Probability Theory

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Monotone functions

Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing function, that is, for any two $x_1, x_2 \in \mathbb{R}$

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$$

The notation $t \uparrow x$ means $t < x, t \to x$; $t \downarrow x$ means $t > x, t \to x$.

1. For each $x \in (-\infty, \infty)$, both *unilateral limits*

$$\lim_{t \uparrow x} f(t) = f(x-) \quad \textit{and} \quad \lim_{t \downarrow x} f(t) = f(x+)$$

exist and are finite.

2. Limits at infinity

$$\lim_{t \uparrow +\infty} f(t) = f(+\infty) \quad \textit{and} \quad \lim_{t \downarrow -\infty} f(t) = f(-\infty)$$

exist; the former could be $+\infty$ and the later $-\infty$.

3. For each x, $f(x)$ is continuous iff

$$f(x+) = f(x) = f(x-)$$

In general, if the equality doesn't hold, we say f has a jump at x.

Indeed, for any monotone function, we have following claim:

Claim .1. *Suppose f monotony increase and has a discontinuity at x_0, suppose*

$$f(x_0) < \lim_{x \downarrow x_0} f(x)$$