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SIMULATION-BASED OPTIMIZATION FOR MULTI-ECHELON INVENTORY SYSTEMS UNDER UNCERTAINTY

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ABSTRACT

Inventory optimization is critical in supply chain management. The complexity of real-world multi-echelon inventory systems under uncertainties results in a challenging optimization problem. We propose a novel simulation-based optimization framework for optimizing distribution inventory systems where each facility is operated with the (r, Q) inventory policy. The objective is to minimize the inventory cost while maintaining acceptable service levels quantified by the fill rates. The inventory system is modeled and simulated, which returns the performance functions. The expectations of these functions are then estimated by the Monte-Carlo method. Then the optimization problem is solved by a cutting plane algorithm. As the black-box functions returned by the Monte-Carlo method contain noises, statistical hypothesis tests are conducted in the iteration.

1 INTRODUCTION

Supply chain optimization is critical for manufacturing companies to maintain economic viability in the current highly competitive global marketplace (Wassick et al. 2012, Grossmann 2005). A supply chain is a network of interconnected entities, such as suppliers, manufactures, distributors, retailers, and customers, which have different functions ranging from procurement of raw materials, transformation of raw materials into products, and distribution of final products to customers (Sarimveis et al. 2008). Supply chain decisions are broadly classified into strategic, tactical, and operational ones (Min and Zhou 2002). Inventory management and control is a critical decision-making in supply chain optimization (Grossmann 2005, You and Grossmann 2008, Papageorgiou 2009). The objective of inventory optimization is to search the optimal parameters for a given inventory control policy so as to minimize the cost while maintaining acceptable service levels (Yue and You 2013, Wan, Pekny, and Reklaitis 2005).

Mathematical programming methods, such as multi-stage stochastic programming, suffer computational complexity (Zapata, Pekny, and Reklaitis 2011, You and Grossmann 2011a, 2008). Because the operations and networks in a supply chain are becoming more complex, analytical approaches are not powerful enough to model complicated real-world supply chains (Peidro et al. 2009). To solve the complicated problems which are intractable to mathematical programming methods, simulation-based optimized methods are developed (Fu 2002). The simulation model is able to capture details which accurately represent the actual stochastic and dynamic inventory system (Shah 2005).

Though a simulation-based optimization method is applicable to a complicated inventory system (Law and McComas 2002, Carson and Maria 1997, Swisher et al. 2000, Andradóttir 1998), there are still a number of challenges. First, the simulation procedure only provides black-box functions which evaluate outputs for given input values. There are no analytical expressions characterizing the input-output relationship. In an inventory optimization problem, not only the objective function but also the constraint functions are black-box. Second, the black-box functions returned by the simulation procedure contain noises and, as a result, simulation-based optimization is not merely optimization with black-box functions (Fu 2002, Azadivar 1999, Fu, Glover, and April 2005). The outputs of the simulation procedure are often expected values over the uncertain parameter region. A computational algorithm has to be adopted to estimate the expectations, resulting in discrepancies between the true expectations and their estimates. The discrepancies need to be considered by the optimization procedure as they can have significant impacts on the solutions.

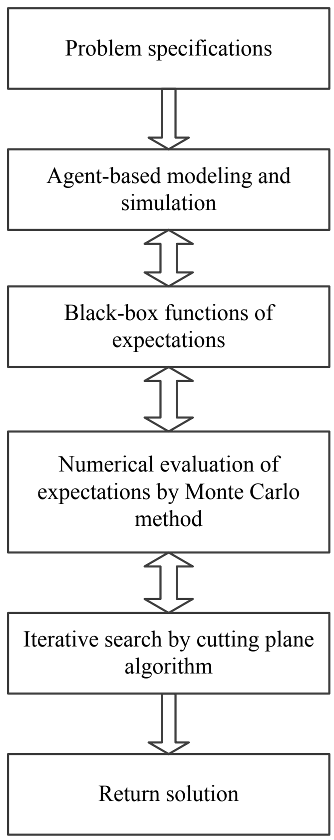

Figure 1: Outline of the simulation-based optimization framework

To address the aforementioned challenges, we propose a novel simulation-based optimization framework which integrates the supply chain simulation, the Monte-Carlo method, a cutting plane algorithm, an experimental design technique, and statistical hypothesis tests. The framework aims to optimize a multi-echelon inventory system under uncertainties. Its main components are visualized in Figure 1. According to the specifications of the multi-echelon system, a simulation model is built, which simulates the inventory system and evaluates the performance measures for given inventory parameters, which define the black-box functions in the objective and the constraints for the optimization procedure. To evaluate the black-box functions, Monte Carlo method is applied which estimates the expectations as the sample averages. The optimization problem with black-box functions is solved by a cutting plane algorithm, which is an iterative search by successively linearizing the black-box functions. The linearized functions are obtained from the first-order response surface models (Zhang, Jiang, and Guo 2009).

2 PROBLEM STATEMENT

There are a great variety of inventory systems and no single method can be applied to all systems. In this work, we focus on the distribution inventory system, which is a divergent network (a tree structure). In the distribution inventory system, a facility has only one predecessor but it can have one or multiple successors. The plant is the root of the tree structure. The inventory of the plant is assumed to be unlimited (You and Grossmann 2010). Each facility, except the plant, is controlled by the (r, Q) policy where r denotes the reorder point and Q is the order quantity (Zipkin 2000). When the demands can be satisfied by the on-hand inventory, the order begins to be processed. A time period is required for preparing, handling, and delivering goods. This period is defined as the order processing time, which is the duration from the moment that the order demand is satisfied by the on-hand inventory, to the moment that the shipment is received (You and Grossmann 2011, You et al. 2011). When the demands are not satisfied by the on-hand inventory, they are backordered.

The objective of inventory optimization is to minimize the inventory cost while satisfying the order demands with acceptable service levels. We focus on the β (or type 2) service level, which is defined as the average fraction of demands delivered immediately from the on-hand inventory (Chan 2003, Chu et al. 2014). This service level is also called the fill rate.

Specifically, the inventory optimization problem is stated as follows.

Assumption
- Divergent distribution network
- (r, Q) inventory control policy
- Service level of fill rate
- Backorder of unsatisfied demand
- Random customer order demands and order processing times

Given
- Network structure
- Time horizon
- Unit holding cost and unit reorder cost
- Minimum service level for each facility
- Probability distributions of random parameters

To determine
- Reorder point and order quantity of each facility

Objective
- To minimize the inventory cost (= holding cost + reorder cost)
- To maintain service levels no less than the minimum values

3 SIMULATION–BASED OPTIMIZATION FRAMEWORK

3.1 Stochastic simulation of multi-echelon system

There are a great variety of inventory systems and no single method can be applied to all systems. In this work, we focus on the distribution inventory system, which is a divergent network (a tree structure). In the distribution inventory system, a facility has only one predecessor but it can have one or multiple successors. The inventory system is driven by the customer order demands, which are random variables following given probability distributions. The lead times are also random variables

The inventory system consists of different facilities, such as the plant, distribution centers, and distributors. The plant is the root of the tree structure. The inventory of the plant is assumed to be unlimited so that the plant can send the shipment once it receives an order. The inventory control of the plant is not taken into consideration. Each facility, except the plant, is controlled by an inventory policy. In this work, we confine the attention to the (r, Q) policy where r denotes the reorder point and Q is the order quantity. The on-hand inventory (or physical inventory) is the amount of stored materials that are immediately available to satisfy the incoming order demands.

Simulating the inventory system returns the performance measures for the given inventory parameters. The performance measures are black-box functions to be used by the optimization procedure to search for the optimal inventory parameters. To facilitate the optimization procedure, the black-box functions evaluated by the simulation are expressed explicitly as

$$INVC = f\left(\mathbf{x},\boldsymbol{\theta}\right) \tag{1}$$

$$FR_i = g_i\left(\mathbf{x},\boldsymbol{\theta}\right),\ \forall i \tag{2}$$

where $INVC$ is the inventory cost and FR_i is the fill rate of facility i. The input data are stacked into the vector of inventory parameters as

$$\mathbf{x} = \left[rp_1,\ \cdots,\ rp_{N_F},\ oq_1,\ \cdots,\ oq_{N_F}\right] \tag{3}$$

and the vector of uncertain parameters as

$$\boldsymbol{\theta} = \left[od_{11},\ \cdots,\ od_{N_F N_T},\ ot_{11},\ \cdots,\ ot_{N_F N_T}\right] \tag{4}$$

The reorder point of facility i is denoted by rp_i and the order quantity is denoted by oq_i. The number of facilities is N_F and the number of time periods is N_T. The customer order demand received by facility i on day t is denoted by od_{it}. As the order processing times are random variables in the inventory system, they are also regarded as inputs of the simulation model as to investigate their effects on the inventory performances. The order processing time for the order sent from facility j to facility i is denoted by ot_{ij}. We should note that the inventory parameters and the uncertain parameters are all variables in the inventory system. However, they are fixed parameters for the simulation model and their values are changed by the optimization procedure outside the simulation procedure.

3.2 Formulation of inventory optimization problem

Inventory optimization aims to minimize the inventory cost while maintaining acceptable service levels. According to the performance functions evaluated by the simulation model, the optimization problem can be formulated. However, besides the inventory parameters, the performance functions in eq. (1) and (2) also depend on the uncertain parameters. As probability distributions of the uncertain parameters are given, the expected performance measures are calculated in the optimization problem.

Using the vector expression in eq. (4), the probability density function of the uncertain parameters is assumed to be

$$\boldsymbol{\theta} \sim p\left(\boldsymbol{\theta}\right) \tag{5}$$

The expected inventory cost is defined as

$$\varphi\left(\mathbf{x}\right) = E_{\boldsymbol{\theta}}\left[f\left(\mathbf{x},\boldsymbol{\theta}\right)\right] \tag{6}$$

where the expectation is calculated by

$$E_{\boldsymbol{\theta}}\left[f\left(\mathbf{x},\boldsymbol{\theta}\right)\right] = \iint_{\boldsymbol{\theta}} f\left(\mathbf{x},\boldsymbol{\theta}\right) p\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta} \tag{7}$$

The expected fill rate of a facility is

$$\psi_i\left(\mathbf{x}\right) = E_{\boldsymbol{\theta}}\left[g_i\left(\mathbf{x},\boldsymbol{\theta}\right)\right],\ \forall i \tag{8}$$

Using the expected performance functions, the inventory optimization problem is formulated as

$$\text{(INV\_OPT)} \qquad \min_{\mathbf{x}} \ \varphi(\mathbf{x}) \tag{9}$$

s.t.

$$\psi_i(\mathbf{x}) \geq fr_i^{\min}, \ \forall i \tag{10}$$

where $fr_i^{\min}$ is the minimum service level of facility i. The problem (INV_OPT) is a nonlinear program (NLP) with continuous decision variables stacked in the vector **x**. However, as the objective function and the constraint functions are all black-box functions, the problem cannot be solved by a numerical solver directly. In addition, the black-box functions are expectations which cannot be calculated analytically, incurring extra difficulties in solving the problem.

3.3 Expectation evaluation by Monte-Carlo method

Expectation evaluation requires calculating a high-dimensional integral as that in eq. (7). Because the performance functions evaluated by the simulation model have no analytical expressions and there are many uncertain parameters, exact calculation of the expectations is not feasible. A general alternative is to numerically calculate the expectations by the Monte-Carlo method that estimates the expectations by averages over a finite number of sampling points (Asmussen and Glynn 2007).

In the Monte-Carlo method, a sample of uncertain parameters is generated, which is a finite set of sampling points defined as

$$\Theta^{(r)} = \left\{\boldsymbol{\theta}^{(r,1)}, \ \boldsymbol{\theta}^{(r,2)}, \cdots, \ \boldsymbol{\theta}^{(r,s)}, \cdots, \ \boldsymbol{\theta}^{(r,N_{MC})}\right\} \tag{11}$$

The sample is denoted by Θ and indexed by r. A sampling point in the sample is indexed by s. The sample size is indicated by N_{MC}. The parameters are sampled according to the probability density function $p(\boldsymbol{\theta})$.

At each sampling point $\boldsymbol{\theta}^{(r,s)}$, the performance functions are evaluated as

$$f^{(r,s)}(\mathbf{x}) = f\left(\mathbf{x}, \boldsymbol{\theta}^{(r,s)}\right) \tag{12}$$

Then the estimated expectations are

$$\hat{\varphi}^{(r)}(\mathbf{x}) = \frac{1}{N_{MC}} \sum_{s=1}^{N_{MC}} f^{(r,s)}(\mathbf{x}) \tag{13}$$

$$\hat{\psi}_i^{(r)}(\mathbf{x}) = \frac{1}{N_{MC}} \sum_{s=1}^{N_{MC}} \psi_i^{(r,s)}(\mathbf{x}) \tag{14}$$

To distinguish the estimated value from true value, the estimated value is expressed with a head.

In this work, a statistical hypothesis test is conducted to investigate the results returned by the Monte-Carlo method. The hypothesis test validates the solution feasibility. As $\hat{\psi}_i^{(r)}(\mathbf{x})$ is an estimate, the inequality $\hat{\psi}_i^{(r)}(\mathbf{x}) \geq fr_i^{\min}$ does not imply $\psi_i(\mathbf{x}) \geq fr_i^{\min}$. To ensure that the latter inequality is satisfied with a given confidence level of $1-\alpha$, the hypothesis test is

$$H_0 : \psi_i(\mathbf{x}) \geq fr_i^{\min}, \ \forall i \tag{15}$$

$$H_1 : \psi_i(\mathbf{x}) < fr_i^{\min}, \ \forall i \tag{16}$$

If the null hypothesis is not rejected, the solution of **x** is feasible.

3.4 Iterative search by cutting plane method

After the expectations are evaluated by the Monte-Carlo method, the optimization problem with the estimated functions becomes

$$\text{(INV\_OPT\_EST)} \qquad \min_{\mathbf{x}} \ \hat{\varphi}(\mathbf{x}) \tag{17}$$

s.t.

$$\hat{\psi}_i\left(\mathbf{x}\right) \geq fr_i^{\min} + dp_i,\ \forall i \tag{18}$$

The safety distances, dp_i, $\forall i$, are added to the service level constraints to compensate the possible discrepancies between the estimated fill rates and the true values. The optimization problem is an NLP with black-box functions and it is solved by the cutting plane algorithm presented in this subsection.

The cutting plane algorithm is an iterative search. It starts from a feasible solution, denoted by $\mathbf{x}^{(p)}$. Explicitly, the vector $\mathbf{x}^{(p)}$ consists of elements as

$$\mathbf{x}^{(p)} = \left[x_1^{(p)},\ \cdots,\ x_m^{(p)},\ \cdots,\ x_{N_X}^{(p)}\right]^T \tag{19}$$

where an element is indexed by m and the dimension of $\mathbf{x}^{(p)}$ is N_X. Then the algorithm linearizes the objective function and the constraint functions at $\mathbf{x}^{(p)}$, which become

$$\hat{\varphi}^{(r)}\left(\mathbf{x}\right) \approx \left(\mathbf{x} - \mathbf{x}^{(p)}\right)^T \mathbf{f}_a^{(p)} + f_b^{(p)} \tag{20}$$

$$\hat{\psi}_i^{(r)}\left(\mathbf{x}\right) \approx \left(\mathbf{x} - \mathbf{x}^{(p)}\right)^T \mathbf{h}_{a,i}^{(p)} + h_{b,i}^{(p)},\ \forall i \tag{21}$$

The coefficients of $\mathbf{f}_a^{(p)}$, $\mathbf{h}_{a,i}^{(p)}$, $f_b^{(p)}$, and $h_{b,i}^{(p)}$ are obtained from first-order response surface models, which are estimated by the first-order response surface models.

After the linearization, the cutting plane algorithm solves a linearized problem of

(INV_LIN$_p$) $$\mathbf{x}^* = \arg\min_{\mathbf{x}}\ \left(\mathbf{x} - \mathbf{x}^{(p)}\right)^T \mathbf{f}_a^{(p)} + f_b^{(p)} \tag{22}$$

s.t.

$$\left(\mathbf{x} - \mathbf{x}^{(q)}\right)^T \mathbf{h}_{a,i}^{(q)} + h_{b,i}^{(q)} \geq fr_i^{\min} + dp_i,\ \forall i,\ q \leq N_Q^{(p)} \tag{23}$$

$$\sum_{m=1}^{N_X} \left| x_m - x_m^{(p)} \right| \leq \delta \tag{24}$$

The objective in eq. (22) is the linearized $\hat{\varphi}^{(r)}\left(\mathbf{x}\right)$ at $\mathbf{x}^{(p)}$ and the functions on the left-hand side of constraint (23) are linearized $\hat{\psi}_i^{(r)}\left(\mathbf{x}\right)$. The first-order response surface models are only local approximations of the original nonlinear functions. Thus, inequality (24) is added to confine the search scope. The absolute values in the constraint can be expressed by linear inequalities. The optimal solution of the linearized problem (INV_LIN$_p$) provides the value in the next iteration.

4 CASE STUDY

To demonstrate the simulation-based optimization framework, we apply it to a case study, which is a two-echelon inventory system shown in Figure 2. The daily customer demands follow the Gaussian distribution with (mean, standard deviation) labeled beside the order icons. The order processing times follow the uniform distribution with [lower bound, upper bound] labeled beside the shipment icons. The minimum service levels for all facilities are set as 95%. The time horizon is 365 days.

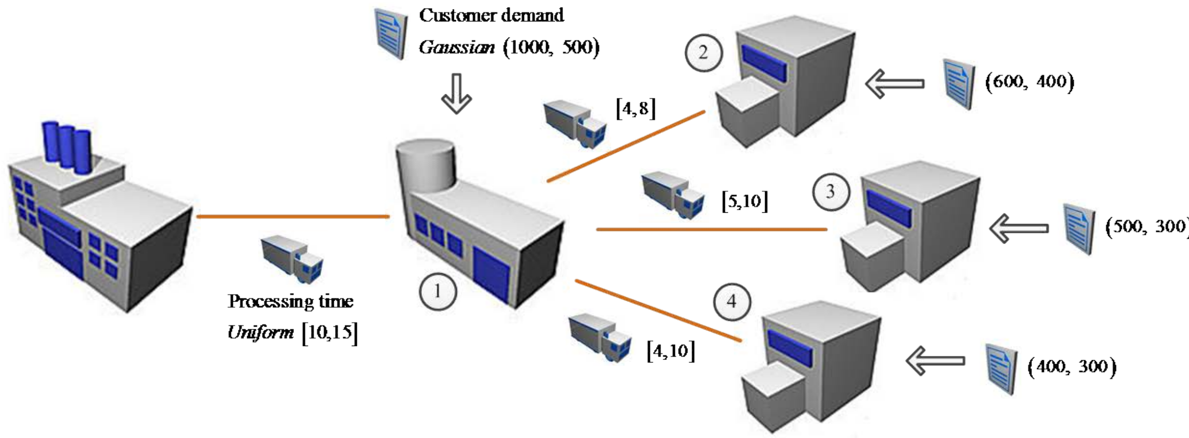

Figure 2: Two-echelon inventory system

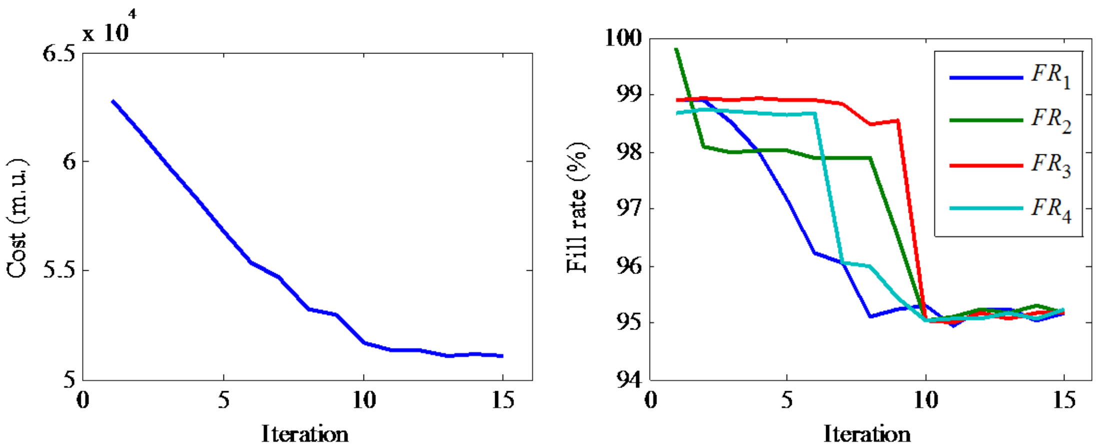

Figure 3: Iterative results of the two-echelon inventory system.

The inventory optimization problem is solved by the simulation-based optimization method. The iterative cutting plane algorithm is applied. The number of sampling points in the Monte-Carlo method is 1,000. The number is sufficiently large to ensure an accurate estimate. It will be seen in Table 2 that the lengths of the 99% confidence intervals using the sample size are all less than 1%. The iterative process is displayed in Figure 3Figure 3. The gap between the objective function values in two adjacent iterations is set as 1%. The optimization algorithm stops after 15 iterations and the computational time is 270.6 seconds.

Table 1: Initial and optimal solutions for the two-echelon inventory system.

	Reorder point				Order quantity			
Facility	1	2	3	4	1	2	3	4
Initial solution	40000	6000	5000	8000	40000	8000	7000	6000
Optimal solution	32606.1	3556.6	3956.7	9289.2	39548.3	9289.2	7025.2	5654.1

The initial and optimal solutions are listed in Table 1. The indices of the reorder points and the order quantities correspond to those of the facilities in Figure 2. The optimal function values are listed in Table 2 along with the radii of the 99% confidence intervals. The range of a confidence interval is small (less than 0.5%), reflecting a small variation in the estimate returned by the Monte-Carlo method.

Table 2: Optimal function values for the two-echelon inventory system.

	Inventory cost	Fill rate			
		Facility 1	Facility 2	Facility 3	Facility 4
Value	51043.6	95.16	95.15	95.16	95.21
99% interval	±108.9	±0.14	±0.13	±0.13	±0.13
Minimum service level	–	95.00	95.00	95.00	95.00

To validate the solution feasibility, Table 2 also lists the minimum service levels and the safety service levels. Every minimum service level is less than the obtained service level minus the radius of the confidence interval. Therefore, the service level constraints are satisfied with the 99% probability. The test shows that all the service level constraints are activated.

5 CONCLUSION

We proposed a simulation-based optimization framework for solving multi-echelon inventory problems. The inventory systems had the divergent structure where each facility was installed with the (r, Q) control policy. The inventory parameters were optimized to minimize the inventory cost while maintaining acceptable service levels quantified by the fill rates. The objective functions and the constraint functions were evaluated by the stochastic simulation. Their expectations were then estimated by the Monte Carlo method using sampling points of the uncertain parameters according to their probability distributions. The estimated expectation functions entered the optimization problem as black-box functions. The problem was solved by the cutting plane algorithm that iteratively linearized the black-box functions using the first-order response surface models.

The simulation-based optimization framework was demonstrated by a case study of a two-echelon system with 4 facilities. In the iterative optimization procedure, the cost objective function decreased while the constraints functions were pushed to the boundaries specified by the minimum service levels. The optimal solution was found in 270.6 seconds.

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