CS 519: Scientific Visualization

Scalar Data Visualization: Isosurfaces

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Some slides adapted from Tao Ju, Washington University St. Louis and Alexandru Telea, *Data Visualization Principles and Practice*

Colormaps: Transfer Functions versus Lookup Tables

Transfer Functions Basic idea

- Map each scalar value $f \in \mathbf{R}$ at a point to a color via an analytical function $c : [0,1] \rightarrow [0,1]^3$

Color Tables Basic Idea

- precompute (sample) c and save results into a table $\{c_i\}_{i=1..N}$
- index table by normalized scalar values

input data — scalar value f

determine input range — scalar value range $[f_{\min}, f_{\max}]$

normalize input to [0,N] — $$i = N \frac{f - f_{\min}}{f_{\max} - f_{\min}}$$

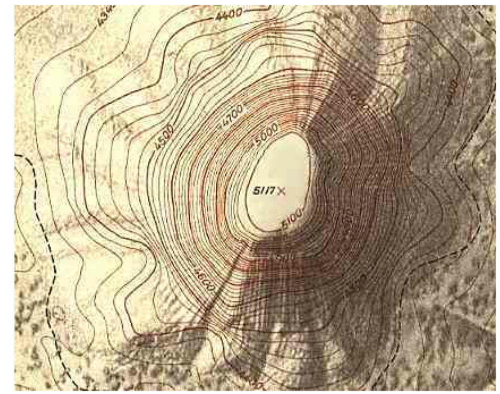
Color mapping ← $c_i = c\left(\frac{i}{N}\right)$ (color table) ← $c : [0,1] \rightarrow [0,1]^3$ (desired color transfer function)

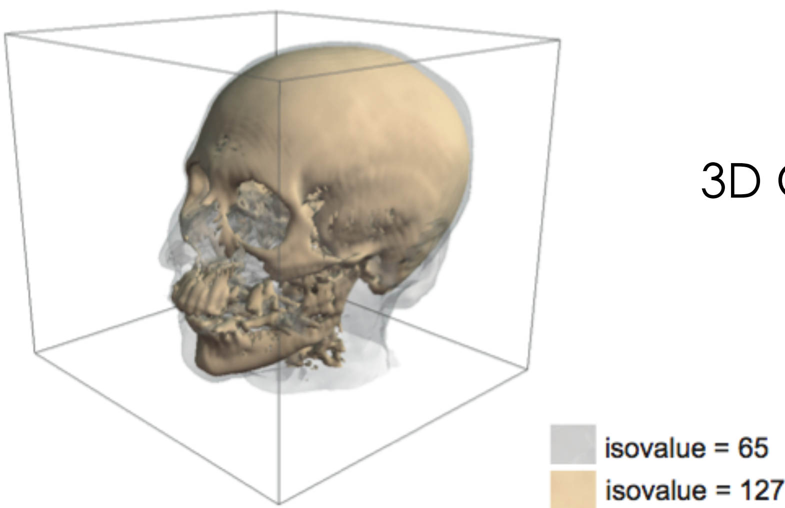
f is color-mapped to c_i

Contouring

Contours have been used for hundreds of years in cartography

- also called *isolines* ('lines of equal value')





3D Contouring: Marching Cubes:

"*Marching cubes: A high resolution 3D surface construction algorithm*", by Lorensen and Cline (1987)

>6000 citations on Google Scholar

Contour Properties

Definition

$$I(f_0) = \left\{ x \in D \middle| f(x) = f_0 \right\}$$

Contours are always closed curves (except when they exit *D*)

- why? Recall that *f* is C^0

Two different contour lines never intersect, thus are nested

- why? What would it mean if a point belonged to two *different* contours

Contours cut D into values smaller resp. larger than the isovalue

- why?

Contour Properties

Contours are always orthogonal to the scalar value's gradient

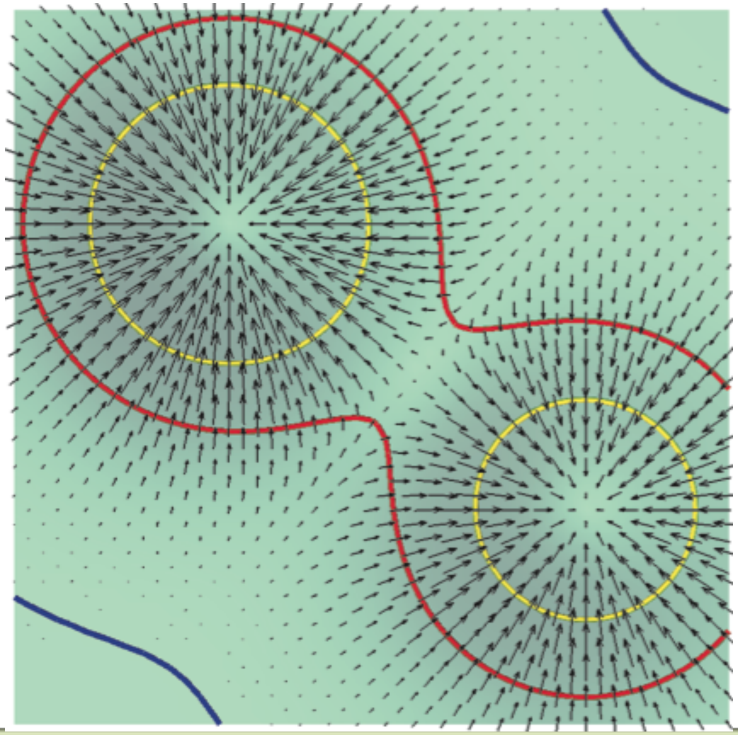
- why? Recall definitions

$$I(f_0) = \{x \in D \mid f(x) = f_0\}$$

contour: $\frac{\partial f}{\partial I} = 0$ since f constant along I

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

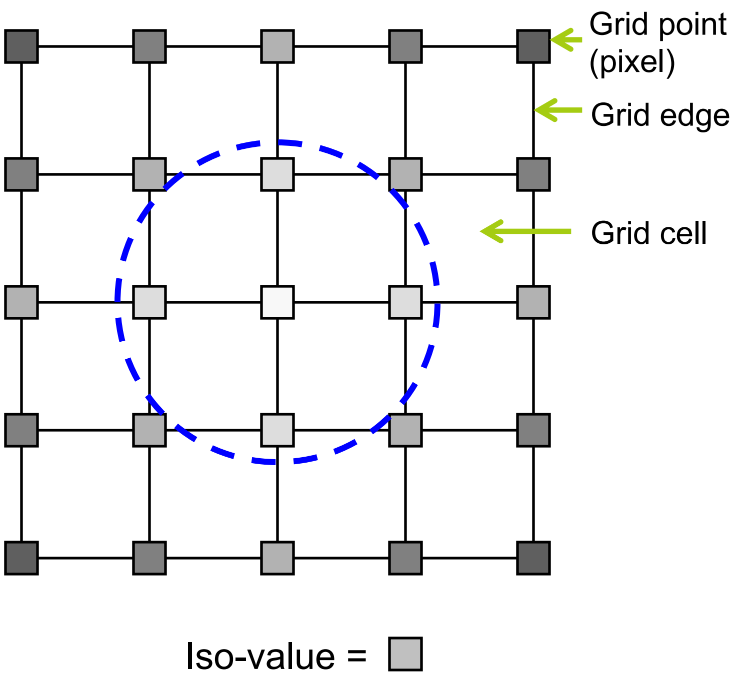
gradient: $\frac{\partial f}{\partial(\nabla f)} = \max$ by definition of gradient direction of greatest increase in f



gradient of a scalar field (drawn with arrows) is orthogonal to contours

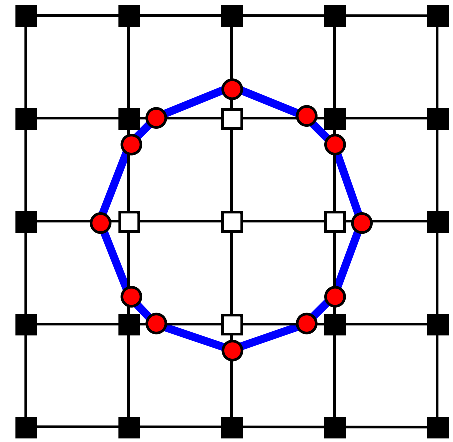
Contouring (On A Grid)

- Input
  - A grid where each grid point (pixel or voxel) has a value (color)
  - An iso-value (threshold)
- Output
  - A closed polyline (2D) or mesh (3D) that separates grid points above or below the iso-value

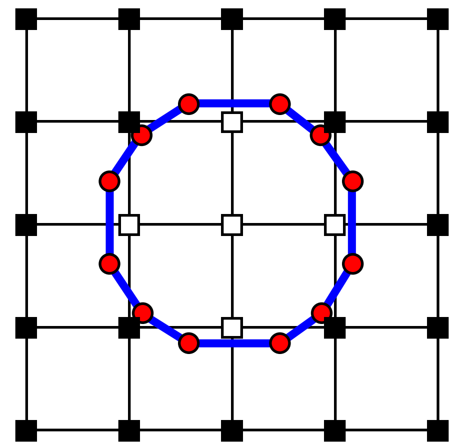


Algorithms

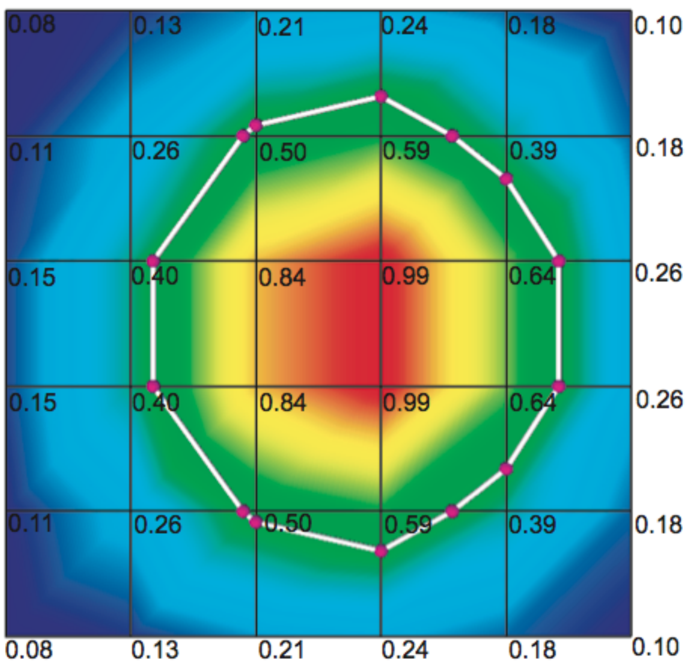
- **Primal methods**
  - Marching Squares (2D), Marching Cubes (3D)
  - Placing vertices on grid edges



- **Dual methods**
  - Dual Contouring (2D,3D)
  - Placing vertices in grid cells



Contouring in 2D

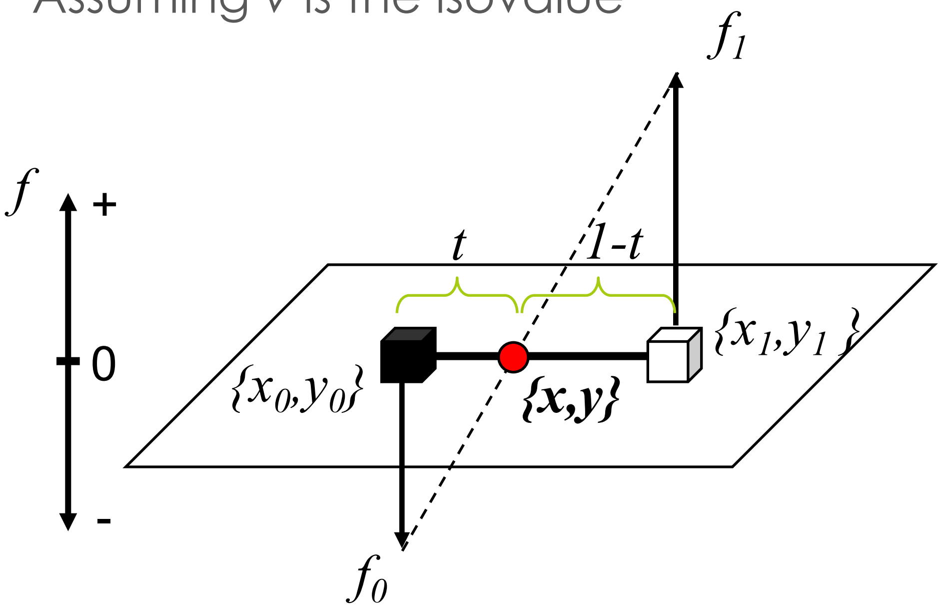


$S = \emptyset$

for(each cell c in D)
{
  for(each edge $e=(p_i,p_j)$ of c)
  {
    if($f_i < v < f_j$)
    {
      $$q = \frac{p_i(v_j - v) + p_j(v - v_i)}{v_j - v_i}$$
      $S = S \cup q$
    }
  }
  connect points in S with lines to build contour;
}

Marching Squares (2D)

- Creating vertices
  - Assuming the underlying, continuous function is linear on the grid edge
  - Linearly interpolate the positions of the two grid points
    Assuming v is the isovalue

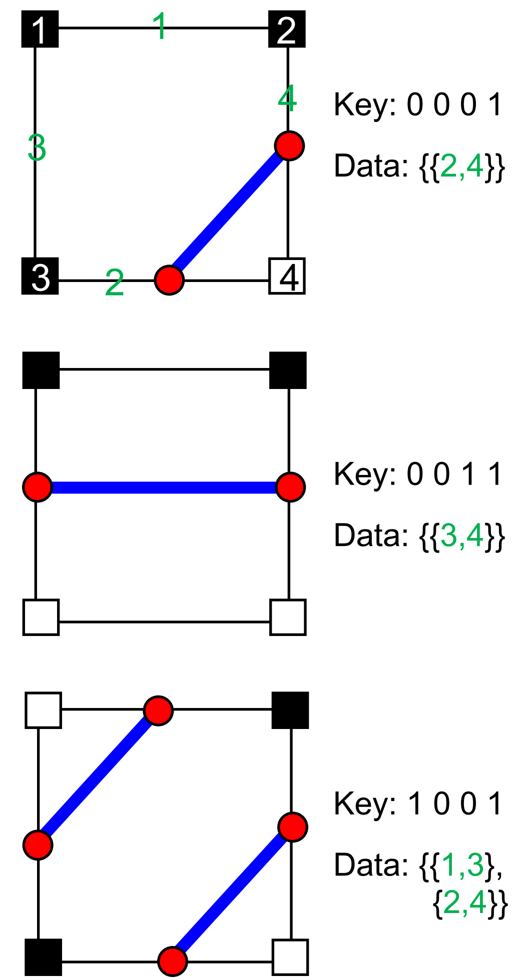


$$t = \frac{v - f_0}{f_1 - f_0}$$

$$x = x_0 + t(x_1 - x_0)$$
$$y = y_0 + t(y_1 - y_0)$$

Marching Squares (2D)

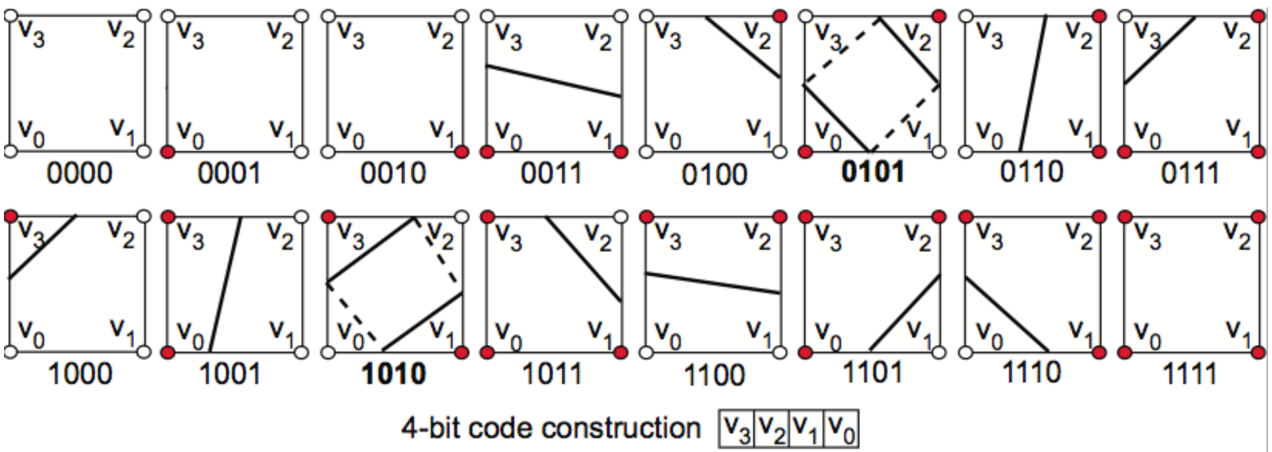
- Connecting vertices by lines
  - Lines shouldn't intersect
  - Each vertex is used once
    - So that it will be used exactly twice by the two cells incident on the edge
- Two approaches
  - Do a walk around the grid cell
    - Connect consecutive pair of vertices
  - Or, using a pre-computed look-up table
    - 2^4=16 entries
    - For each sign combination at the 4 corners, it stores the indices of the grid edges whose vertices make up the lines.



Marching Squares

2D contouring on quad-cell grids

1. Encode inside/outside state of each vertex w.r.t. contour in a 4-bit code



e.g.
inside: $f > f_0$
outside: $f \leq f_0$

2. Process all dataset cells

- for each cell, use codes as pointers into a table with 16 cases
- each case has code to
  - compute the existing edge-contour intersections
  - connect to already-computed contour vertices from previous cells

Marching Squares

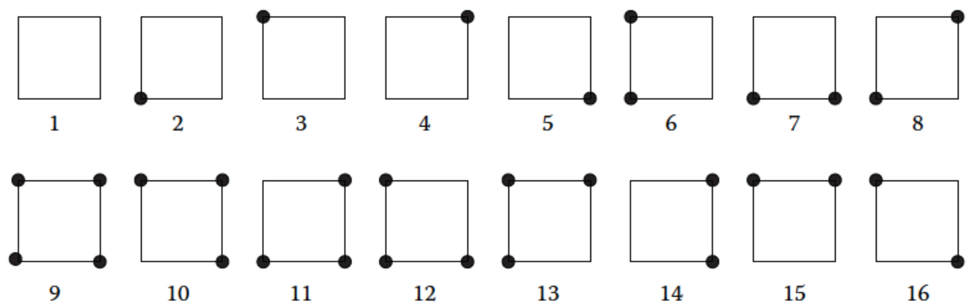


Figure 2.2. Square configurations. Black vertices are positive.

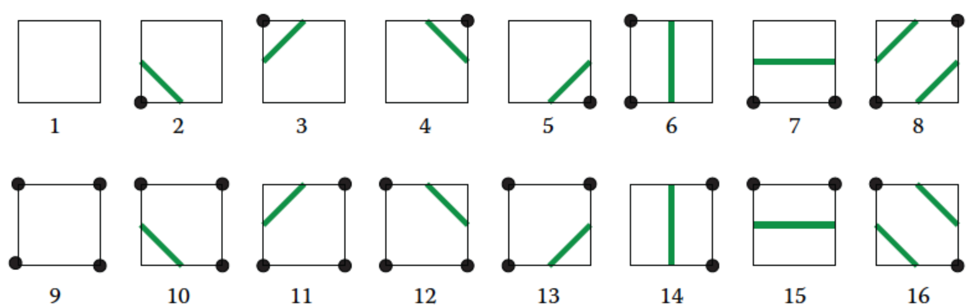
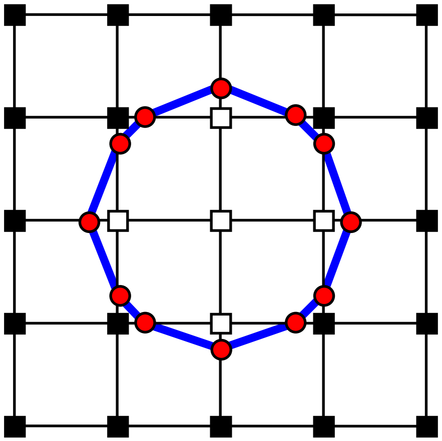


Figure 2.3. Square isocontours. Configurations 1 and 9 have no isocontour. Isocontours for configurations 2–7 and 10–15 are single line segments. Isocontours for configurations 8 and 16 are two line segments.

Implementation Notes

- Avoid computing one vertex multiple times
  - Compute the vertex location once, and store it in a hash table

- When the grid point’s value is same as the iso-value
  - Treat it either as “above” or “below”, but be consistent.



Generate Isocontours for $f_0=5$

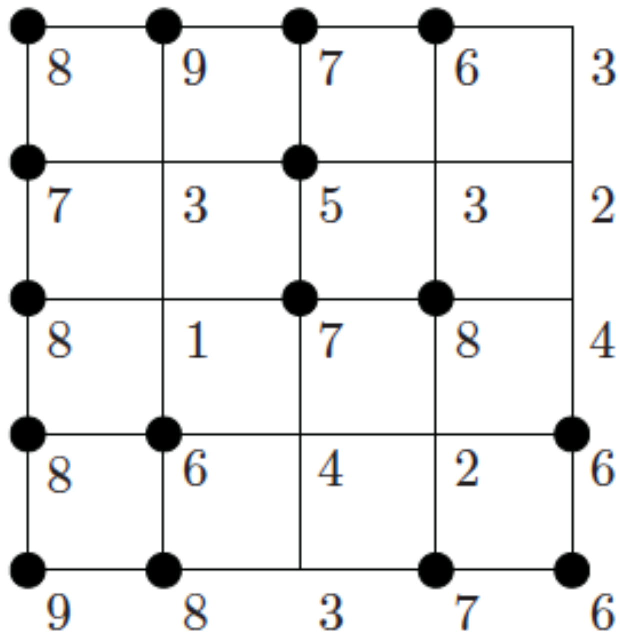
Scalars associated with point to the upper left

Classify points with value 5 as positive

8	9	7	6	3
7	3	5	3	2
8	1	7	8	4
8	6	4	2	6
9	8	3	7	6

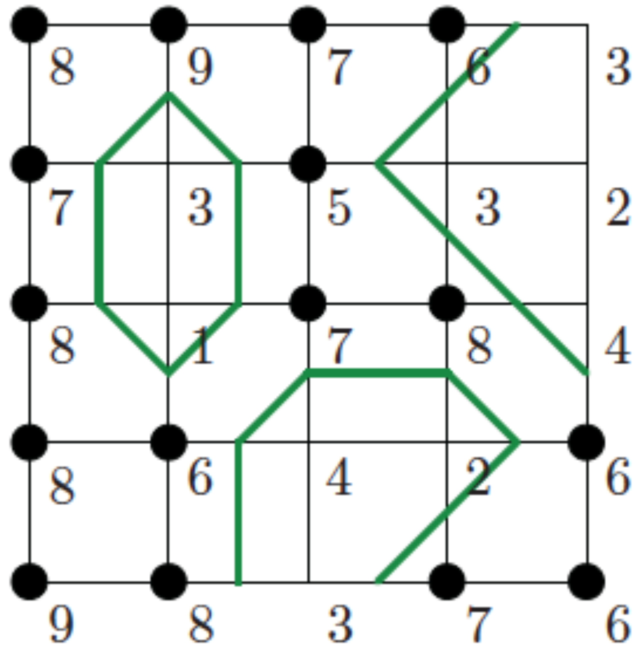
(a) Scalar grid.

+/- Grid for T=5



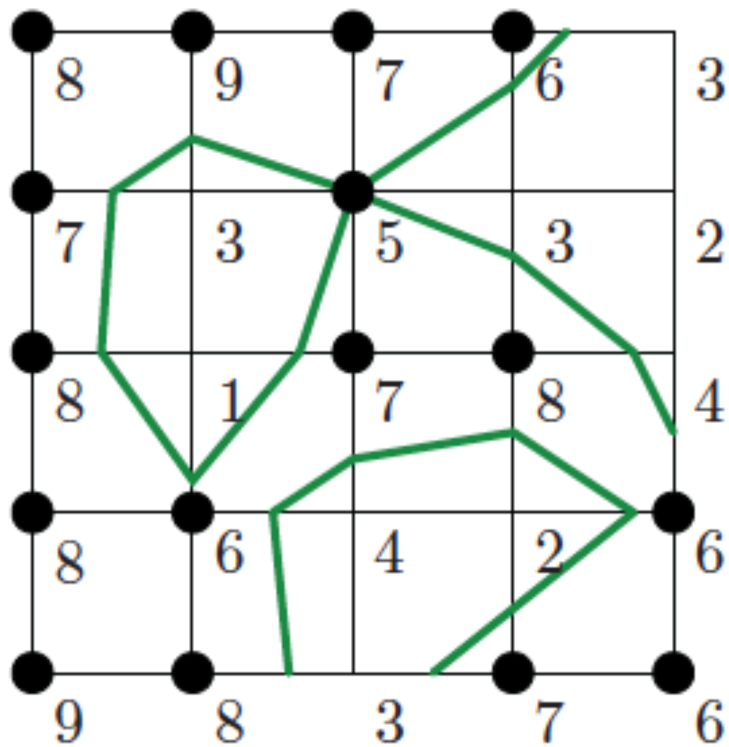
(b) The +/− grid.

Isocontours using midpoints



(c) Midpoint vertices.

Isocontours using Interpolation

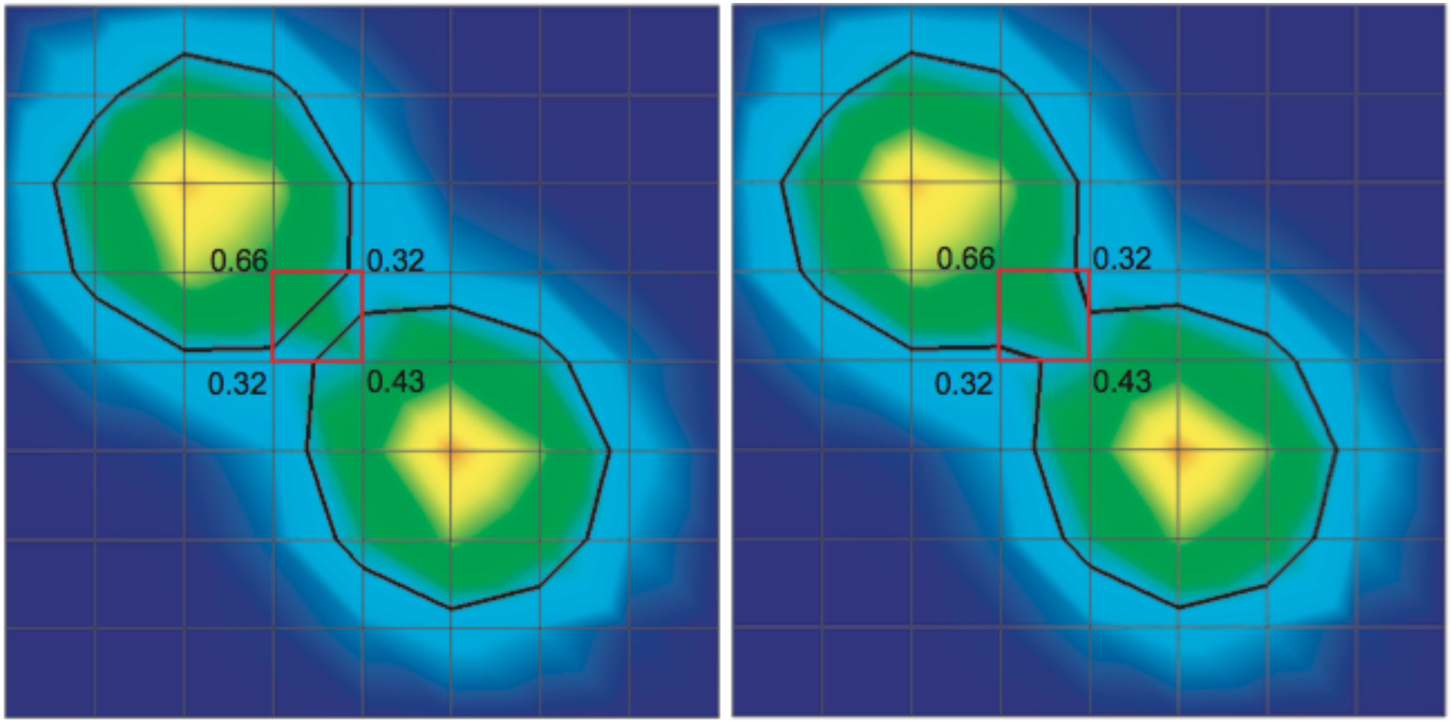


(d) Isocontour.

Contouring Ambiguity

Each edge of the red cell intersects the contour

- which is the right contour result?



Contouring Ambiguity

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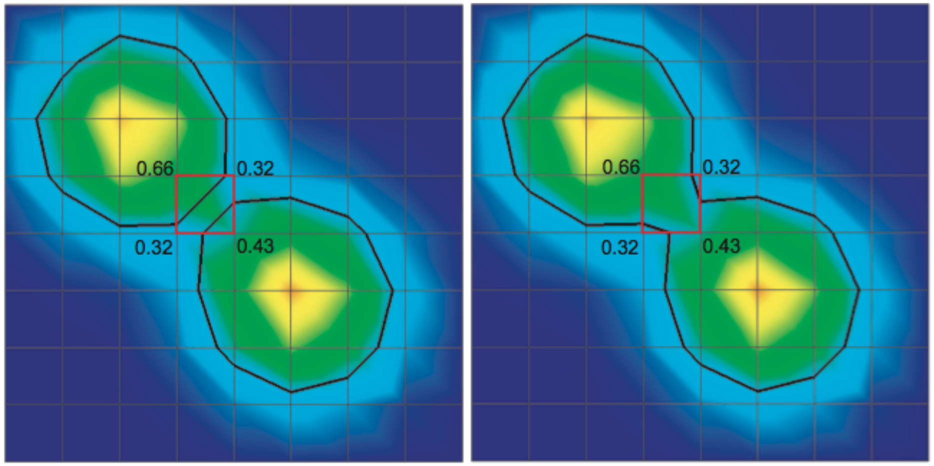
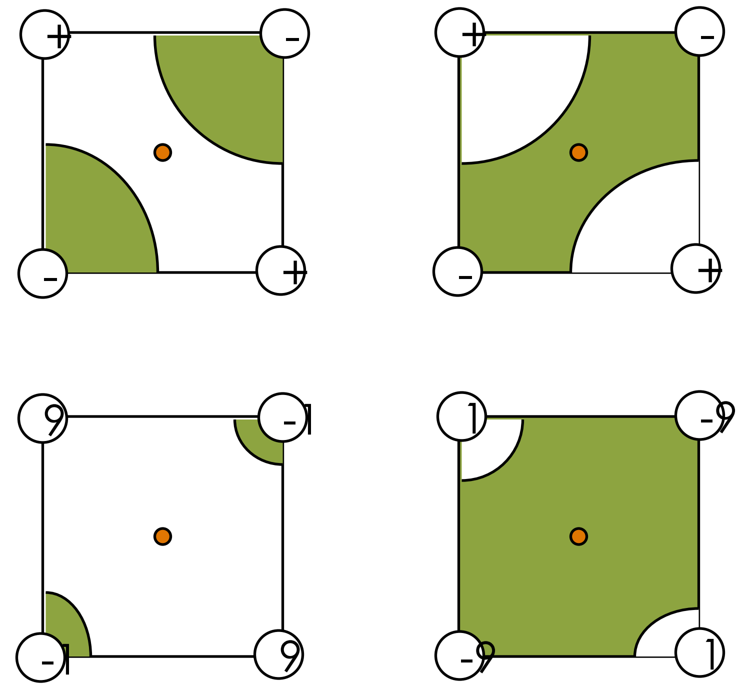


Both answers are equally correct!

- we could discriminate only if we had higher-level information (e.g. topology)
- at cell level, we cannot determine more unless we increase sampling rate

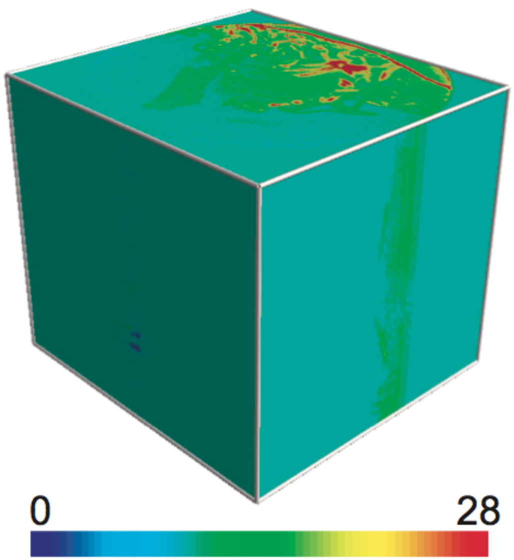
Problem: Ambiguity

- Some cell corner value configurations yield more than one consistent polygon
- In 3-D can yield holes in surface!
- How can we resolve these ambiguities?
- Topological Inference
  - Sample a point in the center of the ambiguous face
  - If data is discretely sampled, bilinearly interpolate samples



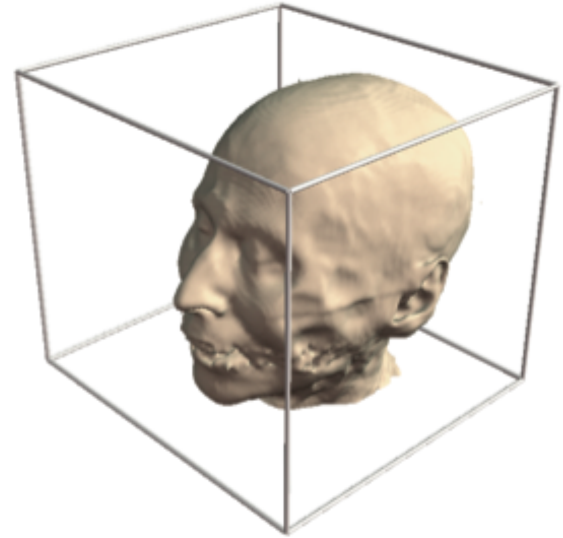
$$p(s,t) = (1-s)(1-t)\,a + s\,(1-t)\,b + (1-s)\,t\,c + s\,t\,d$$

Marching Cubes

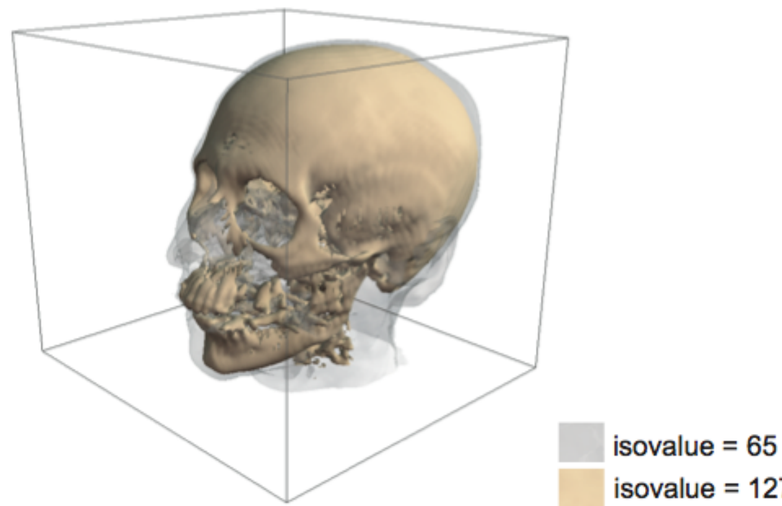


scalar CT volume
(tissue density)

isosurfaces



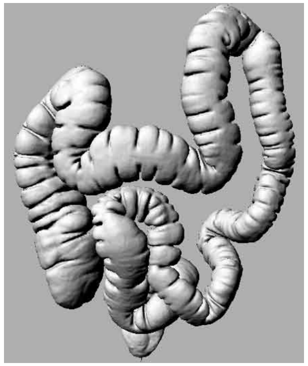
isosurface for scalar value
corresponding to skin



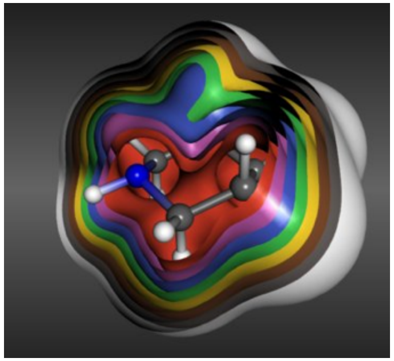
isosurfaces for skin and bone

- extremely simple to use tool
- insightful results

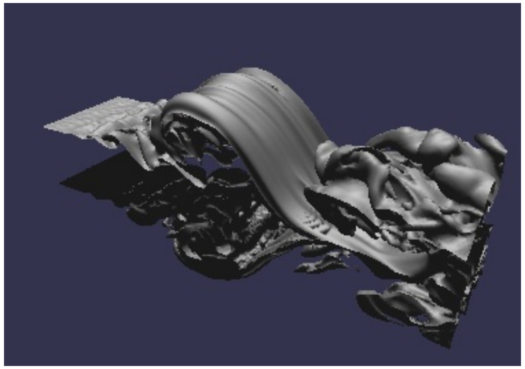
Examples



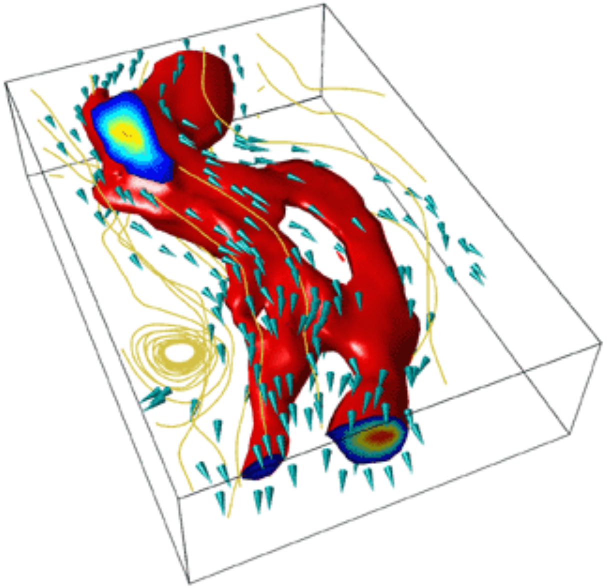
colon (CT dataset)



electron density in molecule



velocity in 3D fluid flow



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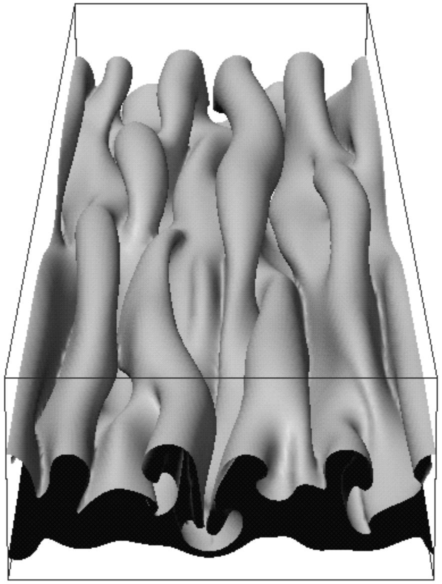
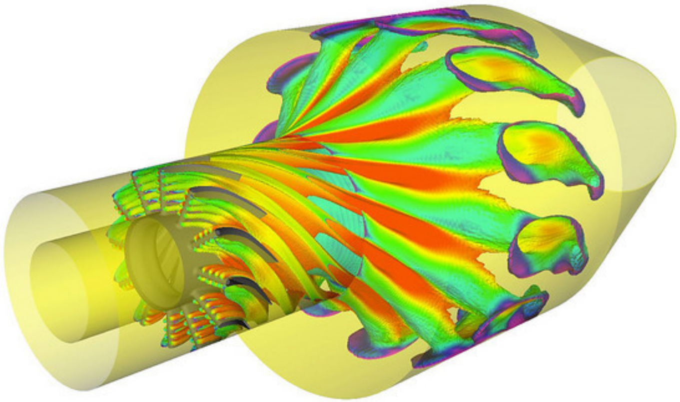


magnetic field in sunspots

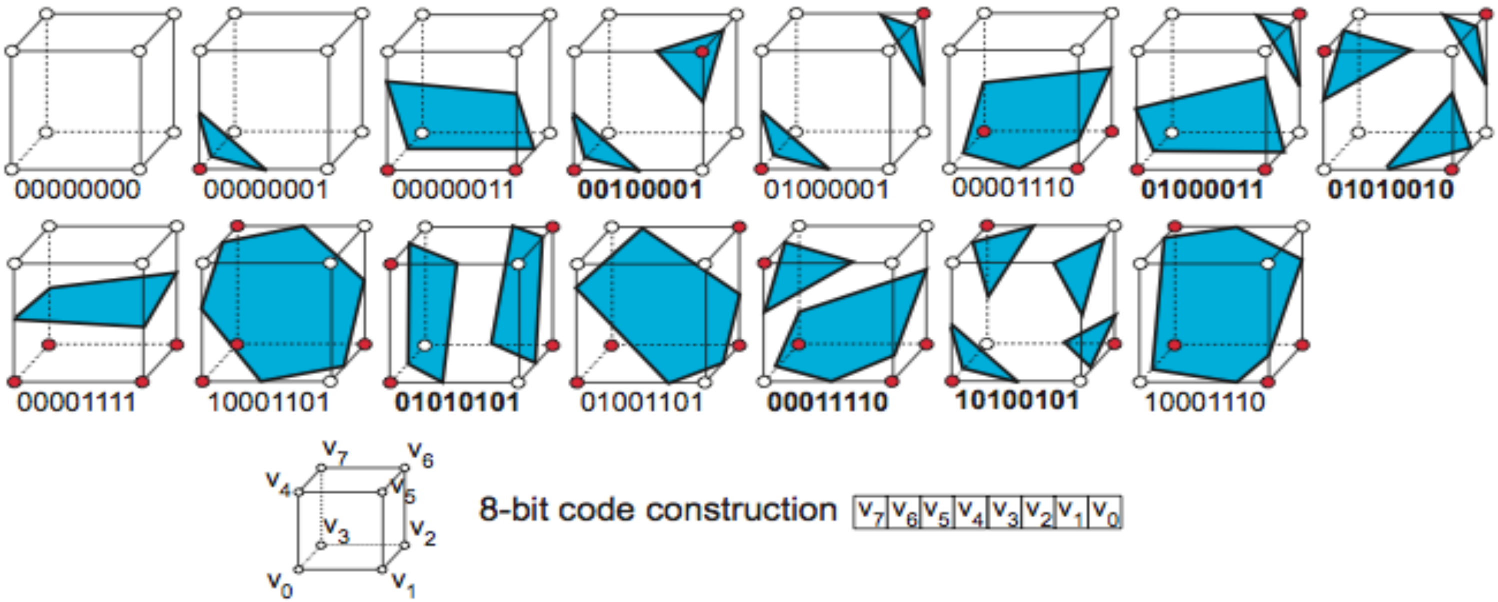


fuel concentration, colored by temperature in jet engine

3D: Marching Cubes

Fast implementation of 3D contouring (isosurfaces) on parallelepiped-cell grids

1. Encode inside/outside state of each vertex w.r.t. contour in a 8-bit code



e.g.
inside: $f > f_0$
outside: $f \leq f_0$

2. Process all dataset cells

- for each cell, use codes as pointers into a jump-table with 15 cases (reduce the 2^8=256 cases to 8 by symmetry considerations)

Marching Cubes

- For each case
  - compute the cell-contour intersection → triangles, quads, pentagons triangulate these on-the-fly → triangle output only

3. Treat ambiguous cases

- 6 such cases (see **bold**-coded figures on previous slide)
- harder to solve than in 2D (need to prevent false cracks in the surface)
- see Sec. 5.3 of the book for algorithmic details

4. Compute isosurface normals

- by face-to-vertex normal averaging
- directly from data

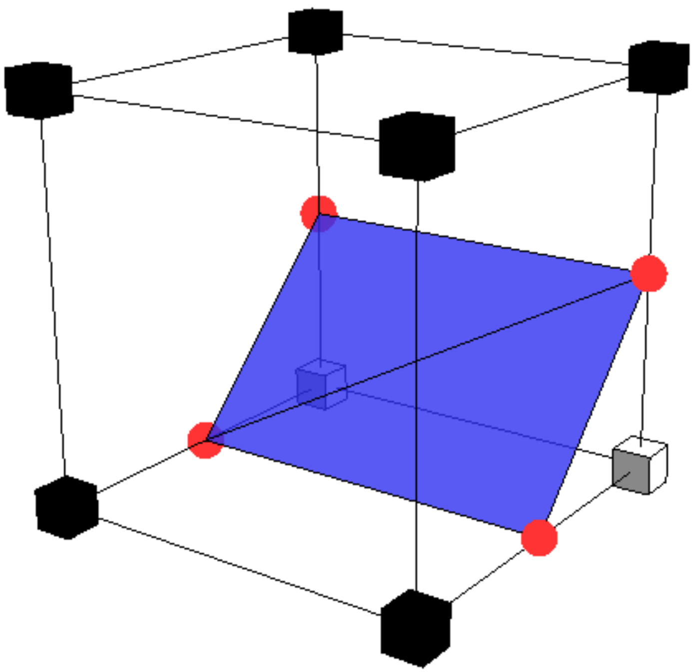
(gradient is normal to contours, see previous slides)

$$\forall x \in I, \, n_I(x) = -\frac{\nabla f(x)}{\|\nabla f(x)\|}$$

5. Draw resulting surface as a (shaded) unstructured triangle mesh

Marching Cubes (3D)

- For each grid cell with a sign change
  - Create one vertex on each grid edge with a sign change (using linear interpolation)
  - Connect vertices into triangles

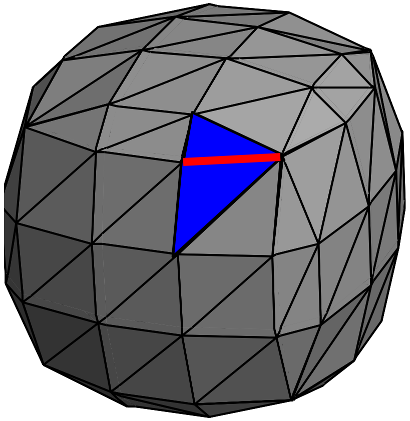


Surface Approximations

- In 2D, the piecewise linear surface approximation is a polyline
- In 3D, it is a triangle mesh

<u>Desired Qualities of the Approximation</u>

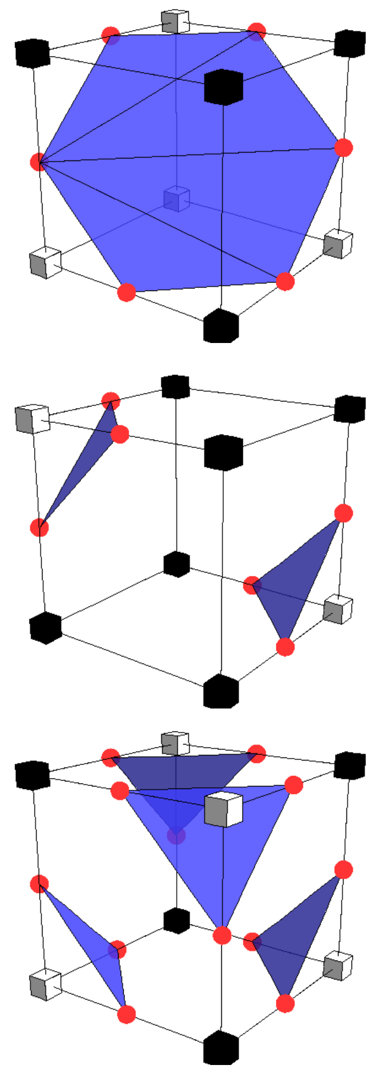
- Closed (with inside and outside)
  - Polyline: a vertex is shared by even # of edges
  - Mesh: an edge is shared by even # of polygons
- Manifold
  - Polyline: a vertex is shared by 2 edges
  - Mesh: an edge is shared by 2 polygons, and a vertex is contained in a ring of polygons
- Non-intersecting



A closed, manifold triangular mesh

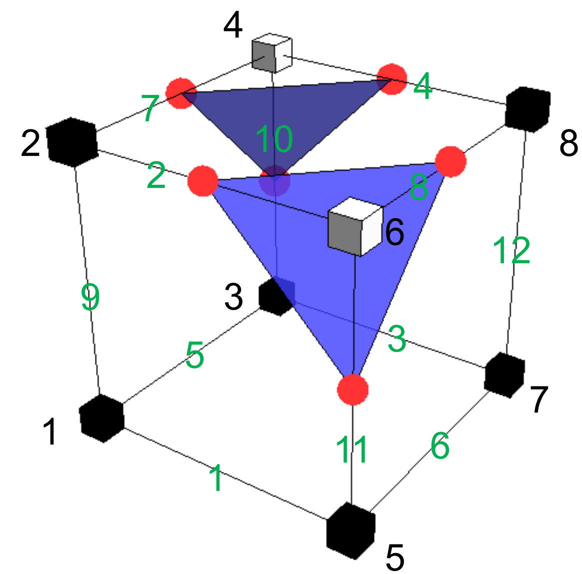
Marching Cubes (3D)

- Connecting vertices by triangles
  - Triangles shouldn't intersect
  - To be a closed manifold:
    - Each vertex used by a triangle "fan"
    - Each mesh edge used by 2 triangles (if inside grid cell) or 1 triangle (if on a grid face)



Marching Cubes (3D)

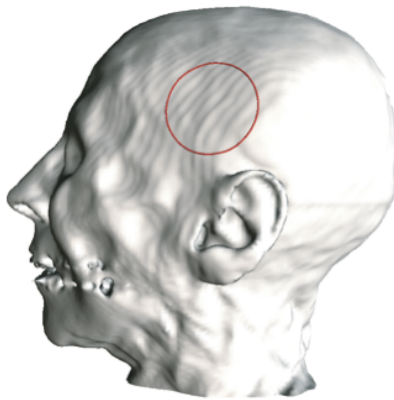
- Connecting vertices by triangles
  - Triangles shouldn't intersect
  - To be a closed manifold:
    - Each vertex used by a triangle "fan"
    - Each mesh edge used by 2 triangles (if inside grid cell) or 1 triangle (if on a grid face)
    - Each mesh edge on the grid face is shared between adjacent cells
- Look-up table
  - 2^8=256 entries
  - For each sign configuration, it stores indices of the grid edges whose vertices make up the triangles

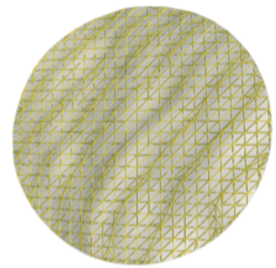


Sign: "0 0 0 1 0 1 0 0"
Triangles: {{2,8,11},{4,7,10}}

Marching Cubes: Artifacts

overview

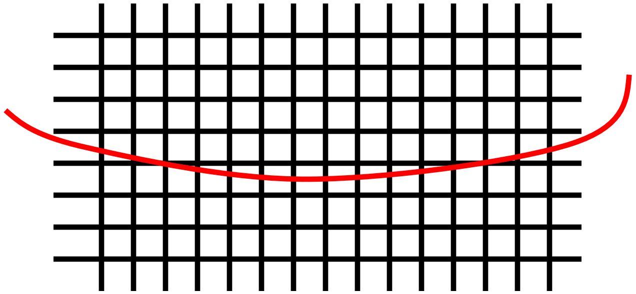




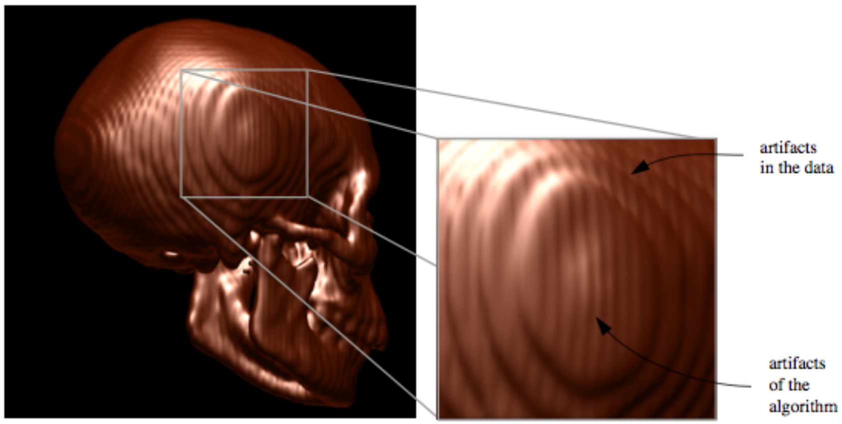
detail

Does this person have wavy wrinkles on his skin?

- these are ‘ringing artifacts’
  - due to the near-tangent orientation of the isosurface w.r.t. finite-resolution volume grid

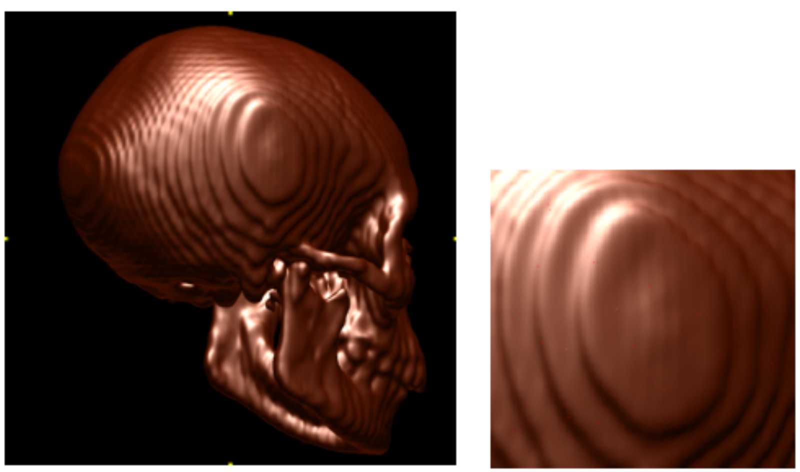


Marching Cubes: Artifacts



Two kinds of artifacts

- from data: cannot remove easily
- from algorithm (due to linear interpolation)



Removing algorithm artifacts

- use higher-order interpolants (e.g. splines)

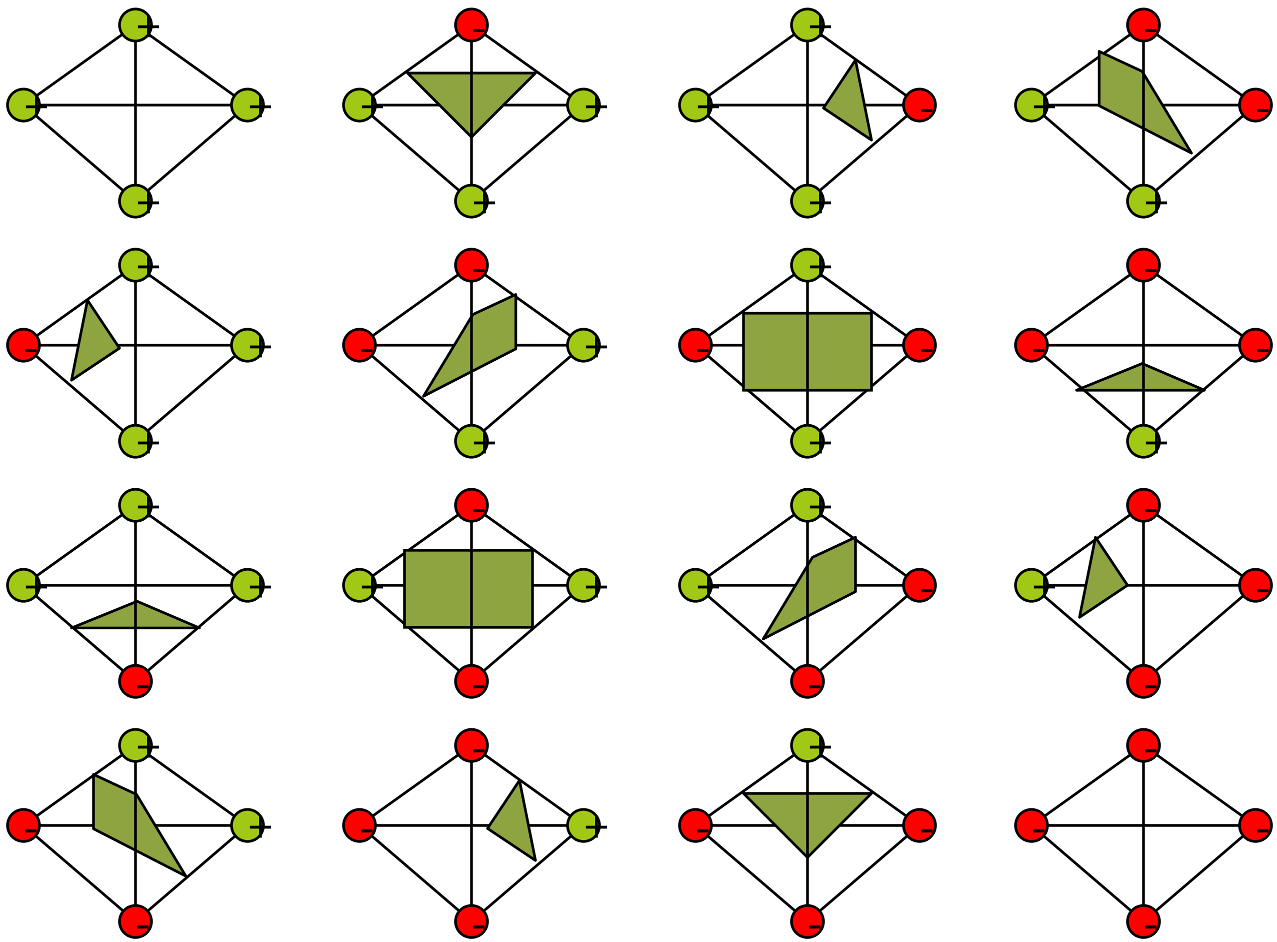
E. C. LaMar, B. Hamann, K. Joy, High-Quality Rendering of Smooth Isosurfaces, JVCA vol. 10, 1999, 79-90

Marching Cubes: Scalability to Big Data Sets

- Running time on a grid with **n** cells?
- How much data must be kept in memory?
- Suitability for parallel processing?

Marching Tet Cases

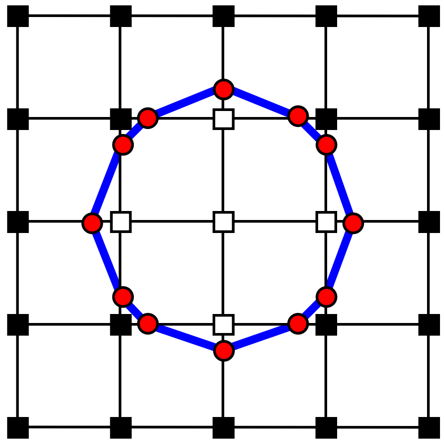
16 in all
3 modulo symmetry



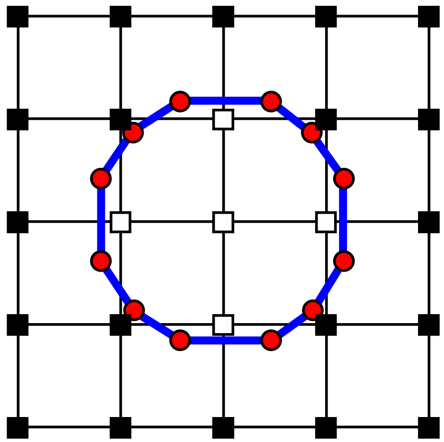
Marching Tetrahedra: Advantages/Disadvantages

Alternative Algorithms

- Primal methods
  - Marching Squares (2D), Marching Cubes (3D)
  - Placing vertices on grid edges

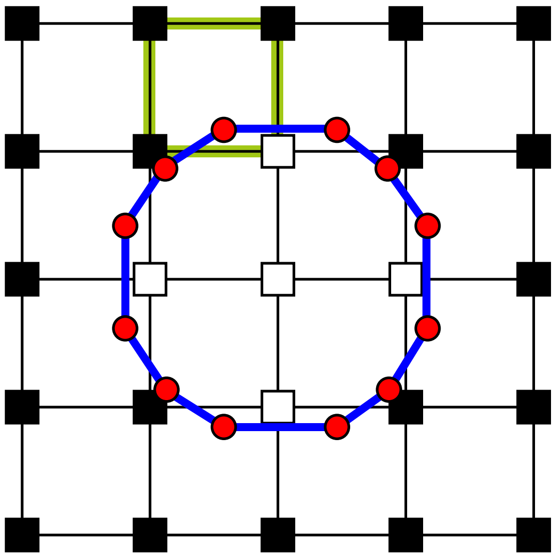


- Dual methods
  - Dual Contouring (2D,3D)
  - Placing vertices in grid cells



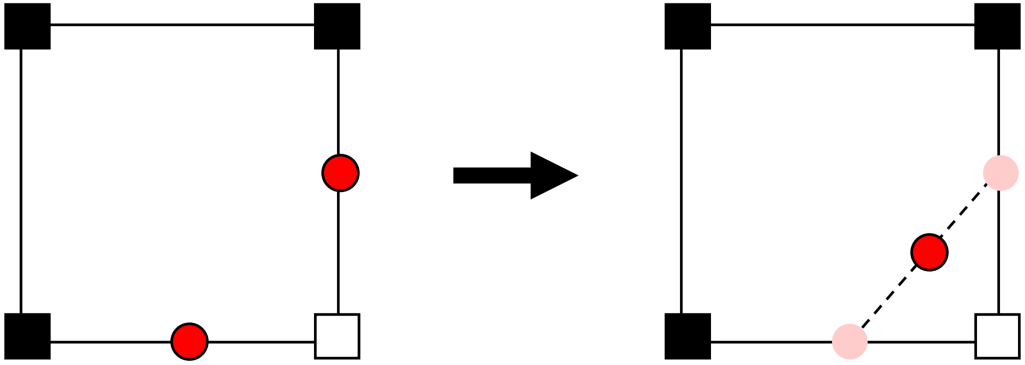
Dual Contouring (2D)

- For each grid cell with a sign change
  - Create one vertex
- For each grid edge with a sign change
  - Connect the two vertices in the adjacent cells with a line



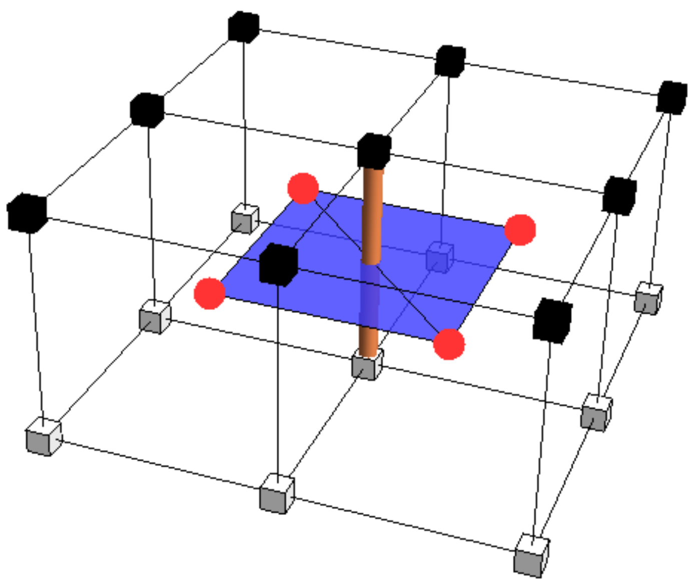
Dual Contouring (2D)

- Creating the vertex within a cell
  - Compute one point on each grid edge with a sign change (by linear interpolation, as in Marching Squares/Cubes)
    - There could be more than two sign-changing edges, so >2 points possible
  - Take the centroid of these points



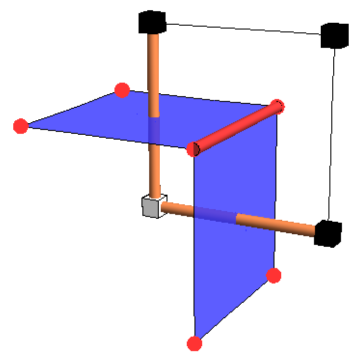
Dual Contouring (3D)

- For each grid cell with a sign change
  - Create one vertex (same way as 2D)
- For each grid edge with a sign change
  - Create a quad (or two triangles) connecting the four vertices in the adjacent grid cubes
  - No look-up table is needed!

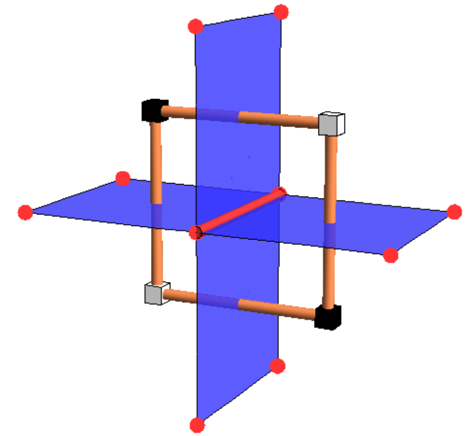


Dual Contouring: Discussion

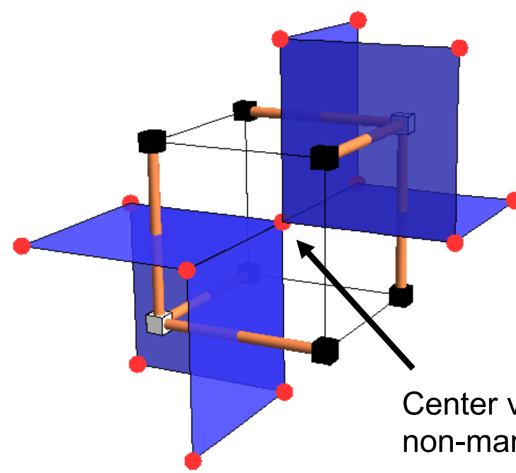
- Result is closed, but possibly non-manifold
  - Each mesh edge is shared by even number of quads
  - An edge may be shared by 4 quads
  - A vertex may be shared by 2 rings of quads
- Can be fixed
  - But with more effort…



Red edge is shared by 2 quads



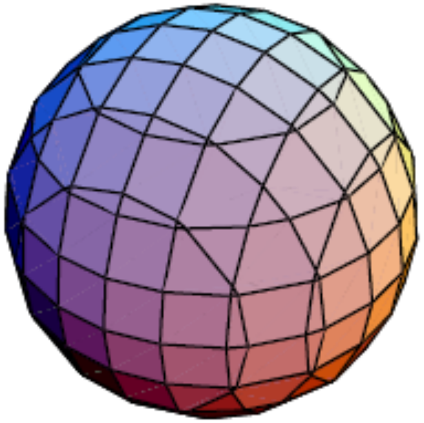
Red edge is shared by 4 quads (non-manifold)

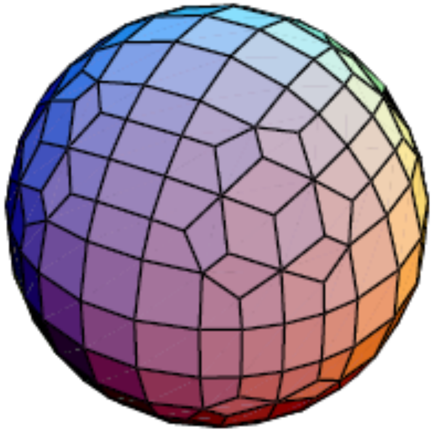


Center vertex is non-manifold

MC vs. DC: Mesh quality

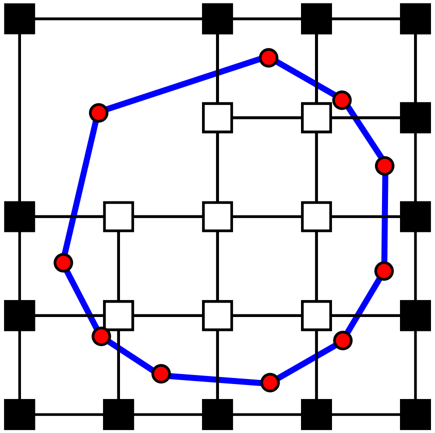
- Marching Cubes
  - ✓ Closed, manifold, and intersection-free
  - ✗ Often generates thin and tiny polygons
- Dual Contouring
  - ✓ Closed and intersection-free
  - ✓ Generates better-shaped polygons
  - ✗ Can be non-manifold


Marching Cubes

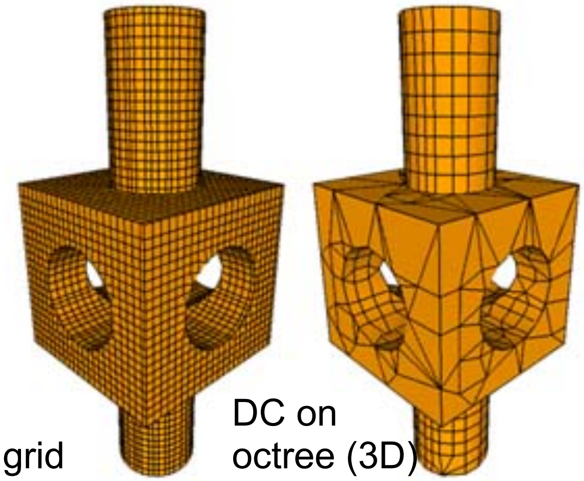

Dual Contouring

MC vs. DC: Implementation

- Marching Cubes
  - ✗ Creating the triangulation table is non-trivial
    - Although table-lookup is straight forward
  - ✗ Restricted to uniform grids
- Dual Contouring
  - ✓ Simple to implement
    - No look-up table is needed
  - ✓ Can be applied to any type of grid
    - Good for generating anisotropic meshes



DC on a Quadtree (2D)



DC on uniform grid

DC on octree (3D)

References

- Marching Cubes:
  - "*Marching cubes: A high resolution 3D surface construction algorithm*", by Lorensen and Cline (1987)
    - >6000 citations on Google Scholar
  - "*A survey of the marching cubes algorithm*", by Newman and Yi (2006)

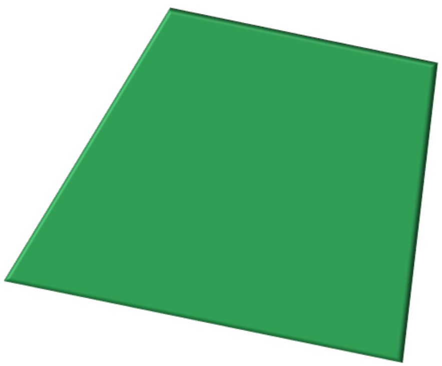
- Dual Contouring:
  - "*Dual contouring of hermite data*", by Ju et al. (2002)
    - >300 citations on Google Scholar
  - "*Manifold dual contouring*", by Schaefer et al. (2007)

Displacement Plots

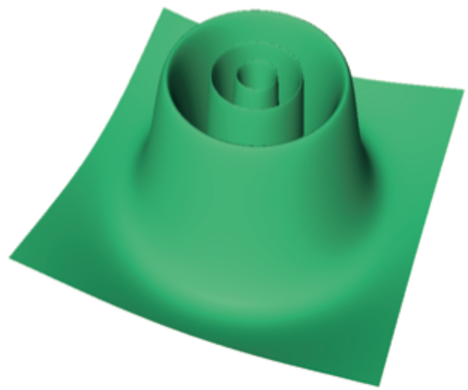
Displace a given surface $S \subseteq D$ in the direction of its normal

Displacement value encodes the scalar data f

$$S_{displ}(x) = x + n(x)f(x), \ \forall x \in S$$



input surface S

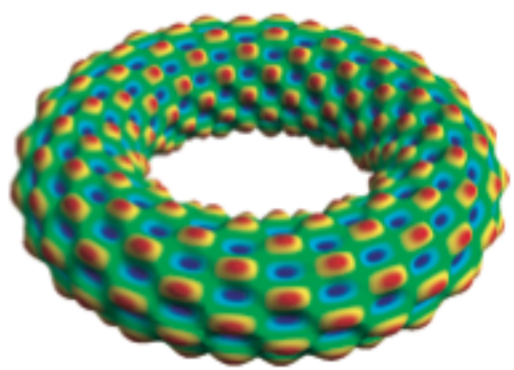


displaced surface S_{displ}

Height plot

- $S = xy$ plane
- displacement always along z

input surface S



displaced surface S_{displ}

Displacement plot

- S = any surface in $\mathbf{R}^3$
- useful to visualize 3D scalar fields