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FLIGHT CONTROL LAW SYNTHESIS FOR A FLEXIBLE AIRCRAFT

F. Kubica and T. Livet

AEROSPATIALE, Aircraft Division, 314 route de Bayonne, 31040 Toulouse, France

Abstract

This paper discusses the design of flight control systems for a highly flexible aircraft, with significant cross coupling between flight mechanics and structural dynamics modes. Because of this coupling, an integrated approach to conceiving flight control laws and guaranteeing aeroelastic stabilization is proposed. Based on a rigid/flexible model which represents the behaviour of the aircraft, it is shown that inclusion of elastic modes in the synthesis gives the possibility of obtaining, at one and the same time, good performance and stability without introducting conventional structural filters. Two synthesis techniques using output feedback are presented : the first one is a mixing of eigenstructure assignment and optimal control based on a notion of compatibility, and the second is the linear quadratic regulator with output feedback.

Nomenclature

Vectors :

b_i : ith column of B
c_i : ith row of C
u_i : left eigenvector
v_i : right eigenvector
w_i : closed-loop input direction
u : control input
x : state vector
y : output vector

Matrices :

A : open-loop state matrix
A_c : closed-loop state matrix
B : control input matrix
C : measurement matrix
G : open-loop plant transfer function
I : identity matrix
K : control gain matrix
P : solution of the algebraic Riccati equation
Q, R : weighting matrices
X : initial autocorrelation of the state

Scalars :

$j : \sqrt{(-1)}$
τ_i : aerodynamic delay of the ith elastic mode
η_i : generalized coordinate of the ith elastic mode
n_z : normal acceleration (g)
g : gravity acceleration
V : forward velocity (m/s)
α : angle of attack (deg)
θ : pitch angle (deg)
q : pitch rate (deg/s)
δq : control surface deflection (deg)
Δ : numerical delay (second)
p : Laplace operator
λ_i : ith eigenvalue
$\underline{\sigma}(E)$: minimum singular value of E
J : performance index
$R_{i,j,k}$: impulse residue of eigenvalue k, for output i and input j
ω : frequency (rads)

Superscripts :

d : desired
r : rigid
e : elastic
t : transpose
- 1 : inverse
* : complex conjugate

Introduction

Aeroservoelasticity (ASE) is the interaction between aerodynamics, flight control systems and structural dynamics. The evolution of modern day aircraft, which includes long fuselages and new materials like composites, makes ASE become a significant phenomenon. Traditionally, passive methods such as increased structural stiffness, mass balancing or speed restriction have been used to reduce it. However, these methods lead to add weight and thus to limit performance and cost-effectiveness. Nowadays, active control systems can be developed easily on aircraft equipped with fly-by-wire and electrically controlled actuators.

Two ways are generally possible to design these control systems. One is to separately conceive a rigid-body and elastic control law, and the other is to design a single integrated control system to achieve both rigid performance and elastic stability. In this paper, the second solution is developed in the case of the longitudinal control of a flexible aircraft. In the first step, a dynamic model is defined, and based on this system, a control law is designed by eigenstructure assignment and it is shown that there is strong interaction with flexible modes, which can lead to instability.

Eigensystem synthesis techniques are practical for flight control systems design because it is very easy to incorporate specifications such as frequency, damping and decoupling in terms of desired closed-loop eigenvalues and eigenvectors[1 2 3]. Unfortunately, eigenstructure assignment using output feedback is not really suitable for flexible mode control because it is difficult to choose the desired eigenspace.

On the other hand, synthesis methods based on optimal theory such as the Linear Quadratic Regulator (LQR) seem to be interesting because they minimize, naturally, the energy associated with elastic modes. They have important robustness properties and they guarantee stability unlike partial eigenstructure assignment, where the uncontrolled modes may be unstable. A limitation of LQR, which implies full state feedback, is that all the states are not generally available, but only the outputs are measured. However, a full state feedback can be designed and then a dynamic observer will estimate the states from the measured outputs : this is the standard Linear Quadratic Gaussian (LQG) procedure[4]. The resulting synthesized law is of the same order as the aircraft plant and the practical implementation requires order reduction. The poor robustness properties obtained and the difficulties of implementation on the whole flight envelope made us abandon this technique. Nevertheless, it seems interesting to mix eigenstructure and optimal synthesis because each of them provides different and complementary advantages.

We present a new methodology, based on compatibility between rigid and flexible modes, which gives the possibility of mixing these two techniques and obtaining robustness stability with performance requirements. Moreover in a second step, we compare this technique with the LQR associated with output feedback and applied on rigid aircraft[5]. We will show how this technique can be extended in the case of flexible aircraft.

Dynamic model and analysis

The state variable representation of flexible aircraft is built by coupling rigid-body and elastic-body models of the aircraft. The aircraft dynamic of the rigid-body model is given by the equations of flight mechanics linearized using small perturbations about a steady-state flight condition :

$$\frac{dx_r(t)}{dt} = A_r.x_r(t) + B_r.u_r(t) \qquad (1)$$

$$A_r \in R^{5x5} \; , \; B_r \in R^{5x1}$$

where :

$$x_r = [V \;\; \alpha \;\; q \;\; \theta \;\; \int n_z]^t$$

$$u_r = [\delta q]$$

The measurement vector is :

$$y_r(t) = C_r.x_r(t) + D_r.u_r(t) \qquad (2)$$

$$C_r \in R^{3x5} \; , \; D_r \in R^{5x1}$$

where :

$$y_r = [n_{zr} \;\; q_r \;\; \int n_{zr}]^t$$

The aeroelastic model is made by :

$$\frac{dx_e(t)}{dt} = A_e.x_e(t) + B_e.u_e(t) \qquad (3)$$

$$A_e \in R^{nxn} \; , \; B_e \in R^{nx1}$$

with :

$$x_e = [\eta_1 \; \dot{\eta}_1 \; \tau_1 \; \eta_2 \; \dot{\eta}_2 \; \tau_2 \ldots \eta_n \; \dot{\eta}_n \; \tau_n]^t$$

$$u_e = [\delta q]$$

where η_i is the generalized coordinate of the ith elastic mode, τ_i the aerodynamic lag terms and n is the number of elastic modes modeled.

The outputs are defined as :

$$y_e(t) = C_e.x_e(t) + D_e.u_e(t) \qquad (4)$$

$$C_e \in R^{3xn} \; , \; D_e \in R^{nx1}$$

where :

$$y_e = [n_{ze} \;\; q_e \;\; \int n_{ze}]^t$$

The number of elastic modes taken into account is chosen to be 10 (so n = 30), which gives a realistic representation of the aircraft structural dynamics.

These two models are coupled by connecting their respective measured outputs at the Same structural points of the aircraft. So the complete model is given by :

$$\frac{dx(t)}{dt} = A.x(t) + B.u(t) \qquad (5)$$

$$y(t) = C.x(t) + D.u(t) \qquad (6)$$

where :

$$\begin{cases} A = \begin{pmatrix} A_r & 0 \\ 0 & A_e \end{pmatrix} \\ B = \begin{pmatrix} B_r \\ B_e \end{pmatrix} \\ C = (C_r \;\; C_e) \\ D = D_r + D_e \end{cases}$$

It is very important to take servo-control modelization and the delays of the control systems into account because of the phase introduced, which is a fundamental point in flexible structure control. The actuator transfer function is chosen, after identification, to be :

$$\frac{\delta q\,(p)}{\delta q_c\,(p)} = \frac{2046 - 8.42p}{2046 + 102.87p + p^2}$$

The delays associated with computation time (Δ=0.1 second) are modelized by Padé approximants to $e^{-p\Delta}$, which match the first few terms of the Taylor's series expansion. The transfer function used is :

$$D\,(p) = \frac{1 - \frac{p\Delta}{2} + \frac{(p\Delta)^2}{12}}{1 + \frac{p\Delta}{2} + \frac{(p\Delta)^2}{12}}$$

Finally, the order of the system is 39 (5 rigid modes, 30 elastic modes and 4 states for actuator and delay modelization).

A first approach is to analyze the influence of dynamics modes by modal analysis technique. The calculation of the system impulse residues shows to what extent flexible modes contribute to the global system dynamics[6]. The residues of a system are related to its dynamics in the following way. For an input j and an output i, we have the following relations :

$$\frac{y_i\,(p)}{u_j\,(p)} = \sum_{k=1}^{n} \frac{c_i.v_k.u_k.b_j}{p - \lambda_k} \qquad (7)$$

For an impulse input with $u_j\,(p) = 1$:

$$y_i\,(p) = \sum_{k=1}^{n} \frac{c_i.v_k.u_k.b_j}{p - \lambda_k} \qquad (8)$$

or :

$$y_i\,(p) = \sum_{k=1}^{n} \frac{R_{i,j,k}}{p - \lambda_k} \qquad (9)$$

$R_{i,j,k}$ is the impulse residue for output i, associated with eigenvalue k, and due to an impulse input j. Considering a flight condition of Mach 0.5 and 200 kts CAS, the main open loop eigenvalues of the aircraft are listed in table 1.

Mode number	Mode	Open-loop poles	Damping	Frequency (Hz)
1	Phugoid	-.001±0.074j	0.02	0.01
2	Short-period	-0.47±0.66j	0.58	0.13
3	Flexible mode	-0.54±7.17j	0.075	1.14
4	Flexible mode	-0.53±15.26j	0.035	2.43
5	Flexible mode	-0.27±16.20j	0.017	2.58
6	Flexible mode	-0.44±17.02j	0.026	2.71
7	Flexible mode	-0.47±18.36j	0.026	2.92
8	Flexible mode	-0.39±20.15j	0.019	3.21
9	Flexible mode	-0.64±24.87j	0.026	3.96
10	Flexible mode	-0.99±26.66j	0.037	4.25
11	Flexible mode	-2.14±29.43j	0.073	4.70
12	Flexible mode	-0.90±36.05j	0.025	5.74

Table 1 : Open-loop poles

The impulse residue magnitudes (expressed as a percentage of the total response) for acceleration and pitch rate outputs are shown in figure 1. Elastic modes (number 3 to number 12) seem to have very important residues compared to phugoid and short-period modes (numbers 1 and 2). It clearly proves that the natural aircraft is strongly flexible, and ignoring the elastic modes could lead to very unsatisfactory results. These must be moderated by the fact that impulse input advantages elastic mode frequencies with respect to classical pilot input. Moreover, sensor location is taken at the pilot's station ahead of the center of gravity, which constitutes a critical case with strong structure deflections and nonminimum phase phenomena. The object of the present paper is not to optimize the sensor location[7 8], but to present efficient synthesis techniques in the case of fixed and restricted sensor positions.

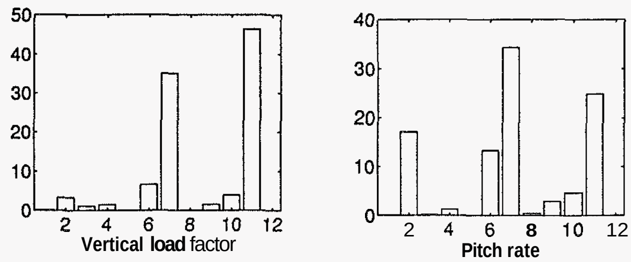

Fig.1 Impulse residue for n_z and q

Performance requirements

In order to design a flight control system which provides the required handling qualities, a control law, based on normal acceleration and pitch rate feedbacks, is defined. An $\int n_z$ feedback (inz), associated with an integrator in the feedforward path, is used to achieve zero steady state error and to assign another closed-loop eigenvalue. The normal acceleration control system is shown in figure 2, where n_{zc} is a reference step input in g's and δq is the control surface deflection in degrees.

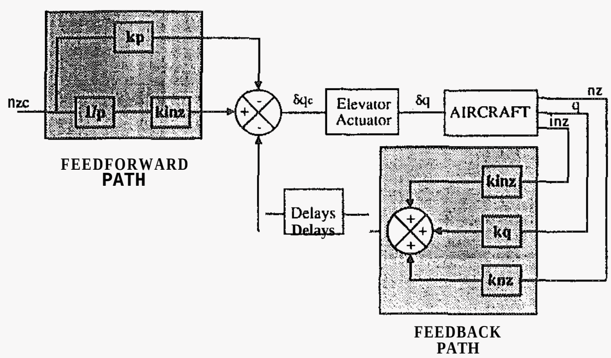

Fig.2 Control law structure

A classical approach is to calculate a control law on a rigid-body aircraft and to verify the aeroelastic stability. Eigenspace techniques are used to design the flight control law. For the system under consideration, the following conditions apply :

a) Three outputs permit the assignment of three closed-loop eigenvalues, with the stipulation that if λ_i is a complex closed-loop eigenvalue, its complex conjugate λ_i^* is also a closed-loop eigenvalue

b) No eigenvector v_i can be altered because of the availability of only one control surface (δq).

The control law is given by :

$$u = - K.y \tag{10}$$

where K is the gain matrix of dimension (1x3). For the closed-loop system, the following relationship applies :

$$(A - BKC).v_i = \lambda_i.v_i \tag{11}$$

or :

$$(\lambda_i.I - A).v_i = - BKC.v_i = - B.w_i \tag{12}$$

So K is given by :

$$K = W.(CV)^{-1} \tag{13}$$

In our example, the desired closed-loop poles are :

$$\begin{cases} -0.6 \pm 0.6j \\ -0.5 \end{cases}$$

The gain matrix K found is :

$$K = [4.10 \quad 0.94 \quad 4.05]$$

The closed-loop time responses to a 0.1-g nzc step command of 8 seconds are shown in figure 3 (time unit in second). considering a rigid model only. Good performance is obtained in terms of damping and time lag. With introduction of the flexible model (figure 4), we can see that the controller downgrades the flexible modes and more particularly mode 6, which is unstable, and mode 7 whose damping is 0.003. The closed-loop modes obtained are listed in table 2. Aerodynamic lag terms are not given because they are not affected by the control law.

In fact, as the frequencies of the aeroelastic modes become lower, flexible modes begin to affect the aircraft dynamics significantly. Traditionally this problem is solved by the addition of low-pass or notch filters in the feedback and feedforward paths. Filters on feedbacks guarantee aircraft stability and filters on pilot input give a good level of comfort in the case of pilot orders which excite elastic modes.

Our objective is to present methodologies that give the possibility of eliminating filters in the feedback path. Indeed, the presence of the filters debases robustness stability at low frequency and adds delays. Moreover, in the case of flexible modes exactly in aircraft control frequencies, filtering techniques will be inapplicable.

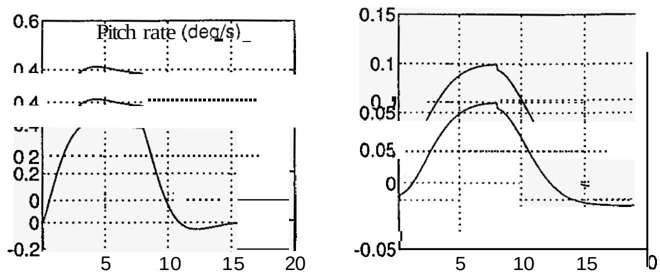

Fig.3 Time responses of the rigid model with eigenstructure assignment synthesis

Mode number	Mode	Closed-loop poles	Damping	Frequency (Hz)
1	Phugoid	-0.5;-0.0159		0.08;0.002
2	Short-period	-0.60±0.60j	0.707	0.13
3	Flexible mode	-0.50±7.15j	0.070	1.14
4	Flexible mode	-0.48±15.24j	0.031	2.43
5	Flexible mode	-0.27±16.19j	0.017	2.58
6	Flexible mode	0.02±16.72j	-0.001	2.66
7	Flexible mode	-0.05±17.74j	0.003	2.82
8	Flexible mode	-0.39±20.14j	0.019	3.21
9	Flexible mode	-0.65±24.94j	0.026	3.97
10	Flexible mode	-0.89±26.60j	0.033	4.24
11	Flexible mode	-1.78±30.28j	0.059	4.83
12	Flexible mode	-0.90±36.04j	0.025	5.74

Table 2 Closed-loop modes obtained with eigenstructure assignment synthesis

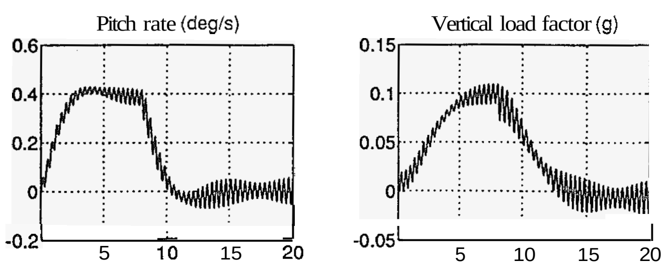

Fig.4 Time responses of the rigid/elastic model with eigenstructure assignment synthesis

Mixed Control

There is no evident solution for coupling optimal control and eigenstructure assignment which involve different calculation proceedings. The fact that desired rigid closed-loop eigenvalues give unstable flexible modes seems to indicate an incompatibility between the closed-loop modes chosen and aeroelastic stability. The following idea is to design a full-state feedback through the linear quadratic technique to obtain good handling qualities. The next step is to look at the compatible rigid and elastic modes found and to try to be as near as possible to this assignment with output feedback associated with optimization.

At the beginning, a linear quadratic design with full state feedback must be achieved. The optimal regulator problem can be formulated by choosing a performance index such as

$$J = \frac{1}{2}\int_0^{\infty} (y^t Q y + u^t R u)\, dt \tag{14}$$

where Q and R are symmetric positive semidefinite weighting matrices. Therefore, the definiteness assumptions on Q and R guarantee that J is nonnegative and lead to a sensible minimization problem. The control law is defined as :

$$u = - K.x \tag{15}$$

where K is given by :

$$K = R^{-1} B^t P \tag{16}$$

with P, the solution of the algebraic Riccati equation (ARE) :

$$A^t P + PA + C^t QC - PBR^{-1} B^t P = 0 \tag{17}$$

The closed-loop system produced by the optimal regulator is guaranteed to be stable. The position of the closed-loop poles is a function of the cost index J to be minimized and, in particular, of the weighting given to output performance and control effort. The usual procedure is to vary Q and R iteratively until performance specifications are met. In fact, for large values of R it is considered more important to minimize control action than output variations and so gain coefficients are very small. On the other hand, if Q is large, strong feedback gains are found in order to obtain fast output regulation. Moreover, some numerical techniques give the possibility of determining weighting matrices to obtain a specification such as time response[9]. On the basis of these properties, it is easy to choose Q and R in order to assign the rigid closed-loop eigenvalues that meet the handling quality requirements. By choosing Q=diag (0.8,1.2,8) and R = 0.5, the closed-loop poles are located in an acceptable region of the complex plane concerning flight control system requirements. Moreover critical flexible modes (6 and 7) are strongly moved into the left-half plane to increase damping ratio. In fact LQR naturally finds optimal closed-loop assignment with compatible rigid and elastic modes and solves the problem of flexible mode eigenspace (see figure 5). Time responses are shown in figure 5 and closed-loop modes obtained with LQR are shown in table 3.

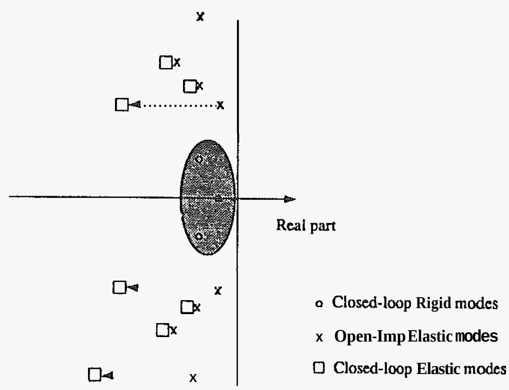

Fig. 5 Effect of LQR on flexible modes

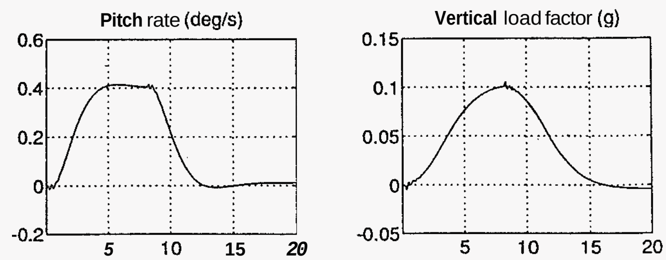

Fig. 6 Time responses obtained with LQR

Mode number	Mode	Closed-loop poles	Damping	Frequency (Hz)
1	Phugoid	-0.62;-0.01		0.1;0.002
2	Short-period	-0.57±0.62j	0.677	0.13
3	Flexible mode	-0.54±7.17j	0.075	1.14
4	Flexible mode	-0.53±15.28j	0.035	2.43
5	Flexible mode	-0.27±16.29j	0.017	2.58
6	Flexible mode	-0.71±17.27j	0.041	2.75
7	Flexible mode	-1.39±17.92j	0.077	2.86
8	Flexible mode	-0.39±20.15j	0.019	3.21
9	Flexible mode	-0.66±24.90j	0.026	3.96
10	Flexible mode	-0.96±26.65j	0.036	4.24
11	Flexible mode	-2.28±29.54j	0.077	4.74
12	Flexible mode	-0.90±36.04j	0.025	5.74

Table 3 Closed-loop poles obtained with LQR

The next step is to obtain a control law with output feedback which is as near as possible to the LQR. The approach chosen is the constrained minimization of the difference between the actual and desired eigenstructure. So we minimize the performance index J (18) with respect to the elements of λ constrained by (19), where $\| () \|$ is the Frobenius norm and p_i are scalar weights :

$$J = \sum_{i=1}^{P} p_i . \| \lambda_i - \lambda_{id} \| \tag{18}$$

$$K_{min} < K < K_{max} \tag{19}$$

Constraints on K are defined for practical implementation reasons such as gain scheduling. The desired eigenstructure chosen consists of the rigid modes and the flexible modes found by LQR. In fact the elastic modes chosen are those which are moved a lot by LQR compared to the open-loop damping. So, in our example the performance index is :

$$\begin{aligned} J = & p_1 . \| \lambda_1 - (-0.62) \| \\ & + p_2 . \| \lambda_2 - (-0.57 \pm 0.62j) \| \\ & + p_3 . \| \lambda_3 - (-0.71 \pm 17.27j) \| \\ & + p_4 . \| \lambda_4 - (-1.39 \pm 17.92j) \| \end{aligned}$$

The mathematical programming problem is then solved numerically using a sequential quadratic programming method, where the search direction is found by solving a subproblem at each iteration[10]. *An* obviously important element in formulating the subproblem is the approximation of the Hessian of the Lagrangian function. By considering a quasi-Newton approximation, we *can* use the Broydon-Fletcher-Shannon-Goldfarb update formula. Because of the iterative nature of the algorithm, an initial gain is needed (K_i). One solution for finding such a gain is to take the unstable gain found by only assigning rigid modes. Naturally, how quickly the algorithm converges is, in part, a function of the initial gain and the tolerance levels set on stopping conditions (chosen as 10^{-8}). In our example let us take $K_i = [4.10 \;\; 0.94 \;\; 4.05]$. After less than 100 iterations, the gain matrix $K_{optimal}$ is found :

$$K_{optimal} = [1.61 \;\; 0.51 \;\; 2.561$$

with $p_1 = p_2 = 10$ and $p_3 = p_4 = 1$.
Time responses are presented in figure 7 and closed-looppoles are listed in table 4.

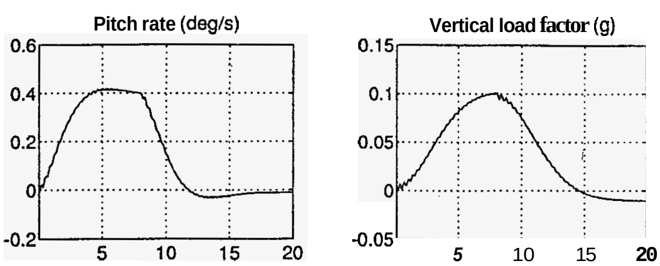

Fig.7 Time responses obtained with mixed control

Mode number	Mode	Closed-loop poles	Damping	Frequency (Hz)
1	Phugoid	-0.42;-0.018		0.07;0.003
2	Short-period	-0.50±0.57j	0.659	0.12
3	Flexible mode	-0.52±7.15j	0.073	1.14
4	Flexible mode	-0.52±15.24j	0.032	2.43
5	Flexible mode	-0.27±16.20j	0.017	2.58
6	Flexible mode	-0.37±16.85j	0.022	2.68
7	Flexible mode	-0.32±18.09j	0.018	2.88
8	Flexible mode	-0.39±20.14j	0.019	3.21
9	Flexible mode	-0.64±24.90j	0.026	3.96
10	Flexible mode	-0.94±26.62j	0.035	4.24
11	Flexible mode	-2.02±29.84j	0.067	4.76
12	Flexible mode	-0.90±36.04j	0.025	5.74

Table 4 Closed-loop poles obtained with mixed control

Good performance is obtained with aeroelastic stability. This technique is closer to pole migration which makes all poles of the system migrate into trapezium in order to achieve both performance and aeroelastic stability[11]. The fact that only compatible poles are considered in the optimization stage of mixed control makes this critical part easier.

Robustness stability of the control law can be evaluated by introducing multiplicative perturbation at the plant input. This type of uncertainty takes actuator errors, delays introduced by discretization or computing, noise... into account. Let $\underline{\sigma}(I + KG)$ be the minimal singular values of the return matrix $[I + KG(j\omega)]$. Gain and phase margins can be computed with the following relations :

$$GM^{+} \geq \frac{1}{(1 - \min(\underline{\sigma}(I + KG(j\omega))))} \tag{20}$$

$$GM^{-} \leq \frac{1}{(1 + \min(\underline{\sigma}(I + KG(j\omega))))} \tag{21}$$

$$|PM| \geq 2 \arcsin\left(\frac{\min(\underline{\sigma}(I + KG(j\omega)))}{2}\right) \tag{22}$$

Figure 8 shows the plot of $\underline{\sigma}(I + KG)$ for the controller **found** with mixed control. The value of **min** $(\underline{\sigma}(I + KG(jw)))$ is - 5.2 dB, which guarantees the following margins :

$$GM^{+} \geq 6.93 \text{ dB}$$

$$GM^{-} \leq -3.8 \text{ dB}$$

$$|PM| \geq 32 \text{ deg}$$

So the control law has good stability robustness. Traditionally, a lot of time and trials are needed to achieve this result. The advantage of the described technique is to give an automated procedure which is very fast and systematic throughout the flight envelope. Indeed, optimal gains for one point can be used as initial gains for the next point.

It is very easy to find a compromise between performance and aeroelastic stability with weights p. The strong point of the method is to respect the physical nature of the system by asking for performance compatible with structural dynamics. This is a basic difference with regard to the eigenstructure theory which assigns eigenvalues and eigenvectors without the notion of energy. Moreover this technique can be extended in order to increase parameter robustness[9].

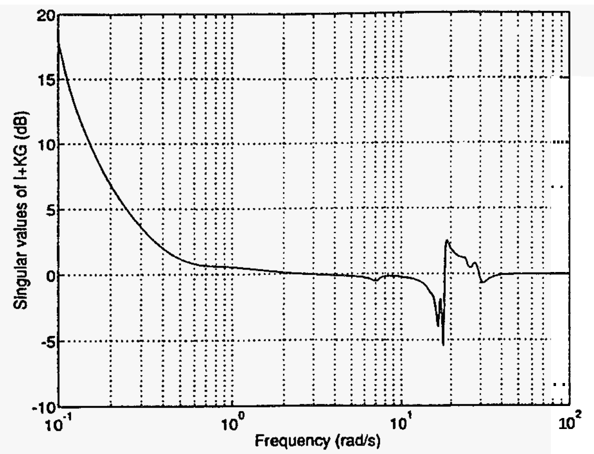

Fig.8 Minimum singular values of the return difference matrix

Linear Quadratic Regulator with output feedback

The objective of this part is to try to obtain the matrix of constant feedback coefficients K directly by the minimization of the quadratic cost :

$$J = \frac{1}{2}\int_0^{\infty} (y^tQy + u^tRu)\, dt \tag{23}$$

So we aim at eliminating the optimization stage of the last part in order to determine K in one step. It is shown that using the Lagrange multiplier approach gives the following necessary conditions for the solution by setting the partial derivatives of the Hamiltonian H with respect to the independent variables equal to zero[12] :

$$\frac{\partial H}{\partial S} = A_c^t P + PA_c + C^t K^t RKC + C^t QC = 0 \tag{24}$$

$$\frac{\partial H}{\partial P} = A_c S + SA_c^t + X = 0 \tag{25}$$

$$\frac{1}{2}\frac{\partial H}{\partial K} = RKCSC^t - B^t PSC^t = 0 \tag{26}$$

with :

$$A_c = A - BKC \tag{27}$$

$$X = x(0)\, x^t(0) \tag{28}$$

The first two conditions are Lyapounov equations, which can be solved using eigenvalue decomposition. As R is positive definite and if CSC^t is nonsingular then K is given by :

$$K = R^{-1} B^t PSC^t (CSC^t)^{-1} \tag{29}$$

The optimal cost is :

$$J = \frac{1}{2} \text{Trace}\,(PX) \tag{30}$$

The numerical technique used for determining the optimal feedback K is an iterative algorithm[13], which gives approximately the same results as the gradient-based routine of Davidon-Fletcher-Powell but with a greater flexibility. To start the algorithm, an initial stabilizing gain K_i is needed. Because of the stability condition, the unstable initial gain found at the beginning of this paper can not be used. So an initial low gain near zero is defined in order to guarantee stability. Let us take K_i = [0.1 0.1 0.11. By considering the initial states uniformly distributed on the unit sphere, we set **X** = I. For given Q and R, the two Lyapounov equations are solved and the gain update direction is evaluated at each iteration by

$$\Delta K = R^{-1} B^t PSC^t (CSC^t)^{-1} - K_k \tag{31}$$

and the new stable gain which makes J decrease is chosen to be :

$$K_{k+1} = K_k + \alpha.\Delta K \tag{32}$$

The selection of Q and R can be made by using, as a first guess, the weighting matrices found in full-state feedback **LQR** design. In our example we take Q = diag (1,1,27) and R = 0.8 in order to obtain acceptable handling qualities. The gain matrix K found is :

K = [0.47 0.012 1.92]

Time responses are presented in figure 9 and closed-loop poles are listed in table 5.

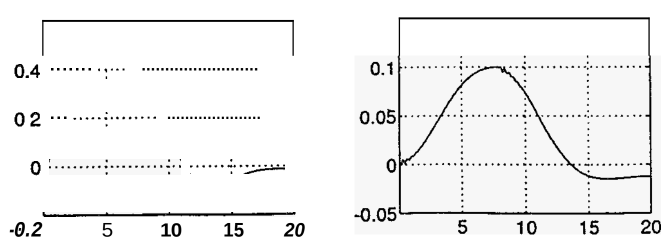

Fig. 9 Time responses obtained with LQR with output feedback

Mode number	Mode	Closed-loop poles	Damping	Frequency (Hz)
1	Phugoid	-0.36;-0.028		0.057;0.004
2	Short-period	-0.32±0.58j	0.483	0.11
3	Flexible mode	-0.53±7.16j	0.074	1.14
4	Flexible mode	-0.54±15.25j	0.035	2.43
5	Flexible mode	-0.28±16.19j	0.017	2.58
6	Flexible mode	-0.55±16.94j	0.032	2.70
7	Flexible mode	-0.58±18.12j	0.032	2.88
8	Flexible mode	-0.39±20.15j	0.019	3.21
9	Flexible mode	-0.62±24.88j	0.025	3.96
10	Flexible mode	-0.99±26.64j	0.037	4.24
11	Flexible mode	-1.98±29.39j	0.067	4.69
12	Flexible mode	-0.90±36.04j	0.025	5.74

Table 5 Closed-loop poles obtained with LQR with output feedback

It is difficult to obtain a good damping ratio for the short period while minimization of J tends to achieve very good robustness concerning flexible modes, whose energy is greater than the energy associated with rigid poles. In fact this method is very efficient for controlling high frequencies but does not give such good performance as mixed control. This is confirmed by the plot of $\underline{\sigma}(I + KG(j\omega))$ on figure 10. The following stability margins are guaranteed:

$$GM^{+} \geq 11.2\,\mathrm{dB}$$

$$GM^{-} \leq -4.73\,\mathrm{dB}$$

$$|PM| \geq 42\,\mathrm{deg}$$

The control law designed is more robust than the previous one, but with average performance. In fact considering the small number of sensors and actuators, LQR with output feedback gives greater importance to elastic modes.

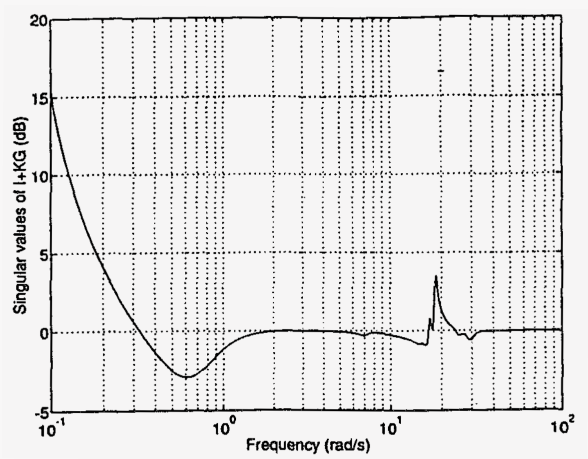

Fig. 10 Minimum singular values of the return difference matrix

Conclusion

In this paper an implementable longitudinal flight control system for a flexible aircraft was designed without introduction of structural filters. Ignoring elastic modes in synthesizing the control law is shown to give unsatisfactory results as regards aeroelastic stability. Inclusion of these modes directly in the synthesis leads to good performance with robustness stability. The first synthesis technique presented is a mixing of eigenstructure assignment and optimal control. On the basis of LQR design, compatible rigid and elastic modes are found and a performance index J is minimized in order to obtain output feedback. This method gave good results with great facility to achieve a compromise between performance and robustness. The second technique aims at obtaining output feedback directly by minimization of a quadratic cost. Results are very positive as regards high frequencies, but performance is very difficult to obtain because of the few degrees of freedom available in our example. To improve low frequency behaviour, sensors can be added or the performance index can be changed[8] in order to give more importance to low frequency modes.

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