CS 519: Scientific Visualization Fall 2015
In-class Worksheet

Meshes

1. The Euler Formula

The Euler Formula states the following relationship for the elements of a closed and connected surface mesh:

$$V-E+F=2(1-G)$$

V is the number of vertices
E is the number of edges
F is the number of faces
G is the genus of the surface (how holes/handles it has)

Show that for a triangle mesh with no holes we have $F \approx 2V$. Hint: each face has 3 edges and each edge is shared by 2 faces.

2. Memory Requirements

Using the fact that $F \approx 2V$, compare the storage requirements for an indexed face mesh and a triangle soup. Assume the mesh has V vertices and a number requires 4 bytes of space. Derive functions for the number of bytes the mesh will require as a function of V.

Laplacian Smoothing

Can be viewed as an iterative averaging process using the following formulation:

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$

$$L(p_i) = \sum_{n_j} \frac{1}{w_j}(n_j - p_i) \text{ and } \lambda \text{ is in } [0,1]$$

with n_j being the neighboring vertices of p_i and w_j a weight

3. **Laplacian Smoothing**

   Consider a linear curve of three vertices: (4,2) to (12, 2) to (16, 2)

   a. Assume the endpoints always stay fixed. What is the position of the middle vertex after 2 iterations of Laplacian smoothing using uniform weights and $\lambda = 1/2$?

   b. If you iterate until convergence, what final position will the middle vertex be in?

   c. What weights in the smoothing formula would result in the middle vertex never moving?

4. **Mesh Simplification**

   Simplify the triangle mesh below using the grid to perform vertex clustering. Use cell centers for the vertex placement