Graduate Statistics Notes
Advanced Probability

Feng Jianting

Contents

1. Measure Theory

1.1. Probability Spaces. A **probability space** is a triple $(\Omega, \mathcal{F}, P)$, where Ω is a set of "outcomes", $\mathcal{F}$ is a set of "events", and $P : \mathcal{F} \to [0,1]$ is a function that assigns probabilities to events. $\mathcal{F}$ is called σ-algebra, which satisties

(1) if $A \in \mathcal{F}$, $A^C \in \mathcal{F}$, and
(2) if $A_i \in \mathcal{F}$ is a countable sequence, then $\cup_i A_i \in \mathcal{F}$.

Without P, $(\Omega, \mathcal{F})$ is a **measurable space**. A **measure** is a nonnegative countably additive set funciton $\mu : \mathcal{F} \to \mathbb{R}$ with

(1) $\mu(A) \geq \mu(\emptyset)$, for all $A \in \mathcal{F}$, and
(2) if A_i is a countable sequence of disjoint sets, then $\mu(\cup_i A_i) = \sum_i \mu(A_i)$

If $\mu(\Omega) = 1$, we call μ is a **probability measure**, denoted by P.

2. Large Number Theory

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