

A Sample Beamer Presentation

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Bates College

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Presentation Outline

A Sample
Beamer
Presentation

Eric Towne

What Can
Happen at a
Critical Point?

What Does
 $g'(c) > 0$
Mean?

Further Work

1 What Can Happen at a Critical Point?

2 What Does $g'(c) > 0$ Mean?

3 Further Work

The Usual Suspects

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You might think that if $f'(0) = 0$ (and f is not a constant function) then at $x = 0$, f must have

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If that's what you think, then you are ...

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A Counterexample

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Further Work

Consider the function

$$f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Let's see what $f'(0)$ is.

Finding $f'(0)$

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By the definition of derivative,

$$f'(0) =$$

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By the definition of derivative,

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

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$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 \sin(1/h) - 0}{h} \end{aligned}$$

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Since $-h \leq h \sin(1/h) \leq h$

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Since $-h \leq h \sin(1/h) \leq h$ and $\lim_{h \rightarrow 0} (-h) = \lim_{h \rightarrow 0} (h) = 0$,

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Since $-h \leq h \sin(1/h) \leq h$ and $\lim_{h \rightarrow 0} (-h) = \lim_{h \rightarrow 0} (h) = 0$, the
Theorem says

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Since $-h \leq h \sin(1/h) \leq h$ and $\lim_{h \rightarrow 0} (-h) = \lim_{h \rightarrow 0} (h) = 0$, the Squeeze Theorem says $f'(0) = 0$.

What Really Happens at $x = 0$?

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But $f(x)$ oscillates wildly as
 $x \rightarrow 0$, so even though
 $f'(0) = 0$, f has neither max,
min, nor inflection point at
 $x = 0$.

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But $f(x)$ oscillates wildly as
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$$y = f(x), y = x^2, y = -x^2$$

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How to Define “Increasing at a Point”?

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It's natural to think that if $g'(c) > 0$ then g must be
“increasing at $x = c$.”

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But what does “increasing at $x = c$ ” really mean?

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It's natural to think that if $g'(c) > 0$ then g must be “increasing at $x = c$.”

But what does “increasing at $x = c$ ” really mean?

A Reasonable Definition

A function g is *increasing at* $x = c$ if there is an open interval $I = (c - \delta, c + \delta)$ such that

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A function g is *increasing at* $x = c$ if there is an open interval $I = (c - \delta, c + \delta)$ such that if $x_1, x_2 \in I$, then $x_1 < x_2 \Rightarrow$

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A Reasonable Definition

A function g is *increasing at $x = c$* if there is an open interval $I = (c - \delta, c + \delta)$ such that if $x_1, x_2 \in I$, then $x_1 < x_2 \Rightarrow g(x_1) < g(x_2)$.

Our Function with a Slight Twist

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Let's modify our function to

$$g(x) = \begin{cases} 0.5x + x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Using the definition of derivative as before, we will find that $g'(0) = 0.5$.

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However, $g(x)$ still oscillates enough as $x \rightarrow 0$ that there is no open interval containing $x = 0$ that satisfies our definition of g increasing at $x = 0$ even though $g'(0) > 0$.

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However, $g(x)$ still oscillates enough as $x \rightarrow 0$ that there is no open interval containing $x = 0$ that satisfies our definition of g increasing at $x = 0$ even though $g'(0) > 0$.

$$y = g(x), y = x^2 + 0.5x, y = x^2 - 0.5x$$

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The function $f(x)$ introduced earlier has other interesting properties, one of which is the fact that while $f'(0)$ exists, $f'(x)$ is discontinuous at $x = 0$.

We leave it to you to work this out for yourself and to explore this interesting function further.

Thank you for your attention today.