How to Be a Good Shooter: Analysis of Projectile Motion

Math Modeling Final Project
Jianqiu Bai

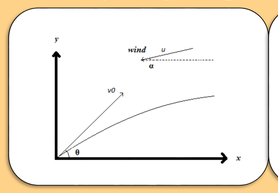
WOOSTER

Abstract

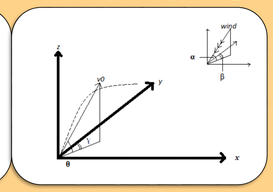
This study investigates the projectile motion associated with air resistance. A 2D and a 3D model are built based on Newton's Second Law. According to the built model, the task is, from experimental data, specifically focusing on computing the drag friction coefficient, and figuring out the relationship between the friction and the object's velocity, and then finding the optimal launch angle.

Introduction

2D Trajectory Graph



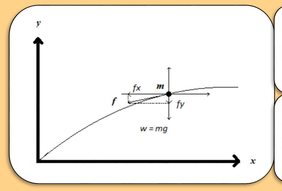
3D Trajectory Graph



- The projectile motion is the motion compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated.
- Launch Velocity – *v0*.
- Launch Angle – θ.
- Friction Velocity – *u*.
- Friction Angle – α.
- Projector Launch Angle – γ. Projector Friction Angle – β.
- Friction Coefficient – *k*.

Model

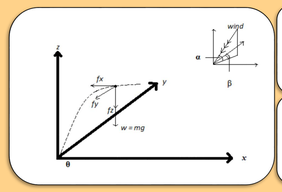
- **Newton's Second Law:** Newton's Second Law states that the net force on an object is the product of the object's mass and acceleration, $F = ma$.
- **2D Model:**



$$\begin{cases} x : F_x = -f_x = m \cdot a_x = -k \cdot (V_x)^n, \\ y : F_y = -f_y - w = m \cdot a_y = -k \cdot (V_y)^n - mg, \end{cases}$$

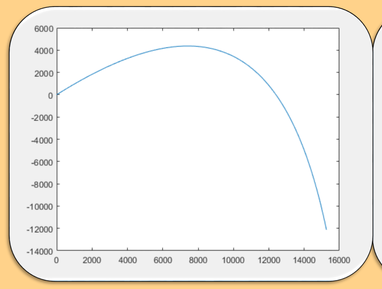
$$\begin{cases} x : x'' = -\frac{k}{m} \cdot (v_x - u \cdot cos(\alpha)), \\ y : y'' = -\frac{k}{m} \cdot (v_y - u \cdot sin(\alpha)) - g. \end{cases}$$

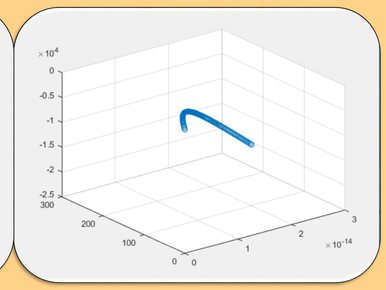
- **3D Model:**



$$\begin{cases} x : x' = v_x, v_x' = -\frac{k}{m} \cdot (v_x - u_x), \\ y : y' = v_y, v_y' = -\frac{k}{m} \cdot (v_y - u_y), \\ z : z' = v_z, v_z' = -\frac{k}{m} \cdot (v_z - u_z) - g. \end{cases}$$

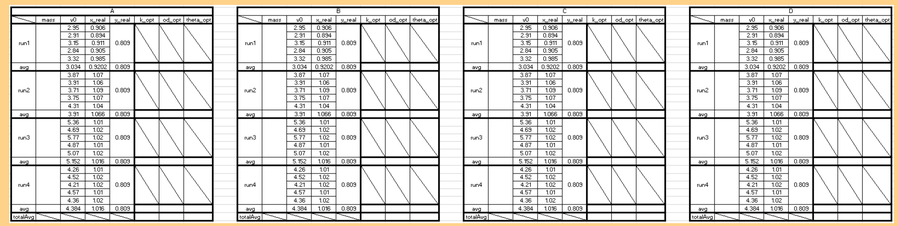
$$\begin{cases} x : x' = v_x, v_x' = -\frac{k}{m} \cdot [v_x - u_0 \cdot cos(\alpha) \cdot cos(\beta)], \\ y : y' = v_y, v_y' = -\frac{k}{m} \cdot [v_y - u_0 \cdot cos(\alpha) \cdot sin(\beta)], \\ z : z' = v_z, v_z' = -\frac{k}{m} \cdot [v_z - v_0 \cdot sin(\alpha)] - g. \end{cases}$$





Methods and Results

- **Data Management:** Recall the task is to find the constant friction coefficient *k* and to see if *k* is the same for any object, and to find the order *n* and lastly to find the optimal angle θ. The data is obviously needed to find the results, and is from *Physics 111 Projectile Motion Lab*. In that lab, students are asked to launch different kinds of balls by a launch machine on a table, and the ball will fall as a projectile motion when it goes to the edge of the table, described as the graph below.



	A						B						C						D					
	mass	v0	x_real	y_real	k_opt	od_opt	theta_opt	v0	x_real	y_real	k_opt	od_opt	theta_opt	v0	x_real	y_real	k_opt	od_opt	theta_opt	v0	x_real	y_real	k_opt	od_opt	theta_opt
run1		2.95	0.906					2.95	0.906					2.95	0.906					2.95	0.906				
		2.91	0.894					2.91	0.894					2.91	0.894					2.91	0.894				
		3.15	0.911		0.809			3.15	0.911		0.809			3.15	0.911		0.809			3.15	0.911		0.809	
		2.84	0.905					2.84	0.905					2.84	0.905					2.84	0.905				
		3.32	0.985					3.32	0.985					3.32	0.985					3.32	0.985				
avg		3.034	0.9202	0.809				3.034	0.9202	0.809				3.034	0.9202	0.809				3.034	0.9202	0.809			
run2		3.87	1.07					3.87	1.07					3.87	1.07					3.87	1.07				
		3.91	1.06					3.91	1.06					3.91	1.06					3.91	1.06				
		3.71	1.09		0.809			3.71	1.09		0.809			3.71	1.09		0.809			3.71	1.09		0.809	
		3.75	1.07					3.75	1.07					3.75	1.07					3.75	1.07				
		4.31	1.04					4.31	1.04					4.31	1.04					4.31	1.04				
avg		3.91	1.066	0.809				3.91	1.066	0.809				3.91	1.066	0.809				3.91	1.066	0.809			
run3		5.36	1.01					5.36	1.01					5.36	1.01					5.36	1.01				
		4.63	1.02					4.63	1.02					4.63	1.02					4.63	1.02				
		5.77	1.02		0.809			5.77	1.02		0.809			5.77	1.02		0.809			5.77	1.02		0.809	
		4.07	1.01					4.07	1.01					4.07	1.01					4.07	1.01				
		5.07	1.02					5.07	1.02					5.07	1.02					5.07	1.02				
avg		5.152	1.016	0.809				5.152	1.016	0.809				5.152	1.016	0.809				5.152	1.016	0.809			
run4		4.26	1.01					4.26	1.01					4.26	1.01					4.26	1.01				
		4.52	1.02					4.52	1.02					4.52	1.02					4.52	1.02				
		4.21	1.02		0.809			4.21	1.02		0.809			4.21	1.02		0.809			4.21	1.02		0.809	
		4.57	1.01					4.57	1.01					4.57	1.01					4.57	1.01				
		4.36	1.02					4.36	1.02					4.36	1.02					4.36	1.02				
avg		4.384	1.016	0.809				4.384	1.016	0.809				4.384	1.016	0.809				4.384	1.016	0.809			
(scatter)																									

- **Find *k*:** By applying the data managed above, assuming the order $n = 1$, *fminresearch* function is utilized in *MatLab* to find the optimal *k*.
- **Find *n*:** After getting the optimal *k* value, the *fminresearch* function is applied again to find the optimal order *n*.
- **Find Optimal θ:** Once both the optimal *k* and optimal *n* are known, the *fminresearch* function is called for the third time to find the optimal launch angle θ.

References

[1] Korn, Granino A. *Advanced Dynamic-system Simulation: Model - replication and Monte Carlo Studies*. New Jersey, 2013.
[2] Kuehn, Kerry. *A Student's Guide through the Great Physics Texts.* Springer, New York. 2015.
[3] Ahmad, Bashir, Hanan Batarfi, Juan J. Nieto, Óscar Otero-Zarraquiños, and Wafa Shammakh. "Projectile Motion via Riemann-Liouville Calculus." *Adv Differ Equ Advances in Difference Equations* 2015.1 (2015): n. pag. Web.
[4] Bernardo, Reginald Christian, Jose Perico Esguerra, Jazmine Day Vallejos, and Jeff Jerard Canda. "Wind-influenced Projectile Motion." *European Journal of Physics* 36, no. 2 (2015): 025016.
[5] Andersen, Poul Winther. "Comment on 'Wind-influenced Projectile Motion'." *European Journal of Physics Eur. J. Phys.* 36, no. 6 (2015): 068003.

Conclusion

- The friction coefficient *k* is 0.1753.
- The order *n* is 2.
- The optimal launch angle θ is 45°.