

**OPRE 6398 Prescriptive Analytics
Homework 2**

**Due 02/15/19
(11:59 p.m.)**

Note: Your homework submission must be typewritten.

Show only the solutions and do not copy the problems in the submission.

1. Read Readings 3 and 4.
2. A small company in Houston, TX, produces two types of noodle: MR and ZT. Each ton of MR made calls for 20 hours of labor and 4 pounds of wheat, whereas each ton of ZT requires 10 hours of labor and 7 pounds of wheat germ. The firm has 90 hours of labor available and 28 pounds of wheat in stock for next week's production. In addition, management wants to make a total of at least one ton of noodle but the amount of ZT should be no more than three times of the amount of MR. The profit margins of MR and ZT are \$300 and \$350 per ton, respectively. It is assumed that all of the noodle to be made next week will be sold.

The decision variables and the LP model in Excel for the product-mix problem are presented below, where the last constraint $3MR - ZT \geq 0$ is converted from an initial constraint $ZT \leq 3MR$:

| MR = Number of tons of macaroni to be produced next week | | | | | |
|--|-----|-----|-----|----|----|
| ZT = Number of tons of ziti to be produced next week | | | | | |
| | 300 | 350 | | | |
| Z | MR | ZT | | | |
| 0 | | | | | |
| | | | LHS | | |
| | 20 | 10 | 0 | <= | 90 |
| | 4 | 7 | 0 | <= | 28 |
| | 1 | 1 | 0 | >= | 1 |
| | 3 | -1 | 0 | >= | 0 |

Both the Answer Report and the Sensitivity Report generated by Solver are shown below, where 1E+30 means ∞ . It is seen that the optimal solution is $(MR^*, ZT^*) = (3.5, 2)$ and the optimal objective function value is $Z^* = 1,750$. In other words, the optimal production plan is to make 3.5 tons of MR and 2 tons of ZT to maximize the total profit at \$1,750.

Microsoft Excel 12.0 Answer Report

Target Cell (Max)

| Cell | Name | Original Value | Final Value |
|--------|------|----------------|-------------|
| \$A\$7 | Z | 0 | 1750 |

Adjustable Cells

| Cell | Name | Original Value | Final Value |
|--------|------|----------------|-------------|
| \$B\$7 | MR | 0 | 3.5 |
| \$C\$7 | ZT | 0 | 2 |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---------|------|------------|------------------|-------------|-------|
| \$D\$9 | LHS | 90 | \$D\$9<=\$F\$9 | Binding | 0 |
| \$D\$10 | LHS | 28 | \$D\$10<=\$F\$10 | Binding | 0 |
| \$D\$11 | LHS | 5.5 | \$D\$11>=\$F\$11 | Not Binding | 4.5 |
| \$D\$12 | LHS | 8.5 | \$D\$12>=\$F\$12 | Not Binding | 8.5 |

Microsoft Excel 12.0 Sensitivity Report

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|--------|------|-------------|--------------|-----------------------|--------------------|--------------------|
| \$B\$7 | MR | 3.5 | 0 | 300 | 400 | 100 |
| \$C\$7 | ZT | 2 | 0 | 350 | 175 | 200 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---------|------|-------------|--------------|----------------------|--------------------|--------------------|
| \$D\$9 | LHS | 90 | 7 | 90 | 50 | 34 |
| \$D\$10 | LHS | 28 | 40 | 28 | 17 | 10 |
| \$D\$11 | LHS | 5.5 | 0 | 1 | 4.5 | 1E+30 |
| \$D\$12 | LHS | 8.5 | 0 | 0 | 8.5 | 1E+30 |

Answer the following questions in light of the two reports presented above without re-running Solver.

- (1) At optimality, how many pounds of wheat germ are unused? Explain your answer. (5 pts.)
- (2) At optimality, by how much does the total amount of noodle made exceed the minimum production level required? Explain your answer. (5 pts.)
- (3) One of the company's employees proposes to work overtime for the firm next week and wants to be paid at an hourly rate of \$10. Should his offer be accepted? Why or why not? (5 pts.)
- (4) If the amount of wheat available decreases by 5 pounds, what will the optimal total profit be and why? (5 pts.)
- (5) If the profit margin per ton of MR decreases by 35%, will the current optimal production plan remain optimal? Why or why not? (10 pts.)

- (6) Will the current optimal production plan change if the profit margin per ton of ZT increases to \$500? Why or why not? (10 pts.)

3. (60 pts.) Find the dual associated with each of the following problems:

a) Minimize $Z = 7x + 9y - 6z$

$$\begin{array}{rcll} \text{subject to:} & -8x + y + 25z & \geq & 17 \\ & 2x - 11z & = & 25 \\ & 14y + z & \leq & 191 \\ & 3x + 10y & \geq & 48 \\ & x & \leq & 0 \\ & y & \geq & 0 \\ & z & \text{UIS} & \end{array}$$

b) Minimize $z = 3x_1 + 2x_2 - 3x_3 + 4x_4$

$$\begin{array}{rcll} \text{subject to:} & x_1 - 2x_2 + 3x_3 + 4x_4 & \leq & 3 \\ & x_2 + 3x_3 + 4x_4 & \geq & -5 \\ & 2x_1 - 3x_2 - 7x_3 - 4x_4 & = & 2 \\ & x_1 \geq 0, & x_4 \leq 0, & \\ & x_2 \text{ and } x_3 \text{ unrestricted} & & \end{array}$$

c) Maximize $z = 2x_1 + x_2 + 3x_3 + x_4$

$$\begin{array}{rcll} \text{subject to:} & x_1 + x_2 + x_3 + x_4 & \leq & 5 \\ & 2x_1 - x_2 + 3x_3 & = & -4 \\ & x_1 - x_3 + x_4 & \geq & 1 \\ & x_1 \geq 0, & x_3 \geq 0, & \\ & x_2 \text{ and } x_4 \text{ unrestricted} & & \end{array}$$