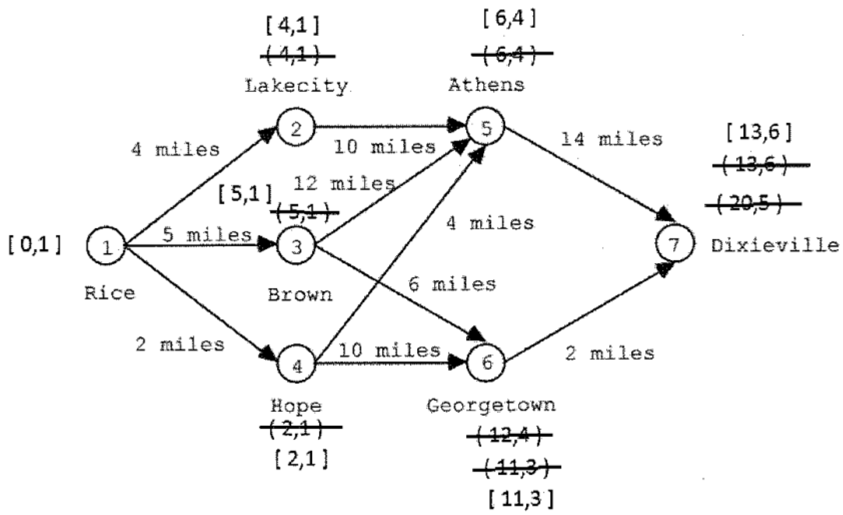
OPRE 6398 Prescriptive Analytics
Solutions to Homework 4

2. Applying Dijkstra's algorithm to the network, we have:



The shortest route from Node 1 (Rice) to each of the other nodes is given below along with the associated distance:

Node	Shortest route	Driving time
2	1-2	4
3	1-3	5
4	1-4	2
5	1-4-5	6
6	1-3-6	11
7	1-3-6-7	13

3. (1) The transportation matrix showing the northwest-corner IFS is presented below:

	X	Y	Z	Supply
A	11 40, 000	5	8	40, 000
B	8 50, 000	9 50, 000	12	100, 000
C	13	10 10, 000	7 70, 000	80, 000
Demand	90, 000	60, 000	70, 000	220, 000 220, 000

TC = 11(40,000) + 8(50,000) + … + 7(70,000) = 1,880,000

Evaluation:

R1 + K1 = 11	R1 = 0	K1 = 11
R2 + K1 = 8		R2 = -3
R2 + K2 = 9		K2 = 12
R3 + K2 = 10		R3 = -2
R3 + K3 = 7		K3 = 9

AY: 5 - (0 + 12) = **-7**
AZ: 8 - (0 + 9) = -1
BZ: 12 - (-3 + 9) = 6
CX: 13 - (-2 + 11) = 4

The current solution is not optimal since -7 < 0 and -1 < 0. To improve it, 40,000 units should be moved to cell AY since a "-" sign appears in each of cells AX and BY and min {40,000, 50,000} = 40,000. The new solution is:

	X	Y	Z	Supply
A	11	5 40,000	8	40,000
B	8 90,000	9 10,000	12	100,000
C	13	10 10,000	7 70,000	80,000
Demand	90,000	60,000	70,000	220,000 \ 220,000

TC = 1,880,000 - 7(40,000) = 1,600,000

Evaluation:

R1 + K2 = 5	R1 = 0	K2 = 5
R2 + K1 = 8		K1 = 4
R2 + K2 = 9		R2 = 4
R3 + K2 = 10		R3 = 5
R3 + K3 = 7		K3 = 2

AX: 11 - (0 + 4) = 7
AZ: 8 - (0 + 2) = 6
BZ: 12 - (4 + 2) = 6
CX: 13 - (5 + 4) = 4

The current solution is optimal since all the evaluations are zero or positive. In conclusion, the optimal shipping plan is as follows with a minimum total shipping cost of $1,600,000.

From	To	Shipment
A	Y	40,000
B	X	90,000
	Y	10,000
C	Y	10,000

Z	70,000

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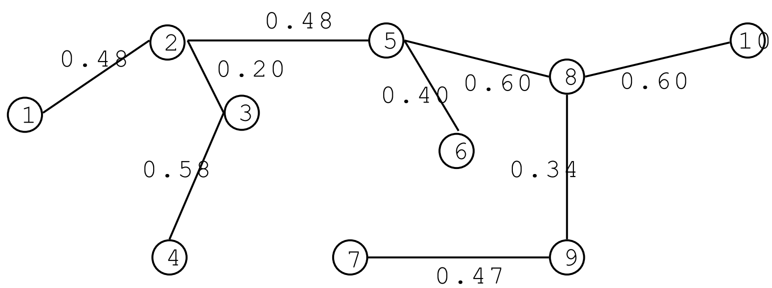
4. Applying the greedy algorithm to the problem leads to the following minimal spanning tree with a minimum amount of cable used at 0.48 + 0.20 + ... + 0.47 = 4.15 (miles):



5. An application of the Ford-Fulkerson algorithm to the problem shows that the maximal flow from the entry to the exit through the pipeline network is (5 + 10 + 7 + 11) - (0 + 6 + 0 + 11) = (9 + 4 + 7) - (4 + 0 + 0) = 16 or 16,000 cubic feet per minute. (Note: The changes to the flow capacities at both ends of each line segment are not shown here since there are many different ways of solving the problem.)