Prescriptive Analytics - HW 3

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1. Readings completed

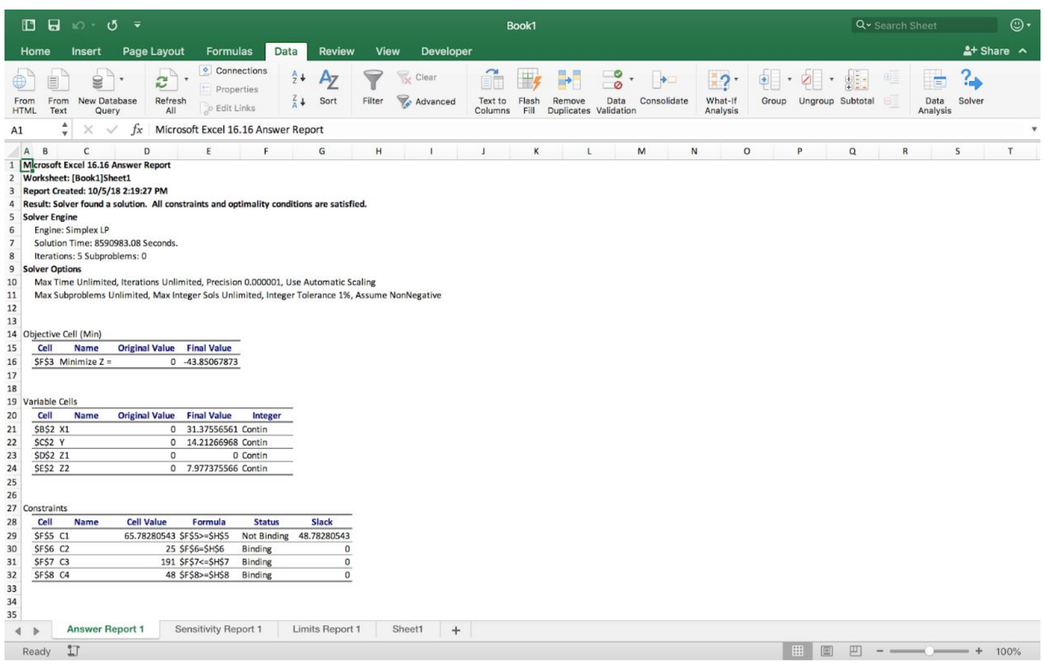
2.

   a. Minimize Z = $-7xp + 9y - 6zn + 6zp$

   Subject to:

$$
\begin{array}{lllll}
8xp + & y + & 25zn - & 25zp & \geq 17 \\
-2xp - & & 11zn + & 11zp & = 25 \\
& 14y + & zn - & zp & \leq 191 \\
-3xp + & 10y & & & \geq 48 \\
& & & x, y, z & \geq 0
\end{array}
$$

   b. Solver Solution:



Microsoft Excel 16.16 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 10/5/18 2:19:27 PM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 8590983.08 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$F$3	Minimize Z =	0	-43.85067873

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$2	X1	0	31.37556561	Contin
$C$2	Y	0	14.21266968	Contin
$D$2	Z1	0	0	Contin
$E$2	Z2	0	7.977375566	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$5	C1	65.78280543	$F$5>=$H$5	Not Binding	48.78280543
$F$6	C2	25	$F$6=$H$6	Binding	0
$F$7	C3	191	$F$7<=$H$7	Binding	0
$F$8	C4	48	$F$8>=$H$8	Binding	0

From the Answer report, we get:

xp = 31.38

y = 14.21

zp = 0
zn = 7.98
The optimal function value is -43.85

c. The optimal solution is as seen below:
x = -31.38
y = 14.21
z = -7.98

Optimal Objective Function Value:
(7*-31.38)+(9*14.21)+(-6*-7.98) = -43.89

d. Maximize Z = 17a + 25b + 191c + 48d

Subject to:	-8a +	2b +		3d	≥ 7
	a +		14c +	10d	≤ 9
	25a -	11b +	c		= -6
				a	≥ 0
				b	UIS
				c	≤ 0
				d	≥ 0

3. Decision variables:
   Let x be the number of ITC courses to be taken in a day.
   Let y be the number of CWS courses to be taken in a day.

AILP:

Maximize Z = 720x + 300y

Subject to:	7.5x +	3y	≤ 56
	6x +	12y	≤ 100
	x, y		≥ 0 and integer

4. Decision variables:
   Let x be the number of drums produced.
   Let y be the number of gallons produced for bulk-orders.

MILP:

Maximize Z = 50x + 1.25y

Subject to:	$16.67x + 0.44y$	≤ 1000
	$13.33x + 0.25y$	≤ 750
	$0.53x + 0.04y$	≤ 80
	x	≥ 0 and integer
	y	≥ 0

5. Decision variables:

   Let x_{ij} be the variables with i = 1, 2, 3, 4 representing the swimmers Gary Hall, Mark Spitz, Jim Mount and Chuck Johnson and j = 1, 2, 3, 4 representing the four different swim strokes; freestyle, breaststroke, butterfly and backstroke for the competition.

   a. ZOLP:

$$
\begin{aligned}
\text{Minimize } Z &= 54x_{11} + 54x_{12} + 51x_{13} + 53x_{14} + 51x_{21} + 57x_{22} \\
&\quad + 52x_{23} + 52x_{24} + 50x_{31} + 53x_{32} + 54x_{33} + 56x_{34} \\
&\quad + 56x_{41} + 54x_{42} + 55x_{43} + 53x_{44}
\end{aligned}
$$

Subject to:	x11 + x12 + x13 + x14	= 1
	x21 + x22 + x23 + x24	= 1
	x31 + x32 + x33 + x34	= 1
	x41 + x42 + x43 + x44	= 1
	x11 + x21 + x31 + x41	= 1
	x12 + x22 + x32 + x42	= 1
	x13 + x23 + x33 + x43	= 1
	x14 + x24 + x34 + x44	= 1
	x11, x12, x12, x14, x21, x22, x23, x24	
	x31, x32, x33, x34, x41, x42, x43, x44	= 0 or 1

b. Solver Solution:

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$S$16		0	207

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$15	x11	0	0	Binary
$D$15	x12	0	0	Binary
$E$15	x13	0	1	Binary
$F$15	x14	0	0	Binary
$G$15	x21	0	0	Binary
$H$15	x22	0	0	Binary
$I$15	x23	0	0	Binary
$J$15	x24	0	1	Binary
$K$15	x31	0	1	Binary
$L$15	x32	0	0	Binary
$M$15	x33	0	0	Binary
$N$15	x34	0	0	Binary
$O$15	x41	0	0	Binary
$P$15	x42	0	1	Binary
$Q$15	x43	0	0	Binary
$R$15	x44	0	0	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$S$17	C1	1	$S$17=$U$17	Binding	0
$S$18	C2	1	$S$18=$U$18	Binding	0
$S$19	C3	1	$S$19=$U$19	Binding	0
$S$20	C4	1	$S$20=$U$20	Binding	0
$S$21	C5	1	$S$21=$U$21	Binding	0
$S$22	C6	1	$S$22=$U$22	Binding	0
$S$23	C7	1	$S$23=$U$23	Binding	0
$S$24	C8	1	$S$24=$U$24	Binding	0
$C$15:$R$15=Binary					

The optimal solution for x_{ij} = (0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0) which translates to Gary Hall should compete in butterfly, Mark Spitz should compete in backstroke, Jim Mount should compete in freestyle and Chuck Johnson should compete in breaststroke. This will give them the optimal time of 207 seconds.

6. Decision variables:

   Let x_i be 1 if a camera covers the stadium area j and 0 otherwise,
   where i = 1, 2, … 12
   and j = 1, 2, … 25

a. A ZOLP for the stadium coverage problem is presented below:

$$
\begin{array}{lll}
\text{Minimize } Z & = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 & \\
 & + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} & \\
\text{Subject to:} & x_1 + x_4 + x_{12} & \geq 2 \\
 & x_3 + x_4 + x_8 + x_{11} & \geq 2 \\
 & x_1 + x_5 & \geq 1 \\
 & x_1 + x_2 + x_{11} & \geq 1 \\
 & x_3 & \geq 1 \\
 & x_1 + x_5 + x_9 + x_{11} + x_{12} & \geq 1 \\
 & x_1 + x_2 & \geq 1 \\
 & x_2 + x_6 + x_{11} & \geq 1 \\
 & x_5 + x_8 & \geq 1 \\
 & x_3 + x_9 & \geq 1 \\
 & x_2 + x_5 + x_{12} & \geq 1 \\
 & x_5 + x_6 & \geq 1 \\
 & x_6 & \geq 1 \\
 & x_6 + x_8 & \geq 1 \\
 & x_6 + x_{12} & \geq 1 \\
 & x_4 + x_7 & \geq 1 \\
 & x_4 & \geq 1 \\
 & x_{10} & \geq 1 \\
 & x_4 + x_7 & \geq 1 \\
 & x_8 & \geq 1 \\
 & x_7 + x_{10} & \geq 1 \\
 & x_9 & = 1 \\
 & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}, x_{11}, x_{12} = 0 \text{ or } 1 &
\end{array}
$$

b. Excel Table:

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12			
	1	1	1	1	0	1	0	1	1	1	0	0			
	1	1	1	1	1	1	1	1	1	1	1	1	8		
C1	1			1								1	2	>=	2
C2			1	1				1			1		3	>=	2
C3	1				1								1	>=	1
C4	1	1									1		2	>=	1
C5			1										1	>=	1
C6	1				1				1		1	1	2	>=	1
C7	1	1											2	>=	1
C8		1				1					1		2	>=	1
C9					1			1					1	>=	1
C10			1						1				2	>=	1
C11		1			1							1	1	>=	1
C12					1	1							1	>=	1
C13						1							1	>=	1
C14						1		1					2	>=	1
C15						1						1	1	>=	1
C16				1			1						1	>=	1
C17				1									1	>=	1
C18										1			1	>=	1
C19				1			1						1	>=	1
C20								1					1	>=	1
C21							1			1			1	>=	1
C22									1				1	=	1

Solver Solution:

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$O$15		0	8

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$14	x1	0	1	Binary
$D$14	x2	0	1	Binary
$E$14	x3	0	1	Binary
$F$14	x4	0	1	Binary
$G$14	x5	0	0	Binary
$H$14	x6	0	1	Binary
$I$14	x7	0	0	Binary
$J$14	x8	0	1	Binary
$K$14	x9	0	1	Contin
$L$14	x10	0	1	Binary
$M$14	x11	0	0	Binary
$N$14	x12	0	0	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$O$16	C1	2	$O$16>=$Q$16	Binding	0
$O$17	C2	3	$O$17>=$Q$17	Not Binding	1
$O$18	C3	1	$O$18>=$Q$18	Binding	0
$O$19	C4	2	$O$19>=$Q$19	Not Binding	1
$O$20	C5	1	$O$20>=$Q$20	Binding	0
$O$21	C6	2	$O$21>=$Q$21	Not Binding	1
$O$22	C7	2	$O$22>=$Q$22	Not Binding	1
$O$23	C8	2	$O$23>=$Q$23	Not Binding	1
$O$24	C9	1	$O$24>=$Q$24	Binding	0
$O$25	C10	2	$O$25>=$Q$25	Not Binding	1
$O$26	C11	1	$O$26>=$Q$26	Binding	0
$O$27	C12	1	$O$27>=$Q$27	Binding	0
$O$28	C13	1	$O$28>=$Q$28	Binding	0
$O$29	C14	2	$O$29>=$Q$29	Not Binding	1
$O$30	C15	1	$O$30>=$Q$30	Binding	0
$O$31	C16	1	$O$31>=$Q$31	Binding	0
$O$32	C17	1	$O$32>=$Q$32	Binding	0
$O$33	C18	1	$O$33>=$Q$33	Binding	0
$O$34	C19	1	$O$34>=$Q$34	Binding	0
$O$35	C20	1	$O$35>=$Q$35	Binding	0
$O$36	C21	1	$O$36>=$Q$36	Binding	0
$O$37	C22	1	$O$37=$Q$37	Binding	0
$C$14:$J$14=Binary					
$L$14:$N$14=Binary					

The optimal solution for x_i = (1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0) which satisfies all the viewer requirements including camera location 9, the ‘blimp’ being covered as well as both the locker rooms (stadium areas 1 and 2) being covered by 2 cameras each. In order to achieve this, 8 cameras need to be used (optimal function value).