Problem 1

(b) $\dot{m}_i = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m_i$

Pseudo steady state: $\dot{m}_i = 0$, $m_i = m^*$

$$0 = r_x\,\bar{u} - (\mu + \theta_m)m^* \implies m^* = \frac{r_x}{(\mu + \theta_m)}\,\bar{u}$$

Gain function: $K_x = \frac{r_x}{\mu + \theta_m}$ \qquad Promoter function: $\bar{u}$

$$r_x = R_E\,R_{x,T}\left(\frac{G}{T_x K_x + (T_x + 1)G}\right) \implies K_x = K_x(G, \theta)$$

$$\bar{u} = \frac{W_1 + W_2 f_I}{1 + W_1 + W_2 f_I}, \quad f_I = \frac{I^n}{K^n + I^n} \implies \bar{u} = \bar{u}(I, K)$$