homework3

a. if we ignore the latency, the problem defined as

$$\max_f U(f) = \sum_{j=1}^{n} \log f_j$$
$$\text{subject to } Rf \preceq c$$

In order to get a convex problem, then we have here

$$\min_f -U(f) = -\sum_{j=1}^{n} \log f_j$$
$$\text{subject to } Rf \preceq c$$

using lagrangian method, we have now

$$L(f, \lambda) = -\sum_{j=1}^{n} U_j\left(f_j\right) + \lambda^T(Rf - c)$$

$$\begin{aligned} g(\lambda) &= \inf_f \left(\sum_{j=1}^{n} -U_j\left(f_j\right) + \lambda^T(Rf - c) \right) \\ &= -\lambda^T c + \sum_{j=1}^{n} \inf_{f_j} \left(-U_j\left(f_j\right) + \left(r_j^T \lambda\right) f_j \right) \end{aligned}$$

---

b. if we ignore latency here, we already know that edge latency is $\frac{1}{c_i - t_i}$, then flow latency shall be defined as

$$l_j = \sum_i \frac{1}{c_i - t_i} \quad \forall i \in R_{i,j=1}$$

Moreover, if we denote here

$$d \in R^m, d_i = \frac{1}{c_i - t_i}$$

Thus

$$l = d^\top R$$

and $L = \max\{l_1, \ldots, l_n\}$ means that we need to find out the biggest value of l, thus we define here

$$\begin{array}{c} \min \max\{l_1, \ldots, l_n\} \\ \text{subject to} \quad Rf \preceq c \end{array}$$

we know that for each d_i,

$$\frac{1}{c_i} \leq d_i = \frac{1}{c_i - t_i}$$

When $f = 0$, the latency is minimal (and we know this is impossible), as I said $l = d^\top R$, the n

$$l = R^T (1/c_1, \ldots, 1/c_m)$$

and

$$L^{\min} = \max\left(R^T (1/c_1, \ldots, 1/c_m)\right)$$

since $l \in \mathbf{R}^n$, $R \in \mathbf{R}^{m \times n}$, $L^{\min}$ is to minimize the maximal entry of the vector l where n is the vertex number and m is the edge number.

---

c. Due to the above inequality, we then have

$$\max\left(R^T (1/(c_1 - t_1), \ldots, 1/(c_m - t_m))\right) \leq L$$

and we know that $t = Rf$ thus we have $t_i = \sum_j R_{i,j} f_j$, therefore we have

$$\sum_{i=1}^{m} \frac{R_{i,j}}{c_i - \sum_j R_{i,j} f_j} \leq L$$

We thus know that the original problem can be divided into two sub-problems such as

①

$$\begin{aligned} &\min L \\ \text{subject to} \quad & L = \max\left(R^T\left(1/(c_1-t_1), \ldots, 1/(c_m-t_m)\right)\right) \\ & Rf \preceq c \\ & -f \preceq 0 \end{aligned}$$

②

$$\begin{gathered} \max \quad \sum_{j=1}^{n} \log f_j \\ \text{subject to } Rf \preceq c \\ -f \preceq 0 \end{gathered}$$

---

d. By using scalarization, we can transform this multi-objective function into a single convex problem

for each flow from a vertex, we calculate its latency by summing up all edges this flow coming cross(indicated by R)

$$\sum_{i=1}^{m} \frac{R_{i,j}}{c_i - \sum_j R_{i,j} f_j}$$

and it shall be less than L,

moreover, we know the objective function here is

$$\max \quad \sum_{j=1}^{n} \log f_j$$

The variable here is f, and we know the constraint is convex on, then the following function

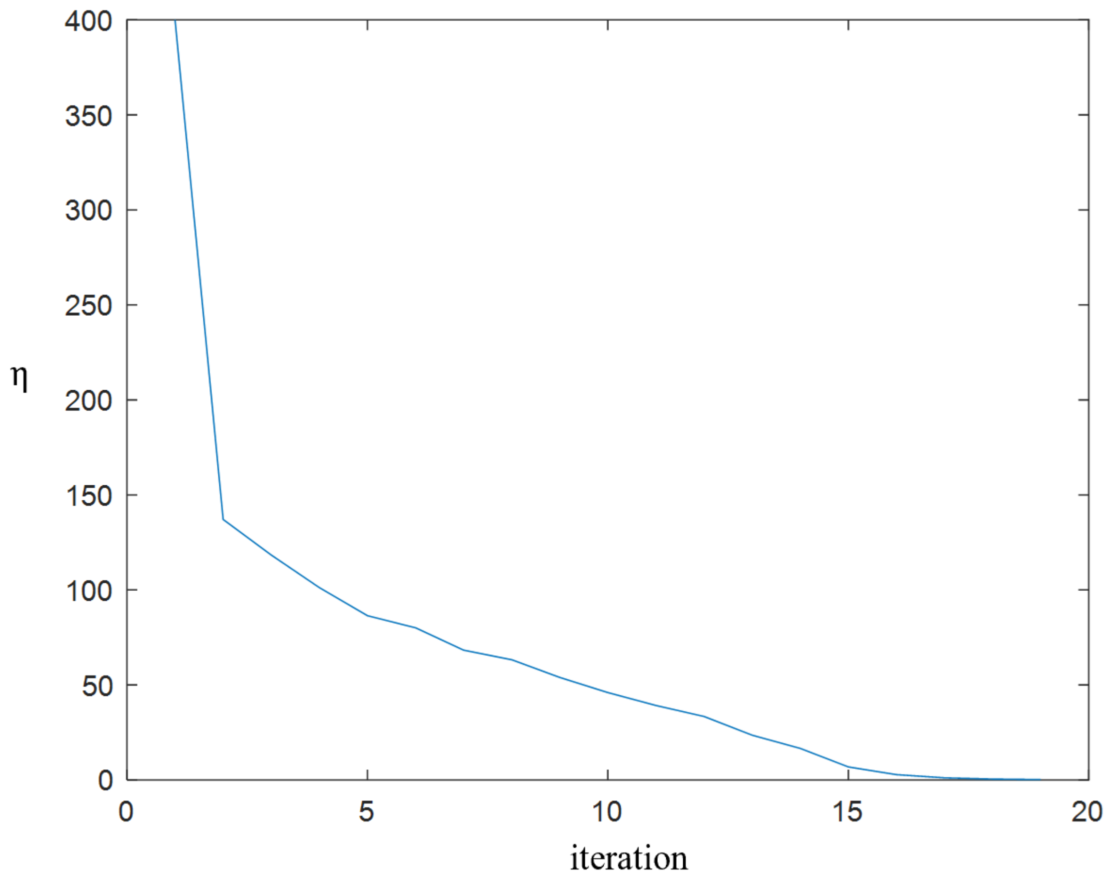
$$
\begin{array}{ll}
\min & -\lambda_1 \sum_{j=1}^{n} \log f_j + \lambda_2 L \\
\text{subject to} & Rf \preceq c \\
& \sum_{i=1}^{m} \frac{R_{i,j}}{c_i - \sum_j R_{i,j} f_j} \leq L \quad \forall j \in [1, n] \\
& -f \preceq 0
\end{array}
$$

is a convex optimization problem.

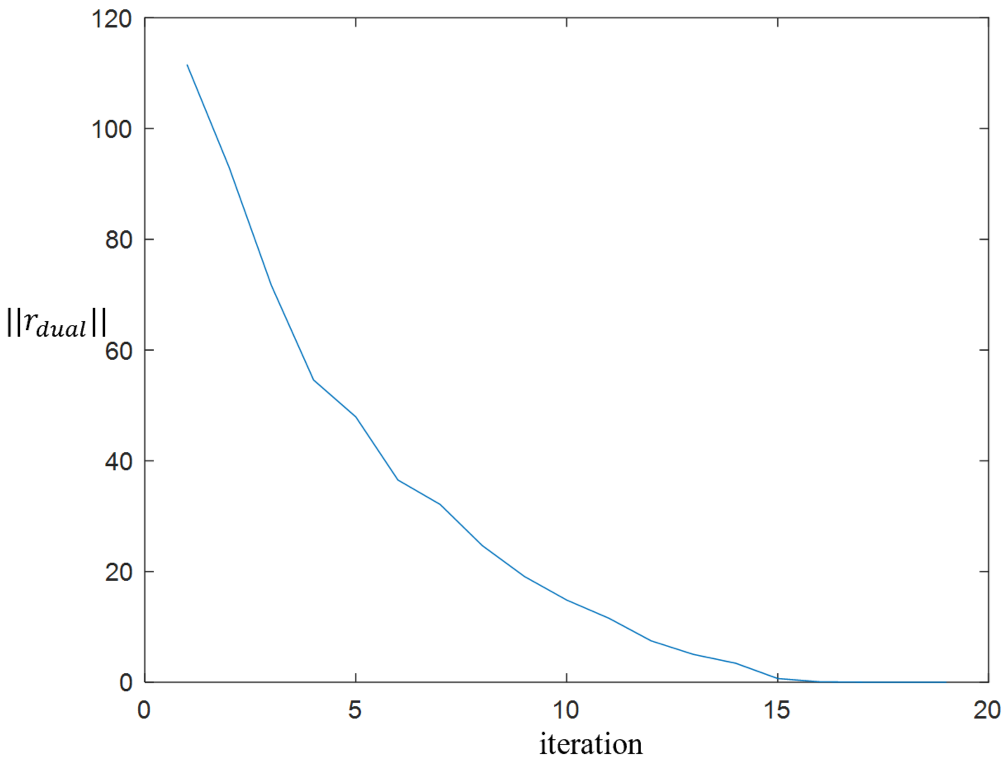
---

e.

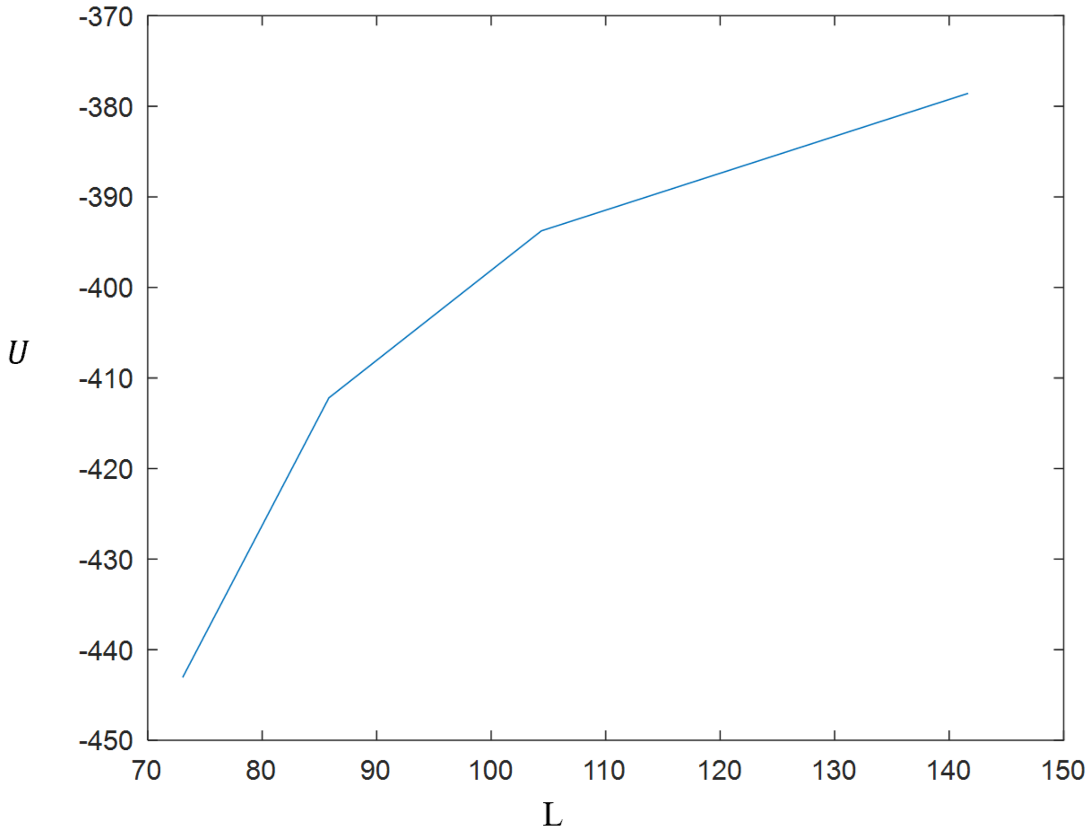
(1) plot η^ versus iteration number, at a fixed $\lambda \in R^2$ (λ is the vector used in scalarization)



(2)plot $||r_{dual}||_2$ versus iteration number, at a fixed λ 2 R2 (λ is the vector used in scalarization)



(3)arying λ, plot the trade-off curve of utility U versus latency L


-370
-380
-390
-400
-410
-420
-430
-440
-450
U
70
80
90
100
110
120
130
140
150
L