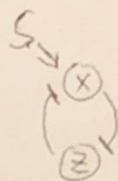Problem 2

a) $\dfrac{d\tilde{X}}{d\tilde{t}} = \dfrac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1 + S + (\hat{Z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{X}$

$\dfrac{d\tilde{Z}}{d\tilde{t}} = \dfrac{\tilde{\alpha}_z}{1 + (\tilde{X}/\tilde{x}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{Z}$

b) $\delta_z = \dfrac{\hat{\delta}_z}{\hat{\delta}_x}$, $t = \tilde{t}\,\tilde{\delta}_x$ (error in eqn. 3) $\alpha_x = \dfrac{\tilde{\alpha}_x}{\tilde{\alpha}_z}$, $\beta_x = \dfrac{\tilde{\beta}_x}{\tilde{\alpha}_z}$

$x_z = \dfrac{\tilde{x}_z \tilde{\delta}_x}{\tilde{\alpha}_z}$, $z_x = \dfrac{\tilde{z}_x \tilde{\delta}_z}{\tilde{\alpha}_z}$, $X = \dfrac{\tilde{X}\tilde{\delta}_x}{\tilde{\alpha}_z}$, $Z = \dfrac{\tilde{Z}\tilde{\delta}_x}{\tilde{\alpha}_z}$

$\dfrac{dX}{dt} = \dfrac{\alpha_x + \beta_x S}{1 + S + (Z/z_x)^{n_{zx}}} - X$

$\dfrac{dZ}{dt} = \dfrac{1}{1 + (X/x_z)^{n_{xz}}} - \delta_z Z$

(c) Steady state: $\dfrac{dX}{dt} = \dfrac{dZ}{dt} = 0$. Using parameters, we got:

In fact, S is the input (signal) while X is the output (expression). However, in math, it is ~~easy~~ easier to get a relationship in the form of $S = f(X)$.

$\dfrac{1.5 + 5S}{1 + S + (Z/0.4)^{2.7}} = X$, $\dfrac{1}{1 + (X/1.5)^{2.7}} = Z$

$\Rightarrow 1.5 + 5S = X\left[1 + S + (Z/0.4)^{2.7}\right]$, let $N = (Z/0.4)^{2.7}$

$\Rightarrow 5S + 1.5S = X(1 + S + N) \Rightarrow 5S - XS = X + XN - 1.5 \Rightarrow S = \dfrac{X + XN - 1.5}{5 - X}$

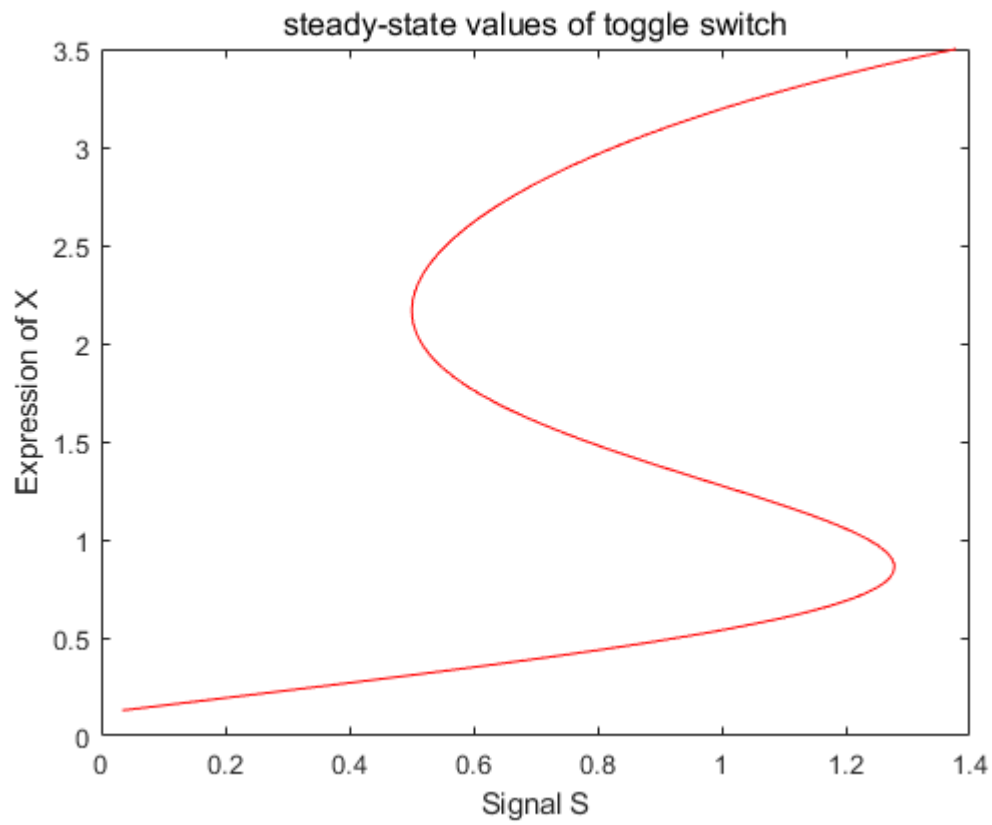Plot the relationship of S and X by Matlab

Conclusion: Figure 1B is reproducible.
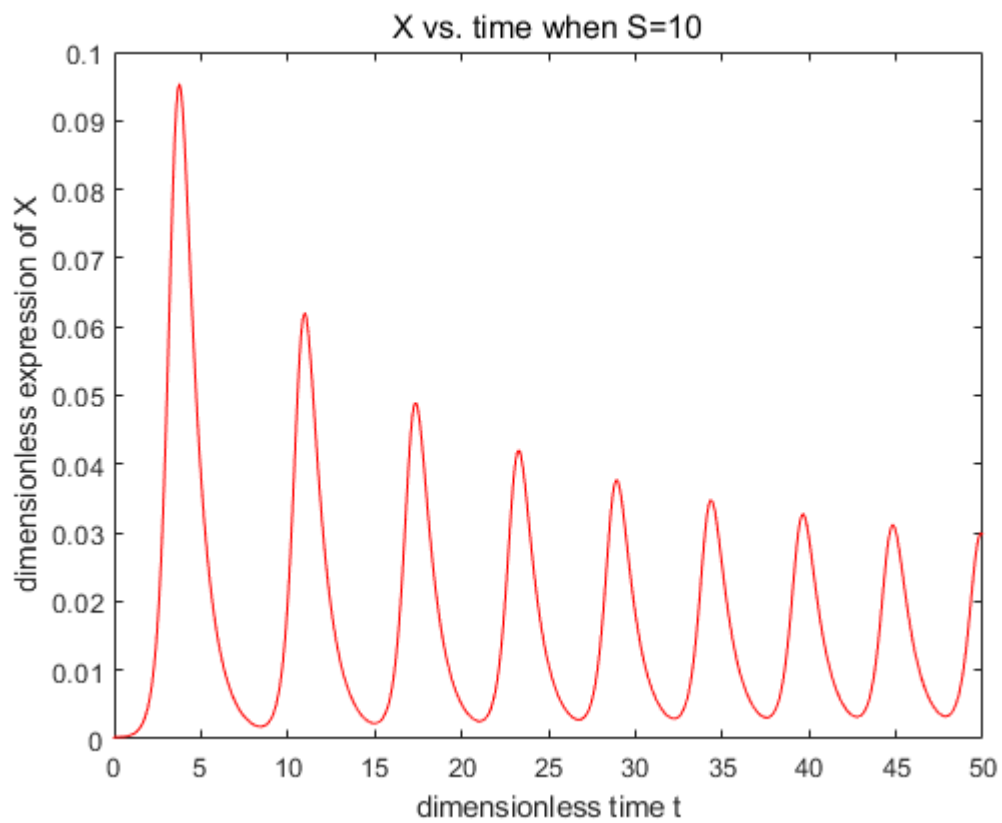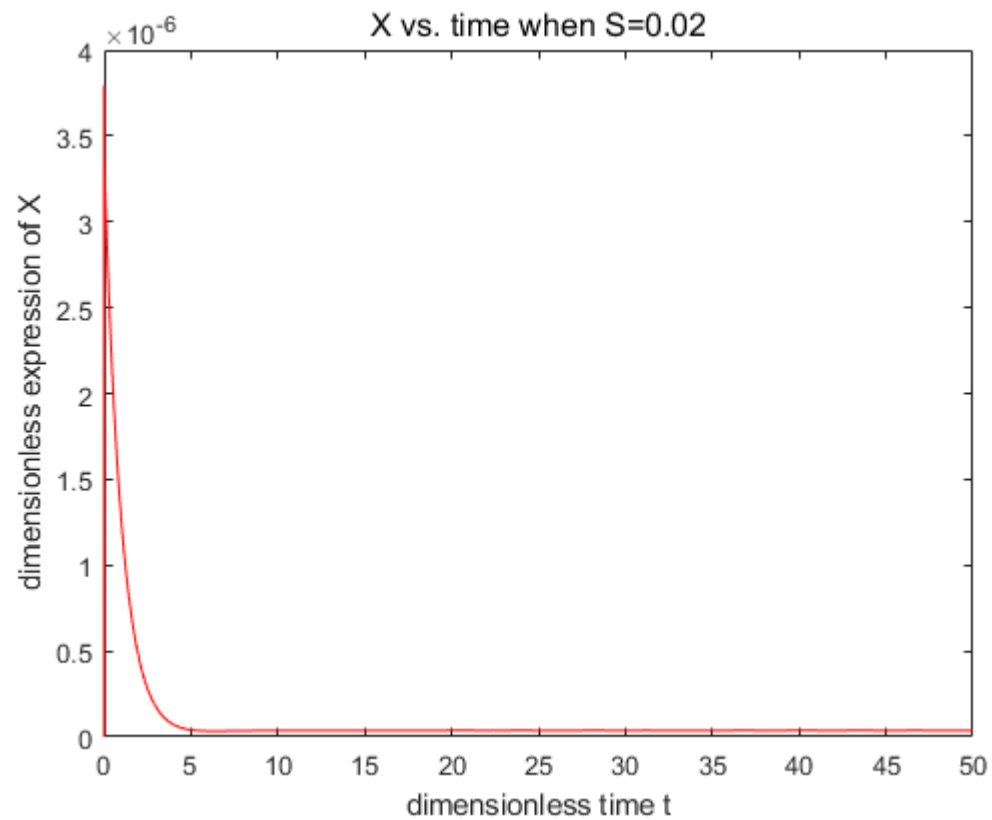
(b) Error in the paper:

$$\delta_Y = \frac{\tilde{\delta}_Y}{\tilde{\delta}_X}, \quad \delta_Z = \frac{\tilde{\delta}_Z}{\tilde{\delta}_X}, \quad t = \tilde{t}\tilde{\delta}_X.$$ (Equation 3)
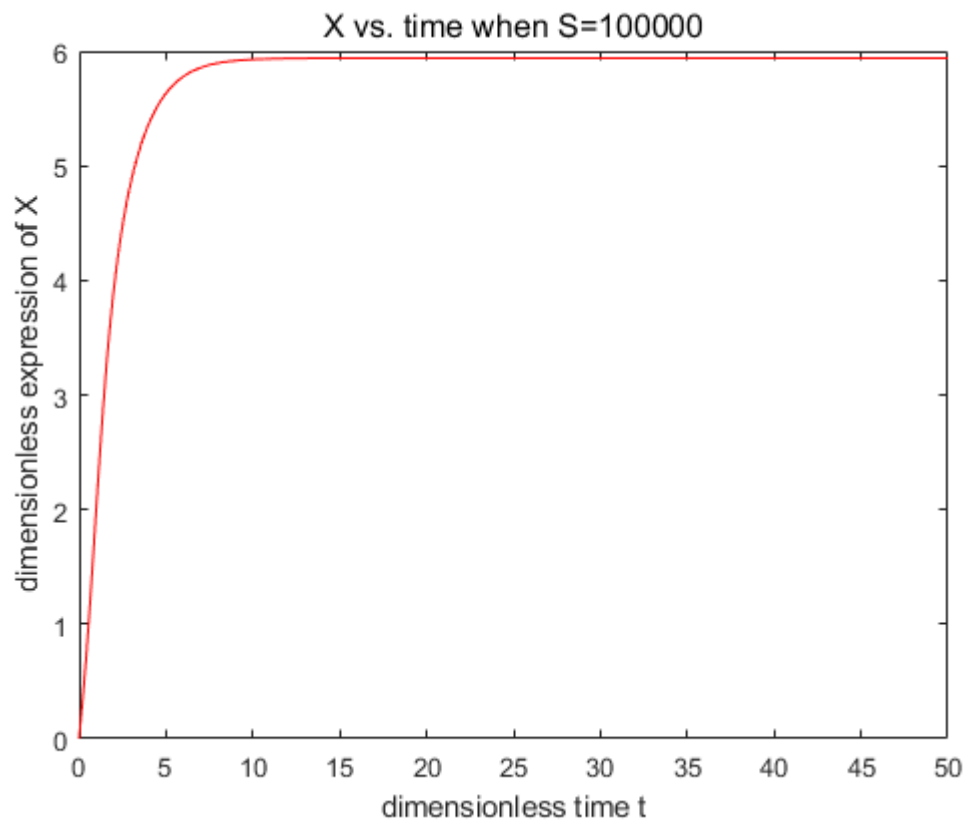
(c) Plot of the toggle switch

steady-state values of toggle switch

Coding file is named as "p2c.m" in this folder.

(d) Plots of X vs. time for the three values of S

X vs. time when S=0.02

X vs. time when S=10

X vs. time when S=100000

Coding files are named "p2d.m" and "acdc.m" in this folder.