# Meshes

#### 1. The Euler Formula

The Euler Formula states the following relationship for the elements of a closed and connected surface mesh:

V-E+F=2	2(1-G)
V-L ' I —	2(I-U)

ASSUME

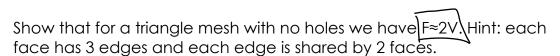
V is the number of vertices

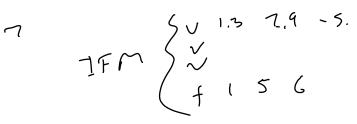
G = 0

**E** is the number of edges

F is the number of faces

**G** is the genus of the surface (how holes/handles it has)





### 2. Memory Requirements

Using the fact that F≈2V, compare the storage requirements for an indexed face mesh and a triangle soup. Assume the mesh has V vertices and a number requires 4 bytes of space. Derive functions for the number of bytes the mesh will require as a function of V.

Noup-reach Fhas 3veitiles w/3 coolds IFM -> Verlices w/3 160 v ds + faces w/3 indices

F(3)(3)(4) -> 2U(3() = 72V bytes

$$TFM \rightarrow V(3)(4) + F(3)(4)$$
12 \( + 2\(\dot(12) -> 36\) \( b\_0 \) tes

#### Laplacian Smoothing

Can be viewed as an iterative averaging process using the following formulation:

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$

$$L(p_i) = \sum_{n_j} \frac{1}{w_j} (n_{j-} p_i)$$
 and  $\lambda$  is in [0,1]

with n<sub>i</sub> being the neighboring vertices of p<sub>i</sub> and w<sub>i</sub> a weight

### 3. Laplacian Smoothing

Consider a linear curve of three vertices: (4,2) to (12, 2) to (16, 2)

a. Assume the endpoints always stay fixed. What is the position of the middle vertex after 2 iterations of Laplacian smoothing using uniform weights and  $\lambda = 1/2$ ?

$$2 + \frac{1}{2}(-\frac{9}{2} + \frac{1}{2}) = 12 - 1 = 1$$

$$2 + \frac{1}{2}(-\frac{7}{2} + \frac{5}{2}) = 11 - \frac{1}{2} = 10.5$$
b. If you iterate until convergence, what final position will the

middle vertex be in?

c. What weights in the smoothing formula would result in the middle

What weights in the smoothing formula would result in the middle vertex never moving?

$$\omega_e: ght bg d: stance \left(-\frac{8}{8} + \frac{1}{4}\right) \rightarrow 0$$

Simplification

## 4. Mesh Simplification

Simplify the triangle mesh below using the grid to perform vertex clustering. Use cell centers for the vertex placement

