CS 519: Scientific Visualization
In-class Worksheet

Fall 2017

Vector Field Visualization

Finding Eigenvalues

You can find the eigenvalues of a matrix M in 2D by finding the roots of the characteristic polynomial. The characteristic polynomial is formed from the determinant $|M - \lambda|$

For example:

If $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ then we have $|M - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2$

Note that this is not a method you should implement in code to find eiegnvalues. It doesn't scale (why?). Alternatives include the QR method or Power Method.

1. Classifying Critical Points

Suppose we have a 2D vector field defined as $v(x) = \langle y^2 + y, x^2 + x \rangle$

a. What is a critical point in the vector field? The point (0,0) since the magnitude of the field is 0 at that point
$\|0^2 + 0, 0^2 + 0\| = 0$

b. What is the Jacobian of the vector field? Recall that in 2D it takes the form of the following matrix:

$$J = \begin{vmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{vmatrix} \begin{bmatrix} 0 & 2y+1 \\ 2x+1 & 0 \end{bmatrix}$$

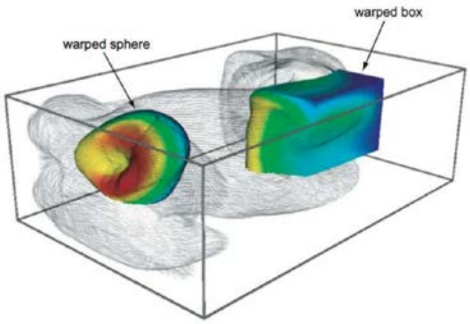
c. Evaluate the Jacobian at the critical point.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

d. What are the eigenvalues of the Jacobian at the critical point?
$\begin{vmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{vmatrix} = \lambda^2 - 1$, so the roots are +1 and -1

e. Classify the critical point based on the eigenvalues. Is it a source, sink, saddle, center, or focus? Saddle

2. Displacement Surfaces



Suppose we place a sphere of radius 1 centered at the origin within a vector field defined by $v(x,y,z) = \langle x^2 + x, y^2 + x, z^2 + x \rangle$

1. If the surface of the sphere is displaced by the vector field using the formula

   $$\boxed{S_{displ} = \{x + \mathbf{v}(x)\Delta t,\ \ \forall x \in S\}}$$

   where is the sphere surface point (1,0,0) moved to by the field if we use a timestep of ½?
   (1,0,0) + ½ <2,1,1> = (2, ½ , ½)

2. We can limit displacement to non-tangential motion by calculating the following displacement

   $$\boxed{S_{displ} = \{x + (\mathbf{v}(x)\mathbf{n}(x))\,\mathbf{n}(x)\Delta t,\ \ \forall x \in S\}}$$

   Using that formula and the same values as part 1, to where is (1,0,0) displaced?
   The normal is <1,0,0> since the tangent plane to the sphere is the x=1 plane.
   $(1,0,0) + \left(\langle 1,0,0\rangle \cdot \langle 2,1,1\rangle \frac{1}{2}\right) = (2,0,0)$