CS 598 PS Problem Set1

Problem 1:

B: Breast cancer
M: Positive mammogram
$P(B \mid M)$

$= \frac{P(B)\ *\ P(M|B)}{P(B)*P(M|B)+(1-P(B))*P(M|\neg B)}$

$= \frac{0.8\%\ *\ 90\%}{0.8\%\ *\ 90\%\ +\ (1\ -\ 0.8\%)\ *\ 7\%}$

$= 9.39\%$

Problem 2:
1.a:
Let matrix X be the given matrix
Let matrix Y equals to K rows, 1 column matrix, which all elements are $\frac{1}{K}$

$$Y = \left.\begin{bmatrix} \frac{1}{K} \\ \frac{1}{K} \\ \vdots \\ \frac{1}{K} \end{bmatrix}\right\} K$$

Thus, the mean image is $vec(X \cdot Y)^{(M)}, X \subset R^{M*N}$

1.b:
Let matrix X be the given matrix
Let matrix Y equals to 2 rows, $M*N$ columns matrix, which element values:

$$Y_{i,j} = \begin{cases} \frac{2}{M*N},\ i\ =\ 1\ and\ M*(k-1)+1 \leq j \leq M*(k-1)+\frac{M}{2},\ 1 \leq k \leq N \\ \frac{2}{M*N},\ i\ =\ 2\ and\ M*(k-1)+\frac{M}{2}+1 \leq j \leq M*(k-1)+M,\ 1 \leq k \leq N \\ 0,\ otherwise \end{cases}$$

$$Y = \begin{bmatrix} \frac{2}{M*N} \cdots 0 \cdots \frac{2}{M*N} \cdots 0 \cdots \\ 0 \cdots \frac{2}{M*N} \cdots 0 \cdots \frac{2}{M*N} \cdots \end{bmatrix}$$

Thus, the average of the top half of all the images and the average of the bottom half can be computed:

$$H = Y \cdot X = \begin{bmatrix} t_1\ t_2\ t_3\ \cdots\ t_K \\ b_1\ b_2\ b_3\ \cdots\ b_K \end{bmatrix}$$

Then, we can get mean of each half part by:

$$M = \begin{bmatrix} \bar{t} \\ \\ \bar{b} \end{bmatrix} = H \cdot \left.\begin{bmatrix} \frac{1}{K} & \frac{1}{K} \\ \vdots & \vdots \\ \frac{1}{K} & \frac{1}{K} \end{bmatrix}\right\} K$$

After we obtain mean of each part, let matrix W be

$$W = \underbrace{\begin{bmatrix} \bar{t} \; \bar{t} \; \bar{t} \; \cdots \; \bar{t} \\ \\ \bar{b} \; \bar{b} \; \bar{b} \; \cdots \; \bar{b} \end{bmatrix}}_{\text{K}}$$

Finally,covariance matrix:
$Cov(t, b) = \frac{1}{K-1}\Sigma_{k=1}^{K}[(t_k - \bar{t})(b_k - \bar{b})^T] = \frac{1}{K-1}(H - W) \cdot (H - W)^T$

2.a:
Step1: using Kronecker product, where I represents a $n * n$ identity matrix and 1_n is an n dimensional vector of 1s, to sum across forth(K) and Third(Color) dimension of the tensor X:

$$(1_K^T \otimes 1_3^T \otimes I_N \otimes I_M)vec(X)$$

Step2: obtain average of all images (all elements times $\frac{1}{3K}$ since we sum them across K and Color dimension):

$$\frac{1}{3K}(1_K^T \otimes 1_3^T \otimes I_N \otimes I_M)vec(X)$$

Step3: vec-transpose the result to obtain M*N matrix:

$$(\frac{1}{3K}(1_K^T \otimes 1_3^T \otimes I_N \otimes I_M)vec(X))^{(M)}$$

2.b:
Step1: using Kronecker product, where I represents a $n * n$ identity matrix and 1_n is an n dimensional vector of 1s, to sum across forth(K) and Third(Color – only red channel) dimension of the tensor X:

$$(1_K^T \otimes [1\ 0\ 0] \otimes I_N \otimes I_M)vec(X)$$

Step2: obtain average of all images (all elements times $\frac{1}{K}$ since we sum them across K and Color(only red channel; green and blue channels are 0s, so it doesn't matter) dimension:

$$\frac{1}{K}(1_K^T \otimes [1\ 0\ 0] \otimes I_N \otimes I_M)vec(X)$$

Step3: vec-transpose the result to obtain M*N matrix:

$$(\frac{1}{K}(1_K^T \otimes [1\ 0\ 0] \otimes I_N \otimes I_M)vec(X))^{(M)}$$

Problem 3:
DFT matrix: $W = (\frac{\omega^{jk}}{\sqrt{N}})_{j,k=0,1...,N-1}, where\ \omega = e^{-2\pi i/N}$

$$W = \frac{1}{\sqrt{N}} \begin{bmatrix} 1\ 1\ 1\ 1 & \cdots & 1 \\ 1\ \omega\ \omega^2\ \omega^3 & \cdots & \omega^{N-1} \\ 1\ \omega^2\ \omega^4\ \omega^6 & \cdots & \omega^{2(N-1)} \\ \vdots\ \vdots\ \vdots\ \vdots & \ddots & \vdots \\ 1\ \omega^{(N-1)}\ \omega^{2(N-1)}\ \omega^{3(N-1)} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

Hann function: $h(n) = \frac{1}{2}(1 - cos(\frac{2\pi n}{N-1}))$
Step1: we can get Hann window with DFT size 1024:

$$T = \frac{1}{\sqrt{N}} \begin{bmatrix} h(0)\ \ h(1)\ \ h(2)\ \ h(3) & \cdots & h(1023) \\ h(0)\ h(1)\omega\ h(2)\omega^2\ h(3)\omega^3 & \cdots & h(1023)\omega^{N-1} \\ h(0)\ h(1)\omega^2\ h(2)\omega^4\ h(3)\omega^6 & \cdots & h(1023)\omega^{2(N-1)} \\ \vdots\ \vdots\ \vdots\ \vdots & \ddots & \vdots \\ h(0)\ h(1)\omega^{(N-1)}\ h(2)\omega^{2(N-1)}\ h(3)\omega^{3(N-1)} & \cdots & h(1023)\omega^{(N-1)(N-1)} \end{bmatrix}$$

Step2: since hop is 512, it is $(1024 * i) * (512 * i + 512)$ matrix, where i is $\frac{input\ size+512}{512}$:

$$M = \begin{bmatrix} T\ 0\ 0 \cdots & 0 \\ 0\ T\ 0 \cdots & 0 \\ \vdots\ \vdots\ \vdots & \ddots\ \vdots \\ 0\ 0\ 0 \cdots & T \end{bmatrix}$$

If input data size can not divided by 512, we have to construct $(512*i+512)*N$

matrix X:

$$X = \begin{bmatrix} 1 & 0 & 0 \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 \cdots & 1 \\ 0 & 0 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 \cdots & 0 \end{bmatrix}$$

Step3: we can obtain the matrix A:

$$A = M \cdot X$$

Plot of the absolute value of the spectrogram matrix: