - Cardinality: $|X|$

- Always: $|Pow(X)| = 2^{|X|}$
- Set of Numbers: P $\subset$ N $\subset$ Z $\subset$ Q $\subset$ R

4. Laws of Sets Operations
   - Commutativity
   - Associativity
   - Distribution
   - Idempotence
   - Identity
   - Double Complementation
   - De morgan Laws: $(A \cup B))^C = A^C \cap B^C$, $(A \cap B))^C = A^C \cup B^C$
5. Cartesian product
   - (a, b): ordered pair
   - $A \times B = \{(a, b) | a \in A, b \in B\}$
6. Formal language
   - Σ: alphabet – a finite, none empty set
   - λ: a empty word
   - Σ^k: set of all words of length k
   - Σ^*: set of all words
   - Σ^+: set of all none empty words

Topic 2: Functions Matrix and Relations

1. Function Definition
   - notation 1: $f : S \to T$
   - notation 2: $f : x \mapsto y$
   - notation 3: f(x) = y
   - every input has an one and only one output
   - Image: Im(f) = $\{f(x), x \in Dom(f)\}$
   - $Im(f) \subset Codom(f)$
   - Composition: $g \circ f = g(f(x))$ where Im(f) $\subset$ Dom(g)
   - Identity: $f \circ Id = Id \circ f = f$
2. Function inverse

- surjective(onto): every output has a related input

$$Im(f) = Codom(f)$$

- injective(one-to-one): every input has an unique output

$$x \neq y \implies f(x) \neq f(y)$$
$$f(x) = f(y) \implies x = y$$

- bijective

$$surjective and injective$$

- inverse

$$f^{-1} : y \to x$$

- $f : D \to C, S_D \subseteq D, S_C \subseteq C$, then:
  $f(S_D) \subseteq C$ is the image, and $f^{\Leftarrow}(S_C) \subseteq D$ is the inverse image
  if $f^{-1}(S_C) = f^{\Leftarrow}(S_C)$

3. Matrix

- M_{mn} m is row and n is column

$$\begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \dots & & & \\ m_{m1} & m_{m2} & \dots & m_{mn} \end{bmatrix}$$

- Transpose M^T
  a matrix is called symmetric if $M^T = M$
- Sum
- product (first row second column)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} & a_{11} * b_{12} + a_{12} * b_{22} + a_{13} * b_{32} \\ a_{21} * b_{21} + a_{22} * b_{21} + a_{13} * b_{31} & a_{21} * b_{12} + a_{22} * b_{22} + a_{23} * b_{32} \end{bmatrix}$$

4. - c

Topic 3: Graph theory

1.

2.

Topic 4: Logic

1.

2.

Topic 5: Induction

1.

2.

Topic 6: Recursion

1.

2.

Topic 7: Running time of programs

1.

2.

Topic 8: Counting

1.

2.

Topic 9: Probability and Expectation

1.

2.