Prescriptive Analytics - HW 2

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1. Readings completed

2. The answers are provided below:

    2.1. Since there is a slack of 0 associated with this constraint, this means that the constraint is binding. As a result, all of the wheat germ is used, and there's no leftover wheat germ.

    2.2. Management wants to make at least 1 ton of noodle. At optimality, 3.5 tons of MR and 2 tons of ZT are made. 3.5 + 2 = 5.5 tons of noodle are made in total. 5.5 - 1 = 4.5 tons of noodle are made in excess.

    2.3. While the constraint has an allowable increase of 50 which means that the number of hours can be increased, the profit from each increase in hour is $7. Therefore, the employee's offer of working for $10 per overtime hour should not be accepted.

    2.4. A decrease of 5 pounds of wheat is within the allowable decrease of 10 pounds. Therefore, the current shadow price of $40/pound remains valid, and the total profit will become $Z^* = \$1,750 - 40(5) = \$1,550$.

    2.5. A 35% decrease in the profit margin for MR means a decrease of 300 * 0.35 = 105 which is outside the allowable decrease range. Therefore, the current production plan will not remain optimal.

    2.6. Even if the profit margin of ZT increases from $350 to $500, the optimal production plan will remain the same since it is within the allowable increase for ZT. (500 - 350 = 150 < 175)

3. Minimize $Z = 7x + 9y - 6z$

subject to:
$-8x + y + 25z \geq 17$
$2x - 11z = 25$
$14y + z \leq 191$
$3x + 10y \geq 48$
$x \leq 0$
$y \geq 0$
z is UIS

3.1. Converting the variables to run through Solver:

Minimize $Z = -7xp + 9y - 6zp + 6zn$

subject to:
$8xp + y + 25zp - 25zn \geq 17$
$-2xp - 11zp + 11zn = 25$
$14y + zp - zn \leq 191$
$-3xp + 10y \geq 48$
$xp, y, zp, zn \geq 0$

3.2. Solver Solution:

	xp	y	zp	zn			
	31.37556561	14.21267	0	7.977376			
Min Z	-7	9	-6	6	-43.8507		
S.T							
C1	8	1	25	-25	65.78281	>=	17
C2	-2	0	-11	11	25	=	25
C3	0	14	1	-1	191	<=	191
C4	-3	10	0	0	48	>=	48

3.3. Optimal Solution:

$x = -31.37$
$y = 14.21$
$z = (zp-zn) = (0-7.97) = -7.97$

Optimal Objective Function Value:

$(7*-31.37)+(9*14.21)+(-6*-7.97) = -43.88$

4. The Duality problems are obtained as follows:

4.1. Minimize $Z = 3x1 + 2x2 - 3x3 + 4x4$

subject to:

$x1 - 2x2 + 3x3 + 4x4$	≤ 3
$x2 + 3x3 + 4x4$	≥ -5
$2x1 - 3x2 - 7x3 - 4x4$	$= 2$
$x1$	≥ 0
$x4$	≤ 0
$x2, x3$	UIS

Maximize $Z = 3y1 - 5y2 + 2y3$

subject to:

$y1 + 2y3$	≤ 3
$-2y1 + y2 - 3y3$	$= 2$
$3y1 + 3y2 - 7y3$	$= -3$
$4y1 + 4y2 - 4y3$	≥ 4
$y1$	≤ 0
$y2$	≥ 0
$y3$	UIS

4.2. Maximize $Z = 2x1 + x2 + 3x3 + x4$

subject to:

$x1 + x2 + x3 + x4$	≤ 5
$2x1 - x2 + 3x3$	$= -4$
$x1 - x3 + x4$	≥ 1
$x1$	≥ 0
$x3$	≥ 0
$x2, x4$	UIS

Minimize $Z = 5y1 - 4y2 + y3$

subject to:

$y1 + 2y2 + y3$	≥ 2
$y1 - y2$	$= 1$
$y1 + 3y2 - y3$	≥ 3
$y1 + y3$	$= 1$
$y1$	≥ 0
$y2$	UIS
$y3$	≤ 0