

Prescriptive Analytics - HW 6

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1. Reading completed
2. Arrival rate (λ): 5 cars/hr
Service time per car ($1/\mu$): 6 mins
Service rate (μ): 10 cars/hr

$$\begin{aligned}L_s &= \lambda / (\mu - \lambda) \\&= 5 / (10 - 5) \\&= 1\end{aligned}$$

The number of cars in the queue is 0.5 per hour which is less than 4 cars per hour. The number of cars in the system is 1 per hour which is also less than 4 cars per hour. Because of this, there will never be a car parked in the no-parking area, so no cars would be fined.

3. Arrival rate (λ): 20 customers/hr
Service time per car ($1/\mu$): 2 mins/customer
Service rate (μ): 30 customers/hr

$$\begin{aligned}\text{a. } P(T \leq 3) &= 1 - e^{-2(3)} = 0.9975 \\P(T \leq 6) &= 1 - e^{-2(6)} = 0.9999\end{aligned}$$

$$P(T \leq 6) - P(T \leq 3) = 0.0024$$

- b. Noontime arrival rate (λ): $60/2.5 = 24$ customers/hr

$$\begin{aligned}P_{>5} &= e^{(\lambda - \mu)t} \\&= e^{(24 - 30)(5/60)} \\&= 0.60653\end{aligned}$$

$$\begin{aligned}P_{<5} &= 1 - P_{>5} \\&= 1 - 0.60653\end{aligned}$$

$$= 0.39347$$

$$0.39347 = 1 - e^{-5\mu}$$

$$\mu = 90/60$$

$$= 1.5 \text{ mins/customer}$$

4. Arrival rate (λ): 17 flights/hr
 Service time per flight ($1/\mu$): 3 mins/flight
 Service rate (μ): 20 flights/hr

Average fuel consumption for flight in air = 10 liters/min
 Fuel cost = \$20/liter

$$\begin{aligned} \text{a. } L_q &= \lambda^2 / [\mu(\mu - \lambda)] \\ &= 17^2 / [20(20 - 17)] \\ &= 289 / (20 \cdot 3) \\ &= 4.8167 \text{ flights} \end{aligned}$$

On average, there are 4.82 flights waiting for permission to land.

$$\begin{aligned} \text{b. } W_q &= L_q / \lambda \\ &= 4.8167 / 17 \\ &= 0.2833 \text{ hrs / flight} \end{aligned}$$

On average, the fuel cost for the flight waiting to land
 $= (0.2833 \cdot 60) \cdot (10 \cdot \$20)$
 $= \$3399.60$

$$\begin{aligned} \text{c. } P_{<n} &= 1 - P_{>n} \\ &= 1 - \rho^{n+1} \\ &= 1 - (17/20)^4 \\ &= 0.4779 \end{aligned}$$

$$\begin{aligned} \text{d. } \rho &= \lambda / \mu \\ &= 0.85 \end{aligned}$$

5. Arrival rate (λ): 3 letters/hr
 Service rate (μ): 4 letters/hr

a. One secretary: $W_s = 1 / (4-3)$
 $= 1 \text{ hr}$

Pooled Secretaries $= 1 / (8-6)$
 $= 0.5 \text{ hrs}$

The Professors would benefit by pooling the secretaries.

- b. It reduces the wait time significantly for Professor A & B, thus allowing their secretaries to become more efficient.