OPRE 6398 Prescriptive Analytics
Solutions to Homework 1

2. Let CN = Number of coconuts to be carried back
   LS = Number of lion skins to be carried back

$$
\begin{array}{lrrcr}
\text{Maximize } Z = & 40CN + & 250LS & & \\
\text{subject to:} & 5CN + & 12LS & \leq & 300 \\
 & 0.125CN + & LS & \leq & 11 \\
 & CN\ , & LS & \geq & 0
\end{array}
$$

3. Let cc = Number of scoops of cottage cheese a new mother eats for breakfast
   se = Number of scoops of scrambled eggs a new mother eats for breakfast

$$
\begin{array}{lrrcr}
\text{Minimize } Z = & 0.26cc + & 0.22se & & \\
\text{subject to:} & 1.5cc + & 2se & \geq & 23 \\
 & 5cc + & 7se & \geq & 49 \\
 & & se & \geq & 4 \\
 & & se & \leq & 8 \\
 & cc\ , & se & \geq & 0
\end{array}
$$

4. Let Qij be the amount of money (in dollars) that Mr. Trump borrows from bank Bi for the purchase of apartment building Aj, i = 1, 2, 3; j = 1, 2, 3, 4. An LP model for the apartment financing problem follows:

$$
\begin{array}{ll}
\text{Minimize } Z = & 0.11Q_{11} + 0.08Q_{12} + 0.10Q_{13} + 0.11Q_{14} + 0.08Q_{21} + 0.09Q_{22} + \\
 & 0.10Q_{23} + 0.08Q_{24} + 0.12Q_{31} + 0.10Q_{32} + 0.10Q_{33} + 0.09Q_{34}
\end{array}
$$

$$
\begin{array}{lrrrrrcr}
\text{subject to:} & Q_{11} + & Q_{12} + & Q_{13} + & Q_{14} & & \leq & 130{,}000 \\
 & Q_{21} + & Q_{22} + & Q_{23} + & Q_{24} & & \leq & 80{,}000 \\
 & Q_{31} + & Q_{32} + & Q_{33} + & Q_{34} & & \leq & 120{,}000 \\
 & Q_{11} + & Q_{21} + & Q_{31} & & & \geq & 50{,}000 \\
 & Q_{12} + & Q_{22} + & Q_{32} & & & \geq & 60{,}000 \\
 & Q_{13} + & Q_{23} + & Q_{33} & & & \geq & 70{,}000 \\
 & Q_{14} + & Q_{24} + & Q_{34} & & & \geq & 130{,}000 \\
 & Q_{11}\ , & Q12\ , & \ldots\ , & & Q_{34} & \geq & 0
\end{array}
$$

5. Let Xi be the number of nurses reporting for duty at the beginning of time segment i, i = 1, 2, ..., 8. A linear program for the personnel scheduling problem follows:

$$
\begin{array}{lrrrrcr}
\text{Minimize } Z = & X1 + & X2 + & X3 + & X4 + & & \\
 & X5 + & X6 + & X7 + & X8 & & \\
\text{subject to:} & X1 + & X7 + & X8 & & \geq & 15 \\
 & X1 + & X2 + & X8 & & \geq & 10 \\
 & X1 + & X2 + & X3 & & \geq & 22 \\
 & X2 + & X3 + & X4 & & \geq & 25 \\
 & X3 + & X4 + & X5 & & \geq & 30 \\
 & X4 + & X5 + & X6 & & \geq & 43 \\
 & X5 + & X6 + & X7 & & \geq & 42 \\
 & X6 + & X7 + & X8 & & \geq & 35 \\
 & X1\ , & X2\ , & \ldots\ , & X8 & \geq & 0
\end{array}
$$

6. Please see the graph on page 5.

7. Please see the graph in page 5.

8. The LP is modified as follows:

Z =	10x +	4y +	0a +	0b -	Mc -	Md		
	x +	0y +	a +	0b +	0c +	0d	=	9
	5x -	10y +	0a -	b +	1c +	0d	=	75
	-2x +	3y +	0a +	0b +	0c +	d	=	12

The initial simplex tableau is as follows:

C_j		10	4	0	0	-M	-M		
	VIS	x	y	a	b	c	d	SQ	
0	a	1	0	1	0	0	0	9	
-M	c	5	10	0	-1	1	0	75	75/10 = 7.5
-M	d	-2	3	0	0	0	1	12	12/3 = 4
Z_j		-3M	-13M	0	M	-M	-M	-87M	
Z_j - C_j		-3M-10	**-13M-4**	0	M	0	0		

The current solution (x, y) = (0, 0) is not optimal since -3M - 10 < 0 and -13M - 4 < 0. Given that -13M - 4 is the most negative number in the "Z_j - C_j" row and 4 < 7.5, we see that the pivot column is "y" ("y" is the entering variable), the pivot row is "d" ("d" is the leaving variable), and the pivot number is 3. The new simplex tableau follows:

C_j		10	4	0	0	-M	-M		
	VIS	x	y	a	b	c	d	SQ	
0	a	1	0	1	0	0	0	9	9/1 = 9
-M	c	35/3	0	0	-1	1	-10/3	35	35/(35/3) = 3
4	y	-2/3	1	0	0	0	1/3	4	
Z_j		-35M/3-8/3	4	0	M	-M	10M/3+4/3	-35M+ 16	
Z_j - C_j		**-35M/3-38/3**	0	0	M	0	13M/3+4/3		

The current solution (x, y) = (0, 4) is not optimal since -35M/3 – 38/3 < 0. Given that -35M/3 – 38/3 is the only negative number in the "Z_j - C_j" row and 3 < 9, we see that the pivot column is "x" ("x" is the entering variable), the pivot row is "x" ("x" is the leaving variable), and the pivot number is 3. The new simplex tableau follows:

C_j		10	4	0	0	-M	-M		
	VIS	x	y	a	b	c	d	SQ	
0	a	0	0	1	3/35	-3/35	2/7	6	6/(3/35) = 70
10	x	1	0	0	-3/35	3/35	-2/7	3	
4	y	0	1	0	-2/35	2/35	1/7	6	
Z_j		10	4	0	-38/35	38/35	-16/7	54	
Z_j - C_j		0	0	0	**-38/35**	M+38/35	M-16/7		

The current solution (x, y) = (3, 6) is not optimal since -38/35 < 0. Given that -38/35 is the only negative number in the "Z_j - C_j" row and 70 is the only positive ratio, we see that the pivot column is "b" ("b" is the entering variable), the pivot row is "a" ("a" is the leaving variable), and the pivot number is 3/35. The new simplex tableau follows:

C_j		10	4	0	0	-M	-M	
VIS		x	y	a	b	c	d	SQ
0	b	0	0	35/3	1	-1	10/3	70
10	x	1	0	1	0	0	0	9
4	y	0	1	2/3	0	0	1/3	10
Z_j		10	4	38/3	0	0	4/3	130
Z_j - C_j		0	0	38/3	0	M	M+4/3	

The current solution (x, y) = (9, 10) is optimal since all the numbers in the "Z_j - C_j" row are positive or zero. In conclusion, the optimal solution is $(x^*, y^*) = (9, 10)$ and the optimal objective function value is $Z^* = 130$.

(2) Running Solver to solve the LP, we obtain the following Answer Report:

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$A$7	z	0	130

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$7	x	0	9	Contin
$C$7	y	0	10	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$10	LHS	9	$D$10<=$F$10	Binding	0
$D$11	LHS	145	$D$11>=$F$11	Not Binding	70
$D$12	LHS	12	$D$12=$F$12	Binding	0

9. Running Solver to solve the LP, we obtain the following Answer Report:

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$7	Z	0	74

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$7	X1	0	6	Contin
$E$7	X2	0	3	Contin
$F$7	X3	0	13	Contin
$G$7	X4	0	9	Contin
$H$7	X5	0	8	Contin
$I$7	X6	0	26	Contin
$J$7	X7	0	8	Contin
$K$7	X8	0	1	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$L$10	LHS	15	$L$10>=$N$10	Binding	0
$L$11	LHS	10	$L$11>=$N$11	Binding	0
$L$12	LHS	22	$L$12>=$N$12	Binding	0
$L$13	LHS	25	$L$13>=$N$13	Binding	0
$L$14	LHS	30	$L$14>=$N$14	Binding	0
$L$15	LHS	43	$L$15>=$N$15	Binding	0
$L$16	LHS	42	$L$16>=$N$16	Binding	0
$L$17	LHS	35	$L$17>=$N$17	Binding	0

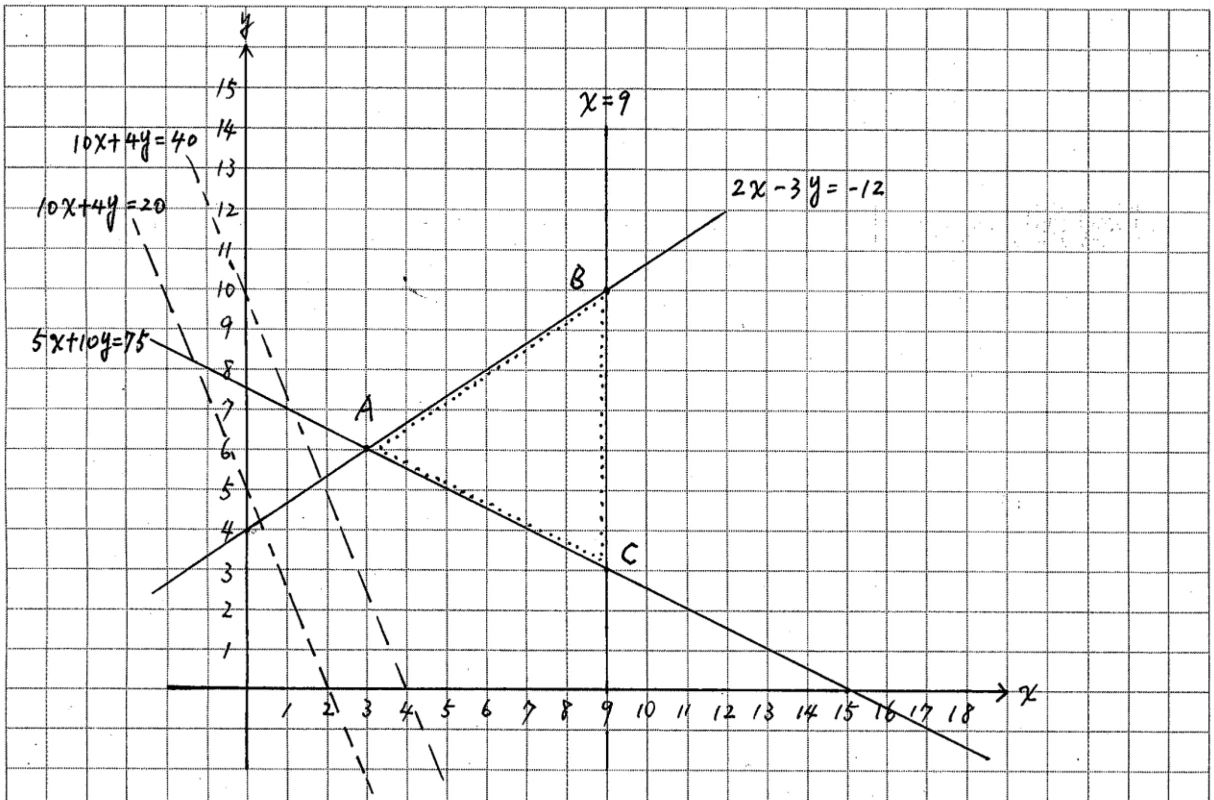
It is seen that the optimal solution is ($X1^*$, $X2^*$, $X3^*$, $X4^*$, $X5^*$, $X6^*$, $X7^*$, $X8^*$) = (6, 3, 13, 9, 8, 26, 8, 1) and the optimal objective function value is $Z^* = 74$. In other words, the nurses should be scheduled as follows to minimize the total number at 74:

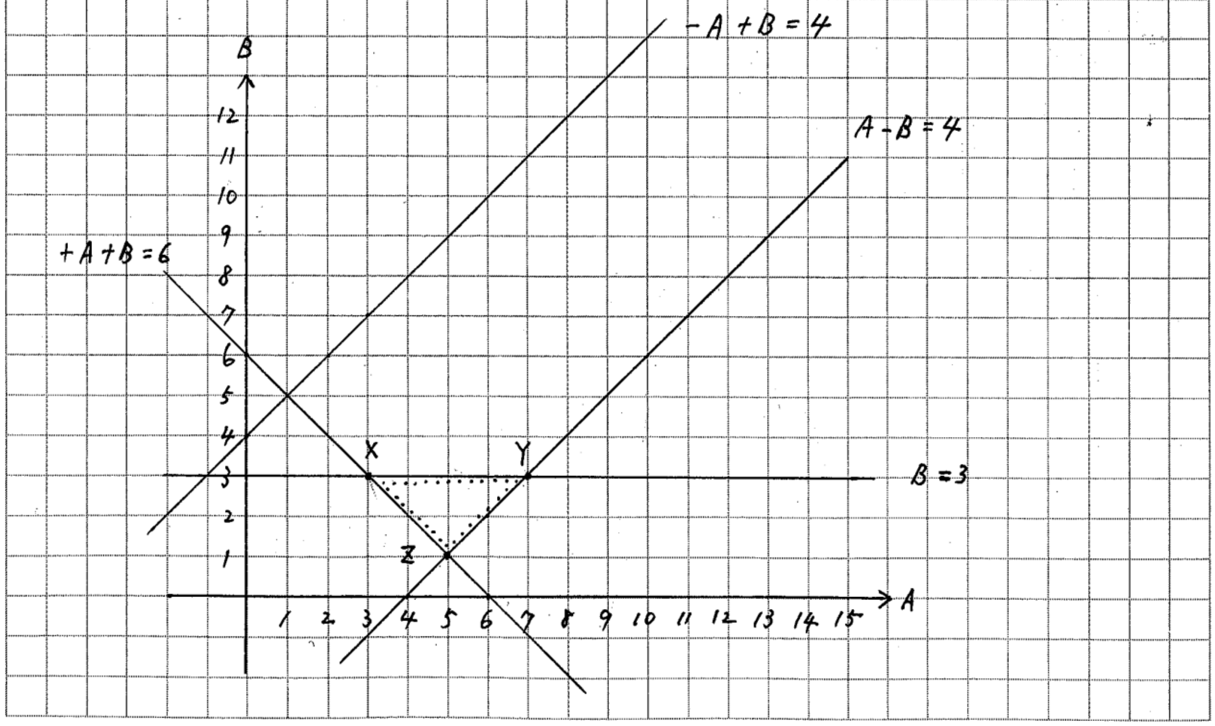
Segment number	1	2	3	4	5	6	7	8
Number of nurses	6	3	13	9	8	26	8	1

10. (1) The LP has multiple optimal solutions.

(2) The LP does not have an optimal solution.

(3) The LP has an unbounded optimal solution.


y
x
x = 9
10x + 4y = 40
10x + 4y = 20
2x − 3y = −12
5x + 10y = 75
A
B
C


B
A
−A + B = 4
A − B = 4
+A + B = 6
B = 3
X
Y
Z