作業

武國寧

0.1 作業

1. 求下列函數的極限

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2}$

   (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{1+x^2+y^2}{x^2+y^2}$

   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{1+x^2+y^2}-1}$

   (d) $\lim\limits_{(x,y)\to(0,0)} (x+y)\sin\dfrac{1}{x^2+y^2}$

   (e) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

2. 討論下列函數在點$(0,0)$的重極限與累次極限

   (a) $f(x,y)=\dfrac{y^2}{x^2+y^2}$

   (b) $f(x,y)=(x+y)\sin\dfrac{1}{x}\sin\dfrac{1}{y}$

   (c) $f(x,y)=\dfrac{x^2y^2}{x^2y^2+(x-y)^2}$

   (d) $f(x,y)=y\sin\dfrac{1}{x}+x\sin\dfrac{1}{y}$

3. 討論下列函數的連續性

   (a) $f(x,y)=\tan(x^2+y^2)$

   (b) $f(x,y)=\lfloor x+y\rfloor$

   (c) $f(x,y)=\begin{cases}\frac{\sin xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0\\ 0, & x^2+y^2=0\end{cases}$

   (d) $f(x,y)=\begin{cases}0, x \text{ is an irrational number}\\ y, x \text{ is a rational number}\end{cases}$