**Chapter 2**

**Example 2.3**

$6x + 9y = 18 \rightarrow (0,2)\ (3,0)$

$6x + 9y = 36 \rightarrow (0,4)\ (6,0)$

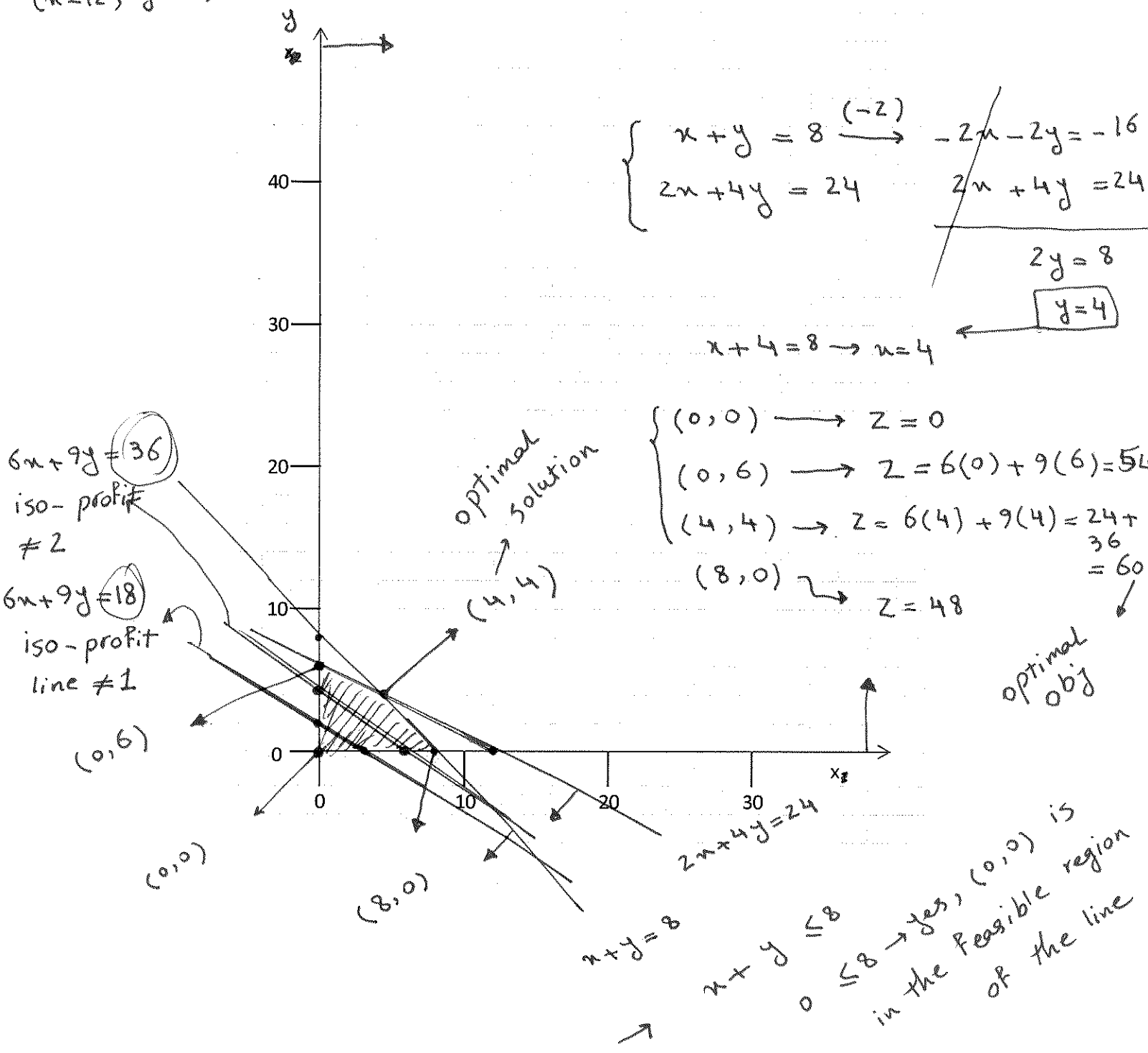
Maximize $Z = 6x + 9y$

subject to:

$x + y \leq 8 \rightarrow x + y = 8 \rightarrow (x=0, y=8)\ (x=8, y=0)$

$2x + 4y \leq 24 \rightarrow (x=0, y=6)\ (x=12, y=0)$

$x, y \geq 0$




$$\begin{cases} x + y = 8 \xrightarrow{(-2)} -2x - 2y = -16 \\ 2x + 4y = 24 \qquad\quad 2x + 4y = 24 \end{cases}$$

$2y = 8$

$y = 4$

$x + 4 = 8 \rightarrow x = 4$

$$\begin{cases} (0,0) \rightarrow Z = 0 \\ (0,6) \rightarrow Z = 6(0) + 9(6) = 54 \\ (4,4) \rightarrow Z = 6(4) + 9(4) = 24 + 36 = 60 \\ (8,0) \rightarrow Z = 48 \end{cases}$$

optimal obj

$x + y \leq 8 \rightarrow$ yes, $(0,0)$ is $0 \leq 8$ in the feasible region of the line

## Ex 2.5

Max $z = 6x + 9y + 0u + 0v$

$x + y + u + 0v = 8$

$2x + 4y + 0u + v = 24$

| | $C_j$ | 6 | 9 | 0 | 0 | |
|---|---|---|---|---|---|---|
| | VIS | $x$ | $y$ | $u$ | $v$ | SQ |
| 0 | $u$ | 1 | 1 | 1 | 0 | 8 → $8/1 = 8$ |
| ← 0 | $v$ | 2 | (4) | 0 | 1 | 24 → $24/4 = 6$ |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 |
| | $Z_j - C_j$ | −6 | −9 | 0 | 0 | |

Max → positive or zero → optimal → $u = 8$, $v = 24$, $x = 0$, $y = 0$, $z = 0$

Min → negative or zero → ~

Row v * $(\frac{1}{4})$ → Row y

Row u − Row y = new Row u

| | $C_j$ | 6 | 9 | 0 | 0 | |
|---|---|---|---|---|---|---|
| | VIS | $x$ | $y$ | $u$ | $v$ | SQ |
| ← 0 | $u$ | 1/2 | 0 | 1 | −1/4 | 2 → $2/\frac{1}{2} = 4$ |
| 9 | $y$ | 1/2 | 1 | 0 | 1/4 | 6 → $6/\frac{1}{2} = 12$ |
| | $Z_j$ | 9/2 | 9 | 0 | 9/4 | 54 |
| | $Z_j - C_j$ | −3/2 | 0 | 0 | 9/4 | — |

$u = 2$, $y = 6$, $x = 0$, $v = 0$, $Z = 54$

$R_x = R_u(2)$

$R_y - R_u =$ new $R_y$

| | $C_j$ | 6 | 9 | 0 | 0 | |
|---|---|---|---|---|---|---|
| | VIS | $x$ | $y$ | $u$ | $v$ | |
| 6 | $x$ | 1 | 0 | 2 | −1/2 | 4 |
| 9 | $y$ | 0 | 1 | −1 | 1/2 | 4 |
| | $Z_j$ | 6 | 9 | 3 | 3/2 | 60 |
| | $Z_j - C_j$ | 0 | 0 | 3 | 3/2 | — |

No Negative value → stop → optimal solution

$x = 4$ $\quad u = 0$

$y = 4$ $\quad v = 0$

$Z = 60$

## Ex 2.6

$M \rightarrow$ large number $= 100$

| | $C_j$ | 16 | 12 | 0 | $M$ | 0 | $M$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | VIS | a | b | c | d | e | f | SQ | |
| $M$ | d | 3 | 5 | -1 | 1 | 0 | 0 | 15 | $15/3 = 5$ |
| ← $M$ | f | 6 | 2 | 0 | 0 | -1 | 1 | 18 | $18/6 = 3$ |
| | | | | | | | | | |
| | $Z_j$ | $9M$ | $7M$ | $-M$ | $M$ | $-M$ | $M$ | $33M$ | |
| | $Z_j - C_j$ | $9M-16$ | $7M-12$ | $-M$ | 0 | $-M$ | 0 | — | |

→ max → positive or zero → stop

Min → negative or zero → stop

$R_f \div 6 \rightarrow R_a$

$R_f(-1/2) + R_d \rightarrow$ new $R_d$

| | $C_j$ | 16 | 12 | 0 | $M$ | 0 | $M$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | VIS | a | b | c | d | e | f | SQ | |
| ← $M$ | d | 0 | 4 | -1 | 1 | 1/2 | -1/2 | 6 | $6/4 = 3/2$ → |
| 16 | a | 1 | 1/3 | 0 | 0 | -1/6 | 1/6 | 3 | $3/(1/3) = 9$ → |
| | | | | | | | | | |
| | $Z_j$ | 16 | $4M+16/3$ | $-M$ | $M$ | $M/2-16/6$ | $-M/2+16/6$ | $6M+48/4$ | |
| | $Z_j - C_j$ | 0 | $4M-20/3$ | $-M$ | 0 | $M/2-16/6$ | $-3/2M+16/6$ | | |

$R_d \div 4 = R_b$

$R_d(-1/12) + R_a \rightarrow$ new $R_a$ $\quad (1 \;\; 0 \;\; 1/12 \;\; -1/24 \;\; -5/24 \;\; 5/24 \;\; 2.5)$

$(0 \;\; 4 \;\; -1 \;\; 1 \;\; 1/2 \;\; -1/2 \;\; 6)(-1/12) + (1 \;\; 1/3 \;\; 0 \;\; 0 \;\; -1/6 \;\; 1/6 \;\; 3) =$

| | $C_j$ | 16 | 12 | 0 | $M$ | 0 | $M$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | VIS | a | b | c | d | e | f | SQ | |
| 12 | b | 0 | 1 | -1/4 | 1/4 | 1/8 | -1/8 | 1.5 | |
| 16 | a | 1 | 0 | 1/12 | -1/24 | -5/24 | 5/24 | 2.5 | |
| | | | | | | | | | |
| | $Z_j$ | 16 | 12 | $-20/12$ | $20/12$ | $-44/24$ | $44/24$ | 58 | |
| | $Z_j - C_j$ | 0 | 0 | $-20/12$ | $-M+20/12$ | $-44/24$ | $-M+44/24$ | — | |

stop → optimal

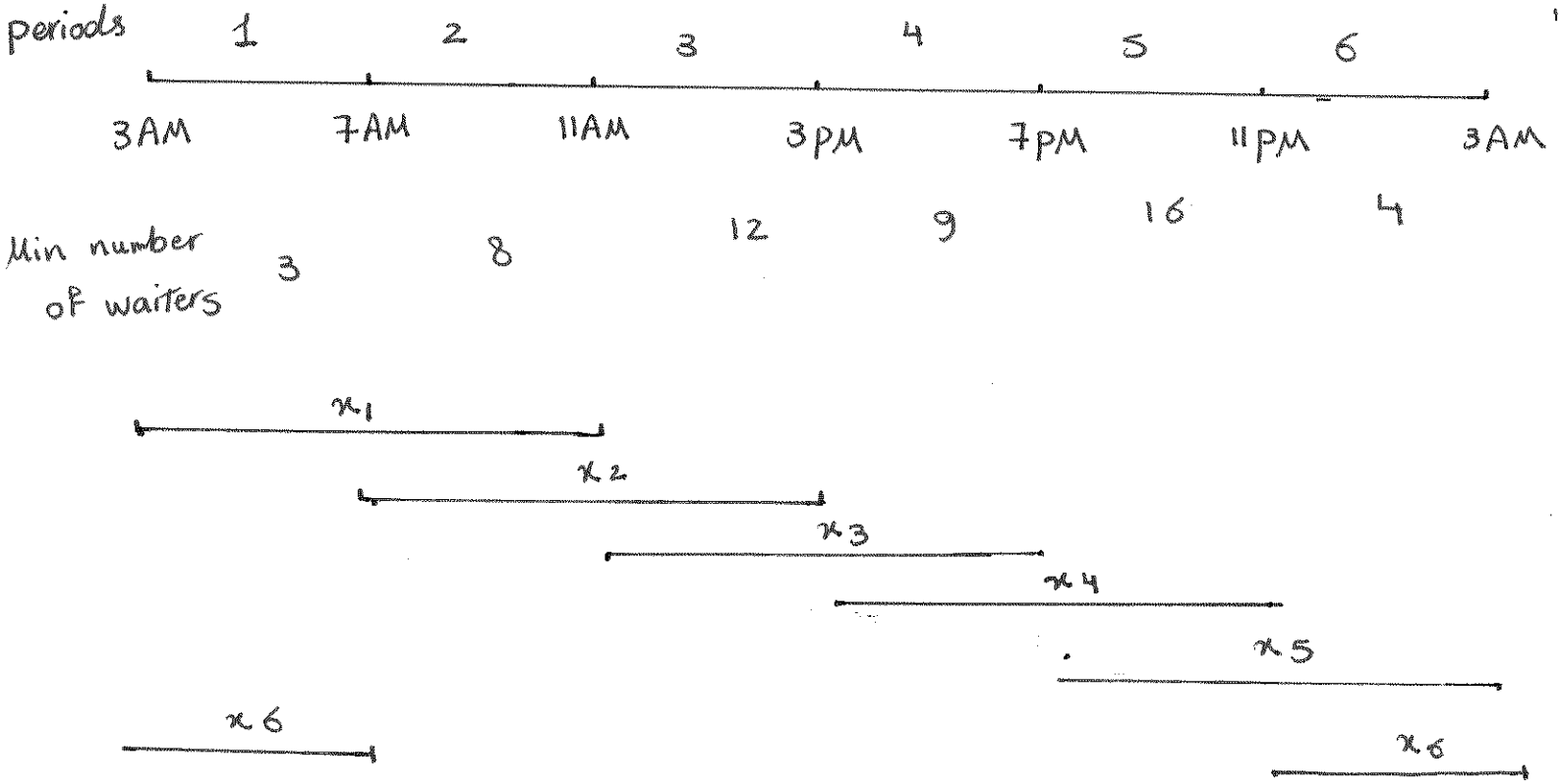
$b = 1.5$ $\quad c = 0$

$a = 2.5$ $\quad d = 0$

$Z = 58$ $\quad e = 0$

$f = 0$

# Ex 2.10




$$\text{Min } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

s.t.

$$x_1 + x_6 \geq 3$$

$$x_1 + x_2 \geq 8$$

$$x_2 + x_3 \geq 12$$

$$x_3 + x_4 \geq 9$$

$$x_4 + x_5 \geq 16$$

$$x_5 + x_6 \geq 4$$

all $x_{ij} \geq 0$