CS 519: Scientific Visualization

Vector Field Simplification

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Some slides adapted Alexandru Telea, *Data Visualization Principles and Practice*

Data Reduction

- Vector field datasets are often quite large
- Most visualization techniques we've discussed are dense
  - They can be difficult to interpret
- So…producing simplified versions of the vector field is useful

- What is one technique we've already seen that does this?
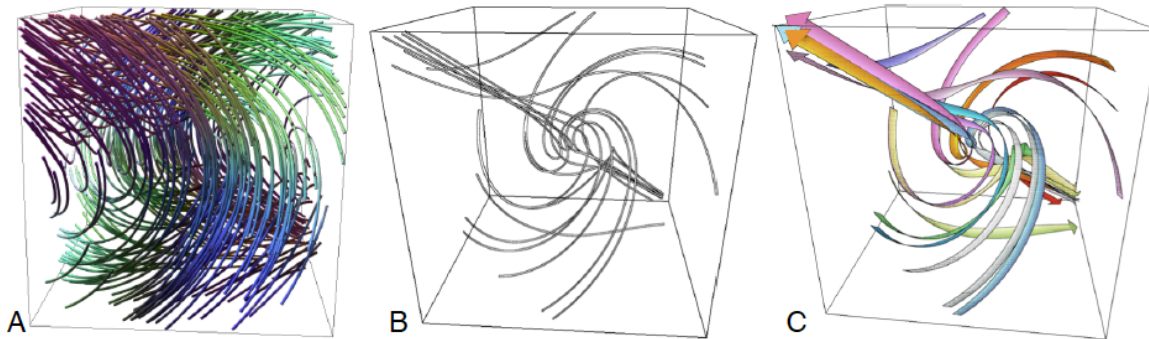
Flowlines can be sparse

A   B   C
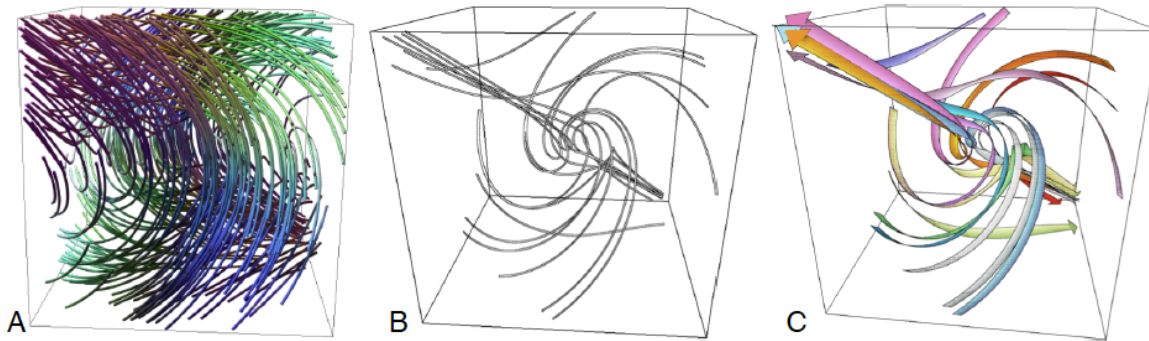
Flow lines can be sparse.

How do we choose which lines to draw?

From *An Illustrative Framework for Visualization of 3D Vector Fields* by Chen et al.
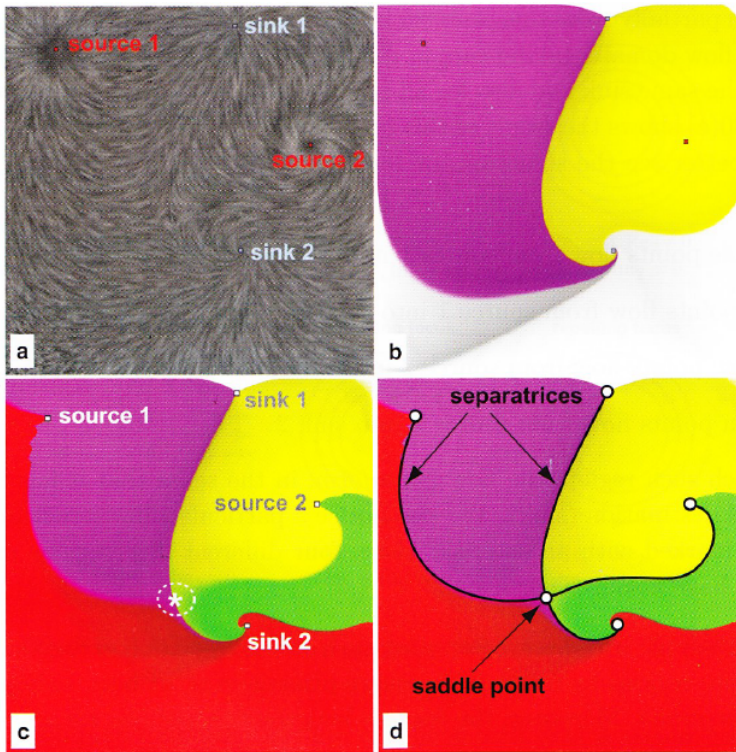
Flowlines can be sparse

How do we choose which lines to draw?

Choose lines which best represent the field behavior…if lots of flowlines act the same way, draw one acting that way….

From *An Illustrative Framework for Visualization of 3D Vector Fields* by Chen et al.

Decomposing a field into regions

We have a field with two sources and two sinks.

We can decompose it into regions in which the flow has a common source and a common sink.

Can do this by densely computing a flowline for each pixel...but there are more analytical alternatives
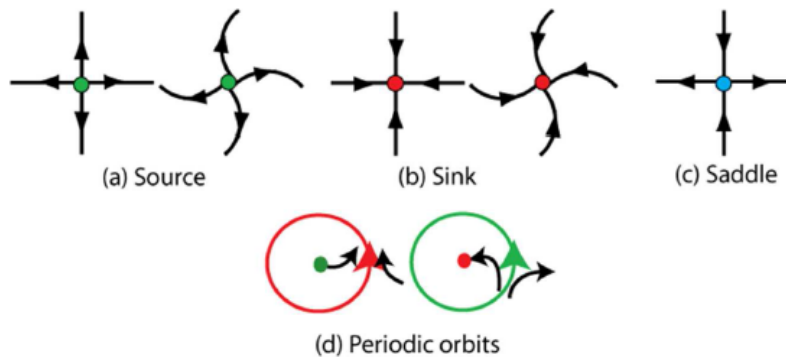
Detecting Critical Points

- A *critical point* is a singularity in field such that $v(x) = 0$.

- Critical points are classified by eigenvalues of the Jacobian matrix, J, of the VF at their position:

  - e.g. in 2d,

$$J = \begin{vmatrix} \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} \\[2em] \dfrac{\partial v_y}{\partial x} & \dfrac{\partial v_y}{\partial y} \end{vmatrix}$$

Detecting Critical Points

$$J = \begin{vmatrix} \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} \\ \\ \dfrac{\partial v_y}{\partial x} & \dfrac{\partial v_y}{\partial y} \end{vmatrix}$$

(a) Source (b) Sink (c) Saddle

(d) Periodic orbits

Find the eigenvectors and eigenvalues of the Jacobian

$\lambda_1 = (R_1, Im_1)$ and $\lambda_2 = (R_2, Im_2)$

- ❏ Source: $R_i > 0$, $Im_i = 0$
- ❏ Sink: : $R_i < 0$, $Im_i = 0$
- ❏ Saddle : $R_1 < 0$, $R_2 > 0$, $Im_i = 0$
- ❏ Center: $R_i = 0$, $Im_1 = -Im_2 \neq 0$
- ❏ Repelling Focus: $R_1 = R_2 > 0$
  $Im_1 = Im_2 \neq 0$
- ❏ Attracting Focus: $R_1 = R_2 < 0$
  $Im_1 = -Im_2 \neq 0$

Detecting Critical Points

Find the eigenvectors and eigenvalues of the Jacobian

Jv= λ v

$$J = \begin{vmatrix} \dfrac{\partial v_x}{\partial x} & \dfrac{\partial v_x}{\partial y} \\[12pt] \dfrac{\partial v_y}{\partial x} & \dfrac{\partial v_y}{\partial y} \end{vmatrix}$$

- Generally:
  - *R > 0* refers to repulsion
  - *R < 0* refers to attraction
    - e.g. a saddle both repels and attracts
  - *I ≠ 0* refers to rotation
    - e.g. a focus and a center

Sectors around critical points

Around a critical point c space can be divided into sectors

- ☐ Parabolic: Streamlines have 1 endpoint at c
- ☐ Elliptic: Streamlines have both endpoints at c
- ☐ Hyperbolic: Streamlines do not pass through c

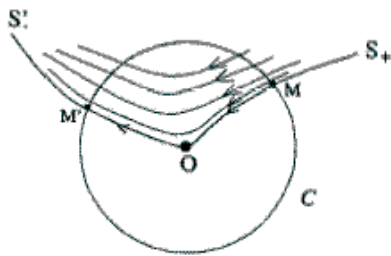Streamlines that form sector boundaries are *separatrices*
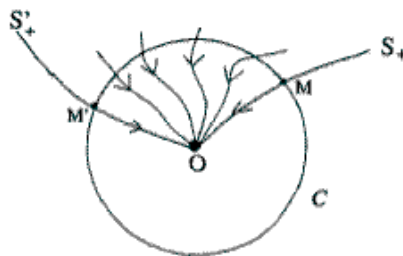

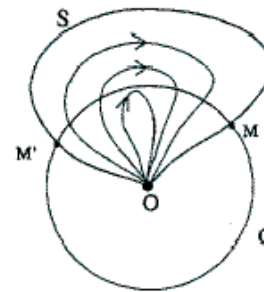Figure 6: Hyperbolic sector   Figure 7: Parabolic sector   Figure 8: Elliptic sector

Detecting Separatrices

Algorithm:

- Compute and classify critical points
- At each saddle point trace two streamlines
  - Use direction of the eigenvector with the positive real eigenvalue
    - Trace one forward and one backward
  - Use direction orthogonal the eigenvector
    - Trace one forward and one backwarD
- Stop tracing when you hit a critical point or domain boundary
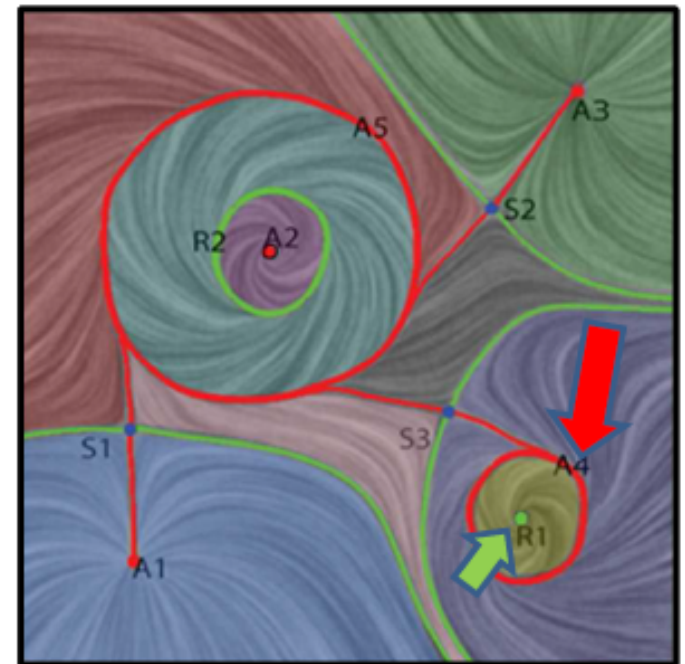
Sectors & Separatrices
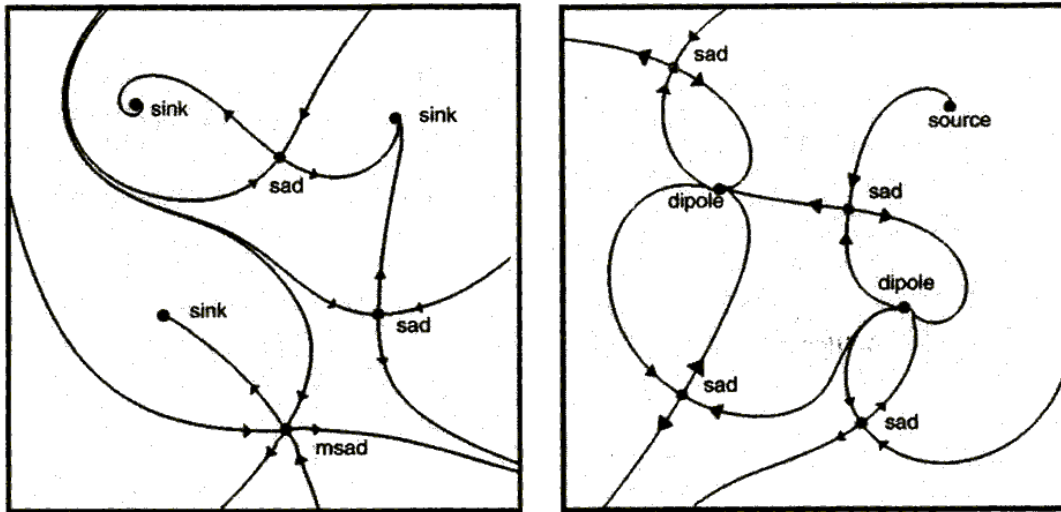
Images: A topology simplification method for 2D vector fields. Xavier Tricoche, Gerik Scheuermann, & Hans Hagen

Entity Connection Graph

- An entity connection graph (or ECG) is an extended topological skeleton which consists of [Chen et al. 2007]
  - *Flow recurrent features*
    (fixed points and periodic orbits)
  - *Connectivity*
    (separatrices and others)

It forms a *topological Graph* .

Three Dimensions

- In 3D, we classify critical points in a similar manner using the 3 eigenvalues of the Jacobian

- Broadly, there are 2 cases:
 - Three real eigenvalues
  - Two complex conjugates & one real

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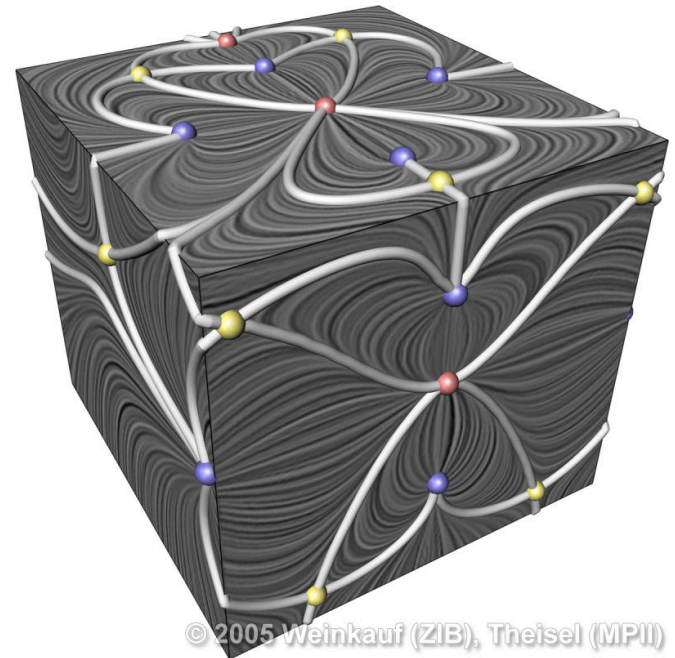
Three Dimensions

- Separatrices now become 2d surfaces and 1d curves.
- Thus topology of first-order critical points will be composed of the critical points themselves + curves + surfaces

Images: Saddle Connectors – An approach to visualizing the topological skeleton of complex 3D vector fields, Theisel, Weinkauf, Hege, and Seidel

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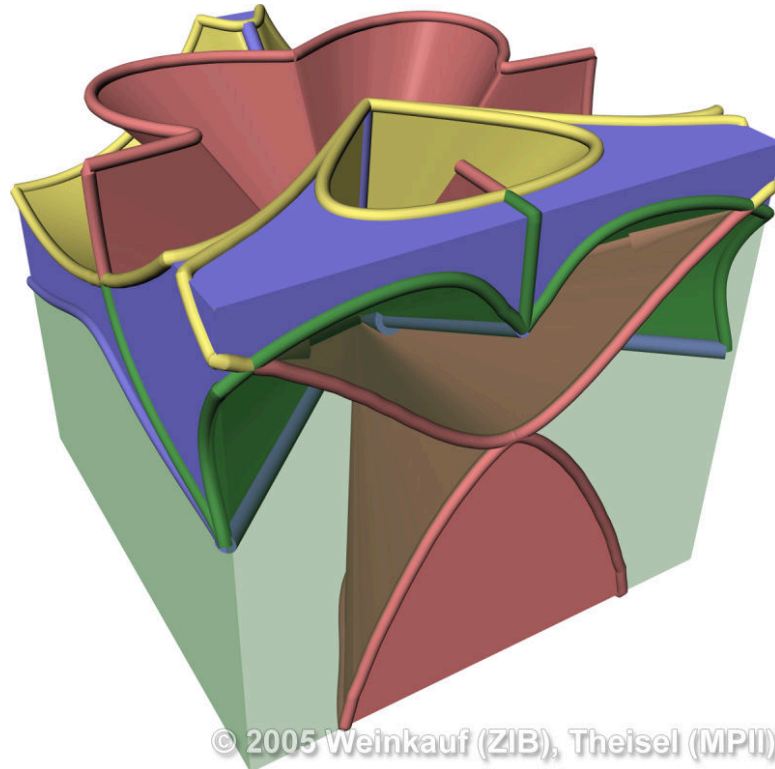
Visualization of Unsteady Flow
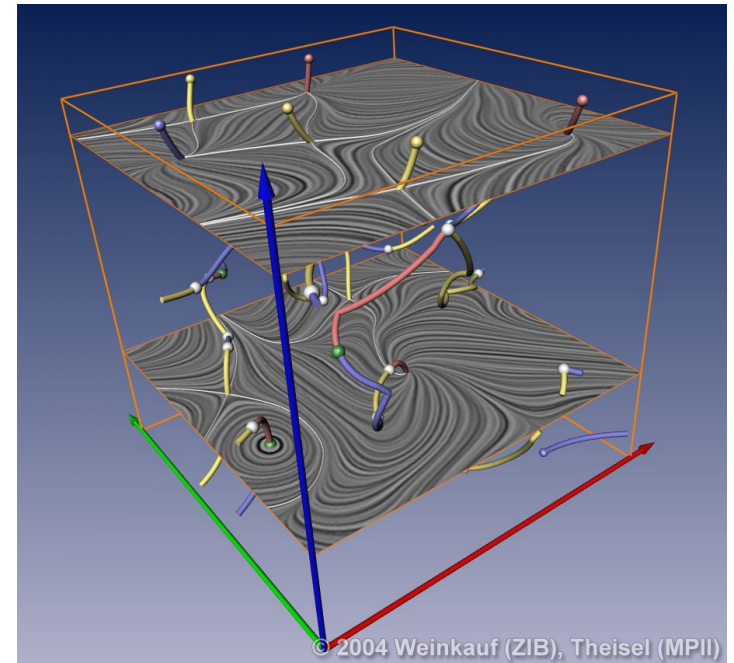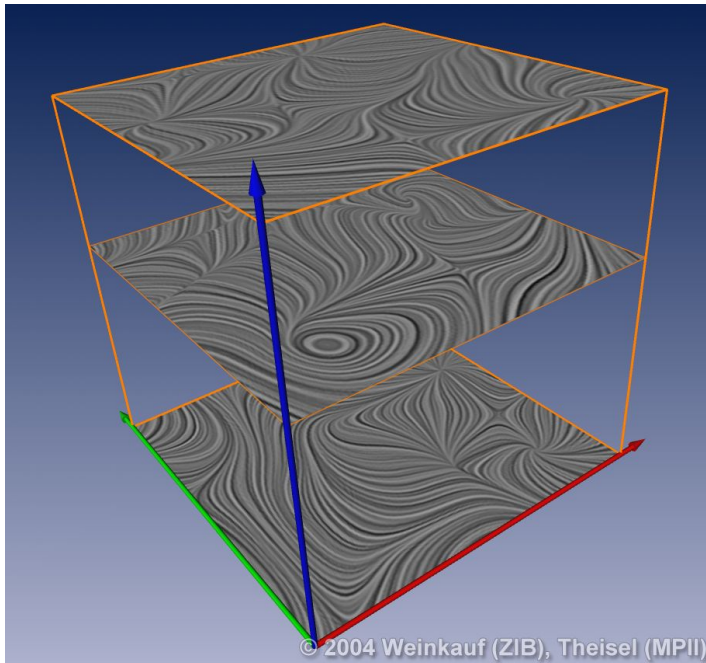
Images: Stream line and path line oriented topology for 2D time-dependent vector fields, Theisel, Weinkauf, Hege, and Seidel

Other Field Simplification Methods

- Top-down Clustering
    - All field points are in a set
    - Representative vector is the average of all vectors in set
    - Split in a way to optimize some error metric
        - e.g. deviation of streamlines in original field from those in simplified field
- Bottom-up
    - Repeatedly merge points in field which exhibit greatest similarity