

Graduate Statistics Notes
Advanced Probability

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1. Measure Theory

1.1. Probability Spaces. A **probability space** is a triple (Ω, \mathcal{F}, P) , where Ω is a set of "outcomes", \mathcal{F} is a set of "events", and $P : \mathcal{F} \rightarrow [0, 1]$ is a function that assigns probabilities to events. \mathcal{F} is called σ -algebra, which satisfies

- (1) if $A \in \mathcal{F}$, $A^C \in \mathcal{F}$, and
- (2) if $A_i \in \mathcal{F}$ is a countable sequence, then $\cup_i A_i \in \mathcal{F}$.

Without P , (Ω, \mathcal{F}) is a **measurable space**. A **measure** is a nonnegative countably additive set function $\mu : \mathcal{F} \rightarrow \mathbb{R}$ with

- (1) $\mu(A) \geq \mu(\emptyset)$, for all $A \in \mathcal{F}$, and
- (2) if A_i is a countable sequence of disjoint sets, then $\mu(\cup_i A_i) = \sum_i \mu(A_i)$

If $\mu(\Omega) = 1$, we call μ is a **probability measure**, denoted by P .

2. Large Number Theory