INFO8006 Introduction to Artificial Intelligence

Exercises 3: Reasoning under uncertainty (part 1)

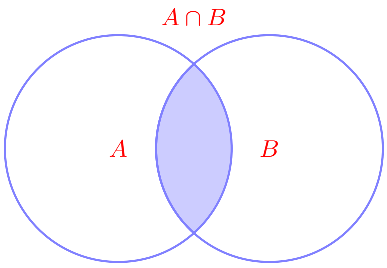
Learning outcomes

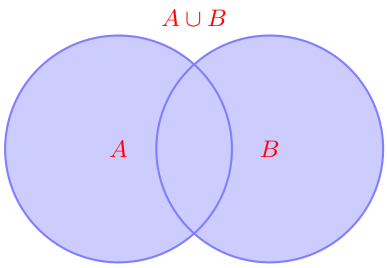
At the end of this exercise session you should be able to:

- Apply the Bayes rule and marginalisation appropriately to compute probabilities.

Exercise 1 (AIMA, Ex 13.4)

Let the two events A, B of the probability space Ω. Would it be rational for an agent to hold the three beliefs $P(A) = 0.4$, $P(B) = 0.3$, and $P(A \vee B) = 0.5$? If so, what range of probabilities be rational for the agent to hold for $A \wedge B$?





Yes it is possible, $P(A \wedge B) = P(A) + P(B) - P(A \vee B) = 0.4 + 0.3 - 0.5 = 0.2$

Exercise 2 (AIMA, Ex 13.8)

Given the probability table of Table 1 compute the following probabilities:

1. P(toothache)
2. P(cavity)
3. P(toothache | cavity)
4. P(cavity | toothache ∨ catch)

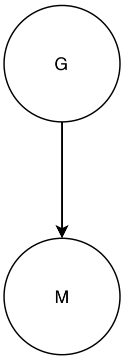
- $P(T = t) = \sum_{i \in \{ct, \neg ct\}} \sum_{j \in \{cv, \neg cv\}} P(T = t, CT = i, CV = j) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2$
- $P(CV = cv) = \sum_{i \in \{ct, \neg ct\}} \sum_{j \in \{t, \neg t\}} P(T = j, CT = i, CV = cv) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$
- $P(T = t | CV = cv) = \frac{P(T=t, CV=cv)}{P(CV=cv)} = \frac{\sum_{i \in \{ct, \neg ct\}} P(T=t, CV=cv, CT=i)}{P(CV=cv)} = \frac{0.108+0.012}{0.2} = 0.6$
- $P(CV = cv | T = t \vee CT = ct) = \frac{P(CV=cv, T=t, CT=\neg ct) + P(CV=cv, T=\neg t, CT=ct) + P(CV=cv, T=t, CT=ct)}{\sum_{i \in \{cv, \neg cv\}} P(CV=i, T=t, CT=\neg ct) + P(CV=i, T=\neg t, CT=ct) + P(CV=i, T=t, CT=ct)} = \frac{0.192}{0.192+0.224} = 0.46$

	Toothache		¬ Toothache	
	catch	¬ catch	catch	¬ catch
cavity	0.108	0.012	0.072	0.008
¬ cavity	0.016	0.064	0.144	0.576

Table 1: Probablity Table of Toothache and Cavity.

Exercise 3: Green Party President (Berkeley CS188 Spring 2014)

It's election season, and the chosen president may or may not be the Green Party candidate. Pundits (experts) believe that Green Party presidents are more likely to legalise marijuana than candidates from other parties, but legalisation could occur under any administration. Armed with the power of probability, the analysts model the situation with the Bayes Net below.



	g	$\neg g$
$P(G)$	.1	.9

	$P(m\|G)$	$P(\neg m\|G)$
g	.667	.333
$\neg g$	.25	.75

1. What is $P(M = m)$, the marginal probability that marijuana is legalised?
   $P(M = m) = P(M = m, G = g) + P(M = m, G = \neg g) = P(M = m|G = g)P(G = g) + P(M = m|G = \neg g)P(G = \neg g) = \frac{2}{3}\frac{1}{10} + \frac{1}{4}\frac{9}{10} = \frac{7}{24}$

2. News agencies air 24/7 coverage of the recent legalisation of marijuana ($M = m$), but you can't seem to find out who won the election. What is the conditional probability $P(G = g|M = m)$ that a Green Party president was elected?
   $P(G = g|M = m) = \frac{P(G=g,M=m)}{P(M=m)} = \frac{P(M=m|G=g)P(G=g)}{P(M=m)} = \frac{\frac{2}{3}\frac{1}{10}}{\frac{7}{24}} = \frac{8}{35}$

3. Fill in the joint probability table over G and M.

G	M	$P(G, M)$
g	m	$\frac{1}{15}$
$\neg g$	m	$\frac{9}{40}$
g	$\neg m$	$\frac{1}{30}$
$\neg g$	$\neg m$	$\frac{27}{40}$

Exercise 4 (AIMA, Ex 13.15)

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease? It is a good news because the conditional probability will drop due to the weak prior value. We have $P(T = t|D = d) = P(T = \neg t|D = \neg d) = 0.99)$ and $P(D = d) = 10^{-4}$ and so:

$$P(D = d|T = t) = \frac{P(T = t|D = d)P(D = d)}{P(T = t|D = d)P(D = d) = P(T = t|D = \neg d)P(D = \neg d)} = \frac{0.99 \times 10^{-4}}{0.99 \times 10^{-4} + (1 - 0.99) \times (1 - 10^{-4})} = 0.0098$$

Exercise 5 ⋆ (AIMA, Ex 13.3)

For each of the following statements, either prove it is true or give a counterexample.

1. If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$
2. If $P(a|b, c) = P(a)$, then $P(b|c) = P(b)$
3. If $P(a|b) = P(a)$, then $P(a|b, c) = P(a|c)$

1. True. By the product rule we know $P(b, c)P(a|b, c) = P(a, c)P(b|a, c)$, which by assumption reduces to $P(b, c) = P(a, c)$. Dividing through by $P(c)$ gives the result.

2. False. The statement $P(a|b,c) = P(a)$ merely states that a is independent of b and c, it makes no claim regarding the dependence of b and c. A counter-example: a and b record the results of two independent coin flips, and $c = b$.

3. False. While the statement $P(a|b) = P(a)$ implies that a is independent of b, it does not imply that a is conditionally independent of b given c. A counter-example: a and b record the results of two independent coin flips, and c equals the xor of a and b.

Exercise 6 $\star$ (AIMA, Ex 13.6)

Prove $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$ from Kolmogorov axioms.

$$P(a \vee b) = p_{a,b} + p_{a,\neg b} + p_{\neg a,b} \tag{1}$$
$$P(a) = p_{a,b} + p_{a,\neg b} \tag{2}$$
$$P(b) = p_{a,b} + p_{\neg a,b} \tag{3}$$
$$P(a \wedge b) = p_{a,b} \tag{4}$$

Supplementary materials

Bayesian or Frequentist ?

AIMA 13.6 The wumpus world revisited
AIMA Chapter 13