CS 519: Scientific Visualization Fall 2015
In-class Worksheet

Meshes

1. The Euler Formula

The Euler Formula states the following relationship for the elements of a closed and connected surface mesh:

V-E+F=2(1-G)

V is the number of vertices
E is the number of edges
F is the number of faces
G is the genus of the surface (how holes/handles it has)

Assume G = 0

Show that for a triangle mesh with no holes we have F≈2V. Hint: each face has 3 edges and each edge is shared by 2 faces.

$V - E + F = 2$

$V - \frac{3F}{2} + F = 2$

$V - F/2 = 2$

$V = F/2 + 2$

$2V = F + 4$

$2V \approx F$

IFM { V 1.3 7.9 -5. ; ~ ; f 1 5 6

2. Memory Requirements

Using the fact that F≈2V, compare the storage requirements for an indexed face mesh and a triangle soup. Assume the mesh has V vertices and a number requires 4 bytes of space. Derive functions for the number of bytes the mesh will require as a function of V.

△ soup → each F has 3 vertices w/ 3 coords

IFM → vertices w/ 3 coords + faces w/ 3 indices

△ soup: F(3)(3)(4) → 2V(36) = 72V bytes

IFM: V(3)(4) + F(3)(4)
12V + 2V(12) → 36V bytes

Laplacian Smoothing

Can be viewed as an iterative averaging process using the following formulation:

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$

$L(p_i) = \sum_{n_j} \frac{1}{w_j}(n_{j-} p_i)$ and λ is in [0,1]

with n_j being the neighboring vertices of p_i and w_j a weight

3. Laplacian Smoothing

Consider a linear curve of three vertices: (4,2) to (12, 2) to (16, 2)

a. Assume the endpoints always stay fixed. What is the position of the middle vertex after 2 iterations of Laplacian smoothing using uniform weights and $\lambda = 1/2$?

x coord: ① $12 + 1/2(-8/2 + 4/2) = 12 - 1 = 11$

② $11 + 1/2(-7/2 + 5/2) = 11 - 1/2 = 10.5$

b. If you iterate until convergence, what final position will the middle vertex be in?

$x = 10$

c. What weights in the smoothing formula would result in the middle vertex never moving?

weight by distance $\left(-\frac{8}{8} + \frac{4}{4}\right) \rightarrow 0$

4. Mesh Simplification

Simplify the triangle mesh below using the grid to perform vertex clustering. Use cell centers for the vertex placement