CS 491 CAP
Advanced Search & Simulation

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Outline

- Bitmasks for Pruning
- Search Order
- Bidirectional Search
- A* Search

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- **Bitmasks for Pruning**
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N Queens

- Find the total number of layouts of n queens on an $n \times n$ chessboard, such that any two queens will not attach each other.
- Bruteforce:
  - Every time, put a queen at (x, y)
  - Mark $(*, y)$ and $(x + \Delta, y + \Delta)$ and $(x + \Delta, y - \Delta)$ as attacked
    - This step is $O(n)$
  - Recursion

Bitmask

- Query the d-th digit of x
  - $(x \gg d)\ \&\ 1$
- Set the d-th digit of x as 1
  - $x \mid (1 \ll d)$
- Set the d-th digit of x as 0
  - $x\ \&\ (\sim(1 \ll d))$
- Only keep the last non-zero digit of x
  - $x\ \&\ (-x)$

N Queens

- ◇Mark columns, diagonals only
- ◇It becomes $O(1)$

- ◇Find the possible position in x-th row
- ◇⇔ Find the non-zero bits

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Sudoku

- 9×9 and 16×16

- No same number in a row
- No same number in a col
- No same number in a sub-square

- Find a solution

1		3				5		9
		2	1		9	4		
			7		4			
3			5		2			6
	6						5	
7			8		3			4
			4		1			
		9	2		5	8		
8		4				1		7

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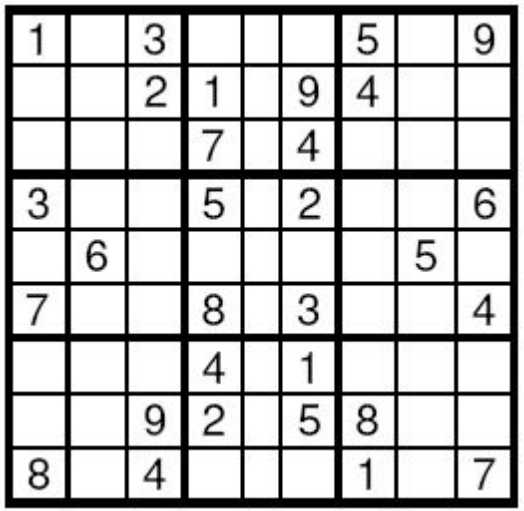
Sudoku - Bruteforce

- Find an empty cell
- Enumerate a possible number to fill
- Mark its row, column, and subsquare
  - Bitmasks could be used again!



1		3				5		9
		2	1		9	4		
			7		4			
3			5		2			6
	6						5	
7			8		3			4
			4		1			
		9	2		5	8		
8		4				1		7

Sudoku - Heuristic

- Find an empty cell
- With the **smallest** number of possible numbers

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Bidirectional search

- Find shortest path given an initial node and a target node.
- Two simultaneous searches
  - Start → … → an overlap! ← … ← Target
- Faster!
  - Suppose both searches expand a tree with branching factor b
  - the distance from start to goal is d
  - Each of the two searches: $O(b^{d/2})$
  - A single search: $O(b^d)$

Bidirectional search

- Requirement: the reversed move is easy to obtain

8-puzzle

- 1 2 3
- x 4 6
- 7 5 8

- How many different layouts?
- Bidirectional BFS is much faster than BFS

k-sum

- Given an array $a[1..n]$ and a target sum s
- Is it possible to find k numbers such that their sum is exactly s?

- $O(n^{\lceil \frac{k}{2} \rceil})$ is desired

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A* search

- $g(x)$ is the current cost from $start$ to x
- Design a heuristic function $h(x)$, which estimates the cost of the cheapest path from x to the goal
  - problem-specific
  - admissible, meaning that it never overestimates the actual cost to get to the nearest goal node
- $f(x) = g(x) + h(x)$
- Use $f(x)$ as the priority

A* search

- Maintain a priority queue Q

- Pick x with the highest priority $f(x)$ from Q
- If x is the goal, $f(x)$ is the answer
- Expand $x \rightarrow y_1, y_2, \ldots, y_k$
- Push all ***unseen*** y's into Q

Dijkstra Revisit

- Dijkstra's algorithm can be viewed as a special case of A*
  - Pick the unseen closet node
  - $g(x)$ is the current shortest distance
  - $h(x) = 0$

15-puzzle

- Goal:
- ```
   1   2   3   4
   5   6   7   8
   9  10  11  12
  13  14  15   X
  ```

- where the only legal operation is to exchange 'x' with one of the tiles with which it shares an edge.


  ```

15-puzzle

```
◇  1  2  3  4     1  2  3  4     1  2  3  4     1  2  3  4
◇  5  6  7  8     5  6  7  8     5  6  7  8     5  6  7  8
◇  9  x 10 12     9 10  x 12     9 10 11 12     9 10 11 12
◇ 13 14 11 15    13 14 11 15    13 14  x 15    13 14 15  x
◇             r->            d->            r->
```

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15-puzzle

- How many different layouts?
- 15!=1,307,674,368,000
- Toooo large for BFS

15-puzzle

- Any heuristic?
- Sum of Manhattan Distances to their destinations

Recommended Readings

- USACO 1.4.1 Search Techniques
- Bidirectional Search
- A* Search
- Exact Cover
- Dancing Links

Q&A