OPRE 6398 Prescriptive Analytics
Solutions to Homework 3

2. Let x = Number of ITC classes to be offered daily
   y = Number of CWP classes to be offered daily

   An AILP for the course scheduling problem follows:

$$
\begin{array}{lrcrcl}
\text{Maximize } Z = & 720x + & 300y & & & \\
\text{subject to:} & 7.5x + & 3y & \le & 56 & \\
& 6x + & 12y & \le & 100 & \\
& x\ , & y & \ge & 0 & \text{and integer}
\end{array}
$$

3. Let D = Number of 55-gallon drums of termite spray to be manufactured and sold
   B = Amount (in gallons) of termite spray in bulk form to be manufactured and sold

   An MILP for the production planning problem is presented below:

$$
\begin{array}{lrcrcl}
\text{Maximize } Z = & 50D + & 1.25B & & & \\
\text{subject to:} & 16.67D + & 0.44B & \le & 1{,}000 & \\
& 13.33D + & 0.25B & \le & 750 & \\
& 0.53D + & 0.04B & \le & 80 & \\
& D & & \ge & 0 & \text{and integer} \\
& & B & \ge & 0 &
\end{array}
$$

4. (1) Let xij be 1 if swimmer i is assigned to swim 100 meters of stroke j and 0 otherwise, i = GH, MS, JM, and CJ; j = FR, BR, BU, and BA. A ZOLP for the swimming race problem is given below:

$$
\begin{array}{lrrrrcl}
\text{Minimize } Z = & 54x11 + & 54x12 + & 51x13 + & 53x14 + & & \\
& 51x21 + & 57x22 + & 52x23 + & 52x24 + & & \\
& 50x31 + & 53x32 + & 54x33 + & 56x34 + & & \\
& 56x41 + & 54x42 + & 55x43 + & 53x44 & & \\
\text{subject to:} & x11 + & x12 + & x13 + & x14 & = & 1 \\
& x21 + & x22 + & x23 + & x24 & = & 1 \\
& x31 + & x32 + & x33 + & x34 & = & 1 \\
& x41 + & x42 + & x43 + & x44 & = & 1 \\
& x11 + & x21 + & x31 + & x41 & = & 1 \\
& x12 + & x22 + & x32 + & x42 & = & 1 \\
& x13 + & x23 + & x33 + & x43 & = & 1 \\
& x14 + & x24 + & x34 + & x44 & = & 1 \\
& x11\ , & x12\ , & \ldots\ , & x44 & = & 0 \text{ or } 1
\end{array}
$$

   (2) The Answer Report from Solver is shown below:

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$J$5	z	0	207

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$K$5	x11	0	0	Binary
$L$5	x12	0	0	Binary
$M$5	x13	0	1	Binary
$N$5	x14	0	0	Binary
$O$5	x21	0	0	Binary
$P$5	x22	0	0	Binary
$Q$5	x23	0	0	Binary
$R$5	x24	0	1	Binary
$S$5	x31	0	1	Binary
$T$5	x32	0	0	Binary
$U$5	x33	0	0	Binary
$V$5	x34	0	0	Binary
$W$5	x41	0	0	Binary
$X$5	x42	0	1	Binary
$Y$5	x43	0	0	Binary
$Z$5	x44	0	0	Binary

It is seen that the optimal solution is ($x11^*$, $x12^*$, $x13^*$, $x14^*$, $x21^*$, $x22^*$, $x23^*$, $x24^*$, $x31^*$, $x32^*$, $x33^*$, $x34^*$, $x41^*$, $x42^*$, $x43^*$, $x44^*$) = (0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0) and the optimal objective function value is $Z^* = 207$. In other words, GH, MS, JM, and CJ should be assigned to swim BU, BA, FR, and BR, respectively, to minimize the total team swimming time at 207 seconds.

5. (1) Let xi be 1 if a camera is set up at location i and 0 otherwise, i = 1, 2, …, 12. A ZOLP for the set covering problem is presented below:

Minimize Z =	x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 + x12			
subject to:	x1 + x4 + x12	≥	2	(Area 1)
	x3 + x4 + x8 + x11	≥	2	(Area 2)
	x1 + x5	≥	1	(Area 3)
	x1 + x2 + x11	≥	1	(Area 4)
	x3	≥	1	(Area 5)
	x1 + x5 + x9 + x11 + x12	≥	1	(Area 6)
	x1 + x2	≥	1	(Area 7)
	x2 + x6 + x11	≥	1	(Area 8)
	x3	≥	1	(Area 9)
	x5 + x8	≥	1	(Area 10)
	x3 + x9	≥	1	(Area 11)
	x2 + x5 + x12	≥	1	(Area 12)
	x3	≥	1	(Area 13)
	x5 + x6	≥	1	(Area 14)
	x6	≥	1	(Area 15)
	x6 + x8	≥	1	(Area 16)
	x6 + x12	≥	1	(Area 17)
	x4 + x7	≥	1	(Area 18)
	x4	≥	1	(Area 19)
	x10	≥	1	(Area 20)

x4 + x7		≥	1	(Area 21)
x10		≥	1	(Area 22)
x8		≥	1	(Area 23)
x7 + x10		≥	1	(Area 24)
x7 + x10		≥	1	(Area 25)
x9		=	1	(Blimp)
x1 , x2 ,	… , x12	=	0 or 1	

(2) The Answer Report from Solver is shown on the next page. It is seen that the optimal solution is (x_1^*, x_2^*, x_3^*, x_4^*, x_5^*, x_6^*, x_7^*, x_8^*, x_9^*, x_{10}^*, x_{11}^*, x_{12}^*) = (1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0) and the optimal objective function value is $Z^* = 8$. In other words, Fox should install a camera in each of locations 1, 2, 3, 4, 6, 8, 9, and 10 to minimize the total number of cameras needed for covering the NFL Super Bowl at 8.

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$A$20	z	0	8

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$20	x1	0	1	Binary
$C$20	x2	0	1	Binary
$D$20	x3	0	1	Binary
$E$20	x4	0	1	Binary
$F$20	x5	0	0	Binary
$G$20	x6	0	1	Binary
$H$20	x7	0	0	Binary
$I$20	x8	0	1	Binary
$J$20	x9	0	1	Binary
$K$20	x10	0	1	Binary
$L$20	x11	0	0	Binary
$M$20	x12	0	0	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$N$22	LHS	2	$N$22>=$P$22	Binding	0
$N$23	LHS	3	$N$23>=$P$23	Not Binding	1
$N$24	LHS	1	$N$24>=$P$24	Binding	0
$N$25	LHS	2	$N$25>=$P$25	Not Binding	1
$N$26	LHS	1	$N$26>=$P$26	Binding	0
$N$27	LHS	2	$N$27>=$P$27	Not Binding	1
$N$28	LHS	3	$N$28>=$P$28	Not Binding	2
$N$29	LHS	3	$N$29>=$P$29	Not Binding	2
$N$30	LHS	1	$N$30>=$P$30	Binding	0
$N$31	LHS	1	$N$31>=$P$31	Binding	0
$N$32	LHS	2	$N$32>=$P$32	Not Binding	1
$N$33	LHS	1	$N$33>=$P$33	Binding	0
$N$34	LHS	1	$N$34>=$P$34	Binding	0
$N$35	LHS	1	$N$35>=$P$35	Binding	0
$N$36	LHS	1	$N$36>=$P$36	Binding	0
$N$37	LHS	2	$N$37>=$P$37	Not Binding	1
$N$38	LHS	1	$N$38>=$P$38	Binding	0
$N$39	LHS	1	$N$39>=$P$39	Binding	0
$N$40	LHS	1	$N$40>=$P$40	Binding	0
$N$41	LHS	1	$N$41>=$P$41	Binding	0
$N$42	LHS	1	$N$42>=$P$42	Binding	0
$N$43	LHS	1	$N$43>=$P$43	Binding	0
$N$44	LHS	1	$N$44>=$P$44	Binding	0
$N$45	LHS	1	$N$45>=$P$45	Binding	0
$N$46	LHS	1	$N$46>=$P$46	Binding	0
$N$47	LHS	1	$N$47=$P$47	Binding	0
$B$20:$M$20=Binary					