

Lecture 6: September 08

Lecturer: Vijay Garg

Scribe: Jiaolong Yu

6.1 Lower Bound on the Number of Shared Memory Location

Theorem 6.1 (Burns and Lynch) Any mutex algorithm that uses only RW on n processes require at least n shared locations.

Proof: Consider 2 processes, say P and Q, in competing for Critical Section. They have only one shared memory location A. Let Q run till it's about to write to A. Let P run and enter critical section. Let Q run again. It enters CS. Mutex violation. Consider 3 processes, say P, Q and R. They have 2 shared memory locations A and B. Let P and Q run till they are about to write to A. Let R run and enter critical section. let P and Q run again. One of them will enter critical section. Mutex violation. With this method we can extend the situation to n processors and prove the theorem. ■

Definition 6.2 *Covering state.* All share variables are about to be overwritten by processes and the shared stats is consistant with no process in CS.

6.2 Fischer's Algorithm

Turn = -1 Means the door is open.

RequestCS:

```
while(ture) {
  while(turn != -1);
  turn = i;
  wait_for_delta_time_units();
  if(turn == i) return;
}
```

ReleaseCS:

```
turn = -1
```

This algorithm can cause mutex violation. One senario:

P_i reads turn = -1

P_j reads turn = -1

P_j sets turn = j

P_j reads turn = j and enter CS

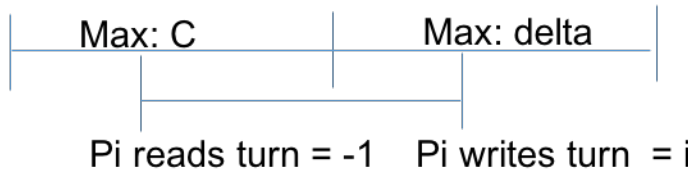
P_i sets turn = i

P_i reads turn = i and enter CS

Theorem 6.3 Assuming $\delta \geq C$, Fischer's Algorithm satisfies mutex. C : maximum time required to close the door, ie. set the turn.

Proof: Let P_i be the processor that enters CS successfully. Assume there is another processor P_j that may enter CS.

P_j reads turn = -1 P_j writes turn = j



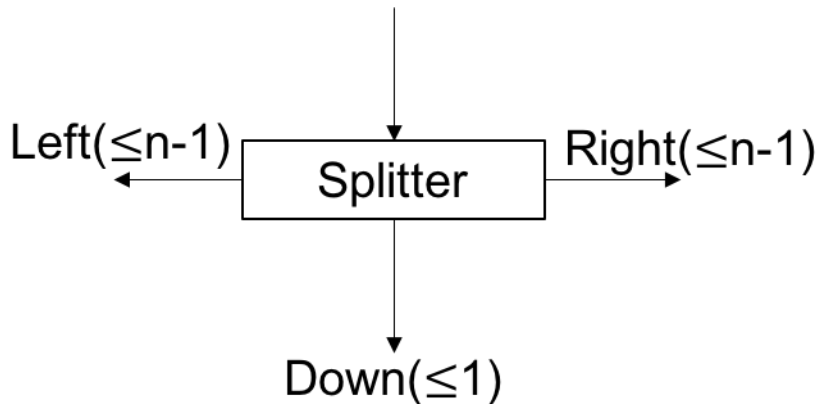
P_j must write turn after P_i reads it to be -1. If it writes before P_i writes turn, after waiting for δ units of time which is longer than C , then P_i must have already finished writing turn, P_j will definitely read turn = i then it will not enter CS. If P_j writes turn after P_i writes turn, then P_i will read turn = j after waiting δ units of time, then it will not enter CS. Proved that only one processor will enter CS ■

6.3 Lamport's Fast Algorithm

This algorithm enables for processors to enter CS fast when there is no contention.

No contention – fast path Contention – slow path

6.3.1 Splitter



```

For every  $P_i$ :
  Variables:
    door: {open, closed}, initially open
    last: pid, initially -1
  last = i;
  if (door == closed) return left;
  else {
    door = closed;
    if (last == i) return down;
  }

```

```

    else return right;
}

```

Claim 6.4 $|left| \leq n - 1$

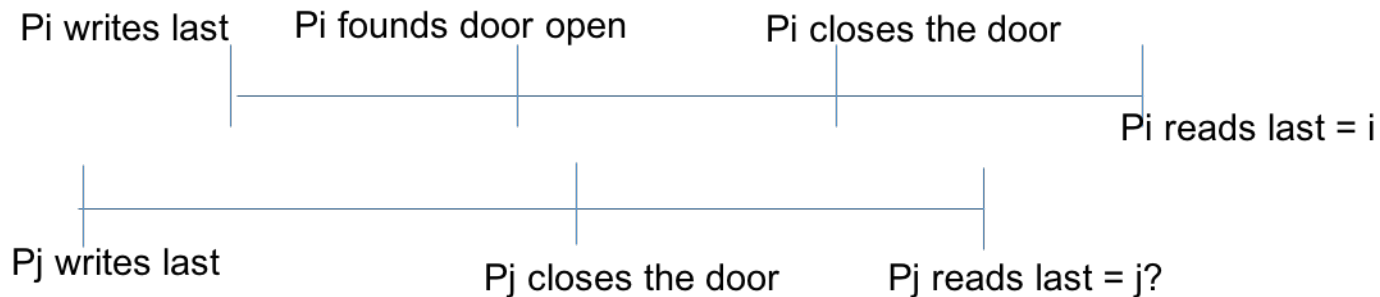
Proof: Someone need to close the door ■

Claim 6.5 $|right| \leq n - 1$

Proof: Consider processor P_i such that $last = i$ then P_i is part of left or down. So as at least one process would not go right. ■

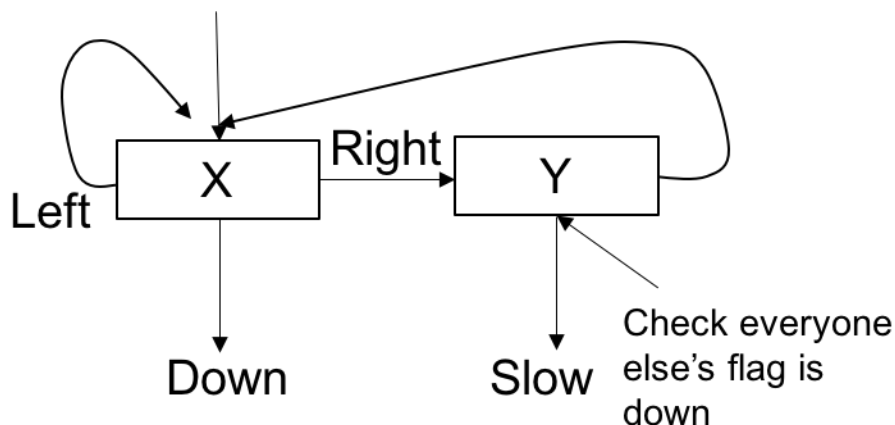
Claim 6.6 $|down| \leq 1$

Proof: Let P_i be the first process to go down. Assume there is another processor P_j that may go down.



As shown in the picture, if P_j does everything before P_i writes `last`, then P_j would be the first one to go down, which conflicts with our assumption. However, P_j must write to `last` before P_i does otherwise P_i would not be able to read `last = i` at the end. P_j must close the door after P_i checks the door, otherwise P_i would have found the door closed, then it cannot go down. Then we have P_i writes to `last` before P_i checks the door, before P_j closes the door, before P_j reads `last`. It means P_j must read `last = i` then it can not go down. ■

6.3.2 Lamport's Fast Algorithm



RequestCS:

```
while (ture) {
    flag[i] = up;
    x = i;
    if (y != -1) { // split left
        flag[i] = down;
        waituntil( y == -1);
        continue;
    } else {
        y = i;
        if (x == i) return; // down
        else { // right
            flag[i] = down;
            waituntil(y = -1);
            continue;
        }
    }
}
```

ReleaseCS:

```
y = -1;
flag[i] = down;
```
