

# Ivy: Safety Verification by Interactive Generalization



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PLDI 2016

<http://microsoft.github.io/ivy/>

# Ivy: Safety Verification by Interactive Generalization

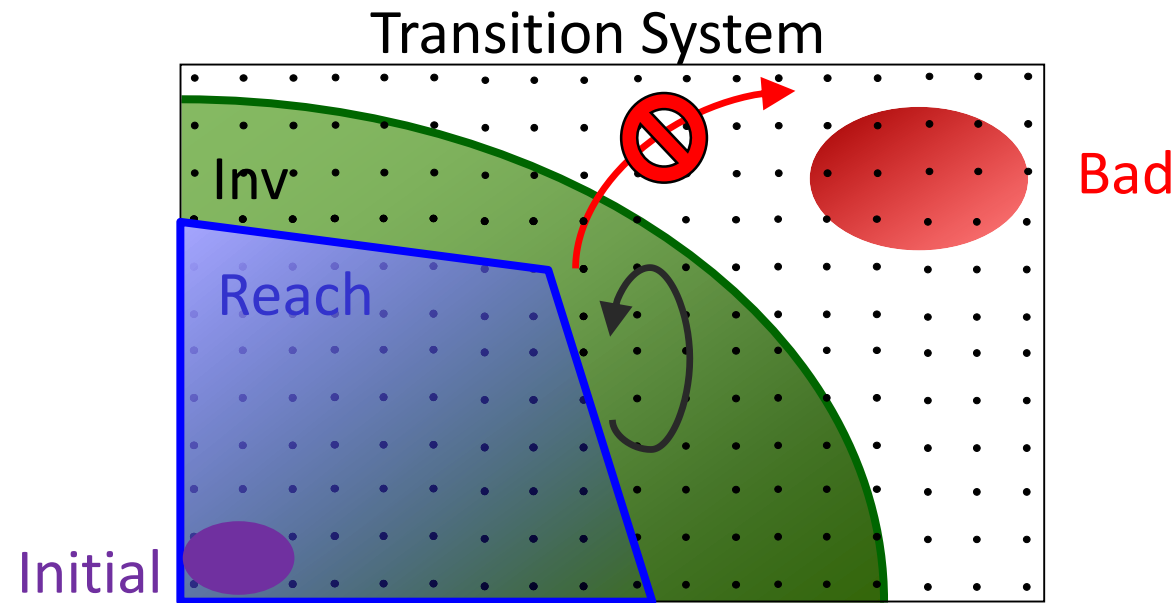
- Verification of distributed systems
- Modeling infinite-state systems in a way which allows decidable automated reasoning (EPR)
- Interactive discovery of inductive invariants

# Ivy: Safety Verification by Interactive Generalization

- Verification of distributed systems
- Modeling infinite-state systems in a way which allows decidable automated reasoning (EPR)
- Interactive discovery of inductive invariants

this talk (mostly)

# Safety of Transition Systems



System  $S$  is safe if no bad state is reachable

System  $S$  is safe iff there exists an inductive invariant  $Inv$  s.t.:

$Init \subseteq Inv$  (Initiation)

if  $\sigma \in Inv$  and  $\sigma \rightarrow \sigma'$  then  $\sigma' \in Inv$  (Consecution)

$Inv \cap Bad = \emptyset$  (Safety)

# Challenges for Deductive Verification

1. Formal specification:
  - Modeling the system
  - Formalizing the safety property
2. Inductive Invariants
  - Hard to specify manually
  - Hard to infer automatically
3. Deduction – Checking inductiveness
  - Undecidability of implication checking
    - Unbounded state, arithmetic, quantifier alternation

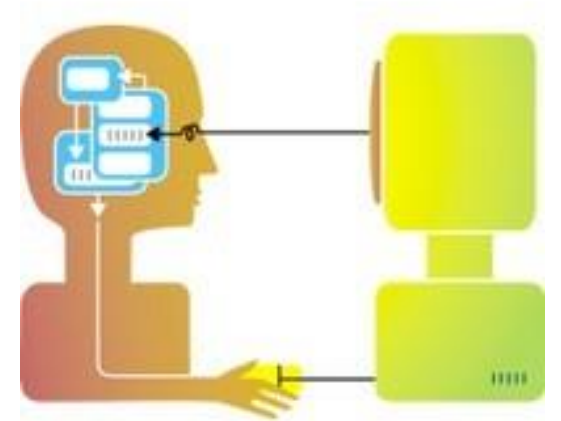
# Existing Approaches for Verification of Infinite-State Systems

- Automated invariant inference
  - Abstract Interpretation
  - Ultimately limited due to undecidability
- Use SMT for deduction with manual program annotations (e.g. Dafny)
  - Requires programmer effort to provide inductive invariants
  - SMT solver may diverge (matching loops, arithmetic)
- Interactive theorem provers (e.g. Coq, Isabelle/HOL)
  - Programmer provides inductive invariant and proves it
  - Huge effort (10-100 lines of proof per line of code)

Usually opaque when failing

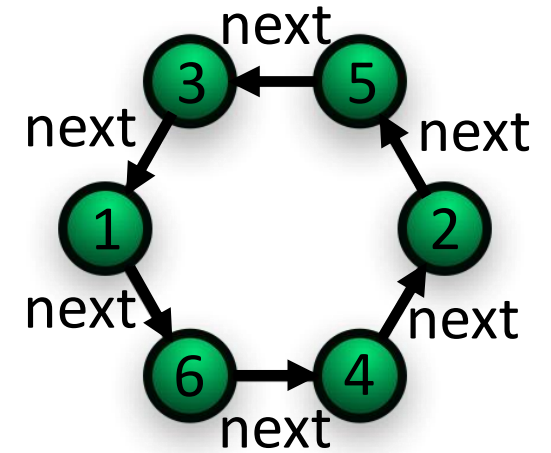
# Our Approach in Ivy

- Restrict the specification language for decidability
  - Deduction is decidable with SAT solvers
  - Challenge: verify complex systems using a restricted language
    - Solution: domain specific axioms
- Finding inductive invariants (still undecidable):
  - Combine automated techniques with human guidance
  - Graphical user interaction
  - Key: generalization from counterexamples to induction
  - Decidability allows reliable automated checks



# Example: Leader Election in a Ring

- Nodes are organized in a ring
- Each node has a unique numeric id
- Protocol:
  - Each node sends its id to the next
  - A node that receives a message passes it (to the next) if the id in the message is higher than the node's own id
  - A node that receives its own id becomes the leader
- Theorem:
  - The protocol selects at most one leader



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[CACM'79] E. Chang and R. Roberts. *An improved algorithm for decentralized extrema-finding in circular configurations of processes*



# Example: Leader Election in a Ring

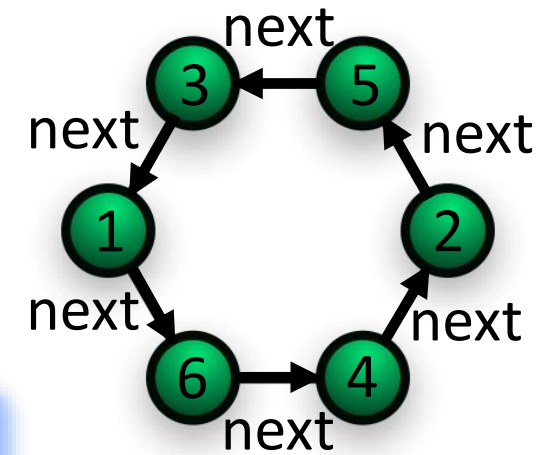
- Nodes are organized in a ring
- Each node has a unique numeric id
- Protocol:

- **Proposition:** This algorithm detects one and only one highest number.

- **Argument:** By the circular nature of the configuration and the consistent direction of messages, any message must meet all other processes before it comes back to its

- **initiator.** Only one message, that with the highest number, will not encounter a higher number on its way

- **Theorem** around. Thus, the only process getting its own message back is the one with the highest number.



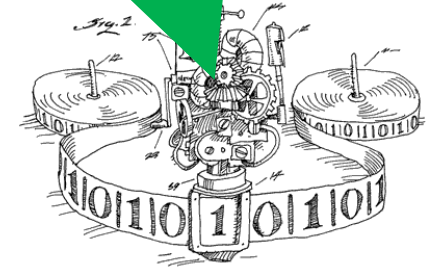
if the id in the

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# Relational Modeling Language (RML)

*I can decide  
inductiveness!*



- Designed to make verification tasks decidable
  - Yet expressive enough to model systems
- Turing-Complete
- Universally quantified inductive invariants are decidable to check
- System state described by finite (unbounded) relations
- No numerics
- Simple (quantifier-free) updates
- Universally quantified axioms (domain specific)
  - Total orders, partial orders, lists, trees, rings, quorums, ...

# Leader Election Protocol (RML)

- $\leq$  (ID, ID) – total order on node id's
- **btw** (Node, Node, Node) – the ring topology
- **id**: Node  $\rightarrow$  ID – relate a node to its unique id
- **pending**(ID, Node) – pending messages
- **leader**(Node) – leader(n) means n is the leader

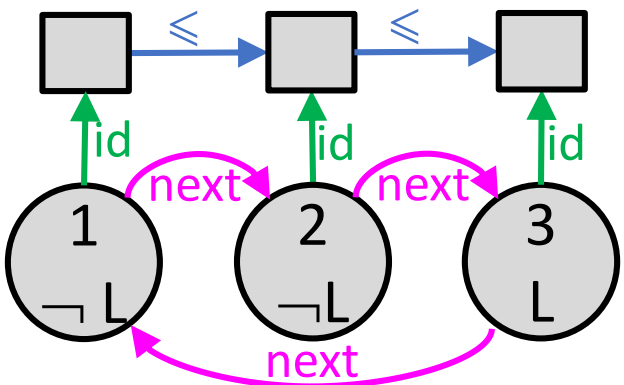
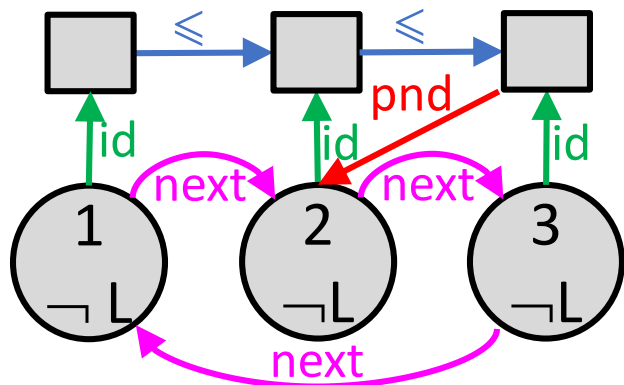
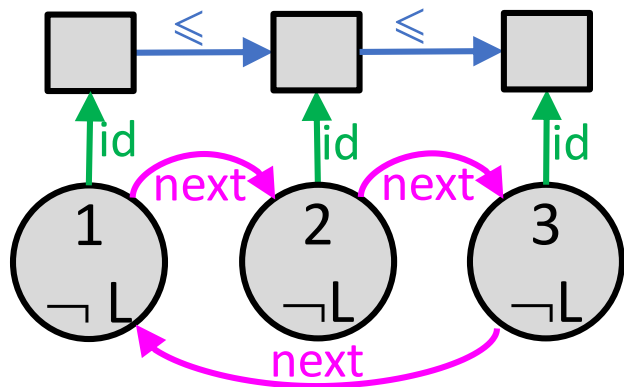
```
action send(n: Node) = {  
    "s := next(n)";  
    pending(id(n), s) := true  
}
```

```
action receive(n: Node, m: ID) = {  
    requires pending(m, n);  
    pending(m, n) := false;  
    if id(n) = m then  
        // found leader  
        leader(n) := true  
    else if id(n)  $\leq$  m then  
        // pass message  
        "s := next(n)";  
        pending(m, s) := true  
}
```

protocol = (send | receive)\*

**next**(a)=b  $\leftrightarrow \forall x: \text{Node}. x=a \vee x=b \vee \text{btw}(a,b,x)$

assert  $I_0 = \forall x,y: \text{Node}. \text{leader}(x) \rightarrow \text{id}(y) \leq \text{id}(x)$

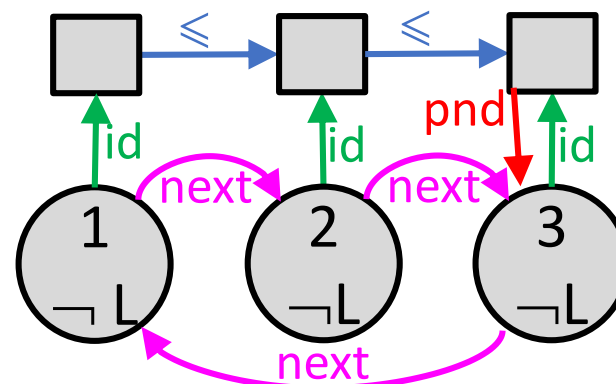
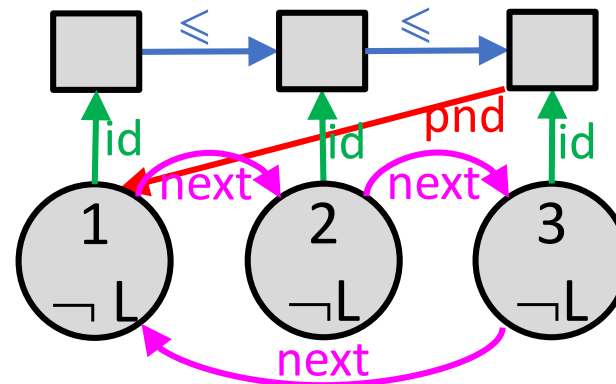


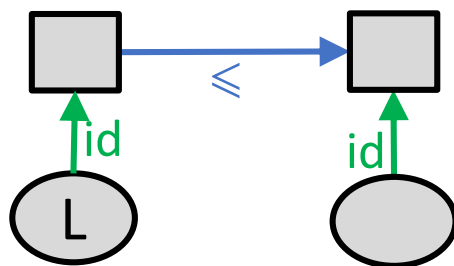
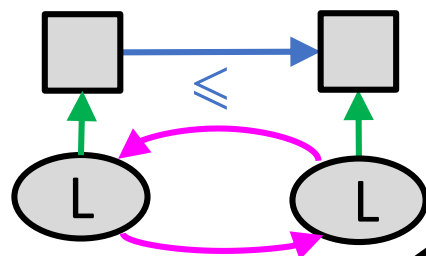
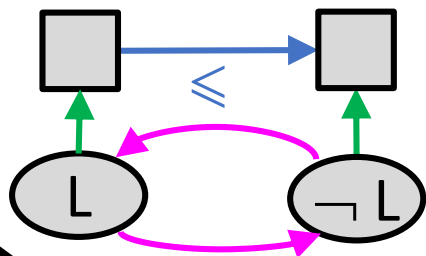
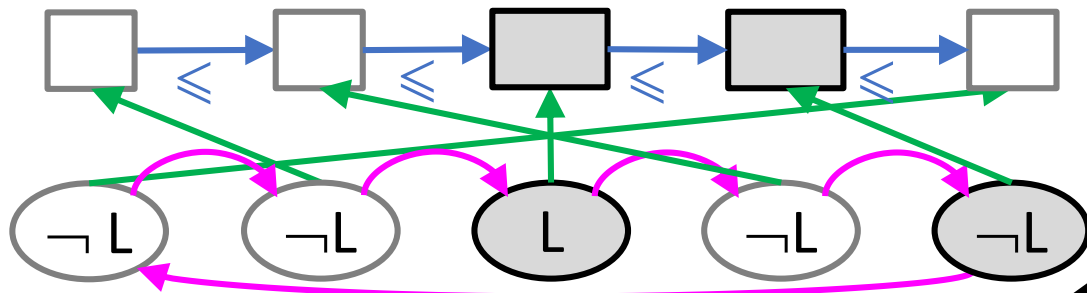
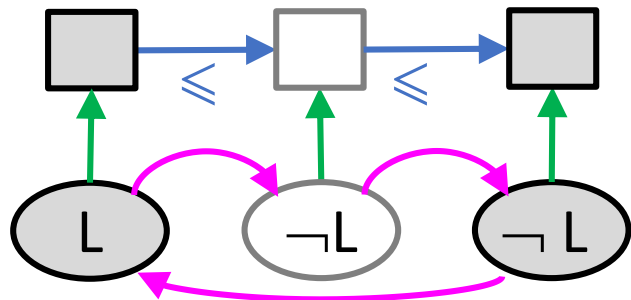
$send(3)$

$rcv(1, id(3))$

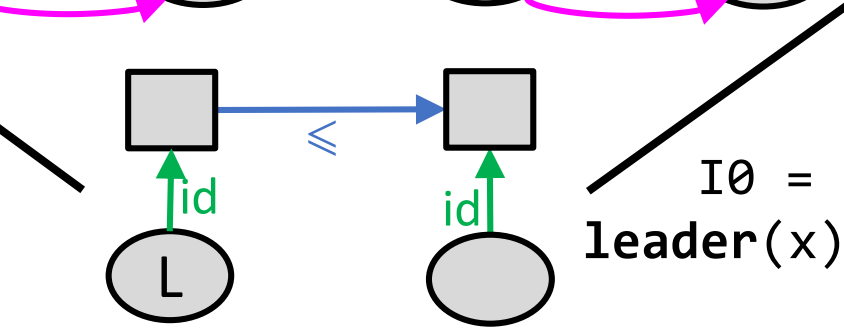
$rcv(2, id(3))$

$rcv(3, id(3))$

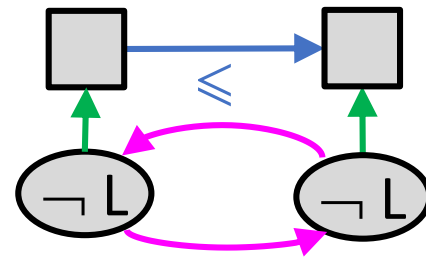


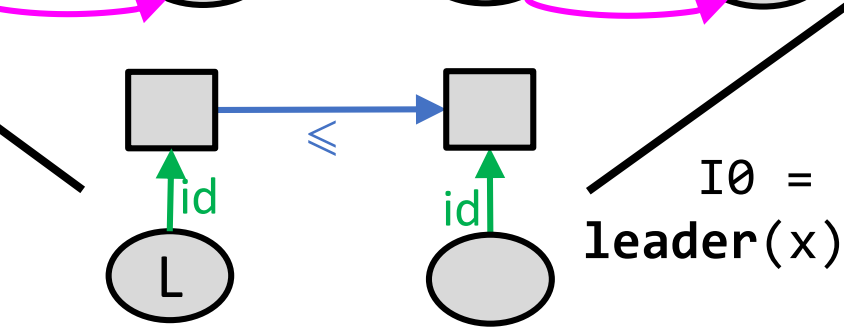


$I0 = \forall x, y: \text{Node.}$   
 $\text{leader}(x) \rightarrow \text{id}(y) \leq \text{id}(x)$

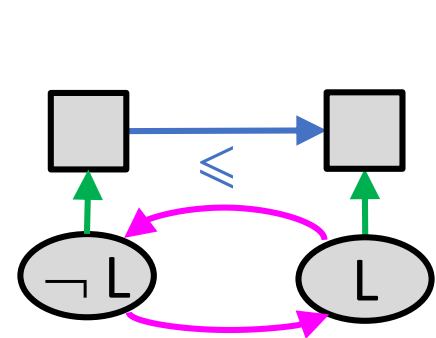


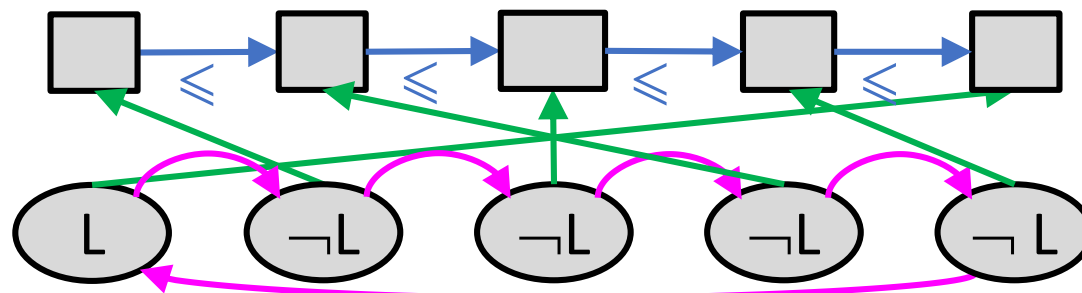
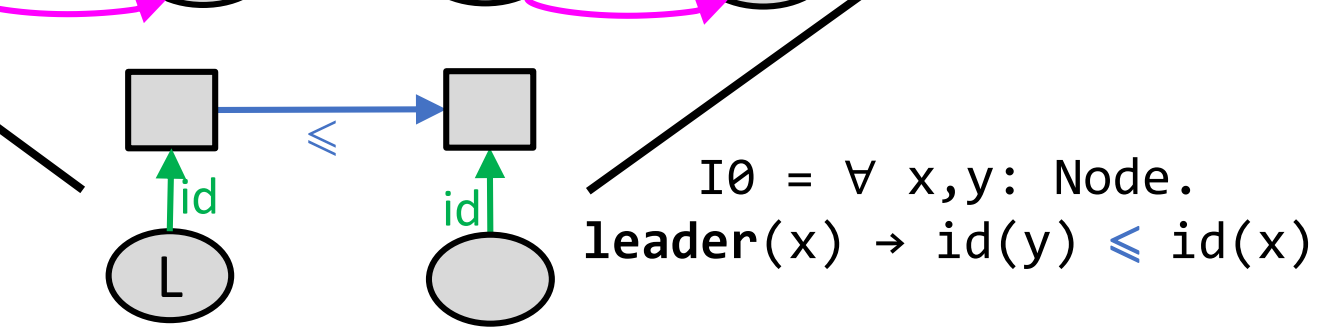
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$I0 = \forall x, y: \text{Node.}$   
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# Inductive Invariant for Leader Election

- $\leq$  (ID, ID) – total order on node id's
- **btw** (Node, Node, Node) – the ring topology
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**Safety property:**  $I_0$

$$I_0 = \forall x, y: \text{Node}. \text{leader}(x) \rightarrow \text{id}(y) \leq \text{id}(x)$$

**Inductive invariant:**  $\text{Inv} = I_0 \wedge I_1 \wedge I_2$

$$I_1 = \forall x, y: \text{Node}. \neg( \text{pending}(\text{id}(x), x) \wedge \text{id}(x) \neq \text{id}(y) \wedge \text{id}(x) \leq \text{id}(y) )$$

$$I_2 = \forall x, y, z: \text{Node}. \neg( \text{btw}(x, y, z) \wedge \text{pending}(\text{id}(y), x) \wedge \text{id}(y) \leq \text{id}(z) )$$

# Inductive Invariant for Leader Election

- $\leq$  (ID, ID) – total order on node id's
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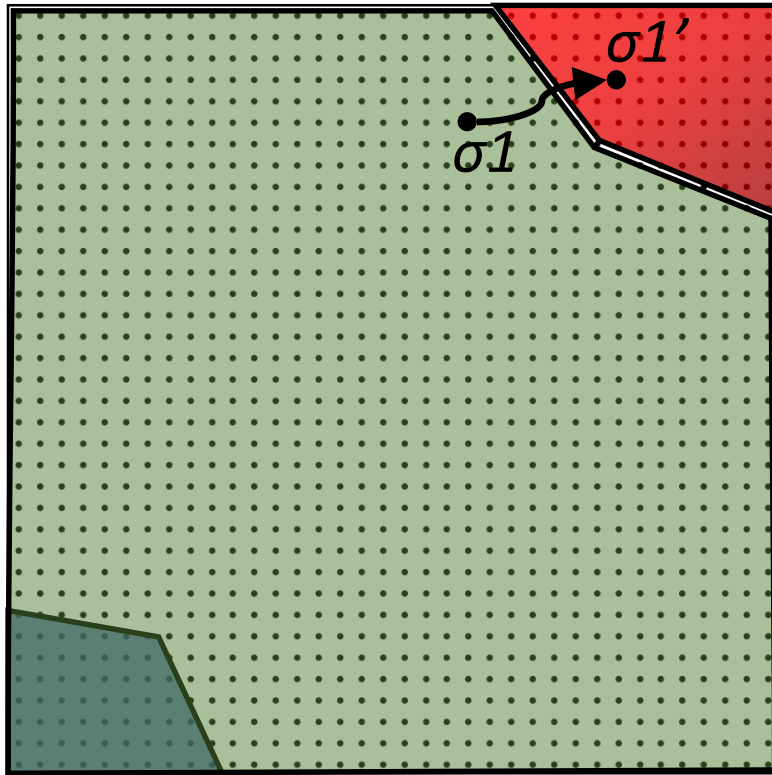
**Inductive invariant:**  $\text{Inv} = I_0 \wedge I_1$

$I_1$

$I_1$

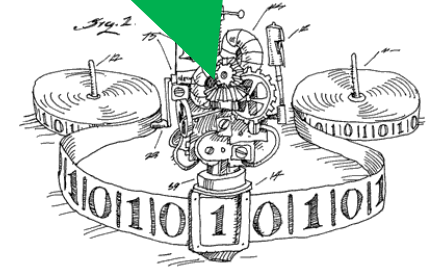
How can we find an inductive invariant without knowing it?

# Invariant Inference In Ivy



$$\text{Inv} = \neg \text{Bad}$$

*I can decide  
inductiveness!*

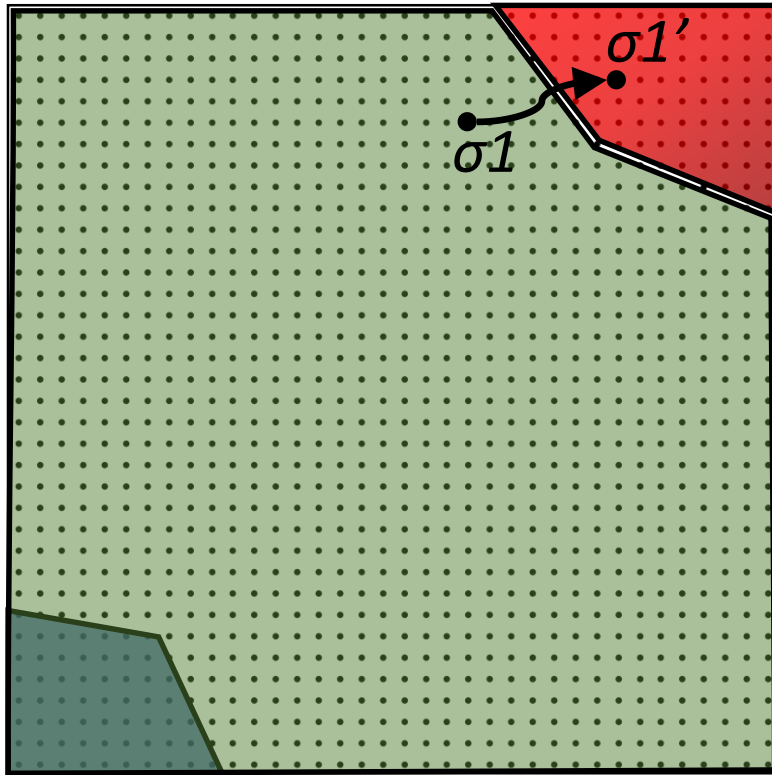


**Check Inductiveness**

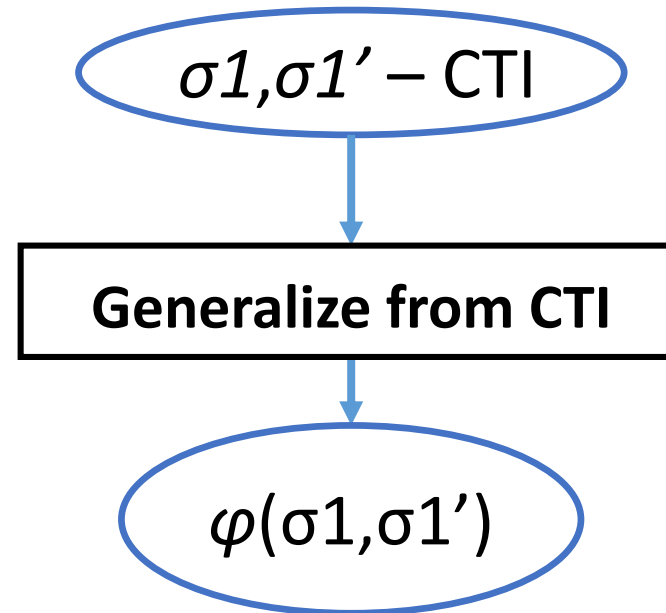


Counterexample To Induction (CTI)

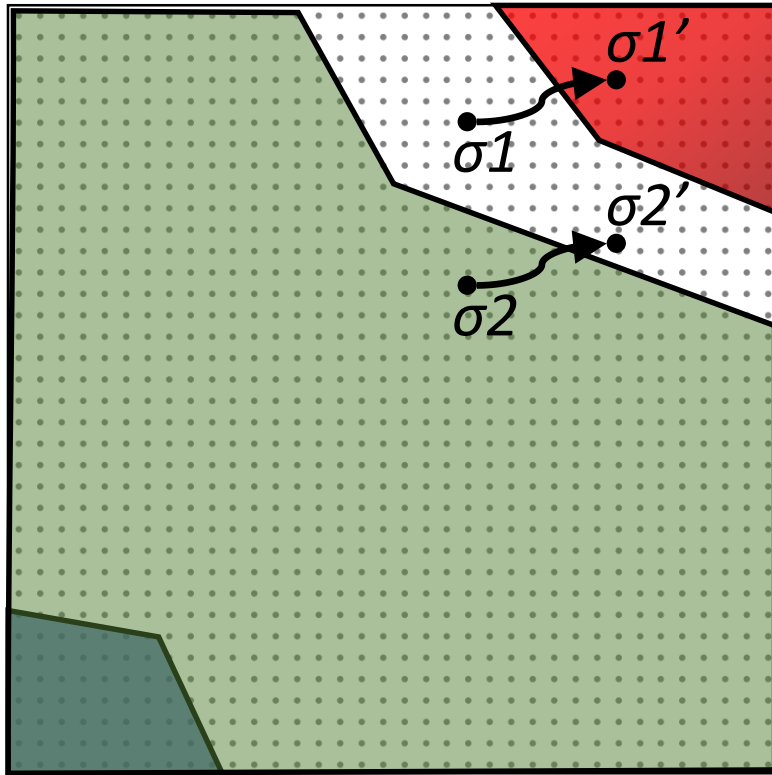
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# Invariant Inference In Ivy



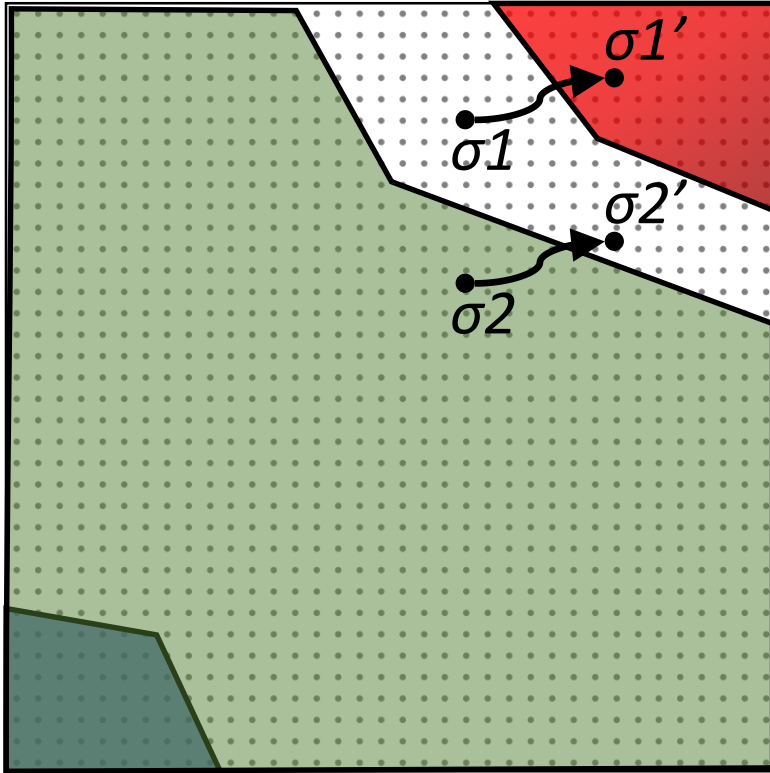
$$\text{Inv} = \neg \text{Bad} \wedge \varphi(\sigma_1, \sigma_1')$$

**Check Inductiveness**

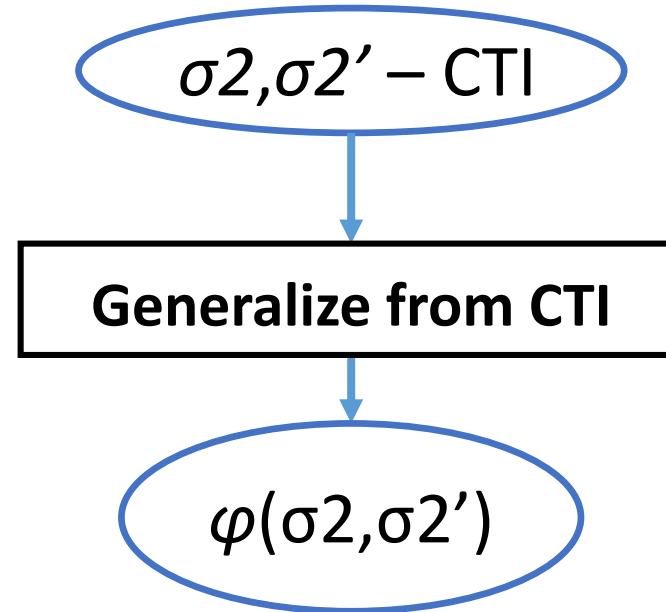


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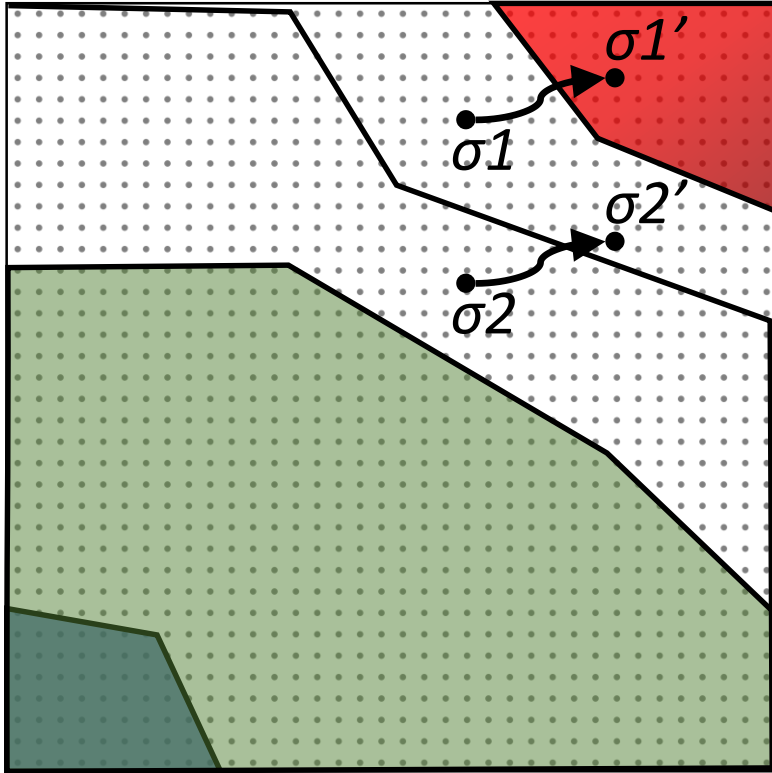
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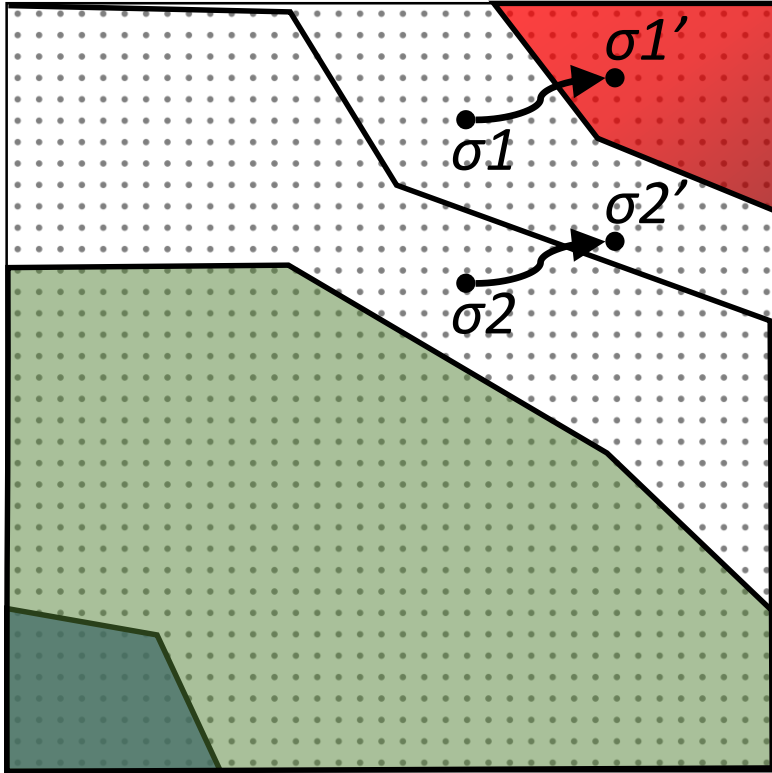


# Invariant Inference In Ivy



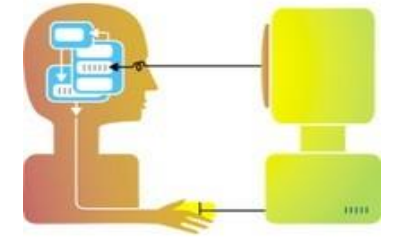
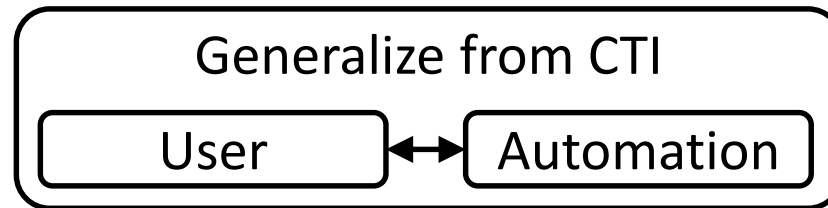
$$\text{Inv} = \neg \text{Bad} \wedge \varphi(\sigma_1, \sigma_1') \wedge \varphi(\sigma_2, \sigma_2')$$

# Invariant Inference In Ivy



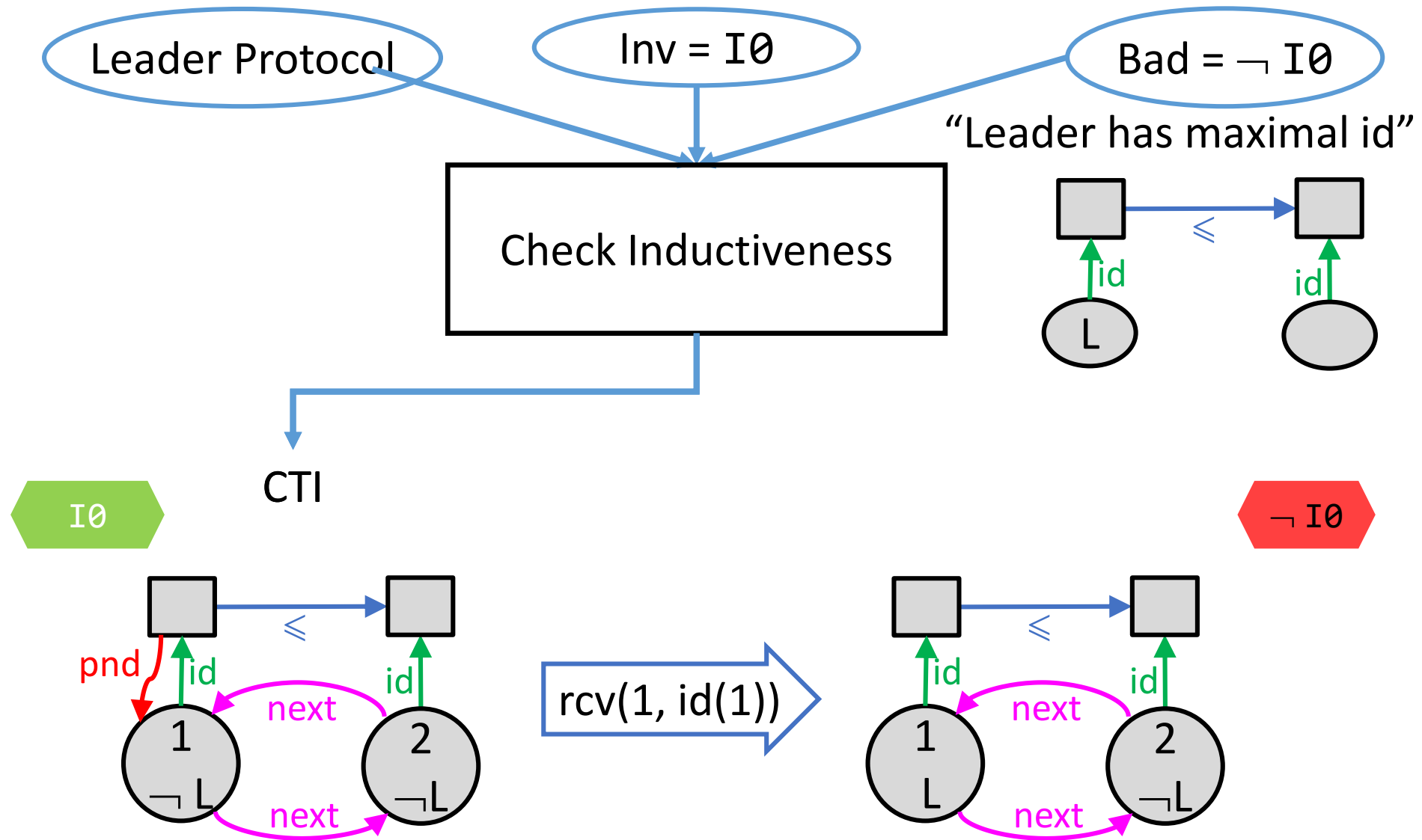
$$\text{Inv} = \neg \text{Bad} \wedge \varphi(\sigma_1, \sigma_1') \wedge \varphi(\sigma_2, \sigma_2')$$

- Key challenge for invariant inference:  
*generalization*
- Ivy's approach: put the user in the loop  
*interactive generalization*

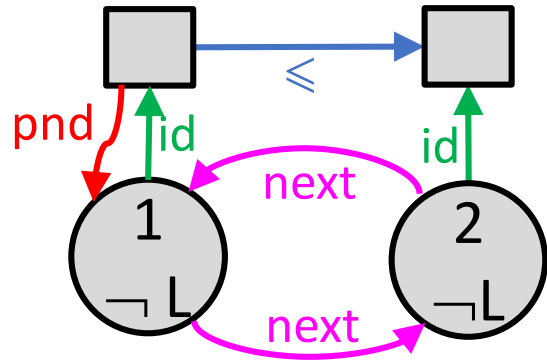




# Ivy: Check Inductiveness (1)



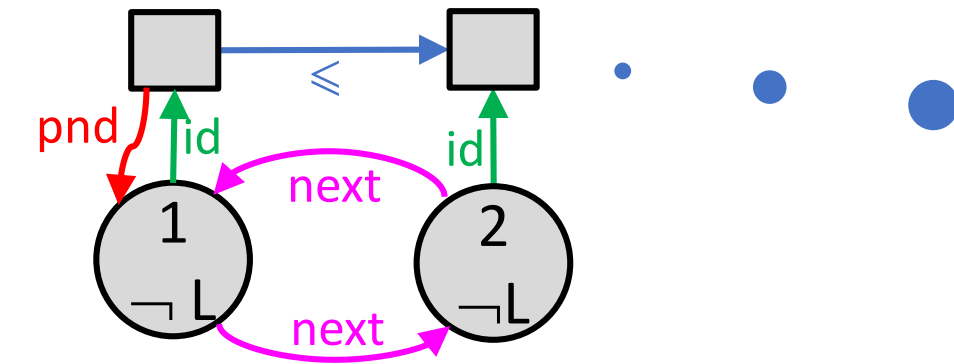
# Ivy: Generalize from CTI (1)



Only the highest id  
can be self pending

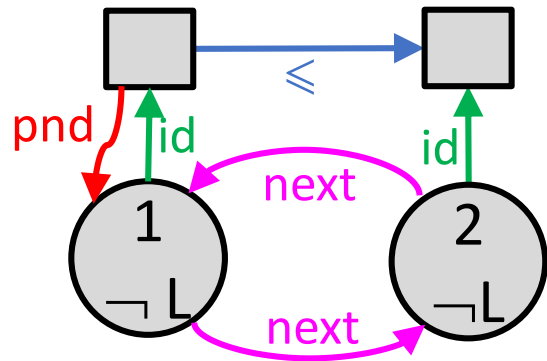
1. Each node sends its id to the next
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# Ivy: Generalize from CTI (1)



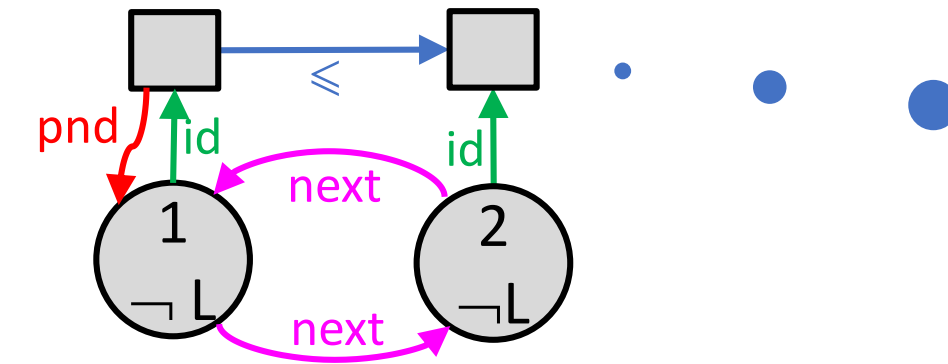
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User's Generalization



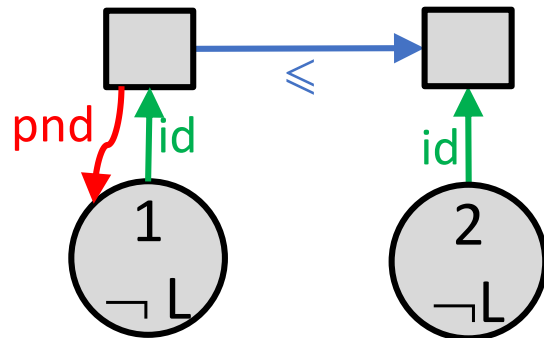
- ☒  $\leq$
- ☒ *btw*
- ☒ *id*
- ☒ *pnd*
- ☒ *L*

# Ivy: Generalize from CTI (1)



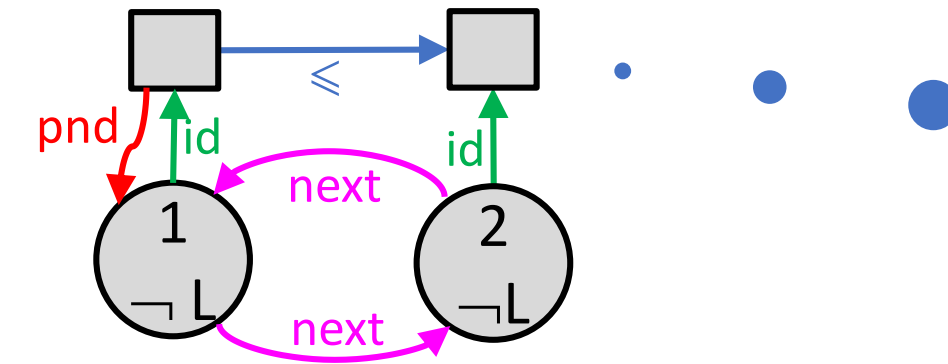
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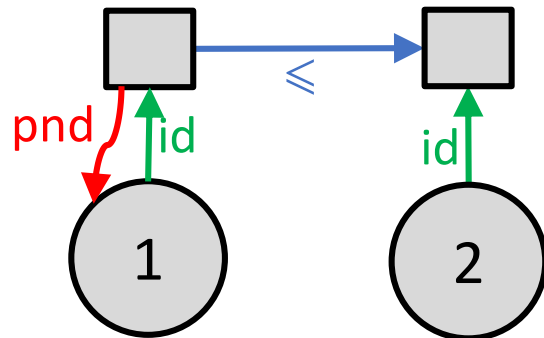
- ☒ ≤
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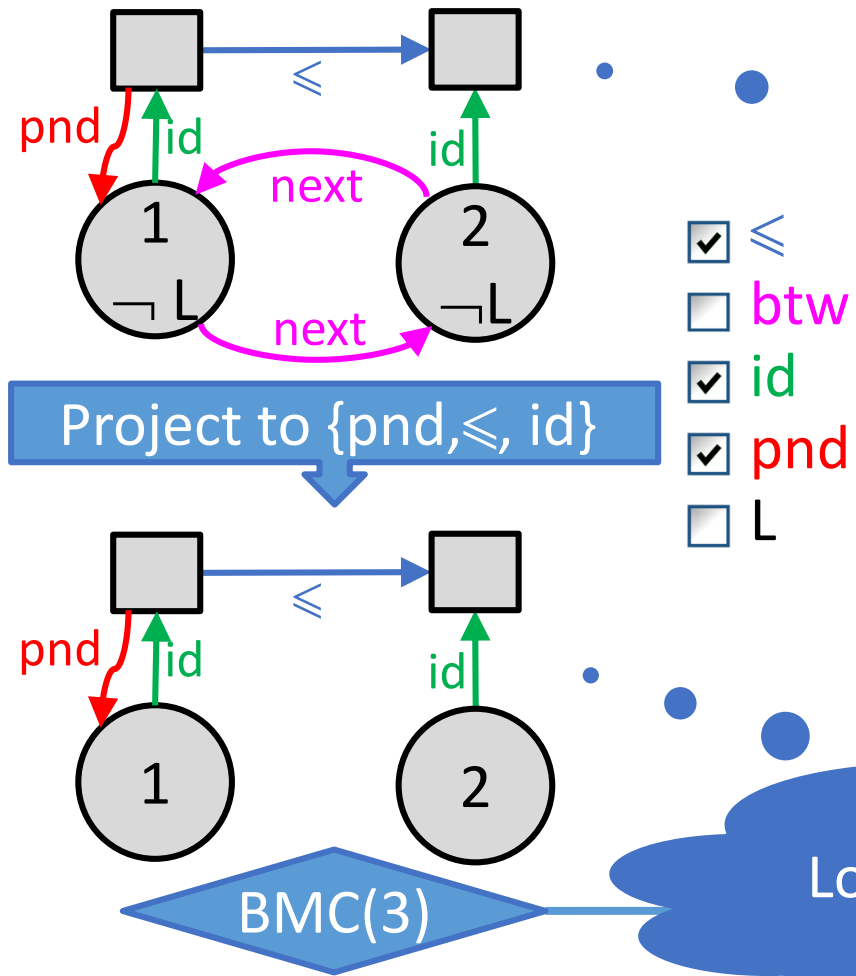
Only the highest id can be self pending

User's Generalization



- ☒  $\leq$
- ☐ btw
- ☒ id
- ☒ pnd
- ☐ L

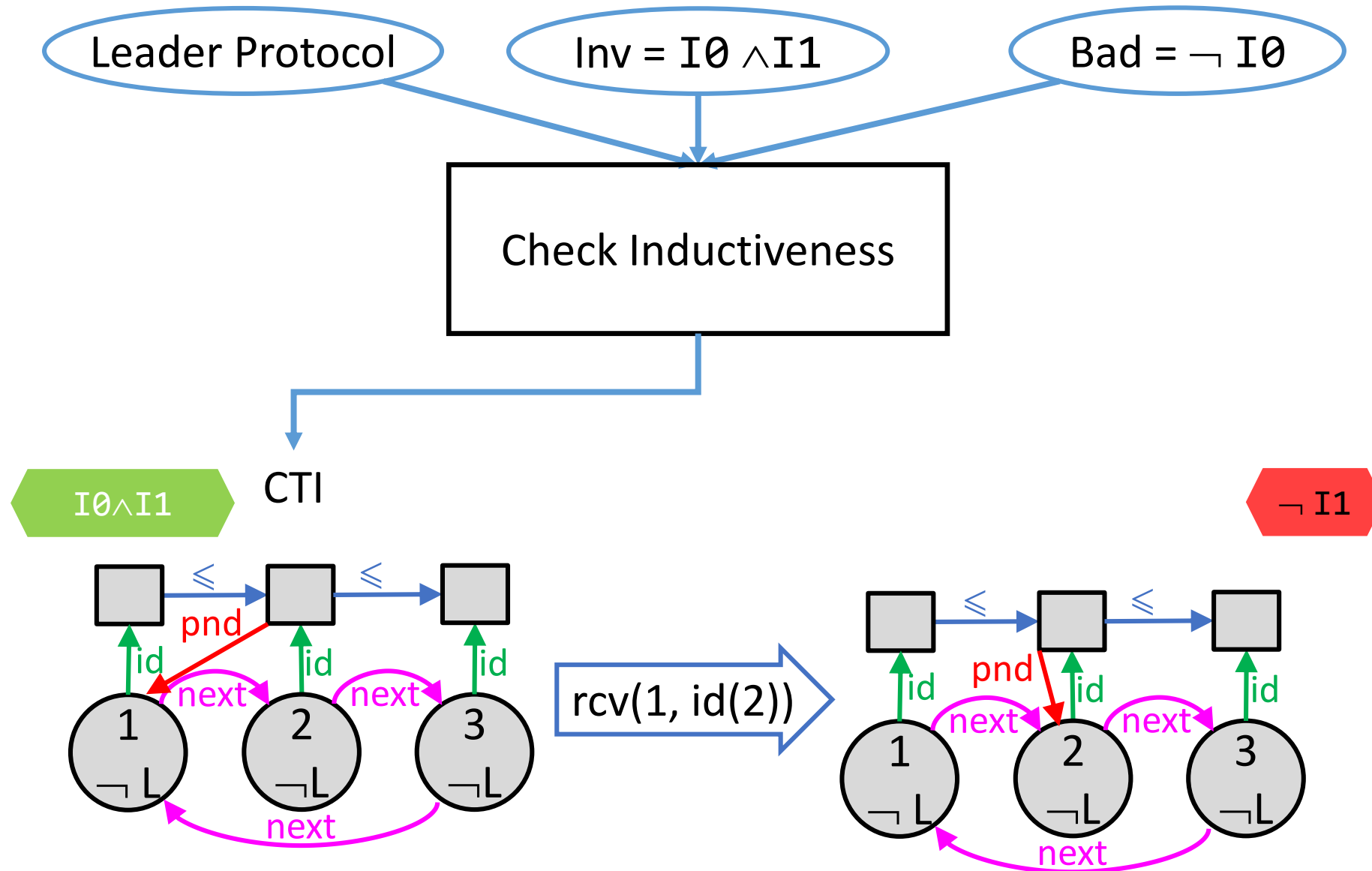
# Ivy: Generalize from CTI (1)



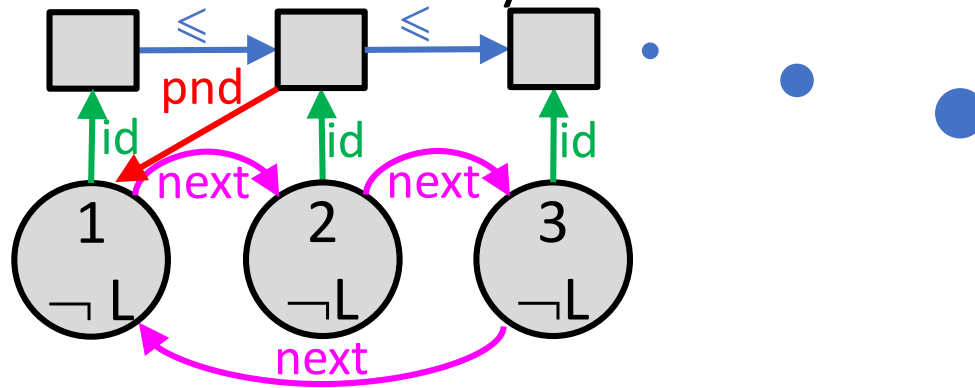
Only the highest id  
can be self pending

Looks good, add to the invariant as I1

# Ivy: Check Inductiveness (2)



## Ivy: Generalize from CTI (2)

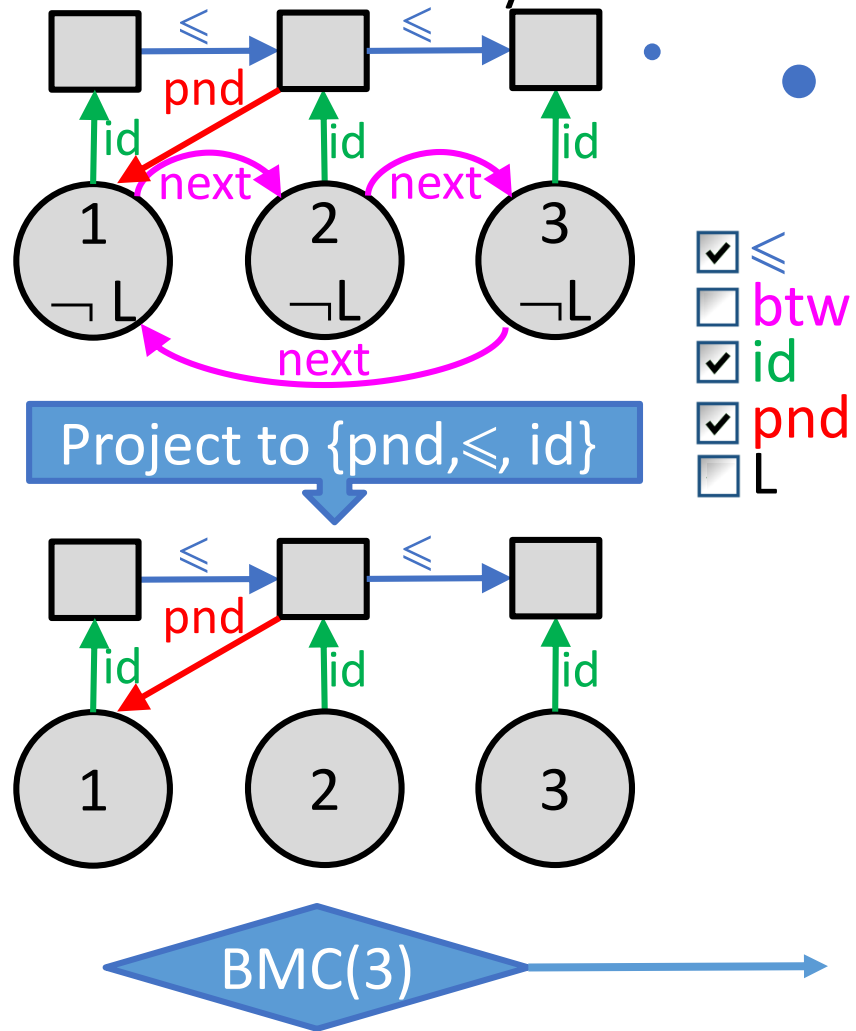


Cannot bypass nodes with higher ids

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# Ivy: Generalize from CTI (2)

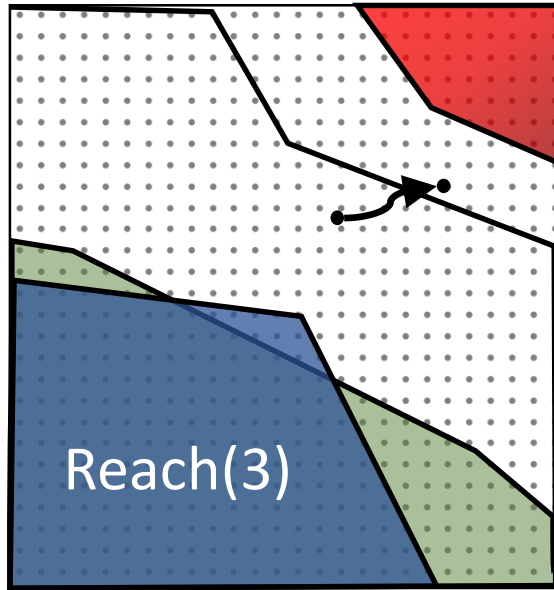


Cannot bypass nodes with higher ids



Counterexample Trace

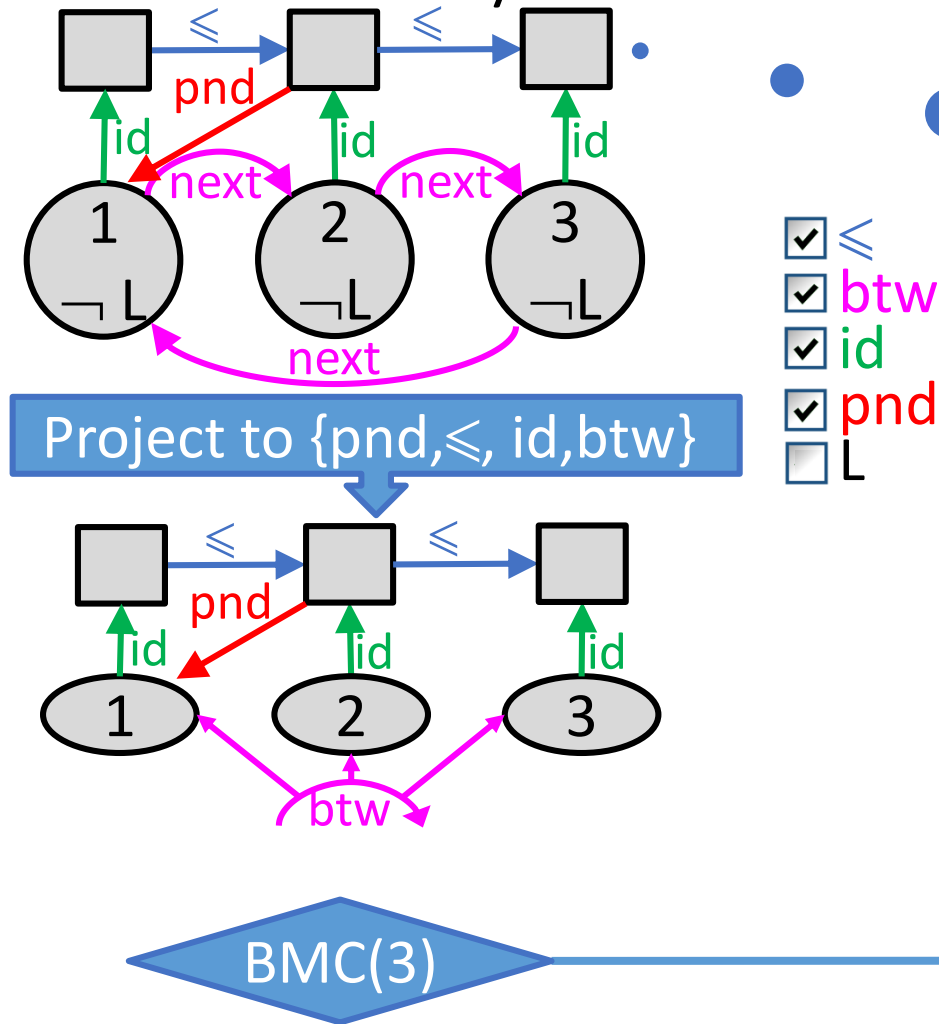
# Ivy: Generalize from CTI (2)



Counterexample Trace

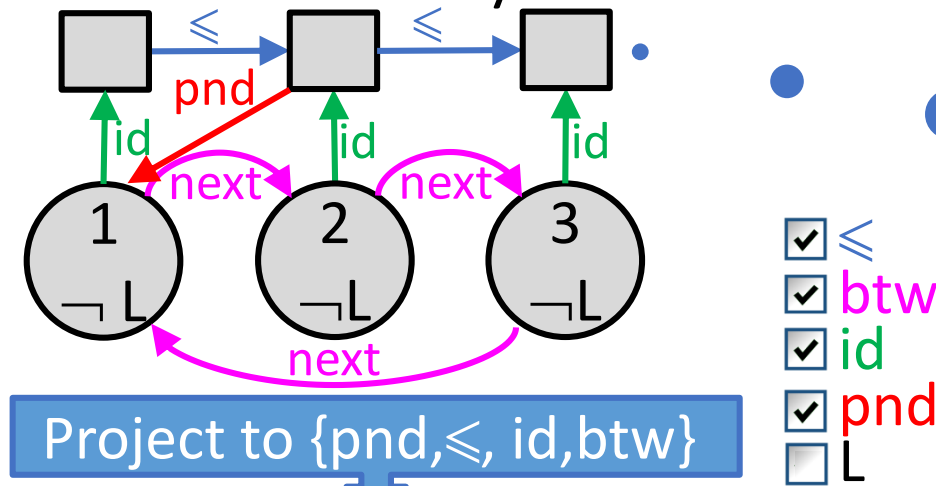


# Ivy: Generalize from CTI (2)



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# Ivy: Generalize from CTI (2)

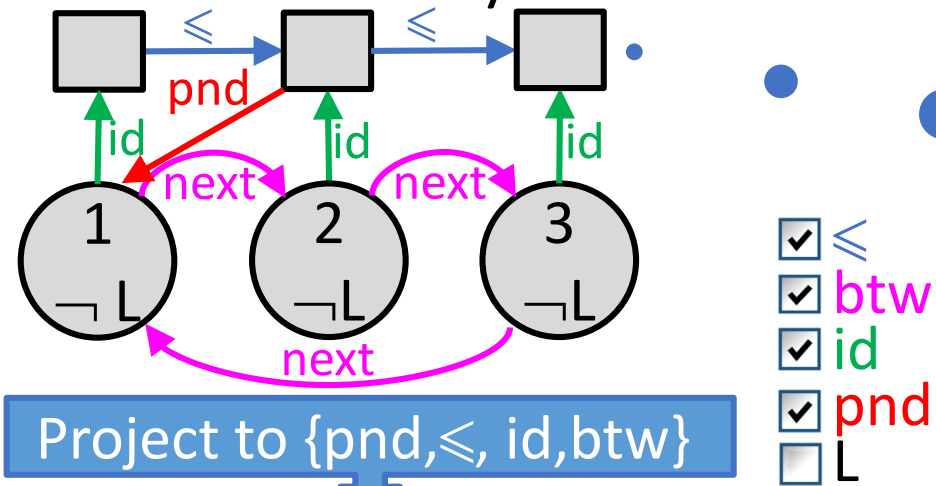


Cannot bypass nodes with higher ids

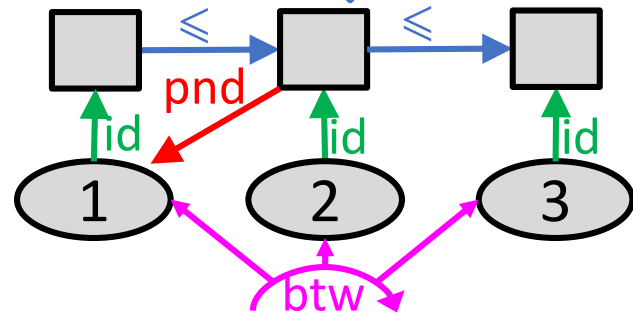


 Proof +  
UNSAT CORE

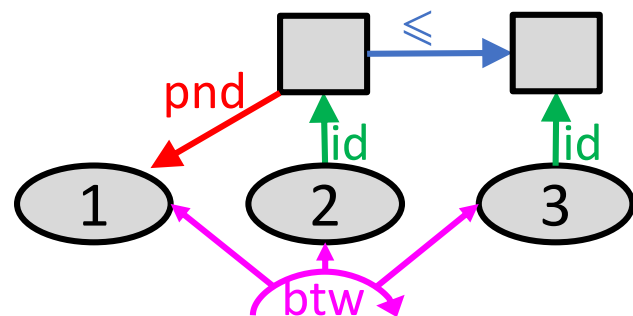
# Ivy: Generalize from CTI (2)



Cannot bypass nodes with higher ids

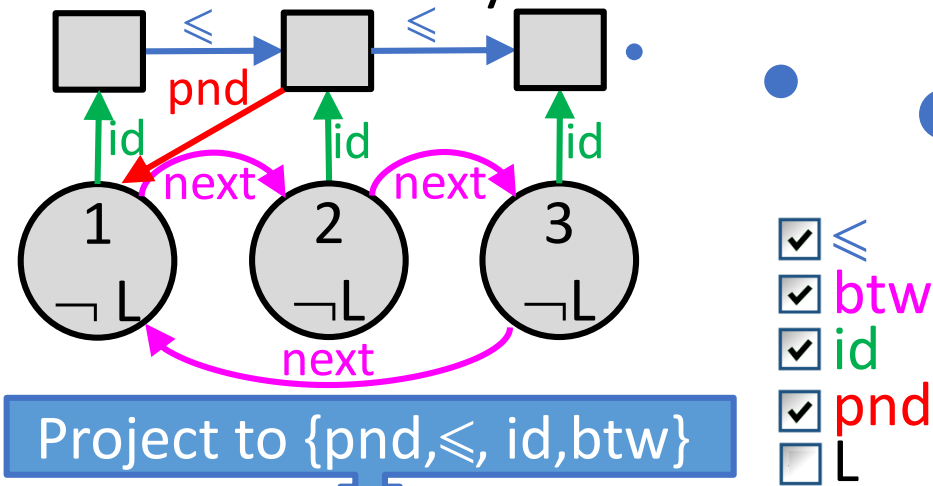


Interp(3)

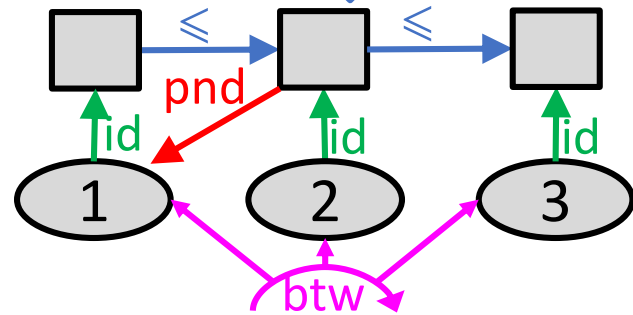


Proof +  
UNSAT CORE

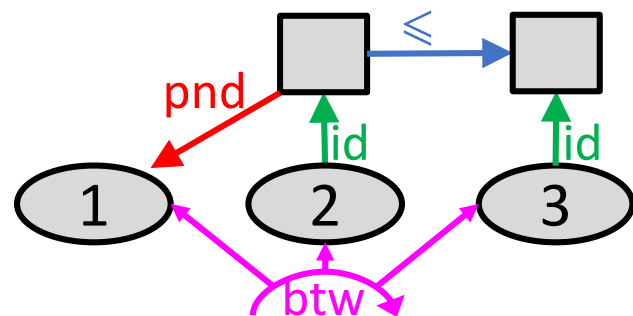
# Ivy: Generalize from CTI (2)



Project to  $\{pnd, \leq, id, btw\}$



Interp(3)

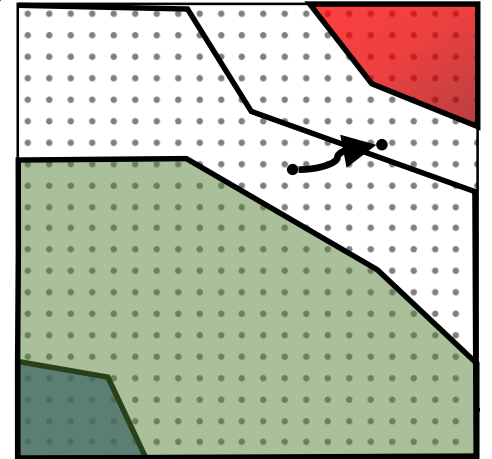
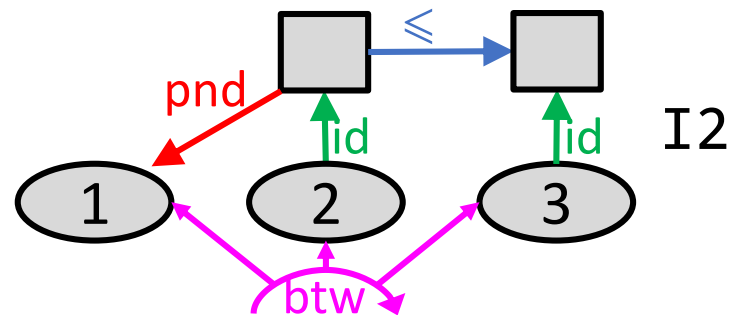
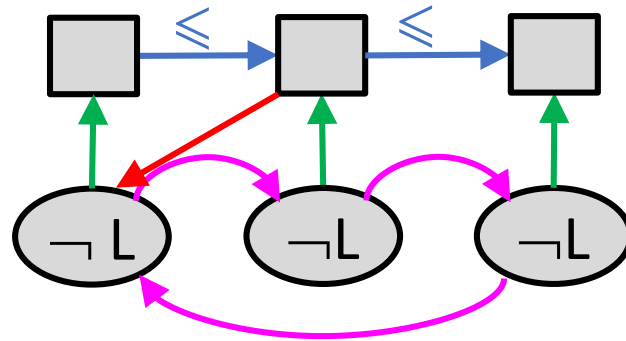
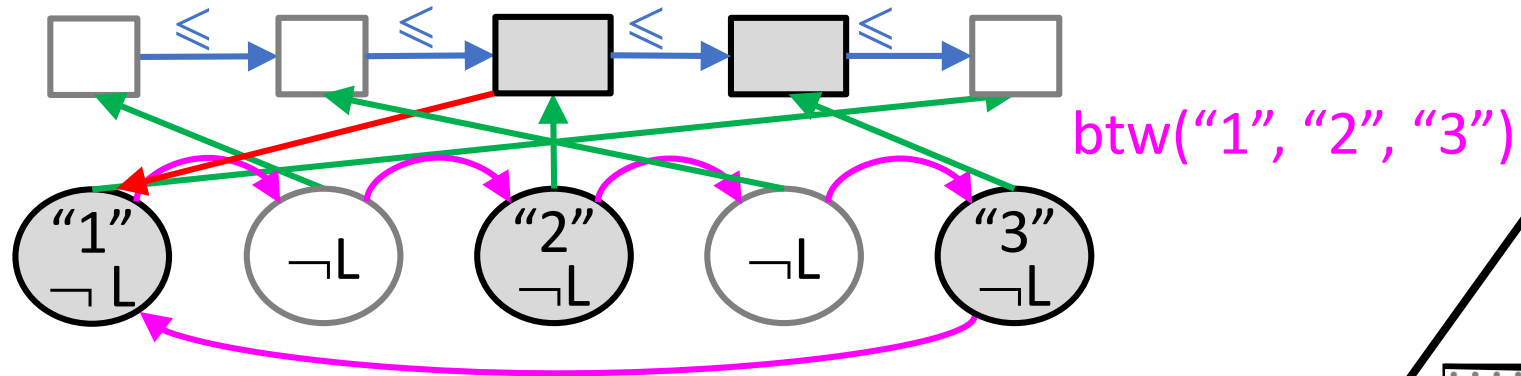


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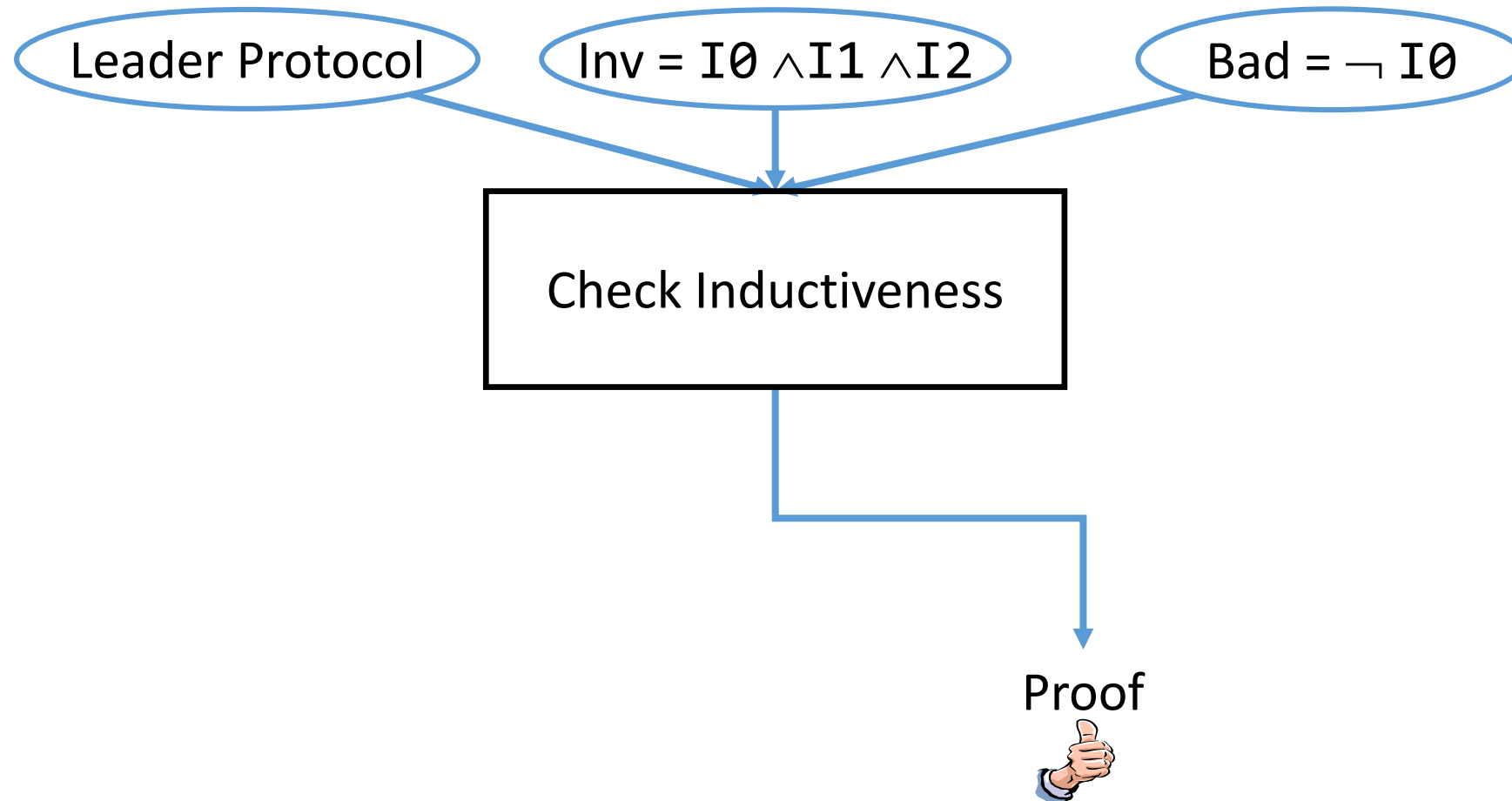
This looks good, add to the invariant as I2

UNSAT CORE

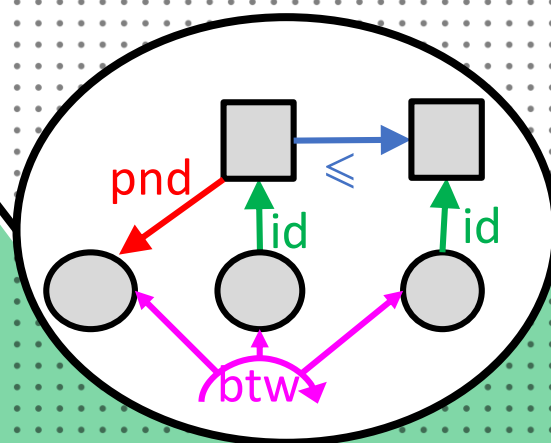
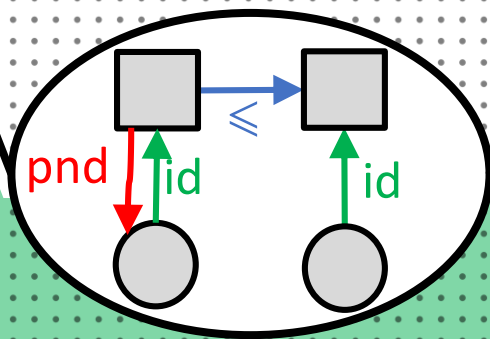
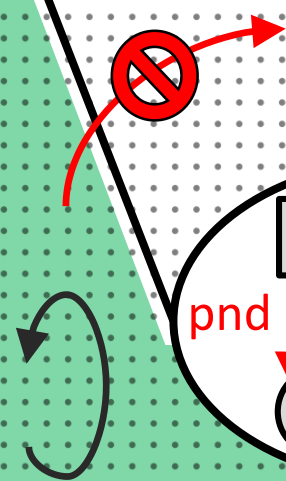
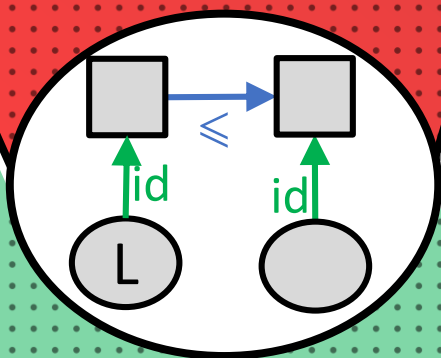
# Generalization with btw



# Ivy: Check Inductiveness (3)







$Init \subseteq Inv$  (Initiation)

if  $\sigma \in Inv$  and  $\sigma \rightarrow \sigma'$  then  $\sigma' \in Inv$  (Consecution)

$Inv \cap Bad = \emptyset$  (Safety)

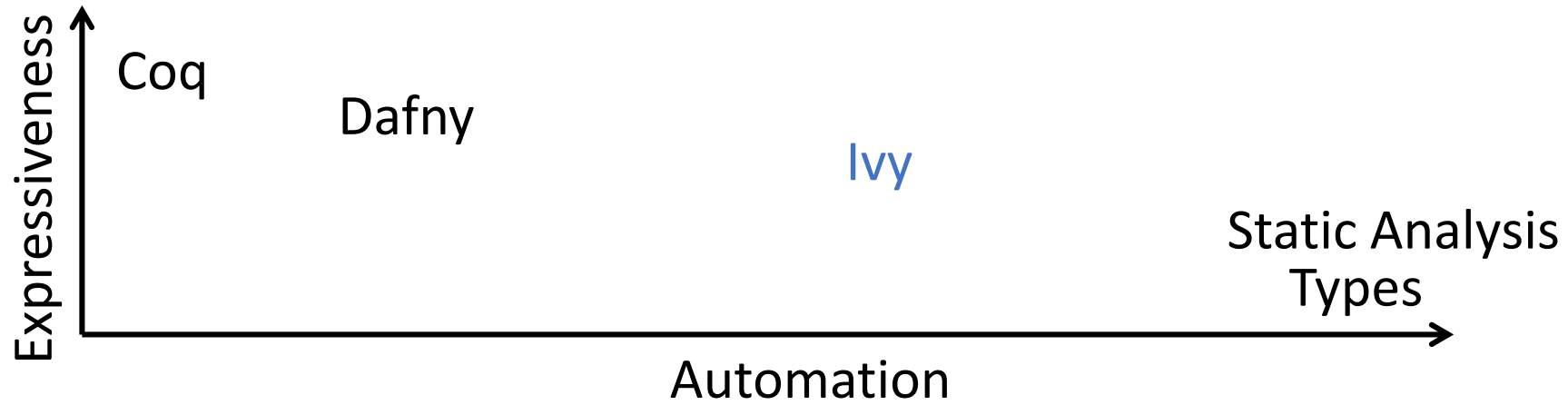
# Completeness and Interaction Complexity

- Any generalization from CTI adds one universally quantified clause
- A universally quantified invariant in CNF with  $N$  clauses, can be obtained by the user in  $N$  generalization steps
  - Assuming the user is optimal
- If the user is sub-optimal, backtracking (weakening) may be needed

# Verified Protocols

Protocol	Model Types	Relations & Functions	Property (# Literals)	Invariant (# Literals)	CTI Gen. Steps
Leader in Ring	2	5	3	12	3
Learning Switch	2	5	11	18	3
DB Chain Replication	4	13	11	35	7
Chord	1	13	35	46	4
Lock Server 500 Coq lines [Verdi]	5	11	3	21	8 (1h)
Distributed Lock 1 week [IronFleet]	2	5	3	26	12 (1h)
Paxos	Work in progress				
Raft					

# Expressiveness vs. Automation



	Coq	Dafny	Ivy	Fully Automatic Static Analysis
Invariant	User	User	User + System	System
Deduction	User	System (Z3) + "User"	System (EPR Z3)	System

# Summary

- RML – modeling language that makes **deduction decidable**
  - Many systems can be verified (axioms for orders, trees, rings, ...)
- **Interactive generalization** for finding inductive invariants
- Application to the domain of **distributed protocols**
- **User intuition and machine heuristics complement each other:**
  - User has intuition that leads to *better generalizations*
  - Machine is better at finding bugs and corner cases
- **Interactive process assists user to gain intuition about the protocol**

I can decide inductiveness!

