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T1 ________________	**42745**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2016
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

- ‘Kowalski, visual of debris at 9’o clock;’
- ‘Half of North America just lost their Facebook.’

Well, the lines above from the movie *Gravity* might be slightly exaggerate, but it would more or less be real if we do nothing about the threatening space debris.

In this paper, we focus on evaluating potential commercial opportunities in removing space debris that is currently revolving around the Earth in low Earth orbit since the growing number of aerospace debris has been a major concern around the world community. Three models are presented to analyze this problem from distinguishing perspectives, namely, the potential economical profits, the probability of collision and the role of the policy maker. We presume that a private firm discussed here can both provide insurance services, covering collision for launched satellites, and remove hazardous space debris. We reduce the spatial debris challenge to a two dimensional problem. Gamma distribution is used to fit the distribution of space debris.

The first of the three models proposed presents a time-dependent evaluation method that projects potential economic profits for 15 consecutive years. The revenue function is determined by the insurance commissions collected from the satellites owner; the cost function can be split into two parts, the debris removing cost, composed by land-based facilities cost and space-based facilities cost, and the potential collision related compensation. The Cobb-Douglas utility function is employed here to estimate the dollar value of probable compensation.

The second of the three models helps to better assess the compensation level by estimating the probability of a collision between the existing debris and lately launched satellites. The dots, or the Matlab generated debris, are distributed in a 1000*1000 matrix, following the gamma p.d.f and some other prerequisites. Then the probability, according to the law of large numbers, is reasonably approximated by multiple run-times.

The third of the three models jumps out of the two parties previously discussed and shifts to a third body, the policy maker. A game theory approach is introduced in this model to help the policy maker better estimate the potential risks and benefits. The payoff matrix is developed and compared under different circumstances to present the effectiveness under pivotal ‘what if’ scenarios.

Admittedly, the potential profits projected in the first model appear to be quite sensitive to the insurance commission determinant. One plausible explanation is that the insurance commission collected each year is the only source of revenue. Also, we did not consider the time value of money.

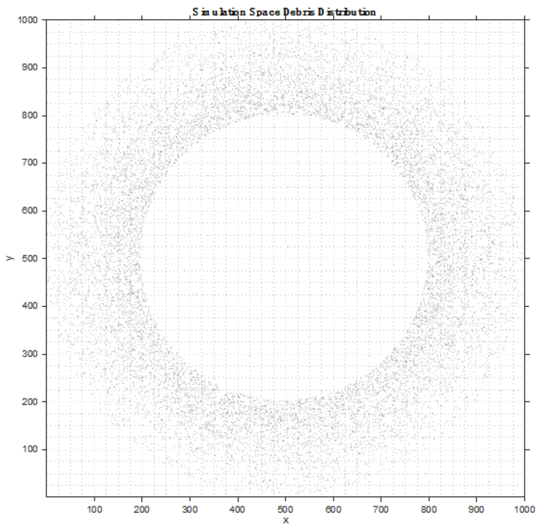
Based on the data and figures returned from the models above, we conclude with confidence that the commercial opportunities do exist if the private firm chooses the land-based and space-based combined alternative. Additionally, the insurance commission determinant shall be set to no less than 4%; and 5% or more is recommended. Policy makers shall implement new industrial standards towards orbit objects’ design and procedures to attain a Pareto improvement.

Executive Summary

We shall first imagine the following scenario, 'Half of North America just lost their Facebook.' The famous line from Gravity may not be exaggerate. Satellites, generally deemed as the main actor in space, are facing tremendous threats caused by space debris currently. If not controlled, the space debris is forecasted to be the main obstacle for further exploration of the universe.

Considering the significance and urgency of this issue, a chorus of furious space specialists pilloried the inaction of some politicians and developed multiple ways to eliminate space debris. Meanwhile, numerous projects to remove space debris have been proposed. To consider the business opportunities and risks in this field, we assume that a private firm involved can both provide insurance services covering collision for launched satellites and remove hazardous space debris; then we develop several models to simulate circumstances and evaluate the feasibility and possible executive blueprint of space debris eliminating projects.

We raised three models from distinguishing perspective. To be precise, the possible commercial profits, the probability of collision between existing debris and lately launched satellites, and the efficiency of public policies. Those models employ knowledge from several subjects, such as mathematics, astrophysics, economy etc.. The simulated debris distribution is as follows:



We classified the prevailing methods of removing space debris into two categories, namely, the land-based alternative and the space-based alternative. From the data and figures returned from the models, the projected profits of a private firm are emphatically analyzed for each alternative.

The land-based alternative requires a huge amount of initial investment and, thus, needs a relatively long period of time to gain a net profit. It also should be noticed that this alternative can control the quantity of debris ideally, with a total raise of less than 1000 in the 15 years consecutive simulation.

On the other side, the space-based alternative requires relatively less initial investment. Yet, the rocket deployment cost incurred when sending a debris remover into low earth orbit is fairly costly. One fatal weakness of this alternative is that the material used to construct, say, a Solar Sail, a kind of space debris capturer, put a lower altitude limit of the debris that can be removed. Hence, this alternative may fail to control the quantity of debris located at low altitude. In the 15 years consecutive simulation, the results prove our theoretical analysis and the debris in inner layer seems to be out of control in the end.

Due to the obvious advantages and weaknesses of each alternative, it makes sense to run this model for a third time with these two alternatives combined, which means we deploy land-based alternative and space-based alternative simultaneously. The result shows that the time needed to earn a profit is noticeably reduced while the total quantity of space debris in the low earth orbit can be controlled with satisfaction.

A game theory analysis is also carried out in the third model. We compare the Nash Equilibrium solution, or the optimal solution, with and without implementing new industrial policies. And the outcome is that by implementing new industrial policies and standards, the policy maker can help the two players reach a better state and promote the social welfare.

Further conducted sensitivity analysis also reveals several interesting points. First, the implementation of new industrial standards toward the design and procedure of orbiting objects can greatly reduce the increasing speed of space debris. If the increasing rate decreases from 5% to 4%, the ending space debris is reduced from 12302 to 9932, net of 2370, which is quite significant. A pattern of diminishing return for each 1% decrease is also shown from data gathered.

The application of recyclable rockets that could be used in space-based alternative does not make a huge difference to the firm’s net profit. We compare the projected profits for the firm by letting the rockets recycle times be 3, 6, 9 and 12. The disparity in accumulated profits caused by different rockets’ lifetime is not significant since the difference between the maximum and the minimum is only 15.4% of the total profits. Further, total profits increased by only 0.38 billion for each additional lifetime increase.

The sensitivity test of the insurance commission parameter shows that this variable can be essential for the firm to make a profit in the early stage of simulations. By setting the variable equal to 4%, the time needed to make a net profit is 5 years while the profits are all positive if we increase that variable by only 1%.

Therefore, we shall formally conclude that the combined alternative is an optimal method to tackle the space debris issue for the private firm. Furthermore, the insurance commission parameter shall be set to equal or greater 4% and is recommended to be equal or above 5% to promote the robustness of the projected net profits. The policy makers are also suggested to implement new industrial standards to reduce space debris, in order to reach a better outcome, as the game theory approach proposed.

Strategies to Eliminating Space Debris:

Approaches from a Time Dependent Evaluation Model

Team #42745

February 1, 2016

Contents

1. Introduction

1.1 Background

Currently, a plentitude of space debris fragments are orbiting the Earth at a velocity of several kilometers per second. There are now roughly over 500,000 pieces of orbital debris, ranging both in size and mass from paint flakes to abandoned satellites, hazardous enough to completely desolate operating satellites upon collision. In face of those threats as well as the collision accident happened in February 2009, a great deal of methods have been proposed to address the issue of space debris, such as small space-based water jets, high energy lasers and specialized satellites designed to sweep up the debris. They possess both unique advantages and unavoidable drawbacks. The costs, risks and benefits for implementing them also vary.

1.2 Restatement of the Problem

To reduce the threat of space debris, we ought to put these high-tech removal methods in to practice. The only way to fulfill this ambition is to seek a commercial opportunity to commercialize that project. But how can we distinguish a commercial opportunity? Commercial opportunity generally consists of four integrated elements as follows.

- A demand or need
- The means to satisfy the demand
- An approach to apply the means to satisfy the need
- An approach to get benefit

Now that the former two elements already exist. Thus, to identify a commercial opportunity is to search for the latter two. In this problem, a time-dependent model shall be constructed to objectively determine the feasibility of a private firm to obtain the two missing elements [1].

2. Existing Methods

A variety of Active Debris Removal (ADR) methods have been raised to cope with the issue of mounting quantity of space debris. Generally, they can be classified into two categories, the space-based and land-based. Now we summarize some typical methods respectively.

2.1 Land-based Methods

Laser Broom

A *laser broom* is a land-based laser beam-powered propulsion system proposed to sweep space junk out of the path of operating artificial satellites such as the ISS. Its axiom is to heat one side of the target object enough to alter its original orbit and accelerate to hit the atmosphere. And the laser broom is supposed to target debris between one and ten centimeters in diameter [2].

One of the most distinguishing advantages of *Laser Broom* is that it can operates efficiently. While its constraints are that it needs high-powered laser launching devices and lacks target tracking accuracy. In addition, this project is not cheap, scientists said it would cost nearly 0.2 billion dollars and needs 2 years to be carried out [9].

Huff-and-Puff

The *huff-and-puff* approach clears orbital debris by firing intensive pulses of atmospheric gases into the track of targeted space trash. This is the notion behind SpaDE, the Space Debris Elimination Initiative, raised by Daniel Gregory Raytheon BBN Technologies in Virginia. Vertical bursts of gas generated within Earth's atmosphere can either be maneuvered at orbiting riffraff to turn its trajectory or cause obstruction to hasten its re-entry [3] [18].

The advantages of the huff-and-puff approach are that it can remove big and small debris unlimitedly and it can also act on many pieces simultaneously when they assemble together. As for the shortcomings, we consider its total cost, most of which lies in the cost of energy. To move the ISS, it costs approximately $2 million per burn. And an earlier estimate of the amount of fuel required to create the atmospheric pulse is up to 500 gallons of gasoline [3] [18].

2.2 Space-based Methods

Snagging and Moving

This approach was first put forward publicly in early 2014 to search satellite debris in a polar orbit at an altitude between 800 and 1,000 kilometers. Currently, some kinds of "capture mechanisms" can pick up the debris, such as harpoons, robotic arms, nets and tentacles [4].

Among these mechanisms, we explore the nets. The ESA has been elaborating a piece of debris, and their newest test involves launching the weighted anti-junk nets in microgravity. The net seems to be a consummate solution for capturing space debris which are relatively static; it also offers an upper hand of envelopment, making sure that fragments don't fly away, forming even more perilous junk [5].

Solar Sail

Solar sails, or the light sails or photon sails, are a form of spacecraft propulsion exerting available radiation pressure from stars to push large ultra-thin mirrors to high speeds [6]. Solar sail craft offer the possibility of low-cost operations combined with long operating lifetimes.
The strengths of *solar sail* are that the material is portable and inexpensive. While its weaknesses are it requires beforehand calculation of the sail's expansion details and the spacecraft's altitude.

3. Terminologies and Notations

3.1 Terminologies

- **Compensation:** a sum of money or material object an insurance firm promises to pay out if certain damage occurs to the insured entity.
- **Revenue:** the inflow of money a business has from its normal business activities.

- **Expense:** an outflow of money a business has from its normal business activities.
- **Fair Value:** rational and unbiased estimate of the potential market price of a good, service, or asset.
- **P.d.f:** the abbreviation for probability density function, is a function that describes the relative likelihood for this random variable to take on a given value.
- **R&D:** research and development
- **Cobb-Douglas utility function:** three dimensional with utility or output measured along the vertical axis and is developed and tested against statistical evidence by Charles Cobb and Paul Douglas during 1927–1947.
- **Gamma distribution:** a two-parameter family of continuous probability distributions in probability theory and statistics.
- **ADR:** active debris removal strategy.
- **LEO:** low earth orbit, an orbit around Earth with an altitude between 160 kilometers and 2,000 kilometers.
- **Game theory:** the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. And it is mainly used in economics, political science, and psychology, as well as logic, computer science, biology and poker.

3.2 Notations Table

The following table provides meanings of important notations used in our models.

Symbols	**Meanings**
N_t	Net income generated from major operating activities in the year of t
R_t	Revenue generated from insurance collected
C_t	Total cost of the private company in year t
N_T	Cumulative net income for the firm
n_t	The number of satellites launched in year t
w	Guaranteed amount for the insured satellite
α	An actuarial adjusting constant
C_{1t}	The cost of removing the space debris in year t
C_{2t}	The cost of potential compensations incurred in year t
p_t	The price or cost of a certain approach in year t
q_t	The quantity of debris a certain approach can trap
$Pr(t)$	Chance of collision between launched satellites and space debris
δ	A compensating parameter determined by w and α
Q_t	The amount of debris in space in year t
ε	The amount of debris each trapper in space can remove.

4. Model One – a Time-dependent Evaluation Model

To better quantitatively evaluate the potential risks and opportunities that a private firm may confront, relating to the subject of space debris, a comprehensive quantitative model is essential and inevitable. Thus, a model based on projected net income is established.

4.1 Assumptions

The following assumptions are listed to simplify this model.

- **All satellites launchers will purchase satellites-crash insurances**. For each satellite launched into low earth orbit, the launchers give a fixed percentage of cost of the satellite to an insurance company. And if, say, an insured orbit object is hit by space debris, compensation determined by the percentage is given to the launcher.

- **The private firm discussed in question is both an insurance company and a space debris eliminating company.** To be exact, we merge the two market parties into one single business entity. The major revenue of the firm originates from the insurance fee collected; the major expense of the firm originates from the space debris removing activities and compensation incurred.

- **The potential commercial opportunities can be quantitatively measured by projected net income.** Net income is often served a significant quantitative benchmark of business performance. Thus forecasted net income is a credible measurement of commercial prospect.

4.2 Establishing the Model

In each year, the firm is involved in both accumulating capital from satellites insurance fee and removing highly threatening space debris. If the firm do not use the capital collected to eliminate space debris, an enormous compensation expense is equitably projected, since the probability that an orbiting object is hit by space debris in elevated each year.

To estimate the commercial prospect of addressing the space debris issue for the merged entity, we simulate the situation for a consecutive of t years. We let $N_1, \ldots, N_t$ denote the net income generated from major operating activities in the year of t. And we let $R_1, \ldots, R_t$ and $C_1, \ldots, C_t$ denote the revenues generated from insurance collected and expenses generated from debris removing activity and compensation incurred in the year of t, respectively. Then, the basic economic equation of net income can be written as,

$$N_t = R_t - C_t$$

The concept of time value of money can be ignored here since we do not conduct horizontal analysis, comparing numbers within the industry at the single time node. The cumulative net income for the firm is the summation of all previous net incomes:

$$N_T = \sum_{i=1}^{t} N_i$$

$$(s.t.\ t \leq t_{max})$$

We will explore the determinants of R_t and C_t in the following sections extensively.

4.2.1 Revenue function

The operating revenue, R_t, as suggested before, is majorly composed by the insurance type payments collected. For a single low earth orbit object launched, the amount of revenue that a firm get is determined by the guaranteed amount, w, and the actuarial adjusting constant, $\alpha\%$ [7]. The total revenue in a given year is then estimated by the single revenue multiplied by number of satellites launched in that year, n_t:

$$R_t = n_t \times w \times \alpha\%.$$

The detailed data of previous launches of orbit objects are collected from Space-Track.org [8]; and the number of satellites launched each year is a pivotal parameter for estimating compensation expense, and thus will be thoroughly discussed in the sub-model later. Here, we simply view it as a known variable.

The guaranteed amount is the cost of assembling and launching an orbit object. Due to the fact that, often, a huge amount of labor and capital are invested to fabricate an orbit object, it is reasonable to let the guaranteed value be equal to the fair value of it.

The actuarial constant is merely an arbitrary given constant, since no such real data could be found. Still, this coefficient will be elaborated in the sensitivity analysis part.

4.2.2 Cost function

The expenses that incur in year t is separated into two components, the cost of eliminating the space debris, C_{1t}, and the cost of potential compensations, C_{2t}. Then, we have:

$$C_t = C_{1t} + C_{2t}$$

The first term, C_{1t}

C_{1t} will vary when the company chooses different methods of removal. Among the numerous approaches discussed in the previous work, the means to get rid of space debris can be divided into two parts, the land-based approach, such as the Laser Broom, Huff and Puff etc., and the space-based approach, such as Snagging and Moving, Solar Sail etc..

Three key distinctions are listed to help better construct the model:

- **Initial cost and following cost.** The land-based approach has a relatively high initial fixed cost for the construction of utilities; the expense of each strike, yet, is relatively low. The laser broom, for instance, has a vast installation cost [9], whereas the energy used for each strike of space debris is minute. Inversely, the space-based approach has a more even cost allocation in regarding to time series, since every time a space rocket is employed to send the space debris catcher into working range.

- **The amount of space debris that can be removed.** It should be noted that the quantity of space debris that each alternative can remove vary.

Let p_t and q_t denote the price of each catcher and quantity of debris a certain approach can capture. The total expense each year is the individual cost combined; the dummy variable, ρ, which can only be 0 or 1, is introduced in this equation to represent whether applying a specific approach or not:

$$C_{1t} = \sum_{each\ type}^{all\ types} (p_t * q_t * \rho)_{each\ type}$$

The second term, C_{2t}

The potential compensations incurred each year for the insurance firm is approximated by the following equation:

$$C_{2t} = Pr(t) * \varepsilon n_{t-1} * \delta$$

$Pr(t)$ is a denotation of the chance that an orbit object launched will collide with a space debris, in year t. If, say, the number of space debris is soaring, the term $Pr(t)$ would increase consecutively, causing the compensation expense to rise; and vice versa. The probability is ascertained carefully in the upcoming sub-model.

We use Cobb-Douglas utility function to determine the variable δ. The general form of Cobb-Douglas utility function is:

$$U(x_1, x_2) = \mu * x_1^c * x_2^d$$

Applying monotonic transformation by letting $= \frac{c}{c+d}$, we derive the new utility function with single variable:

$$U'(x_1, x_2) = \mu' * x_1^k * x_2^{1-k}$$

Let x_1 be the cost of the orbit object, w, which is a constant number, and x_2 be the expected compensation if the collision does occur, which can be represented by the insurance fee, αw. Since most investors are risk-averse and face a diminishing marginal return (DMR), we let k be 0.5 to satisfy that condition. Formally, the δ is written as:

$$\delta = w * \alpha^{\frac{1}{2}} * \mu$$

5. Model Two – A Model to Determine $Pr(t)$

To obtain the probability that a launched satellite may crash with space debris, $Pr(t)$, we need to acquire further understanding about the spatial relationship between the distribution of space debris and spatial geographical location. Hence, a sub-physical model is discussed here.

5.1 Further Assumptions

- **Space debris is uniformly distributed in low earth orbit for a certain range of altitude.** From observable data from NASA, the space junk surrounding the Earth is akin to a several 'well-distributed spherical jacket', covering the Earth layer by layer, thus, we interpret it as uniformly distributed in a certain range [22].

- **Space debris will not be eliminated without human interference.** Here, we simplify the model by assuming that space debris will not be removed by physical factors. Namely, the existing debris will follow the law of gravity and revolve in uniform circular motion.

- **The amount of fragments in a given episodic space are relatively static.** In view of the fact that in each layer the debris is uniformly distributed and is in uniform circular motion perpetually, the amount of space debris in a given length period of time is relatively static.

- **It is the middle sized debris that causes danger to orbiting objects.** Given that space debris can be classified into three categories upon its size, which are the small-sized debris with diameter between 1mm to 1cm, middle-sized debris with diameter between 1cm to 10cm and large-sized debris with diameter greater than 10cm, we regard the middle-sized debris as the only potential danger to satellites, since large-sized debris can be traced and avoided and small-sized debris is not considered fatal [10] [17].

5.2 Some Thoughts

From figure X, which shows the distribution of space debris in low earth orbit, we initially wish to create a three dimensional matrix to represent the situation. Yet, considering the demanding calculation and sophisticated programming skills involved, we instead consider 'smashing' this spatial problem into a two dimensional problem. A two dimensional circular distribution model is representative and can be expanded to three dimensional case, if necessary.

We may take a leaf from the NASA's book. Figure 1 shows the spatial density of debris in accordance with altitude in LEO; Figure 3 is the three dimensional spread of debris, denoted by black dots, in LEO [12]. The shape of the curves in Figure 1 is analogous with the probability density function of gamma distribution, shown in figure 2. Moreover, gamma distribution contains two parameters, the shape parameter α, and rate parameter β, or the inverse scale parameter, which could control the shape of curve. Therefore, gamma distribution is considered as a proper representation and we consider a random variable x, which follows gamma distribution,

$$X \sim Gamma(\alpha, \beta)$$

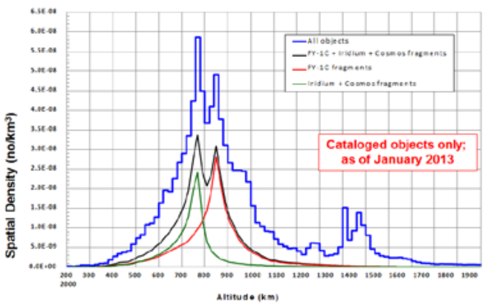


Figure 1 [12]

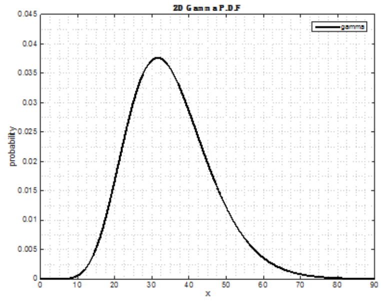
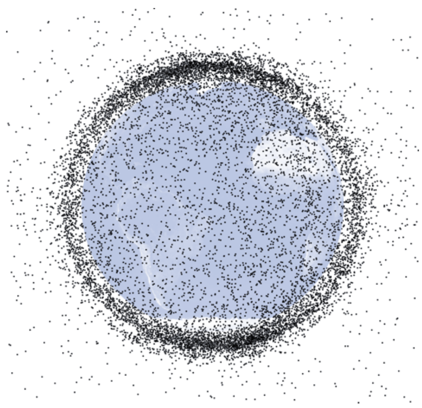


Figure 2

We plot the p.d.f. of the gamma distribution in both XOY coordinates and XOYOZ coordinates. An interesting fact from the Introduction to Physics is that objects revolving around the earth within 10 kilometers to the Earth's surface will slow down due to the existence of atmosphere; furthermore, the objects will be attracted by gravity since the velocity is not enough to sustain circular movement. Thus, we use the diameter of the earth plus the thickness of the atmosphere from the 3D gamma p.d.f. and derive Figure 4.



(*Source: Image of the day, NASA, 2009*)

Figure 3

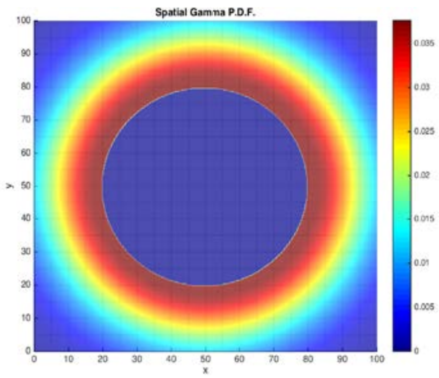


Figure 4

5.3 Establishing the Debris Distribution Model

Here, we establish a 1000*1000 '1-0' matrix; each cell can contain either of 0, a denotation of not being occupied by space debris, or of 1, a denotation of being occupied by space debris. If a particular cell is filled with space junk, we consider it as a dangerous zone and any satellites launched later into that cell is more likely to crash with space debris. Inversely, if a particular cell is free of space debris, we regard it as a safe zone and any satellites launched later into that cell is free from the threat of space junk and will not crash with any debris.

From the figures above, we notice that the debris distribution probability decreases intensively from the inner layer to the outer layer, thus we split it into four consecutive ring-shaped layers; the corresponding integration of the gamma probability density can be calculated easily. Then, we employ random seeds to 'plant' debris into the matrix, with a total of about 15% cell filled with space junk.

Though, committedly, 15% mentioned above is merely derived from an educated-guess process, it still serves as a convincing and reliable estimation. Since we divide the Earth's diameter into 1000 small units, each cell in the matrix may denote a certain small spatial area of the Earth. The total spatial areas surrounding the earth can be approximated by using the area of a square to divide the Earth's whole surface area. Then, the percentage is figured by calculating the entire spatial areas over the entire number of space debris, which is roughly 15%.

The simulated scattering of space debris can be vividly shown in the following Figure 5,

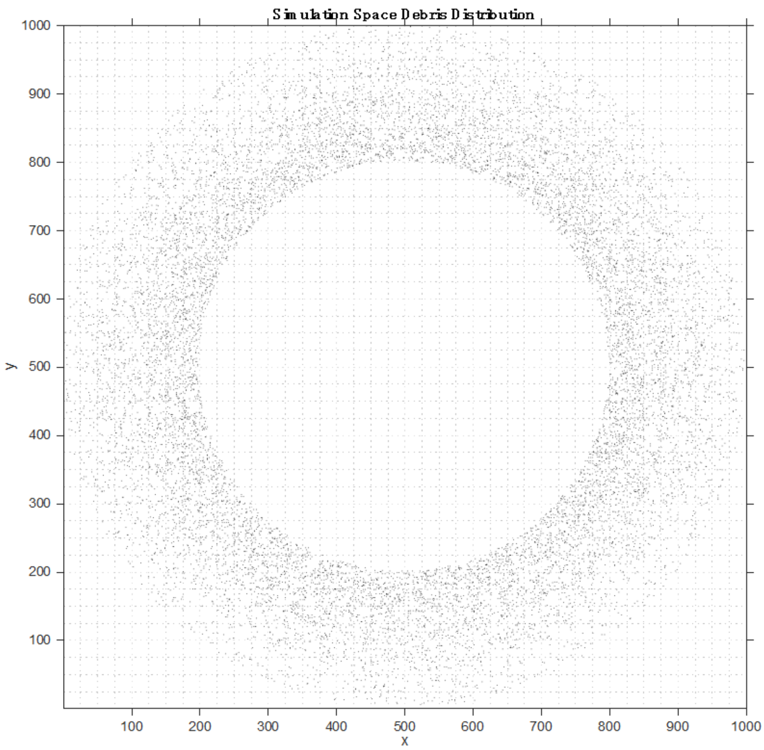


Figure 5

5.4 Space Debris Removal Process

The quantity of debris that can be successfully removed by the private firm is determined by the options of techniques applied, namely, the land type and the spatial type, and the target elimination quantity respectively. The amount of debris pre-determined is, then, removed randomly from this model. However, we shall also consider the fact the optimal work range. We use the distance between the debris in the space and the Earth's surface to standardize the work range; the land type method is roughly between 600-800 kilometers, and the spatial type method is roughly between 800-1000 kilometers [11].

Subsequently, the amount of debris in space, Q_t , is updated by subtracting the amount that has been removed in that year,

$$Q_t = Q_t - (q_{land_t} + \varepsilon * q_{space_t})$$

In which, p_{land_t} and q_{land_t} denote the cost of each removing conduct and the times of conduct in that year, respectively. And ε is a constant that stands for the amount of debris each trapper in space can remove.

5.5 Space Debris Growth Process

The amount of debris that is surged in the space is also an uncertainty that should not be dismissed. Here we use historic statistics of the quantity of space debris to predict future growth trend; and the debris increased by the destruction of Fengyun-1C and Collision between Iridium 33 and Cosmos 2251 is excluded, since those outliers are often deemed as abnormal growth behaviors of the space junk. From the statistics of the growth of the numbers of cataloged objects in earth orbit from NASA [12], we can fit the data by either using a linear growth model or a log growth model. In the very first model, we will use the linear model to fit the data for simplicity and the log model will be discussed later.

The same as the previous removal process discussed above, the quantity of space debris increased is added to the model randomly, with respect of gamma distribution probability density determined before.

5.6 Satellites Launching and Crashing

As our assumptions mentioned, the middle sized space debris cannot be observed and traced by specialists, thus satellites launched face a probability that will collide with space debris. In this part, we simulate the launching process of satellites by human-beings and count the number of times that man-made space craft having a collision with space debris.

The Space-Track documents detailed information about Objects currently in orbit, including object name, launching date, apogee and perigee etc.; those statistical data greatly assist us to make convincing prediction about the number of low earth objects that will be launched in the following years [8]. At the very beginning, we employed the most straight-forward method, the linear increasing mode. The initially quantity of orbiting satellites is set to 110, and this number will increase by 5% each year.

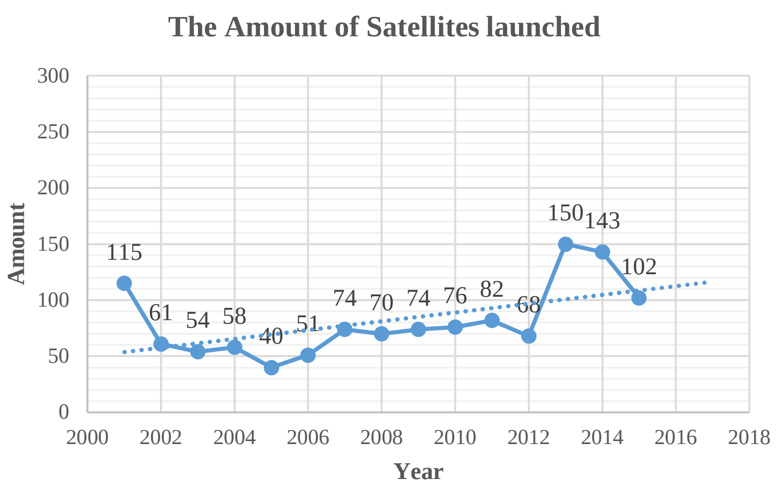


Figure 6 *(Data Source: Space-Track.org)*

Then, those 110 dots are randomly allocated into the debris distribution matrix developed before. When a dot and a space debris happens to be at the same cell, we consider it as a crash. The same logic can be applied inversely, if a satellite is launched into an area free of space junk, we assume that the orbit object is in safe zone and nothing will happen.

Then the $Pr(t)$ is well defined as follows:

$$Pr(t) = \frac{the\ number\ of\ crashes\ in\ year\ t}{the\ total\ launched\ satellites\ in\ year\ t}$$

We may safely assume that the calculated probability, $Pr(t)$, is close to the real probability, b, since the the sample probability will converge to latter one, according to the law of large numbers. And the mathematical expression is:

$$Pr(t) \xrightarrow{p} b$$

6. Results

As we introduced in the very beginning, there are various methods to remove the space debris. We classify them into two basic categories, the land-based and the space-based. *Laser Broom* and *Snagging and Moving* are taken as examples to represent these two categories.

6.1 Land-Based Removal Alternative alone

One distinctive feature for the land-based alternative, as implied before, is that huge amount of initial investment, C_0, is required to construct facilities [9]. Afterwards, each debris removing conduct takes a relatively small amount of cost. Hence, we have the following updated equations,

$$C_{1_t} = C_{land_t} + C_0$$
$$C_{land_t} = p_{land_t} * q_{land_t},$$

With the data related to this alternative [7] [8] [9] [12], we use Matlab to simulate this circumstance from 2016 to 2030; and the results are presented in the following figures,

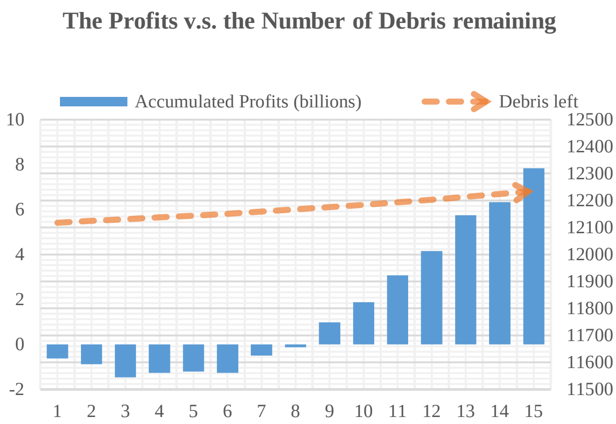


Figure 7

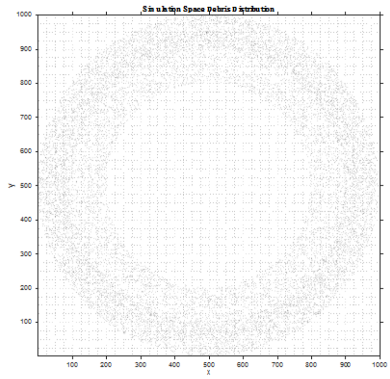


Figure 8

Interpretations: the land-based alternative can successively control the increasing speed of the space debris, with less than 1,000 total upsurge in the 15 years simulation, which appears to have a promising future regarding the space debris elimination plan. However, the main drawback of this single alternative is commercial opportunities is not apparent in the beginning decade and the firm is facing a projected net loss in the first 8 years. The N_1 to N_8 are all negative in the figure above. Thus, we conclude this alternative as an armchair general with no practical possibility for private firms.

6.2 Space-based Removal Alternative alone

Different from the land-based alternative, the space-based approach has considerable lower demand of initial capital input; indeed, it is the rocket deployment cost, C_{space_t}, that account for a substantial proportion of this approach. Thus, we do not take the initial cost into consideration; and the updated equations are written as follows:

$$C_{1_t} = C_{space_t}$$
$$C_{space_t} = p_{space_t} \times q_{space_t}$$

With the data related to this alternative [7] [8] [13], we use Matlab to simulate this circumstance from 2016 to 2030; and the results are presented in the following figures,

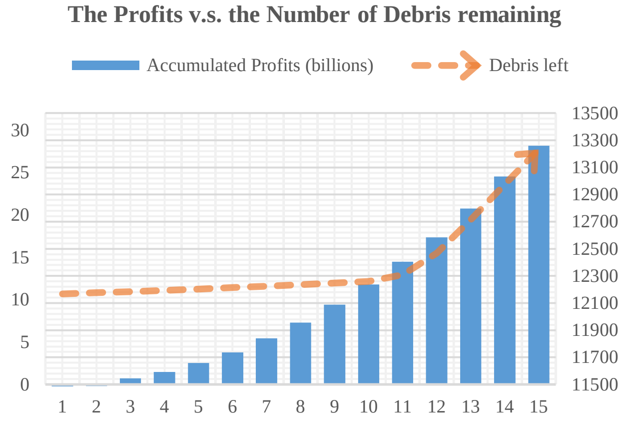


Figure 9

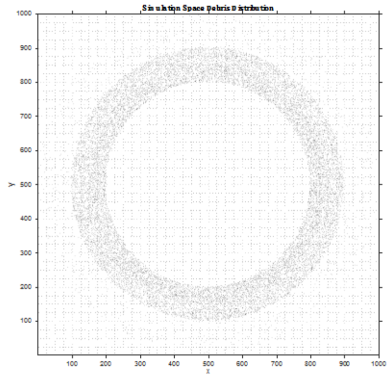


Figure 10

Interpretations: the space-based alternative can ideally control the increasing speed of the space debris in the beginning decade; yet, the amount of debris increased dramatically in the last 5 years since the debris in outer layer has almost been cleared up and it becomes more difficult to meet target quantity for each catcher. The quantity of debris in inner layer, at the same time, increases steadily. Thus, we still consider this alternative as an improper alternative, despite its glowing financial performance.

6.3 The Combination of the two alternatives

As comprehensive explanations in aforementioned parts proposed, it is both sound and critical to run a third approach – the combination of previous alternatives. We shall first derive the updated equations for the total cost, C_{1_t}, and the total quantity of debris that will be removed, Q_t,

$$C_{1_t} = C_{land_t} + C_{space_t} + C_0$$
$$Q_t = Q_t - (q_{land_t} + \varepsilon * q_{space_t})$$

Again, with the data related to this alternative [7] [8] [9] [12] [13], we use Matlab to simulate this circumstance from 2016 to 2030; and the results are presented in the following figures,

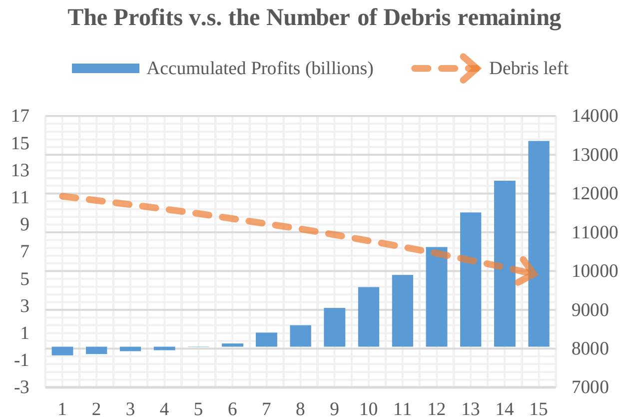


Figure 11

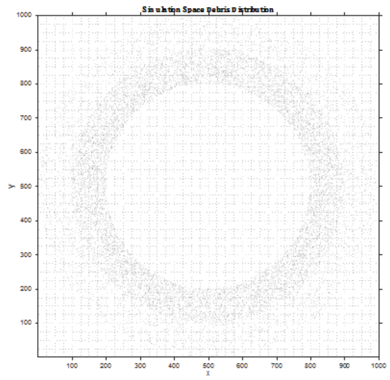


Figure 12

Interpretations: the space and land combined alternative can both perfectly control the quantity of the space debris, with a deduction of more than 2000 in 15 years, and exhibit a glowing financial performance, with an N_{15}, the net profits, of nearly 16 billion, which is the double of the result in the first alternative. Thus, we consider this combined alternative as an appropriate alternative, welcomed by both the firm and the launcher.

7. Some 'What if' Scenarios and Sensitivity Analysis

It is undeniable that some parameters inside the model are set by speculation arbitrarily; and it would be naïve to fix those parameters under ever-changing circumstances. Therefore, a variety of critical 'what if' scenarios are involved, among which new debris related regulations and the application of recyclable rockets speak volume.

7.1 Scenario 1: New Industrial Standards toward Design and Procedures

Based on the standard practice reported by NASA, industrial standards toward the design and procedures to prevent breakups during mission operation and after implemented would have considerable influence on the debris increasing rate [12]. For instance, the automated self-destruction procedures planted inside can slow down a retired orbiting object, which would let it be burned down in atmosphere. This practice can avoid the occurrence of dramatic upsurge of the quantity of space debris caused by collision, say, between retired satellite Iridium 33 and Cosmos 2251.

Therefore, we update the debris increasing rate by decreasing the iteration speed from current 5% to 4%, 3% and 2 %, simulating the different levels of effectiveness of the new regulation; then, we employ a logarithm function to fit the historical data and present the corresponding figures,

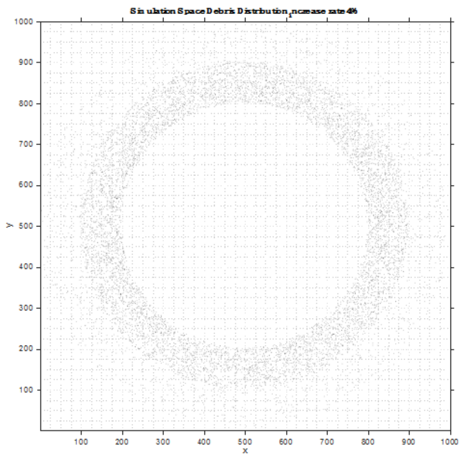


Figure 13, 4%

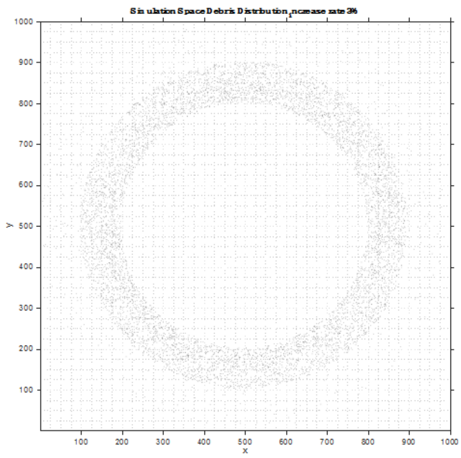


Figure 14, 3%

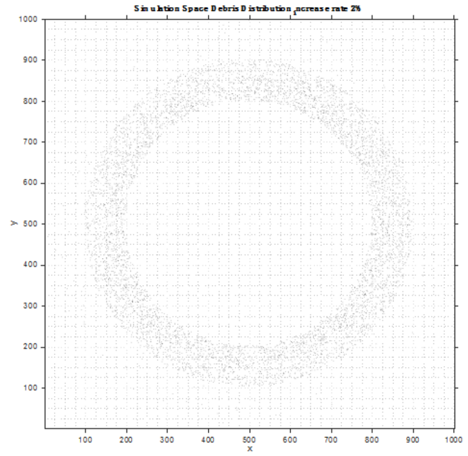


Figure 15, 2%

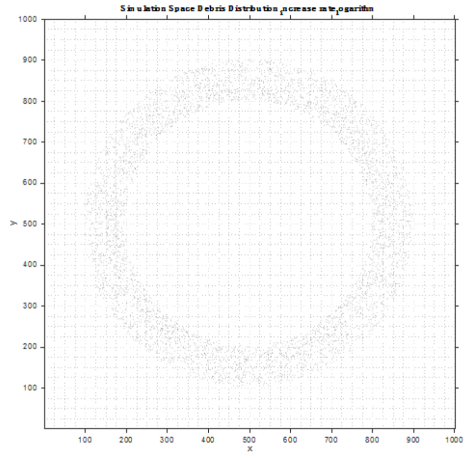


Figure 16, logarithm

The figures presented above are developed by using the combined alternative. The debris in outer layer is removed faster than that the inner layer; and the total debris is decreased by more than 50% of the original level in the logarithm increasing method. From 5% to 4%, the ending space debris is reduced from 12302 to 9932, net of 2370; from 4% to 3%, the ending space debris is reduced from 9932 to 8004, net of 1928; from 3% to 2% the ending space debris is reduced from 8004 to 6374, net of 1670; from 2% to logarithm, the ending space debris is reduced from 6374 to 5313, net of 1061. This pattern makes sense since the less debris left in the space, the harder it is to eliminate them. The logic is similar to the 'man cannot use up all the gas underground'.

Thus, with other variables fixed, we believe it is safe to say that the government regulations toward the design and procedures about the orbit objects have tremendous influence on the debris generation speed and debris spatial distribution.

An interesting point that should be noticed is that, if the debris eliminating firm observes the quantity of debris in outer layer has declined intensively, it would be silly and a waste of money to keep sending debris catchers into LEO. Explicitly, that has become a game theory topic and we will put some thoughts into that problem in further improvements section.

7.2 Scenario 2: Recyclable Rocket

In former discussion, we assume that p_{space_t} includes the cost of both the debris catchers and rockets used to deploy those utilities, which indicates the rockets cannot be recycled. Yet, we noticed the high-tech companies, such as Space X, have devoted a lot into the R&D towards recyclable rockets and have made sounding progress. Hence, we now assume that the technique of recyclable rocket can be used in near future.

We run the model several times to analyze the relationship between the lifetime of a rocket and the firm's projected profits.

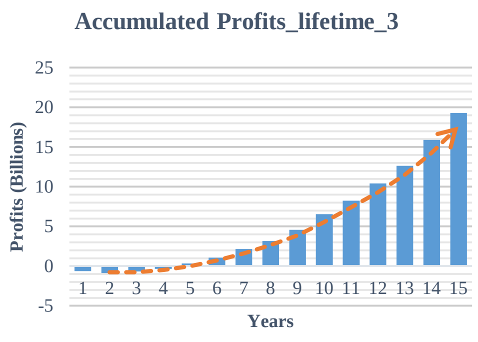


Figure 17

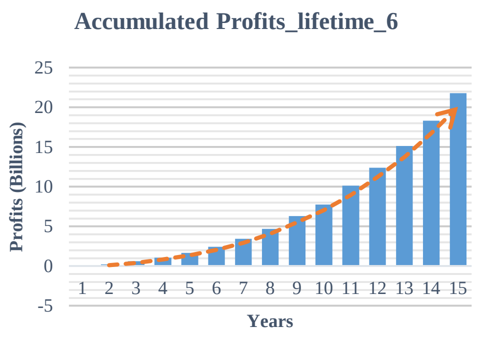


Figure 18

Figure 19

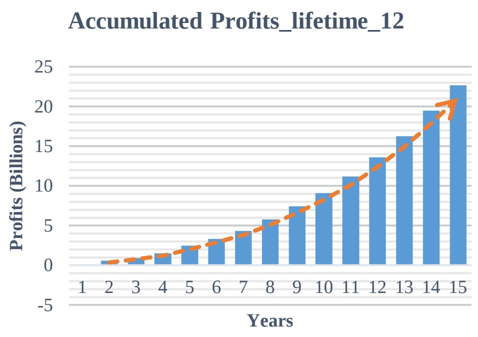


Figure 20

The figures presented above are also developed by using the combined alternative. We compare the projected profits for the firm by letting the rockets recycle time be 3, 6, 9 and 12. The disparity in accumulated profits caused by different rockets' lifetime is not significant since the difference between the maximum and the minimum is only 15.4% of the total profits. Further, total profits are increased by 0.38 billion for each additional lifetime increase. One plausible explanation for this result is that the compensations liability generated at beginning is far higher than the amount of recyclable cost could save, in additional to the sky-high initial investment of land base alternative.

7.3 A Sensitivity Test of α

α is likely to change in long time series. Two acceptable reasons are the insurance company and the satellites launcher can negotiate and the launcher can evaluate the safety of the LEO by observing the remover's prior actions.

Figure 21

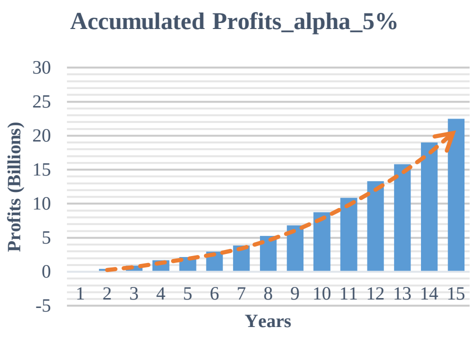


Figure 22

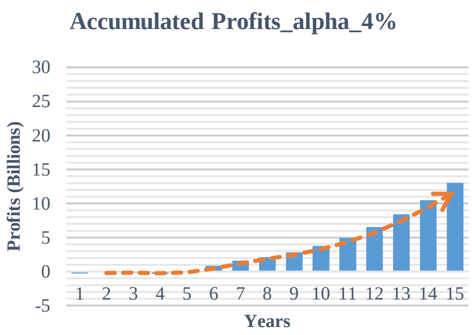


Figure 23

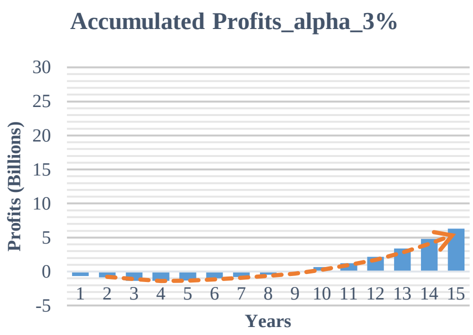


Figure 24

The figures presented above are also developed by using the combined alternative. It is pretty clear that α does play a huge pivotal role in analyzing the accumulated profits. If α set to 3%, the firm will enjoy net income in the first 9 years; and if α is set to 4%, the firm will suffer net loss in the first 5 years. Inversely, when α is set to greater or equal than 5%, the profit model is quite robust and the firm is projected to make net income in all 15 consecutive years. We find that the net profits will increase by an average of 7.4 billion for every percent increase of α. And, the balanced point, the net revenue equals the net loss, would be postponed by about 4 years for every percent decrease of α.

8. A Game Theory Perspective

As we have explained before, two players, namely, the orbit objects launcher and the insurance provider, are involved in this game. We want to analyze the best alternative or combination of alternatives from the prospective of game theory. We assume that it is a perfect information game, which means the two players knows each other's options and can observe other's behavior.

The satellites owner has three choices by having his satellites uninsured, half insured or fully insured. The insurance provider also has three options by choosing to do nothing, remove some debris or remove considerable debris in space. Then we can develop the original matrix by substituting the numbers calculated from previous models.

Player1/2	Do nothing	Remove some	Remove considerably
Uninsured	(-45,0)	(-36,0)	(-24,0)
Half insured	(-31.5,-13.5)	(-27,-9.9)	(-21,-8.4)
Fully insured	(-27,-18)	(-27,-11.7)	(-27,-13.2)

The pure strategy Nash Equilibrium (NE) in this game is either choosing (half insured, remove considerably) or (fully insured, remove some). If player 1 choose fully ensured, player 2 act like a debris eliminating company and remove some debris to maximize its utility. If player 1 choose half ensured, the firm had better to remove considerably since the revenue collected does not exceed the expected compensation. Thus, the first NE is reached. The second NE can be explained similarly.

Player1/2	Do nothing	Remove some	Remove considerably
Uninsured	(-36,0)	(-30,0)	(-21,0)
Half insured	(-25,-9)	(-24,-6.9)	(-19.5,-3.3)
Fully insured	(-27,-9)	(-27,-5.7)	(-27,0.6)

Further, the policy makers can choose to implement new standards, as discussed in 'what if' scenarios; and shift the original Nash Equilibrium Strategy Solutions, or the NESSs, to only one NESS in the new payoff matrix, (half insured, remove considerably). Then the policy makers successfully realize a Pareto improvement.

9. Conclusions

- **The potential commercial opportunity does exist for a private firm.** N_S will always be positive in the end under a variety of settings of parameters. The most plausible alternative, the combination of two existing alternatives, presents a strong commercial prospect. Thus, a firm may take advantage of the market and earn profit by offering a satellite insurance service and eliminating space debris.

- **Recyclable rockets have an immaterial influence on the projected profits.** Though, theoretically, the recyclable rockets can lower the cost, the significance of this factor on net profits is only 15%.

- **The insurance commission set by α is an essential component in the actuarial model.** α should be set to no less than 4% and it is recommended to set α equal or above 5% to strengthen financial robustness.

- **New industrial standards toward the design and procedures are necessary to control the number of space debris.** To utterly solve the space debris issue, we ought to reduce the debris generating rate. The recommended solution is the implementation of new industrial standards, due to its evident effect presented.

10. Strengths and Further Improvements

Strengths:

- The problem is solved by distinguishing approaches and the models employ knowledge from a variety of subjects, such as mathematics, astrophysics, economy etc.

- The sensitivity test is conducted thoroughly, which makes the results more convincing and comprehensive.

- The graphs are depicted accurately and vividly, allowing the results to be understood easily

Some Improvements:

- **Develop a comprehensive three-dimensional model.** In this paper, we use a two dimensional model to estimate $Pr(t)$ in the probability analysis section. A more accurate three-dimensional model can be more helpful to study this problem, for it allowing us to better simulate real case scenario.

- **Find and use detailed relevant data and information.** In our utility evaluation model, some parameters are set, admittedly, arbitrarily, due to lack of relevant data. Those numbers are derived from educated guessing process, which can cause our model to deviate from the real case. Researching and using detailed information can considerably refine our models.

- **Exercise operation research on allocating resources.** The accumulated profits are the capital resources that can be used in the upcoming periods. When analyzing the combined alternative, a more rational allocation of capitals to each approach shall be considered in the perspective of operation research. Also, the optimal timing for starting each alternative is also worth considering.

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