Machine Learning and Computational Physics
Fall 2020

Assignment 5

Due: October 25th 2020, 11:59:59 PM PDT

Physics informed deep networks (PINNs)

In this assignment, you will use feed-forward networks to solve the following advection-diffusion equation

$$au'(x) - \kappa u''(x) = 0 \qquad x \in (0,1) \tag{1}$$
$$u(0) = 0 \tag{2}$$
$$u(1) = 1 \tag{3}$$

The solution of this problem has a boundary layer near $x = 1$, whose gradient depends on the Peclet number $\frac{aL}{\kappa}$ where $L = 1$ is the length of the domain.

1. Find the expression for the exact solution of the above advection-diffusion equation.
2. Write a function that creates a feed-forward network of width=15, depth=8 with input and output dimensions 1. The weights and biases should initialized using `RandomNormal` distribution. All hidden layers should make use of a sine activation function, `tf.math.sin` while no activation should be used in the output layer. Use L2 regularization in all layers with a parameter 1.0e-7.
3. Create an array of N uniformly space nodes in $[0, 1]$, including the boundaries. Train a neural network for $N = 25, 50$, $a = 2, 5, 10$ and $\kappa = 1$ with the following loss function

$$L = \sum_{i=1}^{N} \left(au'[i] - \kappa u''[i]\right)^2 + \lambda_b (u[1]^2 + (u[N] - 1)^2)$$

   which is the sum of the interior residual and a scaled boundary residual. Use $\alpha = 10$ for the training.
4. For each of the 6 neural networks, save the history of the interior residual, boundary residual and the final predicted solution in arrays/lists. Plot them at the end and compare with the exact solution.
5. Now answer the following questions
   (a) Describe the behavior the residual loss functions as a function of N for any fixed a.
   (b) Is it easier or harder to train the network as a is increased? Can you explain why?
   (c) What could have happened if α was set to zero?

Instructions:

- You need to submit your work as a single notebook saved as `A1_FirstName_LastName.ipnyb` (for example `A1_Tommy_Trojan.ipnyb`). You can create this notebook locally (on your computer using Jupyter notebook) or on cloud using Google Colab (which we recommend). If you are using Google Colab, then please make sure that you are signed in to your USC Google account before starting. This will make sharing your saved work little easier.
- At the very beginning of your notebook insert a text cell and write your name and USC id.
- For questions requiring descriptive output (such as question `2d, 2e, 2f` in this assignment) use individual text cell.
- Make sure that your entire notebook runs successfully on Google Colab before submitting it. It is your responsibility to ensure this.
- Once you finish the assignment save it and share it with `dhruvvpa@usc.edu`. (If you are using Google Colab, then the notebook will automatically be saved to your Google Drive. Once you locate it in your Google Drive, right click on it and share it with `dhruvvpa@usc.edu`). While sharing make sure that you enable "editor" option, so that we can run your notebook on our end while grading it.