Vector Field Visualization

Finding Eigenvalues

You can find the eigenvalues of a matrix M in 2D by finding the roots of the characteristic polynomial. The characteristic polynomial is formed from the determinant $|M - \lambda|$

For example:

If $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ then we have $|M - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2$

Note that this is not a method you should implement in code to find eiegnvalues. It doesn't scale (why?). Alternatives include the QR method or Power Method.

1. Classifying Critical Points

Suppose we have a 2D vector field defined as $v(x) = \langle y^2 + y, x^2 - x \rangle$

a. What is a critical point in the vector field?

b. What is the Jacobian of the vector field? Recall that in 2D it takes the form of the following matrix:

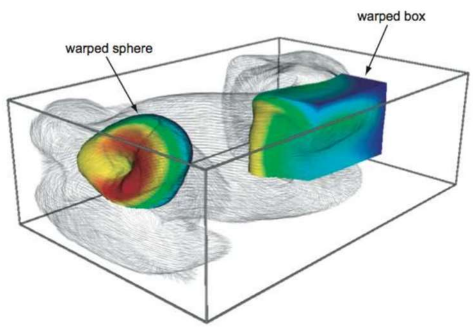
$$J = \begin{vmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{vmatrix}$$

c. Evaluate the Jacobian at the critical point.

d. What are the eigenvalues of the Jacobian at the critical point?

e. Classify the critical point based on the eigenvalues. Is it a source, sink, saddle, center, or focus?

2. Displacement Surfaces



Suppose we place a sphere of radius 1 centered at the origin within a vector field defined by $v(x, y, z) = \langle x^2 + x, y^2 + x, z^2 + x \rangle$

1. If the surface of the sphere is displaced by the vector field using the formula

   $$\boxed{S_{displ} = \{x + \mathbf{v}(x)\Delta t,\ \forall x \in S\}}$$

   where is the sphere surface point (1,0,0) moved to by the field if we use a timestep of ½?

2. We can limit displacement to non-tangential motion by calculating the following displacement

   $$\boxed{S_{displ} = \{x + (\mathbf{v}(x)\mathbf{n}(x))\,\mathbf{n}(x)\Delta t,\ \forall x \in S\}}$$

   Using that formula and the same values as part 1, to where is (1,0,0) displaced?