Prescriptive Analytics - HW 1

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1. Reading Completed

2. Summary of the information provided in the following table:

Discovery	**Weight (pounds)**	**Volume (cubic feet)**	**Unit Price**
Coconut	5	1/8	40
Lion Skin	12	1	250
Availability	300	11	

Decision Variables:

x = Number of coconuts that can be brought back from the trip
y = Number of lion skins that can be brought back from the trip

Objective Function:

$Z = f(x,y) = 40x + 250y$

Constraints:

Structural Constraints:

$5x + 12y \leq 300$
$x/8 + y \leq 11$

Non-Negativity Constraints:

$x \geq 0$
$y \geq 0$

Final LP Model:

Maximize $Z = 40x + 250y$
Subject to: $5x + 12y \leq 300$,
$x/8 + y \leq 11$,
$x, y \geq 0$

3. Summary of the information provided in the following table:

Nutrients	**Cottage Cheese (milligrams)**	**Scrambled Eggs (milligrams)**	**Minimum Requirement**
Vitamin E	1.5	2	23
Iron	5	7	49
Cost per scoop (dollars)	0.26	0.22	

Decision Variables:

x = Scoops of cottage cheese

y = Scoops of scrambled eggs

Objective Function:

$Z = f(x,y) = 0.26x + 0.22y$

Constraints:

Structural Constraints:

$1.5x + 2y \geq 23$

$5x + 7y \leq 49$

$x \geq 4$

$x \leq 8$

Non-Negativity Constraints:

$x \geq 0$

$y \geq 0$

Final LP Model:

$$
\begin{aligned}
\text{Minimize } Z = \; & 0.26x + 0.22y \\
\text{Subject to:} \quad & 1.5x + 2y \geq 23, \\
& 5x + 7y \leq 49, \\
& x \geq 4, \\
& x \leq 8, \\
& x, y \geq 0
\end{aligned}
$$

4. Summary of the information provided in the following table:

Bank	**Interest Rates (%)**				**Maximum Credit (dollars)**
	A_1	A_2	A_3	A_4	
B_1	11	8	10	11	130,000
B_2	8	9	10	8	80,000
B_3	12	10	10	9	120,000
Selling Price (dollars)	50,000	60,000	70,000	130,000	

Decision Variables:

B_1A_1 = Interest Rate provided by bank B_1 for Apartment A_1
B_1A_2 = Interest Rate provided by bank B_1 for Apartment A_2
B_1A_3 = Interest Rate provided by bank B_1 for Apartment A_3
B_1A_4 = Interest Rate provided by bank B_1 for Apartment A_4
B_2A_1 = Interest Rate provided by bank B_2 for Apartment A_1
B_2A_2 = Interest Rate provided by bank B_2 for Apartment A_2
B_2A_3 = Interest Rate provided by bank B_2 for Apartment A_3
B_2A_4 = Interest Rate provided by bank B_2 for Apartment A_4
B_3A_1 = Interest Rate provided by bank B_3 for Apartment A_1
B_3A_2 = Interest Rate provided by bank B_3 for Apartment A_2
B_3A_3 = Interest Rate provided by bank B_3 for Apartment A_3
B_3A_4 = Interest Rate provided by bank B_3 for Apartment A_4

Objective Function:

$$Z = f(B_1A_1,B_1A_2,B_1A_3,B_1A_4,B_2A_1,B_2A_2,B_2A_3,B_2A_4,B_3A_1,B_3A_2,B_3A_3,B_3A_4)$$
$$= 11B_1A_1 + 8B_1A_2 + 10B_1A_3 + 11B_1A_4 + 8B_2A_1 + 9B_2A_2$$
$$+ 10B_2A_3 + 8B_2A_4 + 12B_3A_1 + 10B_3A_2 + 10B_3A_3 + 9B_3A_4$$

Constraints:

Structural Constraints:

$$B_1A_1 + B_1A_2 + B_1A_3 + B_1A_4 \leq 130000$$
$$B_2A_1 + B_2A_2 + B_2A_3 + B_2A_4 \leq 80000$$
$$B_3A_1 + B_3A_2 + B_3A_3 + B_3A_4 \leq 120000$$
$$B_1A_1 + B_2A_1 + B_3A_1 \leq 50000$$
$$B_1A_2 + B_2A_2 + B_3A_2 \leq 60000$$

$$B_1A_3 + B_2A_3 + B_3A_3 \leq 70000$$
$$B_1A_4 + B_2A_4 + B_3A_4 \leq 130000$$

Non-Negativity Constraints:

$$B_1A_1, B_1A_2, B_1A_3, B_1A_4, B_2A_1, B_2A_2,$$
$$B_2A_3, B_2A_4, B_3A_1, B_3A_2, B_3A_3, B_3A_4 \quad \geq 0$$

Final LP Model:

$$\begin{aligned}
\text{Minimize } Z \quad &= \quad 11B_1A_1 + 8B_1A_2 + 10B_1A_3 + 11B_1A_4 + 8B_2A_1 + 9B_2A_2 \\
&+ \quad 10B_2A_3 + 8B_2A_4 + 12B_3A_1 + 10B_3A_2 + 10B_3A_3 + 9B_3A_4 \\
\text{Subject to:} \quad & B_1A_1 + B_1A_2 + B_1A_3 + B_1A_4 \leq 130000, \\
& B_2A_1 + B_2A_2 + B_2A_3 + B_2A_4 \leq 80000, \\
& B_3A_1 + B_3A_2 + B_3A_3 + B_3A_4 \leq 120000, \\
& B_1A_1 + B_2A_1 + B_3A_1 \leq 50000, \\
& B_1A_2 + B_2A_2 + B_3A_2 \leq 60000, \\
& B_1A_3 + B_2A_3 + B_3A_3 \leq 70000, \\
& B_1A_4 + B_2A_4 + B_3A_4 \leq 130000, \\
& B_1A_1, \ldots B_3A_4 \geq 0
\end{aligned}$$

5. Summary of the information provided in the following table:

Segment Number	**Time**	**Number of nurses required**
1	12:00 AM - 3:00 AM	15
2	3:00 AM - 6:00 AM	10
3	6:00 AM - 9:00 AM	22
4	9:00 AM - 12:00 PM	25
5	12:00 PM - 3:00 PM	30
6	3:00 PM - 6:00 PM	43
7	6:00 PM - 9:00 PM	42
8	9:00 PM - 12:00 AM	35

Decision Variables:

Let x_i be the number of nurses required at Segment i

Objective Function:

$Z = f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$

$= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

Constraints:

Structural Constraints:

$x_1 + x_8 \geq 15$

$x_1 + x_2 \geq 10$

$x_2 + x_3 \geq 22$

$x_3 + x_4 \geq 25$

$x_4 + x_5 \geq 30$

$x_5 + x_6 \geq 43$

$x_6 + x_7 \geq 42$

$x_7 + x_8 \geq 35$

Non-Negativity Constraints:

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$

Final LP Model:

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$

Subject to:

$$
\begin{aligned}
x_1 + x_8 &\geq 15 \\
x_1 + x_2 &\geq 10 \\
x_2 + x_3 &\geq 22 \\
x_3 + x_4 &\geq 25 \\
x_4 + x_5 &\geq 30 \\
x_5 + x_6 &\geq 43 \\
x_6 + x_7 &\geq 42 \\
x_7 + x_8 &\geq 35
\end{aligned}
$$

6. The x and y values are formulated below:

$5x + 10y \geq 75$	(15, 0) (0, 7.5)
$2x - 3y \geq -12$	(-6, 0) (0, 4)
$x \geq 9$	(9, 0)

The iso-profit lines are formulated below:

$10x + 4y = 40$	(4, 0) (0, 10)
$10x + 4y = 20$	(2, 0) (0, 5)

The corner points:

$5x + 10y = 75$	Eq1 (x2)
$2x - 3y = -12$	Eq2 (x-5)

Eq1-Eq2: $10x + 20y - 10x + 15y = 150 + 60$

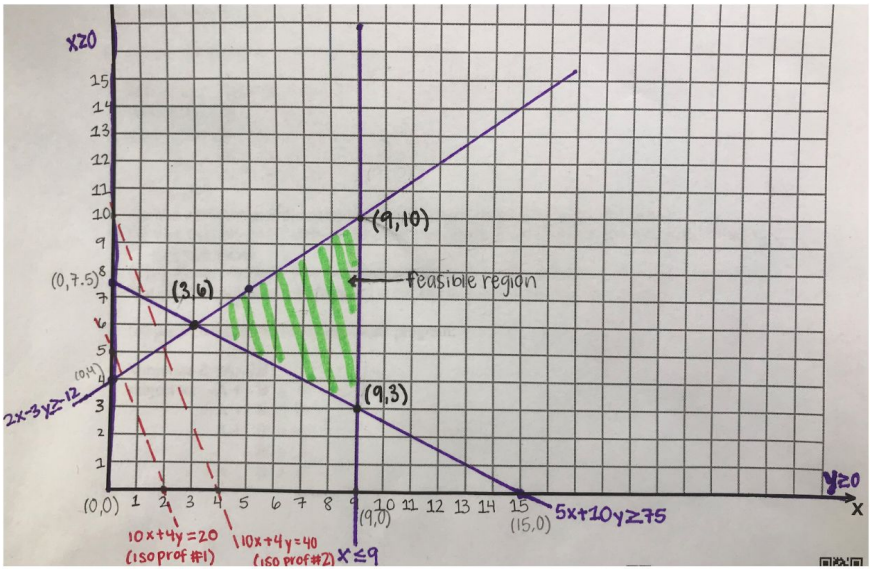
$35y = 210$

$y = 6$

$x = 3$

$2x - 3y = -12$	(Given, x = 9)
$y = 10$	

$5x + 10y = 75$	(Given, x = 9)
$y = 3$	



Optimal Objective Calculation:

$10(3) + 4(6) = 30 + 24 = 54$	(3, 6)
$10(9) + 4(3) = 90 + 12 = 102$	(9, 3)
$10(9) + 4(10) = 90 + 40 = 130$	(9, 10)

Optimal Solution: (9, 10)
Optimal Function Value: 130

7. The A and B values are formulated below:

$-A + B \leq 4$	(-4, 0) (0, 4)
$A - B \leq 4$	(4, 0) (0, -4)
$A + B \geq 6$	(6, 0) (0, 6)
$B \leq 3$	(0, 3)

The iso-profit lines are formulated below:

$A - 2B = 10$	(10, 0) (0, -5)
$A - 2B = 6$	(6, 0) (0, -3)

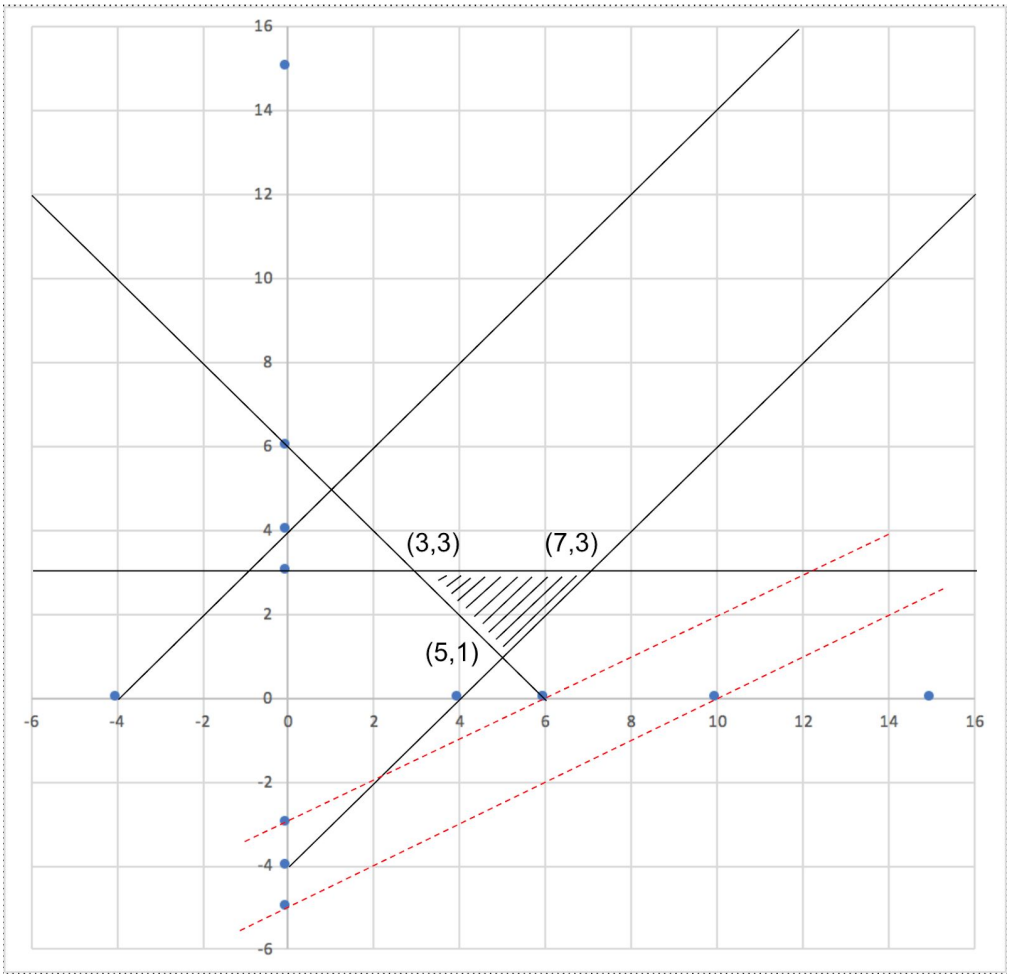
The corner points:

$A - B = 4$
$A + B = 6$

$A = 5, B = 1$

$A + B = 6$	(Given, B = 3)
$A = 3$	

$A - B = 4$	(Given, B = 3)
$A = 7$	



Optimal Objective Calculation:

(3) - 2(3) = -3	(3, 3)
(5) - 2(1) = 3	(5, 1)
(7) - 2(3) = 1	(7, 3)

Optimal Solution: (3, 3)
Optimal Function Value: -3

8. The LP is modified as follows:

(Max) $Z = 10x + 4y + 0C + 0D - ME + MF$

$x \leq 9 \quad \Rightarrow \quad x + C + 0D + 0E + 0F = 9$

$5x + 10y \geq 75 \quad \Rightarrow \quad 5x + 10y + 0C - D + E + 0F = 75$

$-2x + 3y = 12 \quad \Rightarrow \quad -2x + 3y + 0C + 0D + 0E + F = 12$

The initial simplex tableau is as follows:

$\mathbf{C_j}$		10	4	0	0	-M	-M	**SQ**	
VIS		x	y	C	D	E	F		
0	C	1	0	1	0	0	0	9	9
-M	E	5	10	0	-1	1	0	75	75/10
-M	F	-2	3	0	0	0	1	12	4
Z_j		-3M	-13M	0	M	-M	-M		
$Z_j - C_j$		-3M - 10	-13M - 4	0	M	0	0		

The current solution is not optimal since $-13M - 4 < 0$ with y being the pivot column and F will leave since 4 has the least ratio.

$\mathbf{C_j}$		10	4	0	0	-M	-M	**SQ**	
VIS		x	y	C	D	E	F		
0	C	1	0	1	0	0	0	9	9
-M	E	35/3	0	0	-1	1	-10/3	35	3
4	y	-2/3	1	0	0	0	1/3	4	
Z_j		-35M-8/3	4	0	M	-M	10M+4/3		
$Z_j - C_j$		-35M-38/3	0	0	M	0	7M+4/3		

The current solution is not optimal since $(-35M-38)/3 < 0$ with x being the pivot column and E will leave since 3 has the least ratio.

C_j		10	4	0	0	-M	-M	**SQ**	
VIS		x	y	C	D	E	F		
0	C	0	0	1	3/35	-3/35	10/35	6	70
10	x	1	0	0	-3/35	3/35	-10/35	3	
4	y	0	1	0	-2/35	2/35	15/105	6	
Z_j		10	4	0	-38/35	38/35	240/105	54	
$Z_j - C_j$		0	0	0	-38/35	35M+38/35	105M + 240/105		

The current solution is not optimal since -38/35 < 0 with D being the pivot column and C will leave since 70 has the least ratio.

C_j		10	4	0	0	-M	-M	**SQ**	
VIS		x	y	C	D	E	F		
0	D	0	0	35/3	1	-1	10/3	70	
10	x	1	0	1	0	0	0	9	
4	y	0	1	2/3	0	0	1/3	10	
Z_j		10	4	38/3	0	0	4/3	130	
$Z_j - C_j$		0	0	38/3	0	M	3M+4/3		

The current solution (10, 4) is the optimal solution with the optimal function value being 130.

Excel solver was used to verify this solution.

Microsoft Excel 16.16 Answer Report
Worksheet: [Ch 2- solver.xlsx]Ex 2.7
Report Created: 9/11/18 9:55:48 AM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 193274412.88 Seconds.
Iterations: 3 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Solve Without Integer Constraints, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$D$11	Max Z	130	130

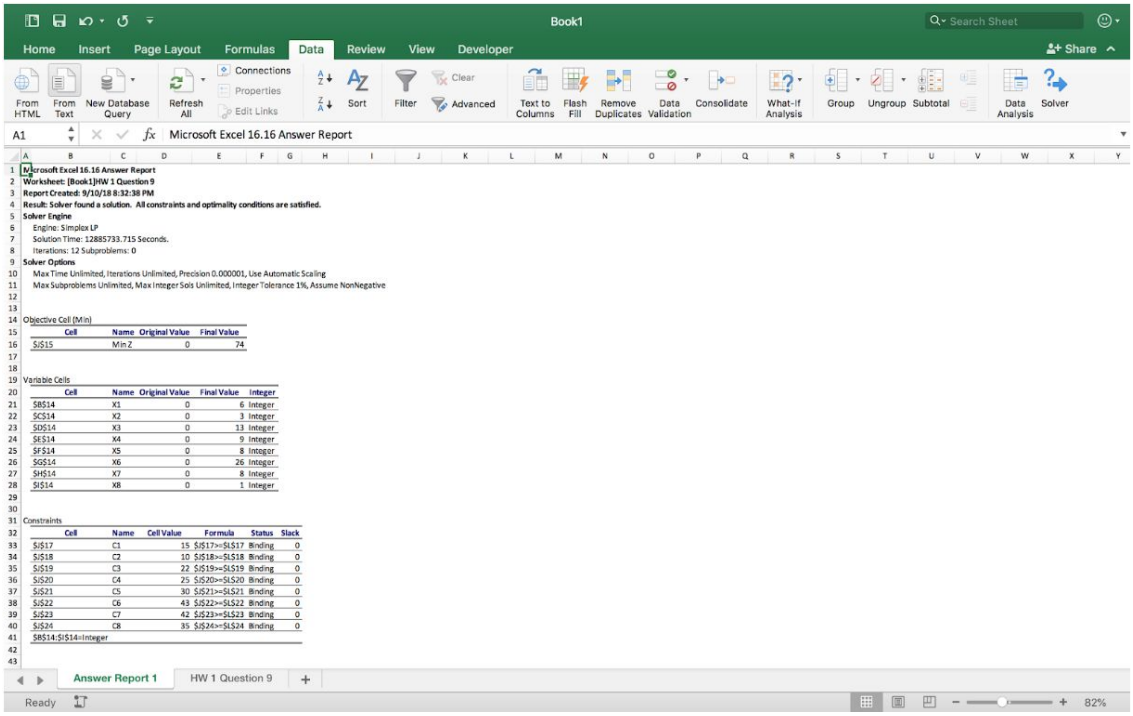
Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$10	x	9	9	Contin
$C$10	y	10	10	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$13	C1	145	$D$13>=$F$13	Not Binding	70
$D$14	C2	12	$D$14=$F$14	Binding	0
$D$15		9	$D$15<=$F$15	Binding	0

9. The solution was arrived at using solver.



Microsoft Excel 16.16 Answer Report
Worksheet: [Book1]HW 1 Question 9
Report Created: 9/10/18 8:32:38 PM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 12885733.715 Seconds.
Iterations: 12 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$I$15	Min Z	0	74

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$14	X1	0	6	Integer
$C$14	X2	0	3	Integer
$D$14	X3	0	13	Integer
$E$14	X4	0	9	Integer
$F$14	X5	0	8	Integer
$G$14	X6	0	26	Integer
$H$14	X7	0	8	Integer
$I$14	X8	0	1	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$17	C1	15	$J$17>=$L$17	Binding	0
$J$18	C2	10	$J$18>=$L$18	Binding	0
$J$19	C3	22	$J$19>=$L$19	Binding	0
$J$20	C4	25	$J$20>=$L$20	Binding	0
$J$21	C5	30	$J$21>=$L$21	Binding	0
$J$22	C6	43	$J$22>=$L$22	Binding	0
$J$23	C7	42	$J$23>=$L$23	Binding	0
$J$24	C8	35	$J$24>=$L$24	Binding	0

$B$14:$I$14=Integer

Answer Report 1 | HW 1 Question 9 | Ready | 82%

The total minimum amount of nurses needed is 74.

<u>Broken Down:</u>

At Segment 1, 6 nurses are reporting for duty.
At Segment 2, 3 nurses are reporting for duty.
At Segment 3, 13 nurses are reporting for duty.
At Segment 4, 9 nurses are reporting for duty.
At Segment 5, 8 nurses are reporting for duty.
At Segment 6, 26 nurses are reporting for duty.
At Segment 7, 8 nurses are reporting for duty.
At Segment 8, 1 nurse is reporting for duty.

10. a. Multiple Optimal Solution
    b. No Optimal Solution
    c. Unbounded Optimal Solution