Announcements

- HW5 (the last HW) will be out tonight

CS6161: Design and Analysis of Algorithms (Fall 2020)

Introduction to Game Theory II

Instructor: Haifeng Xu

Outline

- Recap: Games and Nash Equilibrium
- Computation of Nash Equilibrium
- Computing NE in Two-Player Zero-Sum Games

Game theory studies optimization in settings with multiple strategic agents, each maximizing their own objective

Normal-form representation of two-player games:

- Player A with action set $\{a_1, a_2, \dots, a_n\}$
- Player B with action set $\{b_1, b_2, \dots, b_m\}$
- Player utilities represented in a matrix:

	b_1	b_2	$\dots$	b_m
a_1	(u_{11}, v_{11})	(u_{12}, v_{12})	$\dots$	(u_{1m}, v_{1m})
a_2	(u_{21}, v_{21})	(u_{22}, v_{22})	$\dots$	(u_{2m}, v_{2m})
...				
a_n	(u_{n1}, v_{n1})	(u_{n2}, v_{n2})	$\dots$	(u_{nm}, v_{nm})

Nash Equilibrium

- Player A plays mixed strategy $\boldsymbol{p}(\sum p_i = 1)$ and B plays $\boldsymbol{q}(\sum q_i = 1)$
  - They sample actions independently

Def. A strategy profile $(\boldsymbol{p}, \boldsymbol{q})$ is a Nash equilibrium if

(1) $u(\boldsymbol{p}, \boldsymbol{q}) \geq u(a_i, \boldsymbol{q}) = \sum_j u_{i,j}\, q_j$ for any $a_i \in A$

(2) $v(\boldsymbol{p}, \boldsymbol{q}) \geq v(\boldsymbol{p}, b_j) = \sum_i v_{i,j}\, p_i$ for any $b_j \in B$

Remarks

- Expected utility of strategies $(\boldsymbol{p}, \boldsymbol{q})$ follows standard calculation:
  $u(\boldsymbol{p}, \boldsymbol{q}) = \sum_{ij} u_{i,j}\, p_i q_j$ for A, $v(\boldsymbol{p}, \boldsymbol{q}) = \sum_{ij} v_{i,j}\, p_i q_j$ for B
- The assumption that players sample actions independently is critical
  - If allow correlation, it is called correlated equilibrium
  - Correlation/coordination requires external sources, e.g., traffic light

Recall: Traffic Light Game

B

A

	STOP	GO
STOP	(-3, -2)	(-3, 0)
GO	(0, -2)	(-100, -100)

Traffic light is a "correlation device":

- (green, red) with prob ½, (red, green) with prob ½
- In this case, players' actions are correlated, not independent

Nash Equilibrium

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  $u(\boldsymbol{p}, \boldsymbol{q}) = \sum_{ij} u_{i,j}\, p_i q_j$ for A, $v(\boldsymbol{p}, \boldsymbol{q}) = \sum_{ij} v_{i,j}\, p_i q_j$ for B
- The assumption that players sample actions independently is critical
  - If allow correlation, it is called correlated equilibrium
  - Correlation/coordination requires external sources, e.g., traffic light
- Also called mixed strategy equilibrium

Mixed-Strategy Equilibrium

Theorem (Nash, 1951): Every finite game admits at least one mixed-strategy equilibrium

- E.g., $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is a NE for Rock-paper-scissor

		$1/3$	$1/3$	$1/3$
		Rock	Paper	Scissor
ExpU = 0	Rock	(0, 0)	(-1, 1)	(1, -1)
ExpU = 0	Paper	(1, -1)	(0, 0)	(-1, 1)
ExpU = 0	Scissor	(-1, 1)	(1, -1)	(0, 0)

An Important Property of Nash Equilibrium

Thm.: If $(\boldsymbol{p}, \boldsymbol{q})$ is a NE, then any a_i with $p_i > 0$ must be equally good – they are all optimal actions (also called best responses)

- Will be very useful for computing a NE of a game

		$1/3$	$1/3$	$1/3$
		Rock	Paper	Scissor
ExpU = 0	Rock	(0, 0)	(-1, 1)	(1, -1)
ExpU = 0	Paper	(1, -1)	(0, 0)	(-1, 1)
ExpU = 0	Scissor	(-1, 1)	(1, -1)	(0, 0)

Special Case: Pure Nash Equilibrium (PNE)

- Equilibrium strategy profile is a pure strategy profile
- Example:

	L	M	R
U	$\boxed{4,3}$	5,1	6,2
M	2,1	8,4	3,6
D	3,0	$\boxed{9,6}$	2,5

PNE not necessarily exist, and not unique in general

Special Case: Dominant Strategy Eq.

Definition: Action a_{i^*} is a dominant action for player A if a_{i^*} is better than any other $a_i \in A$, regardless what actions B takes. Formally,

$$u_{i^*,j} \geq u_{i,j}, \qquad \forall i \neq i^*, \forall j$$

Definition: (a_{i^*}, b_{j^*}) is a dominant strategy equilibrium **if** a_{i^*} $[b_{j^*}]$ is a dominant action for A [B].

A \ B	B stays silent	B betrays
A stays silent	-1, -1	-3, 0
A betrays	0, -3	$\boxed{-2, -2}$

Prisoner's Dilemma

- *Betray* is a dominant strategy for both

Special Case: Dominant Strategy Eq.

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Formally,

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Definition: (a_{i^*}, b_{j^*}) is a dominant strategy equilibrium **if** a_{i^*} $[b_{j^*}]$ is a dominant action for A [B].

Dominant strategy equilibrium does not necessarily exist

B

A

	STOP	GO
STOP	(-3, -2)	$\boxed{(-3, 0)}$
GO	$\boxed{(0, -2)}$	(-100, -100)

Traffic Light Game

Important Remark

- Everything described before naturally generalize to many players
- The discussion of two players here is purely for convenience

Outline

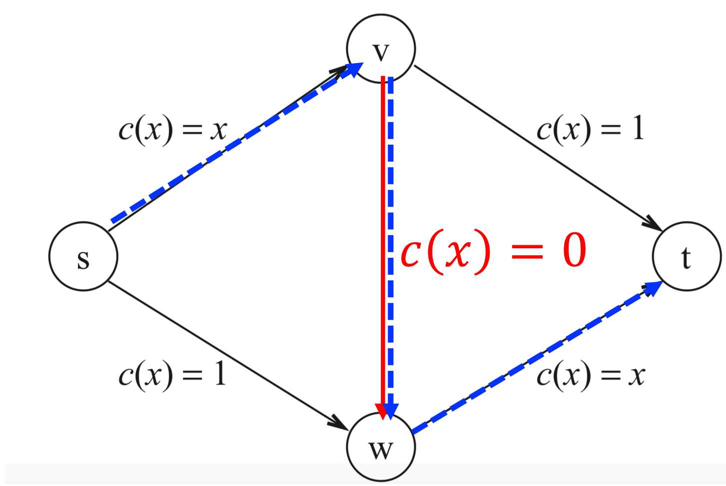
- Recap: Games and Nash Equilibrium
- Computation of Nash Equilibrium
- Computing NE in Two-Player Zero-Sum Games

Why Want to Compute NE?

- In algorithm design, we compute optimal solution for an algorithmic problem
- In games, we compute the equilibrium, which characterize the "optimal solution" for each player
  - There are other notions of equilibrium as well, like correlated equilibrium, coarse correlated equilibrium, etc.
  - But, NE is the most widely used and studied

Why Want to Compute NE?

- Motivation 1: figure out best action against strategic opponents
  - Just like why we want to solve single-agent optimization problem
  - E.g., want to figure out best GO/Poker agent strategy
- Motivation 2: predict where a multi-agent system will stabilize
  - E.g., rock-paper-scissor will stabilize at uniform random strategy?
  - E.g., decide whether government should build a superior highway



Seoul tears down an urban highway and the city can breathe again

By Kamala Rao on Apr 5, 2011

Cross-posted from Sightline's Daily Score blog.

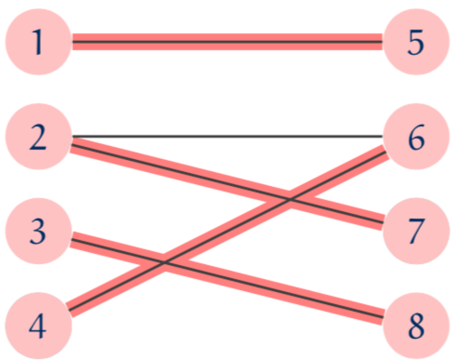
As a sustainability-loving transportation planner, I was thrilled to learn that Dr. Kee Yeon Hwang would be visiting Vancouver and talking about the project that has made Seoul, Korea a legend in urban planning circles: the Cheonggyecheon Restoration Project.

What he and his colleagues accomplished — tearing down a busy, elevated freeway,

This downtown green space in Seoul was once a looming, congested elevated freeway.Photo: Kyle Nishioka

Why Want to Compute NE?

- ➢ Motivation 1: figure out best action against strategic opponents
  - Just like why we want to solve single-agent optimization problem
  - E.g., want to figure out best GO/Poker agent strategy
- ➢ Motivation 2: predict where a multi-agent system will stabilize
  - E.g., rock-paper-scissor will stabilize at uniform random strategy?
  - E.g., decide whether government should build a superior highway
  - E.g., if each self-driving car optimizes the traveling cost for its own driver, would the road be efficient/safe overall?
  - If each node only wants to be matched to a node it prefers, what would the ultimate bipartite matching be like?



Intractability of Finding One NE

Theorem: Computing a Nash equilibrium for any two-player normal-form game is PPAD-hard.

Note: widely believed that PPAD-hard problems cannot be solved in poly time

- Theorem implies no $\mathrm{poly}(mn)$ time algorithm to compute an NE for any input game
- Ok, so what can we hope?
  - If the game has good structures, maybe we can find an NE efficiently
  - For example, zero-sum $u_{i,j}, + v_{i,j} = 0$ for all (i, j), some allocation allocation problems

Intractability of Finding "Best" NE

Theorem: It is NP-hard to compute the NE that maximizes the sum of players' utilities or any single player's utility even in two-player games.

- Proofs of these results for NEs are beyond the scope of this course

- So, probably no hope to have a guaranteed poly-time algorithm to compute a Nash
- Next: will show an $O(2^n)$ time algorithm for computing NE in two-player games
  - Note: this algorithm will only work for two-player games (during my description, you can try to see why)

Let Us Consider A (Much) Simpler Problem

- Suppose, by magic, you know the support of a NE
  - Supp($\boldsymbol{p}$) = $\{a_i : p_i > 0\} = S_A$ contains all actions with positive probabilities
- Previous theorem tells us: any action in the support of a NE must be equally good
- If support (S_A, S_B) of a NE ($S_A \subseteq A,\ S_B \subseteq B$) is given, then NE $(\boldsymbol{p}, \boldsymbol{q})$ is a solution a linear **feasibility problem**:

Find strategy $\boldsymbol{p}, \boldsymbol{q}$ and player utility u, v, such that:

$$\begin{aligned}
&\textstyle\sum_{b_j \in S_B} u_{i,j} \cdot q_j = u, && \text{for every } a_i \in S_A, \\
&\textstyle\sum_{b_j \in S_B} u_{i,j} \cdot q_j \leq u, && \text{for every } a_i \notin S_A, \\
&p_i \geq 0, && \text{for every } a_i \in S_A, \\
&\textstyle\sum_{a_i \in S_A} p_i = 1 &&
\end{aligned}$$

Any $a_i \in S_A$ achieves the same utility u, which is better than any $a_{i'} \notin S_A$

And similar constraints for player B strategy $\boldsymbol{q}$ and utility v

Let Us Consider A (Much) Simpler Problem

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Find strategy $\boldsymbol{p}, \boldsymbol{q}$ and player utility u, v, such that:

$\sum_{b_j \in S_B} u_{i,j} \cdot q_j = u,$ for every $a_i \in S_A$,

$\sum_{b_j \in S_B} u_{i,j} \cdot q_j \leq u,$ for every $a_i \notin S_A$,

$p_i \geq 0,$ for every $a_i \in S_A$,

$\sum_{a_i \in S_A} p_i = 1$

$\boldsymbol{p}$ is indeed a probability supported on S_A

And similar constraints for player B strategy $\boldsymbol{q}$ and utility v

Let Us Consider A (Much) Simpler Problem

Find strategy $\boldsymbol{p}, \boldsymbol{q}$ and player utility u, v, such that:

$$\sum_{b_j \in S_B} u_{i,j} \cdot q_j = u, \qquad \text{for every } a_i \in S_A,$$

$$\sum_{b_j \in S_B} u_{i,j} \cdot q_j \leq u, \qquad \text{for every } a_i \notin S_A,$$

$$p_i \geq 0, \qquad \text{for every } a_i \in S_A,$$

$$\sum_{a_i \in S_A} p_i = 1$$

And similar constraints for player B strategy $\boldsymbol{q}$ and utility v

Claim: Any $\boldsymbol{p}, \boldsymbol{q}$ and u, v feasible to above linear system forms a NE.

Proof: follows easily from definition

- $\boldsymbol{p}$ supports only on best actions, thus must be a best response
- Similar for $\boldsymbol{q}$

Let Us Consider A (Much) Simpler Problem

Find strategy $\boldsymbol{p}, \boldsymbol{q}$ and player utility u, v, such that:

$$\sum_{b_j \in S_B} u_{i,j} \cdot q_j = u, \qquad \text{for every } a_i \in S_A,$$

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$$p_i \geq 0, \qquad \text{for every } a_i \in S_A,$$

$$\sum_{a_i \in S_A} p_i = 1$$

And similar constraints for player B strategy $\boldsymbol{q}$ and utility v

Claim: Any $\boldsymbol{p}, \boldsymbol{q}$ and u, v feasible to above linear system forms a NE.

Remark

- This does not work for $> \mathbf{2}$ players since utilities will not be linear in strategies any more

But….What if we know support of NE?

- Just enumerate all possibilities of supports
  - $2^n \times 2^m$ possible combinations
  - Inevitable in the worst case as we know it is PPAD-hard
- Lesson learned: the difficulty of computing NE lies in finding the correct support sets

Example: 2-player game

(Porter et al., 2004)

	0	q_2	q_3	q_4	0
0	2,3	-1,4	2,4	5,2	1,-1
0	2,2	3,0	4,1	-2,4	1,3
p_3	4,6	7,2	2,-2	4,9	2,1
p_4	9,0	-2,6	6,3	7,0	0,5
p_5	3,2	6,1	2,5	5,3	1,0

$$\left.\begin{aligned} u_{3,2}q_2 + u_{3,3}q_3 + u_{3,4}q_4 &= u \\ u_{4,2}q_2 + u_{4,3}q_3 + u_{4,4}q_4 &= u \\ u_{5,2}q_2 + u_{5,3}q_3 + u_{5,4}q_4 &= u \end{aligned}\right\}$$

player A indifferent between $\{a_3, a_4, a_5\}$

$$\left.\begin{aligned} u_{1,2}q_2 + u_{1,3}q_3 + u_{1,4}q_4 &\leq u \\ u_{2,2}q_2 + u_{2,3}q_3 + u_{2,4}q_4 &\leq u \end{aligned}\right\}$$

player A prefers $\{a_3, a_4, a_5\}$ *to* $\{a_1, a_2\}$

$$p_3 + p_4 + p_5 = 1, p_3, p_4, p_5 \geq 0$$

… same for player B

Example: 2-player game

(Porter et al., 2004)

	0	2/7	3/7	2/7	0	
0	2,3	-1,4	2,4	5,2	1,-1	2
0	2,2	3,0	4,1	-2,4	1,3	2
2/11	4,6	7,2	2,-2	4,9	2,1	4
4/11	9,0	-2,6	6,3	7,0	0,5	4
5/11	3,2	6,1	2,5	5,3	1,0	4
	2	3	3	3	2	

Outline

- Recap: Games and Nash Equilibrium
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(Two-Player) Zero-Sum Games

- Matrix Representation:

	b_1	b_2	$\dots$	b_m
a_1	(u_{11}, v_{11})	(u_{12}, v_{12})	$\dots$	(u_{1m}, v_{1m})
a_2	(u_{21}, v_{21})	(u_{22}, v_{22})	$\dots$	(u_{2m}, v_{2m})
...				
a_n	(u_{n1}, v_{n1})	(u_{n2}, v_{n2})	$\dots$	(u_{nm}, v_{nm})

Zero-sum: $u_{ij} + v_{ij} = 0$ for all outcome (a_i, b_j)

Zero-Sum Games

- Models the strictly competitive scenarios
  - "Zero-sum" almost always mean "2-player zero-sum" games
  - n-player games can also be zero-sum, but not particularly interesting

- Many games are zero-sum
  - Two player poker, GO, Chess, GAN (generative adversarial networks), defender-attacker games …
  - Related to major breakthroughs in recent AI

- $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE for the zero-sum game if: (1) $u(\boldsymbol{p}^*, \boldsymbol{q}^*) \geq u(i, \boldsymbol{q}^*)$ for any $i \in [n]$; (2) $u(\boldsymbol{p}^*, \boldsymbol{q}^*) \leq u(\boldsymbol{p}^*, j)$ for any j $\in [m]$
  - Condition $v(\boldsymbol{p}^*, \boldsymbol{q}^*) \geq v(\boldsymbol{p}^*, j) \Leftrightarrow u(\boldsymbol{p}^*, \boldsymbol{q}^*) \leq u(\boldsymbol{p}^*, j)$
  - Thus, we can "forget" v; and think of player B as minimizing player A's utility

Maximin and Minimax Strategy

- Previous observations motivate the following definitions

Definition. $\boldsymbol{p}^* \in \Delta_n$ is a maximin strategy of player A if it solves

$$\max_{\boldsymbol{p} \in \Delta_n} \min_{j \in [m]} u(\boldsymbol{p}, j).$$

The corresponding utility value is called maximin value of the game.

Notations:

- Bold letters represent vectors
- $\Delta_n = \{\boldsymbol{p}: \sum_i p_i = 1, p_i \geq 0\}$ contains all n-dimensional probabilities

Remarks:

- $\boldsymbol{p}^*$ is player A's best strategy if he was to move first

Maximin and Minimax Strategy

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$$\max_{\boldsymbol{p} \in \Delta_n} \min_{j \in [m]} u(\boldsymbol{p}, j).$$

The corresponding utility value is called maximin value of the game.

Definition. $\boldsymbol{q}^* \in \Delta_m$ is a minimax strategy of player B if it solves

$$\min_{\boldsymbol{q} \in \Delta_m} \max_{i \in [n]} u(i, \boldsymbol{q}).$$

The corresponding utility value is called minimax value of the game.

<u>Remark</u>: $\boldsymbol{q}^*$ is player B's best action if he was to move first

Duality of Maximin and Minimax

Fact. $$\max_{\boldsymbol{p}\in\Delta_n}\min_{j\in[m]} u(\boldsymbol{p},j) \le \min_{\boldsymbol{q}\in\Delta_m}\max_{i\in[n]} u(i,\boldsymbol{q}).$$

That is, moving first is no better.

- Let $\boldsymbol{q}^* = \arg\min_{\boldsymbol{q}\in\Delta_m}\max_{i\in[n]} u(i,\boldsymbol{q})$, so

$$\min_{\boldsymbol{q}\in\Delta_m}\max_{i\in[n]} u(i,\boldsymbol{q}) = \max_{i\in[n]} u(i,\boldsymbol{q}^*)$$

- We have

$$\max_{\boldsymbol{p}\in\Delta_n}\min_{j\in[m]} u(\boldsymbol{p},j) \le \max_{\boldsymbol{p}\in\Delta_n} u(\boldsymbol{p},\boldsymbol{q}^*) \quad = \max_{i\in[n]} u(i,\boldsymbol{q}^*)$$

Duality of Maximin and Minimax

Fact. $\max_{\boldsymbol{p}\in\Delta_n} \min_{j\in[m]} u(\boldsymbol{p}, j) \leq \min_{\boldsymbol{q}\in\Delta_m} \max_{i\in[n]} u(i, \boldsymbol{q}).$

Theorem. $\max_{\boldsymbol{p}\in\Delta_n} \min_{j\in[m]} u(\boldsymbol{p}, j) = \min_{\boldsymbol{q}\in\Delta_m} \max_{i\in[n]} u(i, \boldsymbol{q}).$

- Maximin and minimax can both be formulated as linear program

Maximin

$$\begin{aligned} \text{max} \quad & u \\ \text{s.t.} \quad & u \leq \textstyle\sum_{i=1}^{n} u_{i,j}\, p_i, \quad \forall j \in [m] \\ & \textstyle\sum_{i=1}^{n} p_i = 1 \\ & p_i \geq 0, \qquad \forall i \in [n] \end{aligned}$$

Minimax

$$\begin{aligned} \text{min} \quad & v \\ \text{s.t.} \quad & v \geq \textstyle\sum_{j=1}^{n} u_{i,j}\, p_j, \quad \forall i \in [n] \\ & \textstyle\sum_{j=1}^{m} p_j = 1 \\ & p_j \geq 0, \qquad \forall j \in [m] \end{aligned}$$

- This turns out to be primal and dual LP. Strong duality yields the equation

"Uniqueness" of Nash Equilibrium (NE)

Theorem. In 2-player zero-sum games, $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE if and only if $\boldsymbol{p}^*$ and $\boldsymbol{q}^*$ are the maximin and minimax strategy, respectively.

$\Leftarrow$: if $\boldsymbol{p}^*$ $[\boldsymbol{q}^*]$ is the maximin [minimax] strategy, then $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE

- Want to prove $u(\boldsymbol{p}^*, \boldsymbol{q}^*) \geq u(i, \boldsymbol{q}^*), \forall i \in [n]$

$$\begin{aligned} u(\boldsymbol{p}^*, \boldsymbol{q}^*) &\geq \min_{\mathrm{j}} u(\boldsymbol{p}^*, j) \\ &= \max_{\boldsymbol{p} \in \Delta_n} \min_{\mathrm{j}} u(\boldsymbol{p}, j) \\ &= \min_{\boldsymbol{q} \in \Delta_m} \max_{i \in [n]} u(i, \boldsymbol{q}) \\ &= \max_{i \in [n]} u(i, \boldsymbol{q}^*) \\ &\geq u(i, \boldsymbol{q}^*), \forall i \end{aligned}$$

- Similar argument shows $u(\boldsymbol{p}^*, \boldsymbol{q}^*) \leq u(\boldsymbol{p}^*, j), \forall j \in [m]$
- So $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE

"Uniqueness" of Nash Equilibrium (NE)

Theorem. In 2-player zero-sum games, $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE if and only if $\boldsymbol{p}^*$ and $\boldsymbol{q}^*$ are the maximin and minimax strategy, respectively.

⇒: if $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE, then $\boldsymbol{p}^*$ $[\boldsymbol{q}^*]$ is the maximin [minimax] strategy

- Observe the following inequalities

$$
\begin{aligned}
u_1(\boldsymbol{p}^*, \boldsymbol{q}^*) &= \max_{i \in [m]} u(i, \boldsymbol{q}^*) \\
&\geq \min_{\boldsymbol{q} \in \Delta_m} \max_{i \in [n]} u(i, \boldsymbol{q}) \\
&= \max_{\boldsymbol{p} \in \Delta_n} \min_{\mathrm{j}} u(\boldsymbol{p}, j) \\
&\geq \min_{\mathrm{j}} u(\boldsymbol{p}^*, j) \\
&= u(\boldsymbol{p}^*, \boldsymbol{q}^*)
\end{aligned}
$$

- So the two "≥" must both achieve equality.
  - The first equality implies $\boldsymbol{q}^*$ is the minimax strategy
  - The second equality implies $\boldsymbol{p}^*$ is the maximin strategy

“Uniqueness” of Nash Equilibrium (NE)

Theorem. In 2-player zero-sum games, $(\boldsymbol{p}^*, \boldsymbol{q}^*)$ is a NE if and only if $\boldsymbol{p}^*$ and $\boldsymbol{q}^*$ are the maximin and minimax strategy, respectively.

Corollary.

- NE of any 2-player zero-sum game can be computed by LPs
- Players achieve the same utility in any Nash equilibrium.
  - Player A’s NE utility always equals maximin (or minimax) value
  - This utility is also called the game value

End of Lectures for CS 6161

- More on game theory? The topic courses on Learning and Game Theory in Spring 2021 by Haifeng will cover more
  - Studies relations between machine learning and game theory
- Next Tuesday: a review lecture by TAs
- Happy Thanksgiving – hope everyone had a challenging but fruitful semester ☺

Thank You

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