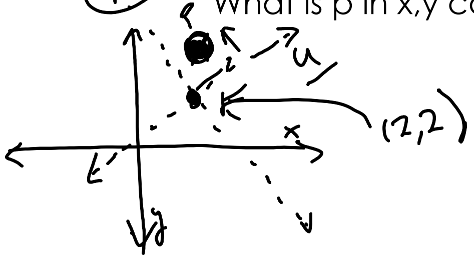
CS 418: Interactive Computer Graphics　　　　　　　　　　　Fall 2016
In-class Worksheet

More on Affine Transformations

1. Suppose we have 2D frame with an origin at (2,2) and basis vectors $u = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \rangle$ $v = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \rangle$ Suppose a point p=(1,1) in u,v coordinates. What is p in x,y coordinates?



$$p_{xy} = 1(\vec{u}) + 1(\vec{v}) + (2,2)$$
$$= \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle + \langle -1/\sqrt{2}, 1/\sqrt{2} \rangle + (2,2)$$
$$= \langle 0, 2/\sqrt{2} \rangle + (2,2)$$
$$= (2, 2 + 2/\sqrt{2})$$

2. Construct a matrix that can convert a point in u,v coordinates to x,y coordinates for the basis described above.

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{e} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

3. What matrix would perform the window-to-viewport transformation specified by the function call `gl.viewPort(0,0,960,640)`? For this question, just ignore the z-coordinate and imagine the NDC coordinates to be transformed are in the form (x, y,1). Express your answer as a 3x3 matrix and express rational numbers as fractions.

> In WebGL, the window to viewport transformation is performed by the WebGL library during primitive assembly. You control it using the call `gl.viewPort(x, y, w, h)`. It specifies the viewport by giving the lower-left coordinate of the viewport with (x, y) and the width and height of the viewport with w and h.

$$\begin{bmatrix} 959/2 & 0 & 0 \\ 0 & 639/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 959/2 & 0 & 959/2 \\ 0 & 639/2 & 639/2 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Suppose we have the following set of transformations that map a point in view (or camera) space to world coordinates. What transformations will map world coordinates to camera space?

$$\left(\begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1} =$$

$$\boxed{\begin{array}{c} (M_1 M_2 \ldots M_{N-1} M_N)^{-1} \\ M_N^{-1} M_{N-1}^{-1} \ldots\ldots M_2^{-1} M_1^{-1} \end{array}}$$

$$\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bonus question:
What is the inverse of matrix that performs a scale transformation?

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \rightarrow \begin{bmatrix} 1/s_x & & 0 & \\ & 1/s_y & & \\ 0 & & 1/s_z & \\ & & & 1 \end{bmatrix}$$