CS 491 CAP
Introduction to Graphs and Search

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Outline

- Graphs
- Adjacency Matrix vs. Adjacency List
- Special Graphs
- Depth-first and Breadth-first Search
- Topological Sort

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Graphs

- Graph is an abstract way of representing connectivity using nodes (vertices) and edges (arcs)
- n nodes, labeled from 1 to n
- m edges connect some pairs of nodes
  - either directed (unidirected) or bidirectional (undirected)
- Nodes and edges can carry some extra information such as weights

Graph Problems

- Shortest path
- Minimum spanning tree
- Matching / Network flow
- 2-SAT
- Graph coloring
- Traveling salesman problem
- …

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Graph storage

- Adjacency matrix
  - bool mat[n][n]
  - mat[u][v] is the indicator that whether there is an edge between node u and v.
- Adjacency list
  - vector<int> adj[n]
  - adj[u] stores a list of nodes which are adjacent to node u.

Matrix v.s. List

- Checking if two nodes are directly connected:
  - Matrix: $O(1)$
  - List: $O(n)$ worst, $O(\frac{m}{n})$ average.
- Memory
  - Matrix: $O(n^2)$
  - List: $O(m+n)$

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Special Graphs

- Tree
  - A connected acyclic graph
  - A connected graph with $n - 1$ edges
  - An acyclic graph with $n - 1$ edges
  - There is exactly one path between every pair of nodes
  - An acyclic graph but adding any edge results in a cycle
  - A connected graph but removing any edge disconnects it
- Bipartite Graph
  - Separate into two groups of nodes such that the edges exist between these two groups only.
- Directed Acyclic Graph (DAG)
  - Nodes have a partial ordering.

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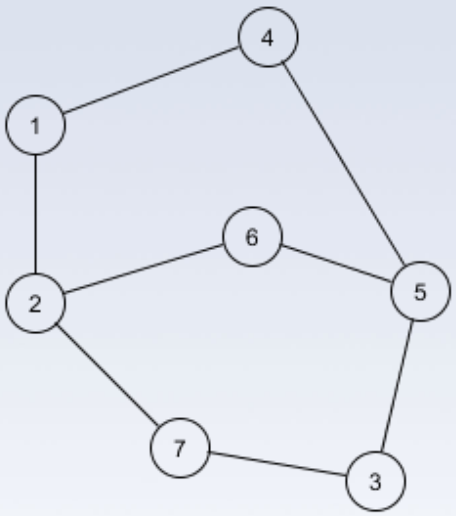
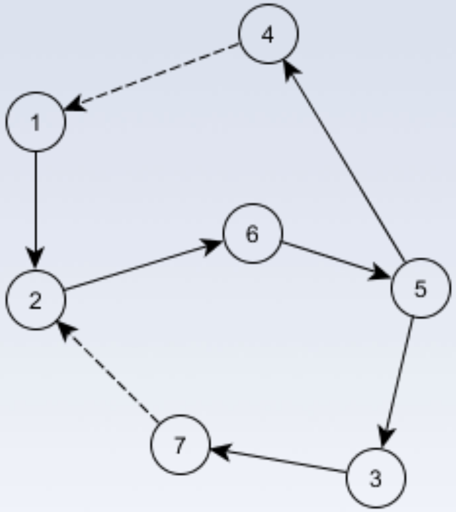
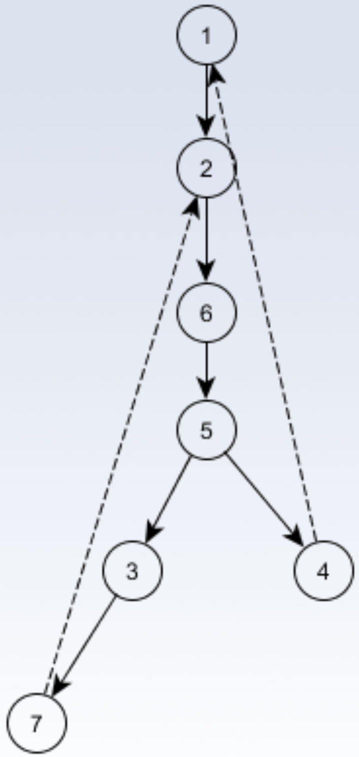
Depth-First Search

- DFS(v): visits all the nodes reachable from v in depth-first order
  - Mark v as visited
  - For each edge v → u:
    - If u is not visited, call DFS(u)

Example







Property

- In undirected graph
  - No cross edges, only tree-edges and back edges

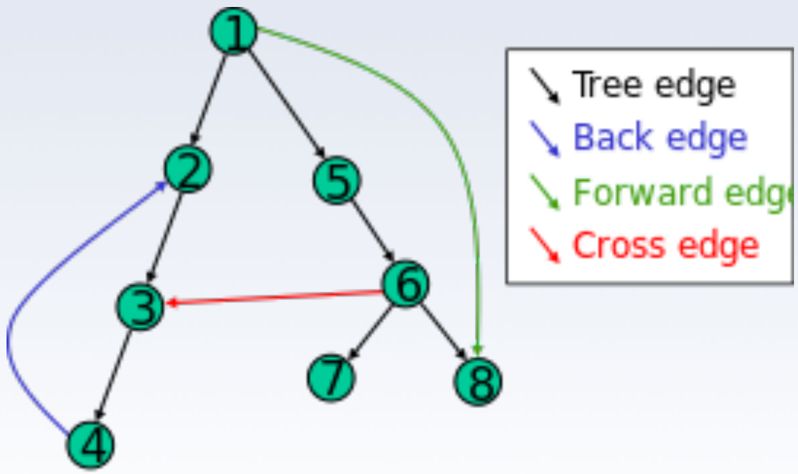


Figure from Wikipedia Depth-first-search

Example Problems

- What is the minimum number of edges should be added to make an undirected graph connected?
- E.g.
- 5 nodes, 3 edges
- 1-2
- 1-3
- 2-3
- Then, the answer is 2.

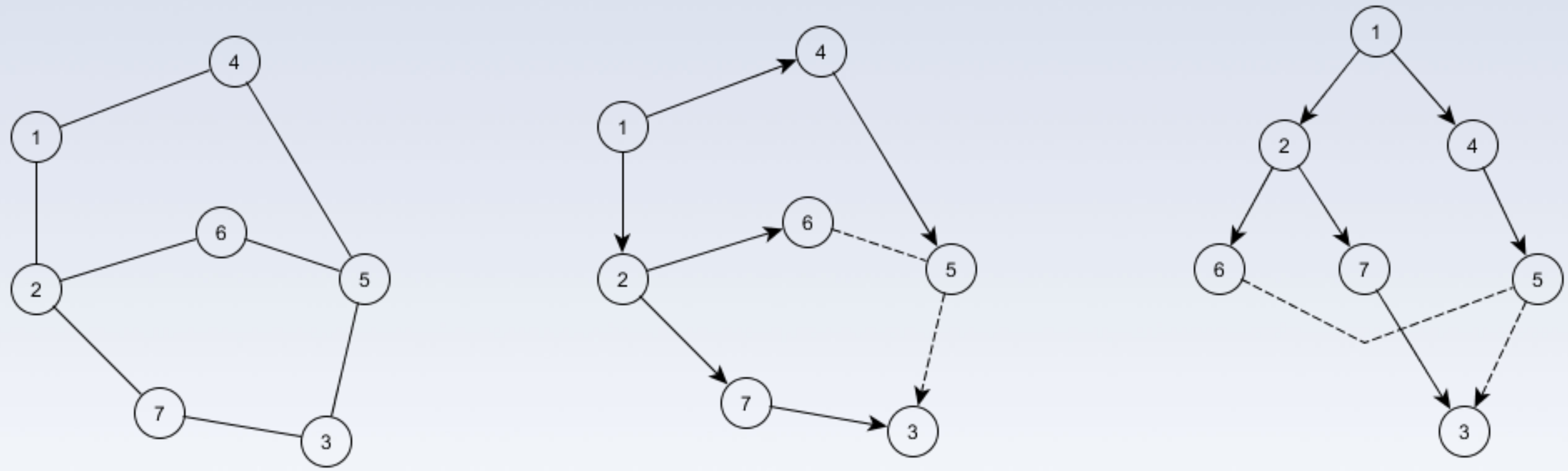
Example Problems: Solution

- Figure out the number of components N
- The answer is $N - 1$

Breadth-First Search

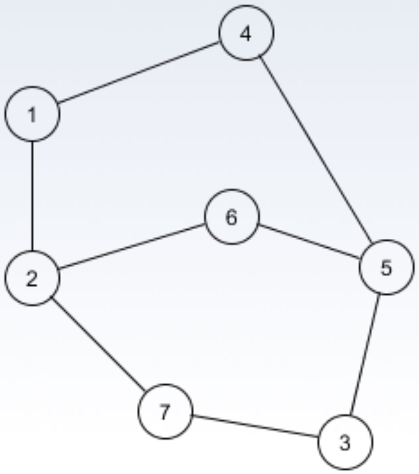
- BFS(v): visits all the nodes reachable from v in breadth-first order
  - Initialize a queue Q
  - Mark v as visited and push it to Q
  - While Q is not empty:
    - Take the front element of Q and call it w
    - For each edge w → u:
      - If u is not visited, mark it as visited and push it to Q

Example



Example Problems

- The distance from node u to node v are defined by the minimum number of edges should be traversed from u to v.
- Find the furthest node from node 1.
- E.g. Answer is 3



Example Problems: Solution

- Depth of the BFS tree

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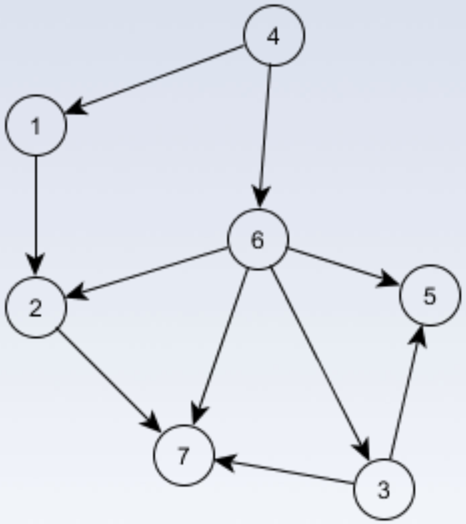
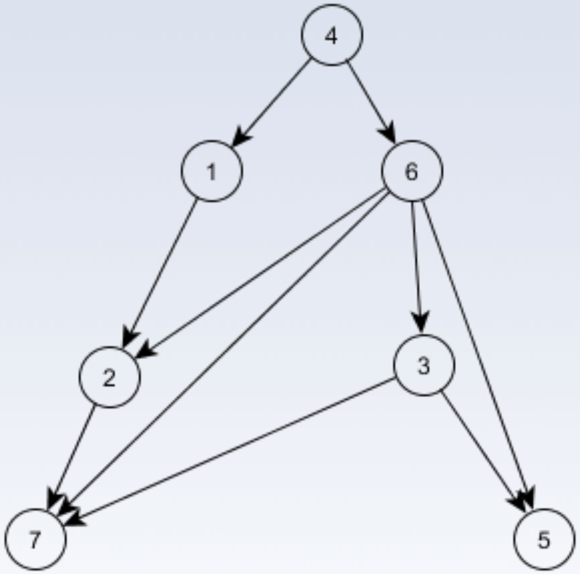
Topological Sort Problem

- Input: a DAG $G = (V, E)$
- Output: an ordering of nodes such that for each edge $u \to v$, u comes before v

Example





- Possible Orders:
  - 4,1,6,2,3,7,5
  - 4,6,3,1,2,5,7
  - …

Topological Sort

- Key Idea
  - Any node without an incoming edge can be the “first element”

Topological Sort

- Precompute the number of incoming edges deg(v) for each node v
- Put all nodes v with deg(v) = 0 into a queue Q
- Repeat until Q becomes empty:
  - Take v from Q
  - For each edge v → u:
    - Decrement deg(u) (essentially removing the edge v → u)
    - If deg(u) = 0, push u to Q
- Time complexity: $\Theta(n + m)$

Example Problems

- N people
- Have known M relations that
  - u is strictly higher than v
- Check whether someone are lying.

Questions?