

Manual for solving recurrences

一、 Environments requirements

- a) Python 3.0+
- b) Imported modules: z3, sympy, numpy, math, re, random, time, copy

二、 Procedures

- For recurrences without conditions:

```
>>From solver_recurr import *
```

```
>>Solve_r(a,b,f(n+1)-k*f(n))
```

$(a \in \mathbb{N}; b \in \mathbb{R}; k \in \mathbb{R}^+)$

```
>>Solve_r(a,b,f(n+1)-k*f(n)+P(n))
```

$(a \in \mathbb{N}; b \in \mathbb{R}; k \in \mathbb{R}^+; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 15)$

```
>>Solve_r(a,b,f(n+1)-k*f(n)+c*E(n))
```

$(a \in \mathbb{N}; b, c \in \mathbb{R}; k \in \mathbb{R}^+; E(n) \text{ is a expontial of } n)$

```
>>Solve_r(a,b,f(n+1)-k*f(n)+c*E(n)+P(n))
```

$(a \in \mathbb{N}; b \in \mathbb{R}; k \in \mathbb{R}^+; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 12;$

$E(n) \text{ is a expontial of } n)$

```
>>From solver_recurr_nonlinear import *
```

```
>>Solve_r(a,b,f(n+1)-k*f(n)+r*f(n)**2)
```

$(a \in \mathbb{N}; b, r \in \mathbb{R}; k \in \mathbb{R}^+)$

```
>>Solve_r(a,b,f(n+1)-k*f(n)+r*f(n)**2+P(n))
```

$(a \in \mathbb{N}; b, r \in \mathbb{R}; k \in \mathbb{R}^+; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7)$

E.g.

>>**Solve_r**(0,1,f(n+1)-2*f(n)-n**3-13)

(This example is to solve the recurrence $f(0)=1$, $f(n+1)=2f(n)+n^3+13$)

- For recurrences with a single condition

>>From **solver_cond_recurr** import *

>>**Solve_cond_r**(a,b,n@c,f(n+1)-f(n)+P(n), f(n+1)-f(n)+G(n))

$(a \in \mathbb{N}; b, c \in \mathbb{R}; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$G(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7; @ \in \{>, \geq, <, \leq\})$

>>**Solve_cond_r**(a,b,n**2@c,f(n+1)-f(n)+P(n), f(n+1)-f(n)+G(n))

$(a \in \mathbb{N}; b, c \in \mathbb{R}; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$G(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7; @ \in \{>, \geq, <, \leq\})$

>>**Solve_cond_r**(a,b,Y(n)@c,f(n+1)-f(n)+P(n), f(n+1)-f(n)+G(n))

$(a \in \mathbb{N}; b, c \in \mathbb{R}; Y(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$G(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7; @ \in \{>, \geq, <, \leq\})$

>>**Solve_cond_r**(a,b,k*f(n)+Y(n)@c,f(n+1)-f(n)+P(n), f(n+1)-f(n)+G(n))

$(a \in \mathbb{N}; b, c, k \in \mathbb{R}; @ \in \{>, \geq, <, \leq\})$

$P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7$ (lower than 7 is better);

$Y(n)$ is a polynomial of n whose degree is at most 7 (lower than 7 is better);
 $G(n)$ is a polynomial of n whose degree is at most 7 (lower than 7 is better);)

E.g.

>> **Solve_cond_r**(0,10,n<9,f(n+1)-f(n)-n**3-13, f(n+1)-f(n)-n**2+11)

(This example is to solve the recurrence $f(0)=10$,
 $f(n+1)=\text{ite}(n<9, f(n)+n^3+13, f(n)+n^2-11)$)

- For recurrences with nested conditions

>> From **solver_nestedCond_recurr** import *

>> **Solve_nestedCond_r**(a,b,n@c,n@d,f(n+1)-f(n)+P(n), f(n+1)-
 $f(n)+G(n)$, $f(n+1)-f(n)+K(n)$)

$(a \in \mathbb{N}; b, c \in \mathbb{R}; P(n)$ is a polynomial of n whose degree is at most 7;

$G(n)$ is a polynomial of n whose degree is at most 7;

$K(n)$ is a polynomial of n whose degree is at most 7; @ $\in \{>, \geq, <, \leq\}$)

>> **Solve_nestedCond_r**(a,b,n**2@c,n**2@d,f(n+1)-f(n)+P(n),
 $f(n+1)-f(n)+G(n)$, $f(n+1)-f(n)+K(n)$)

$(a \in \mathbb{N}; b, c \in \mathbb{R}; P(n)$ is a polynomial of n whose degree is at most 7;

$G(n)$ is a polynomial of n whose degree is at most 7;

$K(n)$ is a polynomial of n whose degree is at most 7; @ $\in \{>, \geq, <, \leq\}$)

>> **Solve_nestedCond_r**(a,b, $Y_1(n)$ @c, Y_2 @d,f(n+1)-f(n)+P(n), $f(n+1)-$
 $f(n)+G(n)$, $f(n+1)-f(n)+K(n)$)

$(a \in \mathbb{N}; b, c \in \mathbb{R}; P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$Y_1(n), Y_2(n) \text{ are polynomial of } n \text{ whose degree is at most } 7$

$G(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7;$

$K(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7; @ \in \{>, \geq, <, \leq\}$

>>Solve_nestedCond_r(a,b,k₁*f(n)+Y₁(n)@c,k₂*f(n)+Y₂@d,f(n+1)-
f(n)+P(n), f(n+1)-f(n)+G(n), f(n+1)-f(n)+K(n))

$(a \in \mathbb{N}; b, c, k_1, k_2 \in \mathbb{R}; @ \in \{>, \geq, <, \leq\}$

$P(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7$ (lower than 7 is better);

$Y_1(n), Y_2(n) \text{ are polynomial of } n \text{ whose degree is at most } 7$ (lower than 7 is better)

$G(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7$ (lower than 7 is better);

$K(n) \text{ is a polynomial of } n \text{ whose degree is at most } 7$ (lower than 7 is better);)

E.g.

>>Solve_nestedCond_r(1,2,n>3,n<=9,f(n+1)-f(n)-n**3-13, f(n+1)-
f(n)-n**2+11, f(n+1)-f(n)+n+1)

(This example is to solve the recurrence $f(1)=2,$
 $f(n+1)=\text{ite}(n>3,\text{ite}(n\leq 9,f(n)+n^3+13, f(n)+n^2-11),f(n)-n-1)$