BUAN/OPRE 6398 Prescriptive Analytics
Homework 1

Due 2/08/19
(11:59 p.m.)

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Note: 1. Your homework submission must be computer typewritten.

2. Please refer to the sample completed homework assignment for the suggested format of your submission.

3. Show only the solutions and do not copy the problems in the submission.

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1. Read Readings 1 and 2.

2. (10 pts.)Serendipity

The three princes of Serendip
Went on a little trip.
They could not carry too much weight;
More than 300 pounds made them hesitate.
They planned to the ounce. When they returned to Ceylon
They discovered that their supplies were just about gone
When, what to their joy, Prince William found
A pile of coconuts on the ground.
"Each will bring 40 rupees," said Prince Richard with a grin
As he almost tripped over a lion skin.
"Look out!" cried Prince Robert with glee
As he spied some more lion skins under a tree.
"These are worth even more--250 rupees each
If we can just carry them all down to the beach."
Each skin weighed 12 pounds and each coconut, five,
But they carried them all and made it alive.
The boat back to the island was very small
11 cubic feet baggage capacity--that was all.
Each lion skin took up one cubic foot
While eight coconuts the same space took.
With everything stowed they headed to sea
And on the way calculated what their new wealth might be.
"Eureka!" cried Prince Robert, "Our worth is so great
That there's no other way we could return in this state.
Any other skins or nut which we might have brought
Would now have us poorer. And now I know what--
I'll write my friend Horace in England, for surely
Only he can appreciate our serendipity."

Formulate the above problem as a linear program that may be used to find the three princes' maximum total new wealth. (Note: The word *serendipity* was coined by the English writer Horace Walpole after a fairy tale entitled *The Three Princes of Serendip*. See http://en.wikipedia.org/wiki/The_Three_Princes_of_Serendip**.**)

3. (10 pts.) The dietitian at a local hospital in Plano, TX, is planning the breakfast menu for the maternity ward patients. She has chosen cottage cheese and scrambled eggs for breakfast. The primary concerns are the vitamin E and iron requirements of new mothers.

   According to the American Medical Association (AMA), new mothers should get at least 23 milligrams of vitamin E and 49 milligrams of iron (fictitious figures) from breakfast. The AMA handbook reports that a scoop of cottage cheese contains 1.5 milligrams of vitamin E and 5 milligrams of iron. An average scoop of scrambled eggs contains 2 milligrams of vitamin E and 7 milligrams of iron. In accordance with the AMA guidelines, new mothers must eat at least 4, but no more than 8, scoops of scrambled eggs for their breakfast.

   The hospital accounting department estimates that a scoop of cottage cheese costs $0.26 and a scoop of scrambled eggs costs $0.22. The dietitian is attempting to determine the optimal breakfast menu that will satisfy all the nutritional requirements and minimize the total cost. Set up an LP for the diet problem, but there is no need to solve it.

4. (10 pts.) Catch 22 Realty, Inc., has identified four small apartment buildings (A1, A2, A3, and A4) in Richardson, TX, in which it would like to invest. Mr. Trump, an assistant manager at the firm, approaches three local banks (B1, B2, and B3) for financing. Each bank has placed a credit ceiling on how much it will lend Mr. Trump in total. Moreover, each loan officer has set a different interest rate on each of the four properties. Relevant information has been summarized in the table below:

	Interest rate (%)				
Local bank	A1	A2	A3	A4	Maximum credit
B1	11	8	10	11	$130,000
B2	8	9	10	8	$80,000
B3	12	10	10	9	$120,000
Selling price	$50,000	$60,000	$70,000	$130,000	

   Since each of the four apartment buildings is equally attractive as an investment to Mr. Trump, he decides to purchase all of them. Formulate the problem as a linear program for Mr. Trump that can be used to determine the financing plan with the minimum total interest cost.

5. (10 pts.) Dr. Maureen Becker, the head administrator at Jefferson County Hospital, must determine a staffing schedule to make sure that there are enough nurses on duty throughout the day. She has broken the 24-hour period into eight three-hour segments and the demand for nurses during each of them segments is shown below:

Segment number	Time	Number of nurses required

1	12:00 A.M. - 3:00 A.M.	15
2	3:00 A.M. - 6:00 A.M.	10
3	6:00 A.M. - 9:00 A.M.	22
4	9:00 A.M. - 12:00 P.M.	25
5	12:00 P.M. - 3:00 P.M.	30
6	3:00 P.M. - 6:00 P.M.	43
7	6:00 P.M. - 9:00 P.M.	42
8	9:00 P.M. - 12:00 A.M.	35

A nurse reports for duty at the beginning of one of the eight segments and works nine consecutive hours. As an assistant to Dr. Becker, you are asked to formulate for her a linear program that may be used to determine the work schedule with the minimum number of nurses to meet the daily requirements.

6. (10 pts.) Consider the following linear program:

$$\begin{aligned} \text{Maximize } Z &= 10x + 4y \\ \text{subject to: } & x \leq 9 \\ & 5x + 10y \geq 75 \\ & 2x - 3y \geq -12 \\ & x, y \geq 0 \end{aligned}$$

(1) Use a ruler to plot the constraints and indicate the feasible region for the LP.
(2) Plot two iso-profit lines to determine the optimal solution and the maximum objective function value.

7. (10 pts.) Consider the following linear program:

$$\begin{aligned} \text{Minimize } Z &= A - 2B \\ \text{subject to: } & -A + B \leq 4 \\ & A - B \leq 4 \\ & A + B \geq 6 \\ & B \leq 3 \\ & A, B \geq 0 \end{aligned}$$

(1) Use a ruler to plot the constraints and indicate the feasible region for the LP.
(2) Use the corner-point method to find the optimal solution and the minimum objective function value.

8. (20 pts.) Consider the following linear program:

$$\begin{aligned} \text{Maximize } Z &= 10x + 4y \\ \text{subject to: } & x \leq 9 \\ & 5x + 10y \geq 75 \\ & -2x + 3y = 12 \end{aligned}$$

$$x \ , \quad y \geq 0$$

(1) Use the simplex method to solve the above LP. What are the optimal solution and the optimal objective function value? Be sure to show your complete work similar to the solutions to Examples 2.15.

(2) Repeat (1) by running Solver. Be sure to copy and paste the Answer report at the appropriate place in your homework submission.

9. (10 pts.) Run Solver to solve the nurse scheduling problem in Question 5 of Homework 1. Be sure to copy and paste the Answer report at the appropriate place in your homework submission and provide interpretation of the results (i.e., the minimum number nurses needed and their work schedule).

10. (10 pts.) Consider the following three LPs. Which of them has no optimal solution, has an unbounded optimal solution, or has multiple optimal solutions? Simply answer the questions and there is no need to submit the graphs.

(1)

Maximize $Z = 15x + 10y$

subject to:
$$\begin{aligned} x + y &\leq 50 \\ 3x + 2y &\leq 120 \\ x \ , \quad y &\geq 0 \end{aligned}$$

(2)

Maximize $Z = 15x + 10y$

subject to:
$$\begin{aligned} x + y &\leq 50 \\ 3x + 2y &\leq 120 \\ x &\geq 30 \\ y &\geq 20 \\ x \ , \quad y &\geq 0 \end{aligned}$$

(3)

Maximize $Z = 2x - y$

subject to:
$$\begin{aligned} x - y &\leq 1 \\ 2x + y &\geq 6 \\ x \ , \quad y &\geq 0 \end{aligned}$$