发现定理:

一． 对于$f(x)=y$, $f(n+1)=k*f(n)$, x,y,k为常数,$k\neq 1$:

$$\mathrm{f(n)}=\frac{y*k^{n}}{k^{x}}=y*k^{n-x}$$

二． 对于$f(x)=y$, $f(n+1)=k*f(n)+c$, x,y,k,c为常数,$k\neq 1$:

$$\mathrm{f(n)}=(y+\frac{c}{k-1})*k^{n-x}-\frac{c}{k-1}$$

三． 对于$f(x)=y$, $f(n+1)=f(n)$, $x,y,(k=1)$为常数:

$$\mathrm{f(n)=y}$$

四． 对于$f(x)=y$, $f(n+1)=f(n)+c$, $x,y,(k=1),c$为常数:

$$\mathrm{f(n)=cn-xc+y}$$

五． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n$, $x,y,(k=1),a$为常数:

$$\mathrm{f(n)}=\frac{a}{2}n^{2}-\frac{a}{2}n-\frac{ax(x-1)}{2}+y$$

六． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n+c$, $x,y,(k=1),a,c$为常数:

$$\mathrm{f(n)}=\frac{a}{2}n^{2}-\frac{a}{2}n-\frac{ax(x-1)}{2}-xc+cn+y$$

$$\mathrm{f(n)}=\frac{a}{2}n^{2}+\left(-\frac{a}{2}+c\right)n-\frac{ax(x-1)}{2}-xc+y$$

七． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^{2}$, $x,y,(k=1),a$为常数:

$$\mathrm{f(n)}=\frac{a}{3}n^{3}-\frac{a}{2}n^{2}+\frac{a}{6}n-\frac{ax(x-1)(2x-1)}{6}+y$$

八． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^{2}+c$, $x,y,(k=1),a,c$为常数:

$$\mathrm{f(n)}=\frac{a}{3}n^{3}-\frac{a}{2}n^{2}+\frac{a}{6}n-\frac{ax(x-1)(2x-1)}{6}-xc+cn+y$$

$$\mathrm{f(n)}=\frac{a}{3}n^{3}-\frac{a}{2}n^{2}+\left(\frac{a}{6}+c\right)n-\frac{ax(x-1)(2x-1)}{6}-xc+y$$

九． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^{2}+b*n+c$, $x,y,(k=1),a,b,c$为常数:

$$\mathrm{f(n)}=\frac{a}{3}n^{3}-\frac{a}{2}n^{2}+\frac{a}{6}n-\frac{ax(x-1)(2x-1)}{6}+\frac{b}{2}n^{2}-\frac{b}{2}n-\frac{bx(x-1)}{2}-xc+cn+y$$

$$\mathrm{f(n)}=\frac{a}{3}n^{3}+(\frac{b}{2}-\frac{a}{2})n^{2}+(\frac{a}{6}-\frac{b}{2}+c)n-\frac{ax(x-1)(2x-1)}{6}-\frac{bx(x-1)}{2}-xc+y$$

十． 对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^{3}$, $x,y,(k=1),a$为常数:

$$\mathrm{f(n)}=\frac{a}{4}n^4-\frac{a}{2}n^3+\frac{a}{4}n^2-\frac{ax^2(x-1)^2}{4}+y$$

十一．对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^3+c$, $x,y,(k=1),a,c$为常数:

$$\mathrm{f(n)}=\frac{a}{4}n^4-\frac{a}{2}n^3+\frac{a}{4}n^2-\frac{ax^2(x-1)^2}{4}-xc+cn+y$$

$$\mathrm{f(n)}=\frac{a}{4}n^4-\frac{a}{2}n^3+\frac{a}{4}n^2+cn-\frac{ax^2(x-1)^2}{4}-xc+y$$

十二．对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^3+b*n^2+d*n+c$, $x,y,(k=1),a,b,c,d$为常数

$$\mathrm{f(n)}=\frac{a}{4}n^4-\frac{a}{2}n^3+\frac{a}{4}n^2-\frac{ax^2(x-1)^2}{4}+\frac{b}{3}n^3+(\frac{d}{2}-\frac{b}{2})n^2+(\frac{b}{6}-\frac{d}{2}+c)n-\frac{bx(x-1)(2x-1)}{6}-\frac{dx(x-1)}{2}-xc+y$$

$$\mathrm{f(n)}=\frac{a}{4}n^4+(\frac{b}{3}-\frac{a}{2})n^3+(\frac{a}{4}+\frac{d}{2}-\frac{b}{2})n^2+(\frac{b}{6}-\frac{d}{2}+c)n-\frac{ax^2(x-1)^2}{4}-\frac{bx(x-1)(2x-1)}{6}-\frac{dx(x-1)}{2}-xc+y$$

十三．对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^4$, $x,y,(k=1),a$为常数:

$$\mathrm{f(n)}=\frac{a}{5}n^5-\frac{a}{2}n^4+\frac{a}{3}n^3-\frac{a}{30}n-\frac{ax(x-1)[6(x-1)^3+9(x-1)^2+x-2]}{30}+y$$

十四．对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^4+c$, $x,y,(k=1),a,c$为常数:

$$\mathrm{f(n)}=\frac{a}{5}n^5-\frac{a}{2}n^4+\frac{a}{3}n^3-\frac{a}{30}n-\frac{ax(x-1)[6(x-1)^3+9(x-1)^2+x-2]}{30}-xc+cn+y$$

$$\mathrm{f(n)}=\frac{a}{5}n^5-\frac{a}{2}n^4+\frac{a}{3}n^3+(-\frac{a}{30}+c)n-\frac{ax(x-1)[6(x-1)^3+9(x-1)^2+x-2]}{30}-xc+y$$

十五．对于$f(x)=y$, $f(n+1)=f(n)+\mathrm{a}*n^4+b*n^3+d*n^2+e*n+c$, $x,y,(k=1),a,b,c,d,e$为常数

$$\mathrm{f(n)}=\frac{a}{5}n^5-\frac{a}{2}n^4+\frac{a}{3}n^3-\frac{a}{30}n-\frac{ax(x-1)[6(x-1)^3+9(x-1)^2+x-2]}{30}+\frac{a}{4}n^4+(\frac{b}{3}-\frac{a}{2})n^3+(\frac{a}{4}+\frac{d}{2}-\frac{b}{2})n^2+(\frac{b}{6}-\frac{d}{2}+c)n-\frac{ax^2(x-1)^2}{4}-\frac{bx(x-1)(2x-1)}{6}-\frac{dx(x-1)}{2}-xc+y$$

$$\mathbf{f(n)} = \frac{a}{5}n^5 + \left(\frac{b}{4} - \frac{a}{2}\right)n^4 + (\frac{a}{3} + \frac{d}{3} - \frac{b}{2})n^3 + (\frac{b}{4} + \frac{e}{2} - \frac{d}{2})n^2 + (\frac{d}{6} - \frac{e}{2} - \frac{a}{30} + c)n$$

$$- \frac{ax(x-1)[6(x-1)^3 + 9(x-1)^2 + x - 2]}{30} - \frac{bx^2(x-1)^2}{4}$$

$$- \frac{dx(x-1)(2x-1)}{6} - \frac{ex(x-1)}{2} - xc + y$$