

INFO8006 Introduction to Artificial Intelligence

Exercises 5: Reasoning over time

Learning outcomes

At the end of this exercise session you should be able to:

- Define a Markov Model
- Define what is Filtering/Prediction/Smoothing/Most Likely Explanation in general and how it is done in the context of Markov Model
- Explain the simplified matrix representation of HMM (Hidden Markov Model).

Exercise 1: Umbrella World (AIMA, Ex 15.2)

In this exercise, we examine what happens to the probabilities in the umbrella world in the limit of long time sequences.

R_{t-1}	$P(r_t R_{t-1})$	R_t	$P(u_t R_t)$
T	0.7	T	0.9
F	0.3	F	0.2

Table 1: Transition and Observation probability tables.

1. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases monotonically toward a fixed point. Calculate this fixed point.
2. Now consider forecasting further and further into the future, given just the first two umbrella observations. Compute the exact value of this fixed point.

Exercise 2: The coin

You are in a room containing a table, on this table are placed 3 very precious biased coins (named A , B and C). Suddenly another person enters the room and takes the coins. He throws a coin 4 times and then tells you the following information: First he tells you that he drew the first coin uniformly at random and did not throw it. Then, to select the next coin for each following throw, he either kept the same coin with a probability $2/3$ or replaced it by another coin with equal probabilities. When you entered the room you inspected the coins and noticed that the coins A , B and C have a head probability of 80%, 50% and 20% respectively. The result of the throws are head, head, tail and head. If you answer right to his questions he will give you the coins. His four questions are as follows:

1. Provide a hidden Markov model that describes the sequence of throws.
2. What are the probabilities of the last coin given the sequence of evidence ?
3. What are the probabilities of the first coin chosen given the sequence evidence? And of the first coin thrown?
4. What is the most likely sequence of coins thrown?

★ Exercise 3: September 2019 (AIMA, Ex: 15.13 + 15.14)

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Answer the following questions:

1. Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Provide the conditional probability tables.
2. Then reformulate the dynamic Bayesian network as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.
3. For the evidence values $e_1 = \text{'not red eyes, not sleeping in class'}$, $e_2 = \text{'red eyes, not sleeping in class'}$ and $e_3 = \text{'red eyes, sleeping in class'}$ compute the following conditional probability distributions:
 - (a) $P(\text{EnoughSleep}_t | e_{1:t})$ for $t = 1, 2, 3$.
 - (b) $P(\text{EnoughSleep}_t | e_{1:3})$ for $t = 1, 2, 3$.

Supplementary materials

Berkeley Handout 6

