

2D Transformational Geometry

For the following questions, assume we are working in 2D using homogenous coordinates.

- Which of the following sets of transformations commute?
 - ☒ A rotation and a uniform scaling
 - ☐ A rotation and a squash (non-uniform scaling)
 - ☐ A rotation and a translation
 - ☒ Two translations
- Create a single matrix that encodes the following transformations:
 - Translate by +2 in X
 - Scale by a factor of 4 uniformly

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad p = p'$$

$S_{4,4}$ (labeled "2nd") $T_{2,0}$ (labeled "happens first")

$$\begin{bmatrix} 4 & 0 & 8 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Imagine you had a line segment with endpoints (2,1) and (4,1). Can you construct a transformation matrix that will rotate the segment by 90 degrees around its midpoint?

Solution is to:

$$T_{3,1} \quad R_{90} \quad T_{-3,-1}$$

(1) Translate midpoint to origin
 (2) Rotate
 (3) Translate back to original position

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$