OPRE 6398 Prescriptive Analytics
Solutions to Homework 5

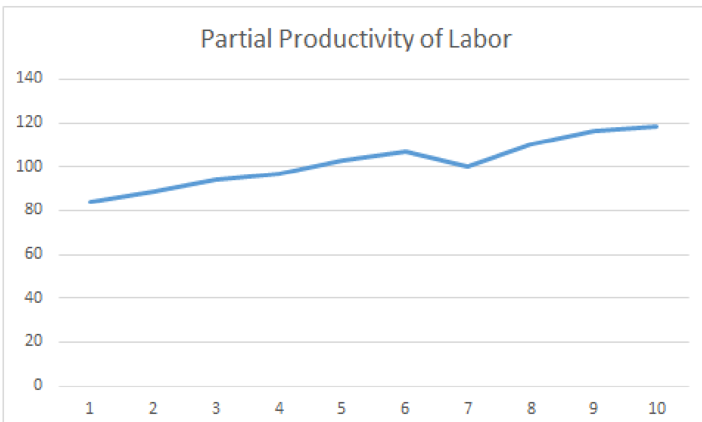
2. (1) $F_6 = A_5 = 1{,}390{,}000$ units

   (2) $F_6 = (1{,}390{,}000 + 1{,}168{,}500 + 1{,}198{,}400)/3 = 3{,}756{,}900/3 = 1{,}252{,}300$ units

   (3) $F_6 = 0.4(1{,}390{,}000) + 0.3(1{,}168{,}500) + 0.15(1{,}198{,}400) + 0.1(1{,}545{,}200) + 0.05(1{,}356{,}800) = 1{,}308{,}670$ units

   (4) Given $F_3 = 1{,}146{,}400$ and $\alpha = 0.8$, we have $F_4 = 1{,}146{,}400 + 0.8(1{,}198{,}400 - 1{,}146{,}400) = 1{,}188{,}000$, $F_5 = 1{,}188{,}000 + 0.8(1{,}168{,}500 - 1{,}188{,}000) = 1{,}172{,}400$, and $F_6 = 1{,}172{,}400 + 0.8(1{,}390{,}000 - 1{,}172{,}400) = 1{,}346{,}480$ units.

3. (1) Let Year 2003 = Period 1, Year 2004 = Period 2, and so on. A plotting of the data below reveals an upward linear rend in the time series. This observation justifies the use of the trend projection method.



   (2)

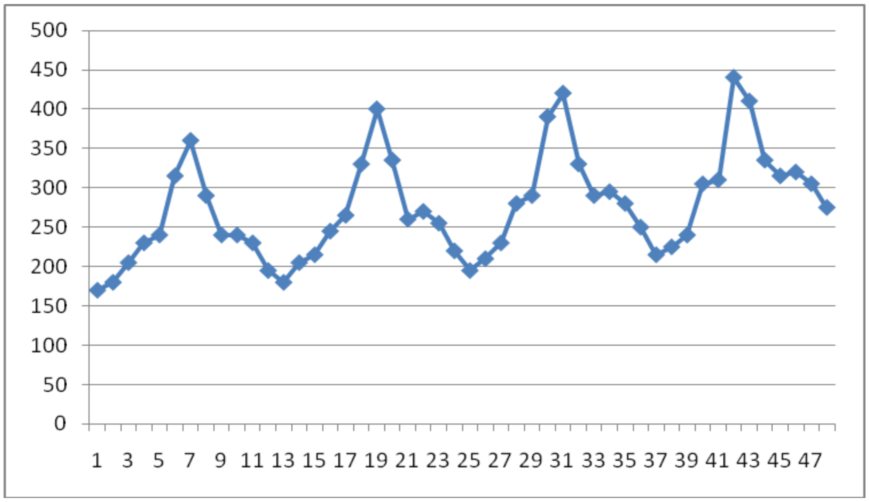
t	A_t	t^2	tA_t
1	84.0	1	84.0
2	88.6	4	177.2
3	93.8	9	281.4
4	97.1	16	388.4
5	102.9	25	514.5
6	107.0	36	642.0
7	100.0	49	700.0
8	110.2	64	881.6
9	116.2	81	1,045.8
10	118.2	100	1,182.0
55	1,018.0	385	5,896.9

$n = 10$
$b = [10(5{,}896.9) - 55(1{,}018.0)]/[10(385) - (55)^2] = 2{,}979/825 \approx 3.6109$
$a = [1{,}018.0 - 3.6109(55)]/10 = 819.4005/10 \approx 81.9401$

Thus, the linear trend model is $F_t = 81.9401 + 3.6109t$. It follows that the forecast of the partial productivity of labor for 2013 is $F_{11} = 81.9401 + 3.6109(11) = 121.66$.

(3) The slope of the best-fit trend line $F_t = 81.9401 + 3.6109t$ is $b = 3.6109$, which means that the U.S. partial productivity of labor increased by an average of 3.6109 per year during the time period of 2003 - 2012.

4. (1) Let January 2012 = Period 1, February 2012 = Period 2, …, December 2015 = Period 48. The time series is plotted below. It is seen that the air pollution levels increase with a seasonal pattern occurring on a monthly basis.



(2) The trend projection method with seasonal adjustments should be used to provide normal forecasts of air pollution levels in the future since the historical data exhibits both an upward trend and a monthly cycle.

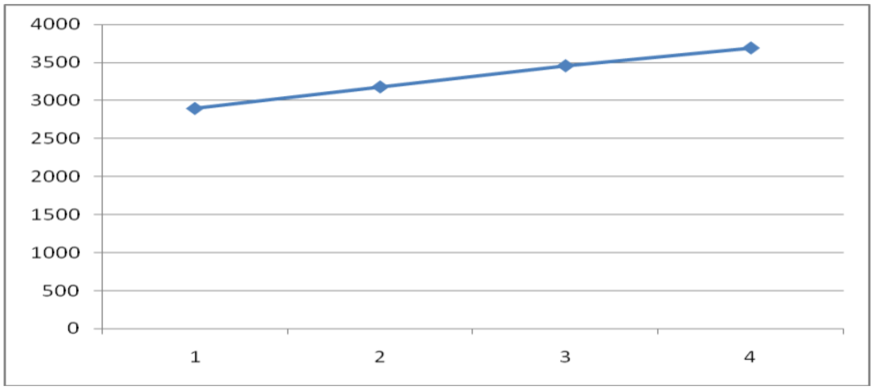
(3) To begin, the following table is prepared:

	Month												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
2012	172	178	207	228	242	313	362	288	242	238	232	193	2,895
2013	178	207	213	247	263	332	398	337	258	272	253	222	3,180
2014	197	208	232	278	292	388	422	328	292	293	282	248	3,460
2015	213	227	238	307	308	442	408	337	313	322	303	277	3,695
Total	760	820	890	1,060	1,105	1,475	1,590	1,290	1,105	1,125	1,070	940	13,230

The air pollution levels for the past four years are summarized below, Periods 1, 2, 3, and 4 represent Years 2012, 2013, 2014, and 2015, respectively

Period	1	2	3	4
Demand	2,895	3,180	3,460	3,695

A simple plotting of the annual data is shown below:



Since there is an upward linear trend present in the time series, the trend projection method should be used to provide normal forecasts in the future.

t	A_t	t^2	tA_t
1	2,895	1	2,895
2	3,180	4	6,360
3	3,460	9	10,380
4	3,695	16	14,780
10	13,230	30	34,415

$n = 4$

$$b = \frac{4(34{,}415) - 10(13{,}230)}{4(30) - (10)^2} = \frac{5{,}360}{20} = 268$$

$$a = \frac{13{,}230 - 268(10)}{4} = \frac{10{,}550}{4} = 2{,}637.5$$

Thus, the best-fit trend line is $F_t = 2{,}637.5 + 268t$. It follows that the normal forecast of the air pollution level in Year 2016 is $F_5 = 2{,}637.5 + 268(5) = 3{,}977.5$.

To compute the seasonal indexes, we see from the table at the beginning of (3) that $D_1 = 760$, $D_2 = 820$, …, $D_{12} = 940$, and $D_1 + D_2 + \ldots + D_{12} = 13{,}230$. It follows that

$$SI_1 = \frac{760}{13{,}230} \approx 0.0574,\ SI_2 = \frac{820}{13{,}230} \approx 0.0620,\ SI_3 = \frac{890}{13{,}230} \approx 0.0673,\ SI_4 = \frac{1{,}060}{13{,}230} \approx 0.0801,$$

$$SI_5 = \frac{1{,}105}{13{,}230} \approx 0.0835,\ SI_6 = \frac{1{,}475}{13{,}230} \approx 0.1115,\ SI_7 = \frac{1{,}590}{13{,}230} \approx 0.1202,\ SI_8 = \frac{1{,}290}{13{,}230} \approx 0.0975,$$

$$SI_9 = \frac{1{,}105}{13{,}230} \approx 0.0835,\ SI_{10} = \frac{1{,}125}{13{,}230} \approx 0.0850,\ SI_{11} = \frac{1{,}070}{13{,}230} \approx 0.0809,\ SI_{12} = \frac{940}{13{,}230} \approx 0.0711,$$

The seasonally adjusted forecasts of air pollution levels for the 12 months in 2016 are:

Jan: $SAF_1 = SI_1 \times F_5 = 0.0574 \times 3{,}977.5 \approx 228$

Feb: $SAF_2 = SI_2 \times F_5 = 0.0620 \times 3{,}977.5 \approx 247$

Mar: $SAF_3 = SI_3 \times F_5 = 0.0673 \times 3{,}977.5 \approx 268$

Apr: $SAF_4 = SI_4 \times F_4 = 0.0801 \times 3{,}977.5 \approx 319$

May: $SAF_5 = SI_5 \times F_5 = 0.0835 \times 3{,}977.5 \approx 332$

Jun: $SAF_6 = SI_6 \times F_5 = 0.1115 \times 3{,}977.5 \approx 444$

Jul: $SAF_7 = SI_7 \times F_5 = 0.1202 \times 3{,}977.5 \approx 478$

Aug: $SAF_8 = SI_8 \times F_5 = 0.0975 \times 3{,}977.5 \approx 388$

Sep: $SAF_9 = SI_9 \times F_5 = 0.0835 \times 3{,}977.5 \approx 332$

Oct: $SAF_{10} = SI_{10} \times F_5 = 0.0850 \times 3{,}977.5 \approx 338$

Nov: $SAF_{11} = SI_{11} \times F_5 = 0.0809 \times 3{,}977.5 \approx 322$

Dec: $SAF_{12} = SI_{12} \times F_5 = 0.0711 \times 3{,}977.5 \approx 283$

5 (1) The naive forecasts are $F_2 = 5{,}100$, $F_3 = 4{,}900$, ..., and $F_6 = 4{,}700$.

An application of the three-month simple moving average approach shows that $F_4 = (5{,}200 + 4{,}900 + 5{,}100)/3 \approx 5{,}066.6667$, $F_5 = (5{,}000 + 5{,}200 + 4{,}900)/3 \approx 5{,}033.3333$, and $F_6 = (4{,}700 + 5{,}000 + 5{,}200)/3 \approx 4{,}966.6667$.

Finally, based on the simple exponential smoothing method with $\alpha = 0.6$ and $F_1 = 5{,}100$, we have $F_2 = F_1 + \alpha(A_1 - F_1) = 5{,}100 + 0.2(5{,}100 - 5{,}100) = 5{,}100$, $F_3 = 5{,}100 + 0.2(4{,}900 - 5{,}100) = 5{,}060$, $F_4 = 5{,}060 + 0.2(5{,}200 - 5{,}060) = 5{,}088$, $F_5 = 5{,}088 + 0.2(5{,}000 - 5{,}088) = 5{,}070.4$, and $F_6 = 5{,}070.4 + 0.2(4{,}700 – 5{,}070.4) = 4{,}996.32$. All of the results are summarized below:

		Naïve		SMA		SES	
t	A_t	F_t	e_t	F_t	e_t	F_t	e_t
1	5,100					5,100.00	0.00
2	4,900	5,100	-200			5,100.00	-200.00
3	5,200	4,900	300			5,060.00	140.00
4	5,000	5,200	-200	5,066.6667	-66.6667	5,088.00	-88.00
5	4,700	5,000	-300	5,033.3333	-333.3333	5,070.40	-370.40
6	5,300	4,700	600	4,966.6667	333.3333	4,996.32	303.68

(2) Naive: MAD = $(|-200| + |300| + |-200| + |-300| + |600|)/5 = 1{,}600/5 = 320.0000$
SMA: MAD = $(|-66.6667| + |-333.3333| + |333.3333|)/3 \approx 733.3333/3 \approx 244.4443$
SES: MAD = $(|0.00| + |-200.00| + \ldots + |303.68|)/6 \approx 1{,}102.08/6 = 183.6800$

Based on the criterion of MAD, we see that the simple exponential smoothing method provides the most accurate forecasts since $183.68 < 244.4443 < 320.0000$.

(3) Based on the simple exponential smoothing method, the forecasted time requirement in Week 7 is $F_7 = F_6 + \alpha(A_6 - F_6) = 4{,}996.32 + 0.2(5{,}300 - 4{,}996.32) \approx 5{,}057.056$ or about 5,057 hours.

(4) Naive: MSE = $[(-200)^2 + (300)^2 + (-200)^2 + (-300)^2 + (600)^2]/5 = 620{,}000/5 = 124{,}000.0000$
SMA: MSE = $[(-66.6667)^2 + (-333.3333)^2 + (333.3333)^2]/3 \approx 226{,}666.6267/3 \approx 75{,}555.5422$
SES: MSE = $[(0.00)^2 + (-200.00)^2 + \ldots + (303.68)^2]/6 \approx 296{,}761.7024/6 \approx 49{,}460.2837$

Based on the criterion of MSE, we see that the simple exponential smoothing method provides the most accurate forecasts since 49,460.2837 < 75,555.5422 < 124,000.0000.

6. (1) The naive forecasts are F_2 = 1,703, F_3 = 1,720, F_4 = 1,649, and F_5 = 1,686.

(2) An application of the 2-hour simple moving average method leads to F_3 = (1,720 + 1,703)/2 = 1,711.5, F_4 = (1,649 + 1,720)/2 = 1,684.5, and F_5 = (1,686 + 1,649)/2 =1,667.5.

(3) All of the results obtained above are summarized below:

		Naïve		SMA	
t	A_t	F_t	e_t	F_t	e_t
1	1,703				
2	1,720	1,703	17		
3	1,649	1,720	-71	1,711.5	-62.5
4	1,686	1,649	37	1,684.5	1.5
5	1,718	1,686	32	1,667.5	50.5

Naive: MAD = (|17| + |-71| + |37| + |32|)/4 = 157/4 = 39.2500
SMA: MAD = (|-62.5| + |1.5| + |50.5|)/3 = 114.5/3 ≈ 38.1667

Based on the criterion of MAD, we see that the simple moving average method is more accurate since 38.1667 < 39.2500.

(4) Naive: MSE = $[(17)^2 + (-71)^2 + (37)^2 + (32)^2]/4 = 7{,}723/4 = 1{,}930.7500$
SMA: MSE = $[(-62.5)^2 + (1.5)^2 + (50.5)^2]/3 = 6{,}458.75/3 \approx 2{,}152.9167$

Based on the criterion of MSE, we see that the naïve approach is more accurate since 1,930.7500 < 2,152.9167.

(5) Another measure of aggregate forecast errors should be introduced as a tie-breaker to determine which of the two forecasting technique is more accurate.