

Lab Problems Week 8: Deterministic and Stochastic Trends

Thomas Yang and Jianhua Mei

September 19, 2023

The specification for a general ARIMA($p, 1, q$) model is

$$\Delta y_t = \sum_{i=1}^p \pi_i \Delta y_{t-i} + \sum_{j=0}^q \alpha_j \epsilon_t + \delta_t$$

where $\alpha_0 = 1$.

- If you decide there is a constant only then

$$\delta_t = a_0$$

- If you decide there is a constant and a trend then

$$\delta_t = a_0 + a_2 t$$

- If you decide there is a constant, a trend and a quadratic trend, then

$$\delta_t = a_0 + a_2 t + a_3 t^2$$

- If you decide there are no deterministic trend, then

$$\delta_t = 0$$

Testing for Trends

1. We conduct the ADF test (τ_τ) using the following equation, and ϕ tests to select parts for the deterministic trend.

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum \beta \Delta y_{t-1} + \epsilon_t \quad (1)$$

2. If we reject $\gamma = 0$, we conclude the series has no unit root.
3. If we fail to reject $\gamma = 0$, we can conclude the series has at least one unit root. We continue further with ϕ_2 and ϕ_3 tests for the option of a deterministic trend in addition to the stochastic trend.

Tutorial Questions The file `usdata.csv` contains 209 observations on:

- r_t = the overnight Federal Funds Rate for the US (*ffr*);
- and y_t = log real per capita GDP (*gdp*).

1. For r_t

- (a) Plot the time series plot.
- (b) Conduct the appropriate tests for deterministic and stochastic components using ADF. Transform the series as needed before continuing.
- (c) Plot the ACF and PACF to suggest AR and MA terms needed.
- (d) Propose three possible models.
- (e) Estimate the proposed models.
- (f) Which model do you choose? Why?

2. Repeat for y_t .