作業

Guoning Wu

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1 解答題

設

$$f(x) = \begin{cases} x^2, & x \geq 3 \\ ax + b, & x < 3 \end{cases}$$

試確定a, b的值，使f在$x = 3$處可導。

2 解答題

求下列曲線在指定點處的切線，法線方程。

(1) $y = \dfrac{x^2}{4}, P(2, 1)$

(2) $y = \cos x, P(0, 1)$

3 解答題

求下列函數的導數

(1) $f(x) = |x|^3$

(2) $f(x) = \begin{cases} x + 1, & x \geq 0 \\ 1, & x < 0 \end{cases}$

4 解答題

設函數

$$f(x) = \begin{cases} x^\alpha \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

試問：

(1) α為何值時，函數在$x = 0$點連續；

(2) α為何值時，函數在$x = 0$點可導.

5 求下列函數的導數

(1) $y = 3x^2 + 2$

(2) $y = \dfrac{1 - x^2}{1 + x + x^2}$

(3) $y = x^n + nx$

(4) $y = \dfrac{x}{m} + \dfrac{m}{x} + 2\sqrt{x} + \dfrac{2}{\sqrt{x}}$

(5) $y = x^3 \log_3 x$

(6) $y = e^x \cos x$

(7) $y = (x^2 + 1)(3x - 1)(1 - x^3)$

(8) $y = \dfrac{\tan x}{x}$

(9) $y = \dfrac{x}{1 - \cos x}$

(10) $y = \dfrac{1 + \ln x}{1 - \ln x}$

(11) $y = x\sqrt{1 - x^2}$

(12) $y = (x^2 - 1)^3$

(13) $y = \left(\dfrac{1 + x^2}{1 - x}\right)^3$

(14) $y = \ln(\ln x)$

(15) $y = \ln(\sin x)$

(16) $y = \ln(x + \sqrt{1 + x^2})$

(17) $y = \ln \left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$

(18) $y = (\sin x + \cos x)^3$

(19) $y = (\sin x^2)^3$

(20) $y = \arcsin(\sin^2 x)$

(21) $y = x^{x^x}$

(22) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(23) $y = \sin(\sin(\sin x))$

(24) $y = \sin \left(\dfrac{x}{\sin(\frac{x}{\sin x})} \right)$

(25) $y = (x - a_1)^{a_1} (x - a_2)^{a_2} \cdots (x - a_n)^{a_n}$

6 求下列函數在指定點的高階導數

(1) $f(x) = 3x^3 + 4x^2 - 5x - 9$，求 $f'''(1), f^{(4)}(x)(1)$

(2) $f(x) = \dfrac{x}{\sqrt{1+x^2}}$，求 $f''(0), f''(1), f''(-1)$

7 求下列函數的高階導數

(1) $f(x) = x \ln x$，求 $f''(x)$

(2) $f(x) = e^{-x^2}$，求 $f'''(x)$

(3) $f(x) = \ln(1+x)$，求 $f^{(5)}(x)$

(4) $f(x) = x^3 e^x$，求 $f^{(10)}(x)$

8 解答題

設 f 為二階可導函數，求下列函數的二階導數

(1) $f(\ln x)$

(2) $f(x^n)$

(3) $f(f(x))$

9 解答題

求下列函數的 n 階導數

(1) $y = \ln x$

(2) $y = a^x (a > 0, a \neq 1)$

(3) $y = \dfrac{1}{x(1-x)}$

(4) $y = \dfrac{\ln x}{x}$

(5) $y = \dfrac{x^n}{1-x}$

10 解答題

求下列參數方程所確定的函數的二階導數

(1) $\begin{cases} x = a\cos^3 t \\ y = a\sin^3 t \end{cases}$

(2) $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$