# Lecture 8A: Binary Dependent Variable

*Introduction ot Econometrics,Fall 2018*

**Zhaopeng Qu**

**Nanjing University**

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# Introduction to limited dependent variable

# Introduction

- So far the dependent variable (Y) has been continuous:
  - testscore
  - average hourly earnings
  - GDP growth rate
- What if Y is discrete?
  - Y= get into college, or not; X = parental income.
  - Y= person smokes, or not; X = cigarette tax rate, income.
  - Y= mortgage application is accepted, or not; X = race, income, house characteristics, marital status ...
- Binary outcomes models:
  - Logit Probability Model(LPM)
  - Logit
  - Probit

# The Linear Probability Model(LPM)

# The linear probability model

- If a outcome variable is **binary**,then the expecation of it is

$$E[Y] = 1 \times Pr(Y = 1) + 0 \times Pr(Y = 0) = Pr(Y = 1)$$

- Then we have the probability of Y conditional on X

$$E[Y|X_{1i}, ..., X_{ki}] = Pr(Y = 1|X_{1i}, ..., X_{ki})$$

# The linear probability model

- The conditional expectation equals the probability that $Y_i = 1$ conditional on $X_{1i}, ..., X_{ki}$ :

$$E[Y|X_{1i}, ..., X_{ki}] = Pr(Y = 1|X_{1i}, ..., X_{ki}) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta$$

- The population coefficient $\beta_j$ equals the change in the probability that $Y_i = 1$ associated with a unit change in $X_j$.

$$\frac{\partial Pr(Y_i = 1|X_{1i}, ..., X_{ki})}{\partial X_j} = \beta_j$$

- $\beta_j$ is *the change in the probability that* $Y = 1$ *associated with a unit change in* $X_j$

# The linear probability model

- Almost all of the tools of Multiple OLS regression can carry over to the LPM model.
  - Assumptions are the same as for general multiple regression model.
  - The coefficients can be estimated by OLS.
  - t-statistic and F-statistic can be constructed as before.
  - the errors of the LPM are **always heteroskedastic**, so it is essential that heteroskedasticity-robust s.e. be used for inference.
  - $R^2$ is not a useful statistic now.

# The linear probability model

- Advantages of the linear probability model:
  - easy to estimate
  - Coeffcient estimates are easy to interpret
- Disadvantages of the linear probability model
  - Predicted probability can be above 1 or below 0!(it doesn't make sense)
  - Error terms are heteroskedastic

# An Example: Mortgage applications

- Most individuals who want to buy a house apply for a mortgage at a bank.
- Not all mortgage applications are approved.
- What determines whether or not a mortgage application is approved or denied?
- Boston HMDA data: a data set on mortgage applications collected by the Federal Reserve Bank in Boston.

| Variable | Description | Mean | SD |
|---|---|---|---|
| deny | $= 1$ if application is denied | 0.120 | 0.325 |
| pi_ratio | monthly loan payments / monthly income | 0.331 | 0.107 |
| black | $= 1$ if applicant is black | 0.142 | 0.350 |

# An Example: Mortgage applications

- Does the payment to income ratio affect whether or not a mortgage application is denied?

$$\widehat{deny} = -0.080 + 0.604\ P/I\ ratio$$
$$(0.032)(0.098)$$

- The estimated OLS coefficient on the payment to income ratio $\hat{\beta}_1 = 0.60$
- The estimated coefficient is significantly different from 0 at a 1% significance level.
- How should we interpret $\hat{\beta}_1$ ?
  - An original one: "payments/monthly income ratio increase *1*,then **probability being denied** will also increase *0.6*."
  - More reasonable one: "payments/monthly income ratio increase *0.1(10%)*,then probability being denied will also increase *0.06(6%)*".

# An Example: Mortgage applications

- What is the effect of race on the probability of *denial*, *holding constant the P/I ratio?* To keep things simple, we focus on differences between black applicants and white applicants.

$$\widehat{deny} = -0.091 + 0.559\ P/I\ ratio + 0.177 black$$
$$(0.029)\ (0.089)\qquad\qquad\qquad (0.025)$$

- The coefficient on black, *0.177*, indicates that an African American applicant has a *17.7%* higher probability of having a mortgage application denied than a white applicant, holding constant their payment-to-income ratio.
- This coefficient is significant at the 1% level (the t-statistic is 7.11).

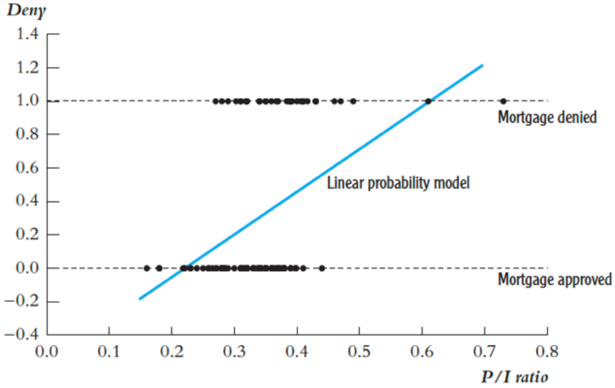
# LPM: shortcomings

- Always suffer heteroskedasticity.
  - Always use heteroskedasticity robust standard errors!
- While in LPM model,the predicted probability can be below 0 or above 1!

# Mortgage applications: Predicted value

**FIGURE 11.1** Scatterplot of Mortgage Application Denial and the Payment-to-Income Ratio

Mortgage applicants with a high ratio of debt payments to income (*P/I ratio*) are more likely to have their application denied (*deny* = 1 if denied, *deny* = 0 if approved). The linear probability model uses a straight line to model the probability of denial, conditional on the *P/I ratio*.



# Nonlinear probability model

# Introduction

- Intuition: Probabilities should not be less than 0 or greater than 1
- To address this problem,consider nonlinear probability models

$$\begin{aligned} Pr(Y_i = 1|X_1, ...X_k) &= G(Z) \\ &= G(\beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}) \end{aligned}$$

- where $Z = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}$ and $0 \leq g(Z) \leq 1$

# Logit and Probit

- Two types nonlinear functions
  1. Probit
  $$G(Z) = \Phi(Z) = \int_{-\infty}^{z} \phi(Z)dZ$$
  2. Logit
  $$G(Z) = \frac{1}{1+e^{-Z}}$$

# Probit Model

- Probit regression models the probability that $Y = 1$

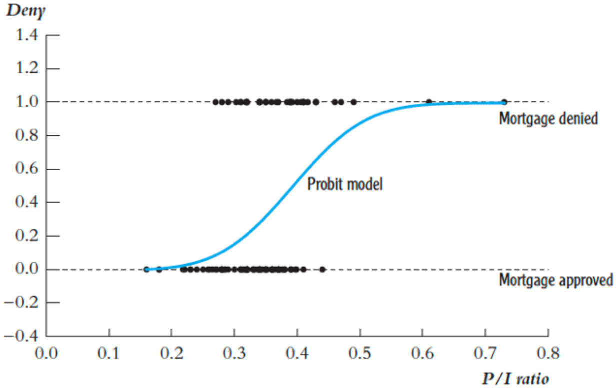
$$Pr(Y_i = 1|) = \Phi(\beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i})$$

- Using the cumulative standard normal distribution function $\Phi(Z)$ and $0 \leq \Phi(Z) \leq 1$
- Since $\Phi(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1.

# Probit Model

**FIGURE 11.2** Probit Model of the Probability of Denial, Given *P/I Ratio*

The probit model uses the cumulative normal distribution function to model the probability of denial given the payment-to-income ratio or, more generally, to model $\Pr(Y = 1 | X)$. Unlike the linear probability model, the probit conditional probabilities are always between 0 and 1.



# Probit Model

- evaluated at $Z = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}$
- The coefficient $\beta_1$ is the change in the $z$-value arising from a unit change in $X_1$, holding constant $X_2, ..., X_k$.
- The effect on the predicted probability of a change in a regressor is computed by
  1. computing the predicted probability for the initial value of the regressors,
  2. computing the predicted probability for the new or changed value of the regressors,
  3. taking their difference.

# Probit Model with one regression

- Suppose the probit population regression model with only one regressors, $X_1$

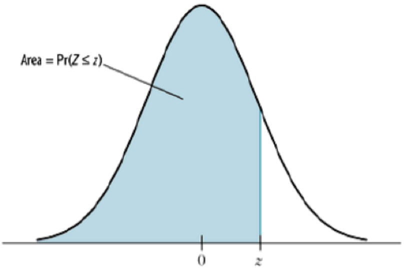
$$Pr(Y = 1|X_1) = \Phi(Z) = \Phi(\beta_0 + \beta_1 X_1)$$

- Suppsoe the estimate result is $Z = -2 + 3X_1$,
- And we want to know the probability that $Y = 1$ when $X_1 = 0.4$
- Then $z = -2 + 3 \times 0.4 = -0.8$
- So the probability $Pr(Y = 1) = Pr(z \leq -0.8) = \Phi(-0.8)$

# Probit Model

- $Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-.8) = 0.2119$

**TABLE 1** The Cumulative Standard Normal Distribution Function, $\Phi(z) = Pr(Z$ " $z)$




| | Second Decimal Value of z | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **z** | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |

# Probit Model with multiple regressors

- Suppose the probit population regression model with two regressors, $X_1$ and $X_2$,

$$Pr(Y = 1|X_1, X_2) = \Phi(\beta_0 + \beta_1 X_1 + \beta_2 X_2)$$

- Suppose $\beta_0 = -1.6, \beta_1 = 2 \; and \; \beta_2 = 0.5$. And If $X_1 = 0.4 \; and \; X_2 = 1$ then

$$z = -1.6 + 2 \times 0.4 + 0.5 \times 1 = -0.3$$

- So the probability $Pr(Y = 1) = Pr(z \leq -0.3) = \Phi(-0.3) = 0.38$

# Example: Mortgage Applications

- Mortgage denial (deny) and the payment-toincome ratio (P/I ratio)

$$\widehat{Pr(deny = 1|P/I\ ratio)} = \Phi(-2.19 + 2.97P/I\ ratio)$$

- *What is the change in the predicted probability* that an application will be deniedif P/I ratio incrreases from *0.3 to 0.4*?
- The probability of denial when $P/I\ ratio = 0.3$

$$\Phi(-2.19 + 2.97 \times 0.3) = \Phi(-1.3) = 0.097$$

- The probability of denial when $P/I\ ratio = 0.4$

$$\Phi(-2.19 + 2.97 \times 0.4) = \Phi(-1.0) = 0.159$$

- The estimated change in the probability of denial is $0.159 - 0.097 = 0.062$

# Effect of a change in X: Marginal Effects

- For nonlinear models, the ME varies with the point of evaluation
  - *Marginal Effect at a Representative Value* (MER):ME at $X = X^*$ (at representative values of the regerssors)
  - *Marginal Effect at Mean* (MEM): ME at $X = \bar{X}$(at the sample mean of the regressors)
  - *Average Marginal Effect* (AME): average of ME at each $X = X_i$ (at sample values and then average)

# Example: Mortgage applications: marginal effect

- Because the probit regression function is nonlinear, the effect of a change in X depends on the starting value of X.

$$\frac{\partial Pr(deny = 1|P/I\ ratio)}{\partial P/I\ ratio} = \Phi(-2.19 + 2.97P/I\ ratio) \times 2.97$$

- *Marginal Effect at Mean* (MEM):(at the sample mean of the regressors: $P/I\ ratio_{mean} = 0.331$

$$\frac{\partial Pr(deny = 1|P/I\ ratio)}{\partial P/I\ ratio}_{at\ mean} = \Phi(-2.19 + 2.97 \times 0.331) \times 2.97$$

# Special Case: The explanatory variable is discrete.

- If $xj$ is a discrete variable, then we should not rely on calculus in evaluating the effect on the response probability.
- Assume $X_2$ is a dummy variable, then partial effect of $X_2$ changing from 0 to 1:

$$G(\beta_0+\beta_1 X_{1,i}+\beta_2\times 1+...+\beta_k X_{k,i})-G(\beta_0+\beta_1 X_{1,i}+\beta_2\times 0+...+\beta_k X_{k,i})$$

# Example: Mortgage applications: Race

# Example: Mortgage applications: Race

- Mortgage denial (deny) and the payment-toincome ratio (P/I ratio) and race

$$\widehat{Pr(deny = 1|}P/I\ ratio) = \Phi(-2.26 + 2.74P/I\ ratio + 0.71black)$$

- The probability of denial when $black = 0$,thus whites(non-blacks) is

$$\Phi(-2.26 + 2.74 \times 0.3 + 0.71 \times 0) = \Phi(-1.43) = 0.075$$

- The probability of denial when $black = 1$,thus blacks is

$$\Phi(-2.26 + 2.74 \times 0.3 + 0.71 \times 1) = \Phi(-0.73) = 0.233$$

- so the difference between whites and blacks at $P/Iratio = 0.3$ is $0.233 - 0.075 = 0.158$, which means probability of denial for blacks is 15.8% higher than that for whites.

# Logit Model

- Logit regression models the probability that Y = 1
- Using the cumulative standard logistic distribution function

$$Pr(Y_i = 1|Z) = \frac{1}{1 + e^{-Z}}$$

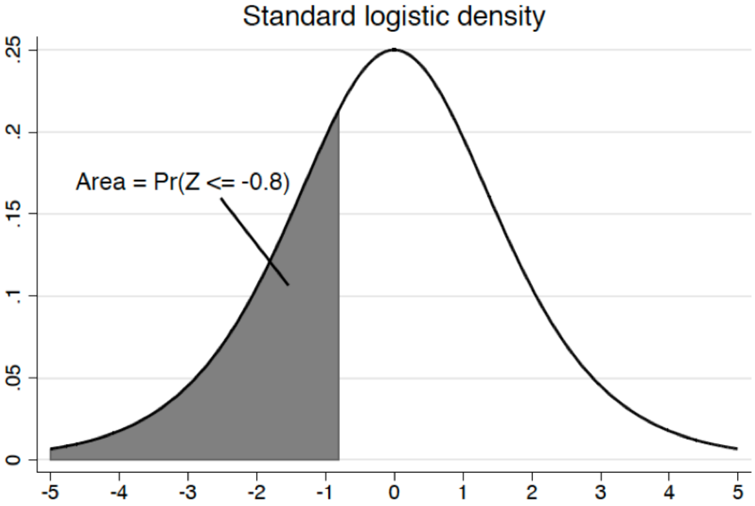
- evaluated at $Z = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}$
- since $F(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1.

# Logit Model

- Suppose we have only one regressor X and $Z = -2 + 3X_1$
- We want to know the probability that $Y = 1$ when $X_1 = 0.4$
- Then $Z = -2 + 3 \times 0.4 = -0.8$
- So the probability $Pr(Y = 1) = Pr(Z \leq -0.8) = F(-0.8)$

# Logit Model

- $Pr(Y=1) = Pr(Z \leq -0.8) = \frac{1}{1+e^{0.8}} = 0.31$



# Example: Mortgage applications

- Logit Model: Mortgage denial (deny) and the payment-toincome ratio (P/I ratio) and race

$$\widehat{Pr(deny = 1|}P/I\ ratio) = F(\underset{(0.35)}{-4.13} + \underset{(0.96)}{5.37}P/I\ ratio + \underset{(0.15)}{1.27}black)$$

## Example: Mortgage applications: Race

- The predicted denial probability of a white applicant with $P/I\ ratio = 0.3$ is

$$\frac{1}{1+e^{-(-4.13+5.37\times 0.3+1.27\times 0)}} = 0.074$$

- The predicted denial probability of a black applicant with $P/I\ ratio = 0.3$ is

$$\frac{1}{1+e^{-(-4.13+5.37\times 0.3+1.27\times 1)}} = 0.222$$

# How to estimate Logit and Probit models

- nonlinear in the independent variables(X).
  - these models can be estimated by OLS
- While Logit and Probit models are nonlinear in the coefficients $\beta_0, \beta_1, ..., \beta_k$
  - these models can NOT be estimated by OLS
- The method used to estimate logit and probit models is **Maximum Likelihood Estimation** (MLE).

# MLE estimator in practice

- There is no simple formula for the probit and logit MLE, the maximization must be done using **numerical algorithm** on a computer.
- Because regression software commonly computes the MLE of the estimate coefficients, this estimator is easy to use in practice.
- The MLE is consistent and normally distributed in large samples.

# Statistical inference based on the MLE

- Because the MLE is normally distributed in large samples, statistical inference about the probit and logit coefficients based on the MLE proceeds in the same way as inference about the linear regression function coefficients based on the OLS estimator.
- That is, hypothesis tests are performed using the **t-statistic** and **95% confidence intervals** are formed as 1.96 standard errors.
- Tests of joint hypotheses on multiple coefficients use the **F-statistic** in a way similar to that discussed for the linear regression model.
- F-statistic and Chi-squared stistic

$$F_{stat} \longrightarrow \frac{\chi_q^2}{q}$$

where q is the number of restrictions being tested.

# Measures of Fit

- R2 is a poor measure of fit for the linear probability model. This is also true for probit and logit regression.
- Two measures of fit for models with binary dependent variables

1. *fraction correctly predicted*

- If $Y_i = 1$ and the predicted probability exceeds $50\%$ or if $Y_i = 0$ and the predicted probability is less than $50\%$, then $Y_i$ is said to be correctly predicted.

# Measures of Fit

2. **The pseudo-R2**

- The $pseudo - R^2$ compares the value of the likelihood of the estimated model to the value of the likelihood when none of the Xs are included as regressors.

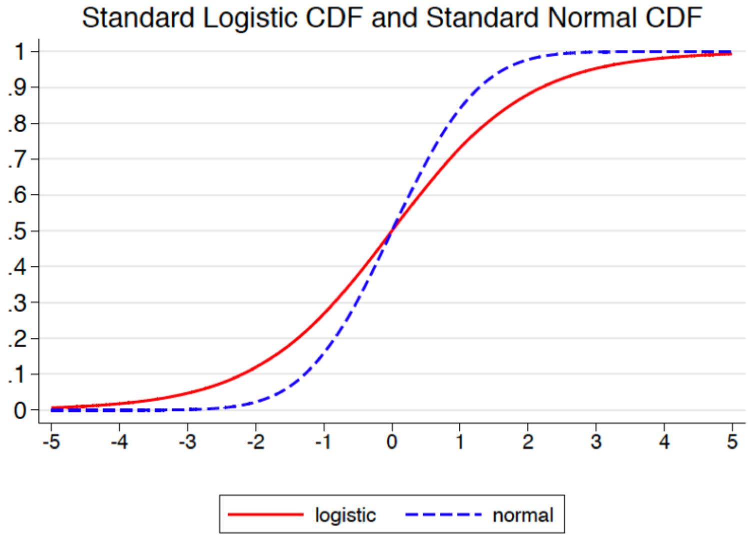
$$pseudo - R^2 = 1 - \frac{ln(f_{probit}^{max})}{ln(f_{bernoulli}^{max})}$$

- $f_{probit}^{max}$ is the value of the maximized probit likelihood (which includes the X's)
- $f_{bernoulli}^{max}$ is the value of the maximized Bernoulli likelihood (the probit model excluding all the X's).

# Comparing the LPM,Probit and Logit

- All three models: *linear probability, probit, and logit* are just approximations to the unknown population regression function $E(Y|X) = Pr(Y = 1|X)$.
    - LPM is easiest to use and to interpret, but it cannot capture the nonlinear nature of the true population regression function.
    - Probit and logit regressions model this nonlinearity in the probabilities, but their regression coefficients are more difficult to interpret.
- So which should you use in practice?
    - *There is no one right answer*, and different researchers use different models.
    - *Probit and logit regressions frequently produce similar results.*

# Logit v.s. Probit



# Comparing the LPM,Probit and Logit

- The marginal effects and predicted probabilities are much more similar across models.
- Coefficients can be compared across models, using the following rough conversion factors (Amemiya 1981)

$$\hat{\beta}_{logit} \simeq 4\hat{\beta}_{ols}$$
$$\hat{\beta}_{probit} \simeq 2.5\hat{\beta}_{ols}$$
$$\hat{\beta}_{logit} \simeq 1.6\hat{\beta}_{probit}$$

# Example: Mortgage Applications(short regression)

| Dependent variable: *deny* = 1 if mortgage application is denied, = 0 if accepted | | | |
|---|---|---|---|
| regression model | LPM | Probit | Logit |
| *black* | 0.177*** | 0.71*** | 1.27*** |
| | (0.025) | (0.083) | (0.15) |
| *P/I ratio* | 0.559*** | 2.74*** | 5.37*** |
| | (0.089) | (0.44) | (0.96) |
| *constant* | -0.091*** | -2.26*** | -4.13*** |
| | (0.029) | (0.16) | (0.35) |
| difference Pr(*deny*=1) between black and white applicant when *P/I ratio*=0.3 | 17.7% | 15.8% | 14.8% |

# Example: Mortgage Applications(long regression)

**TABLE 11.1** Variables Included in Regression Models of Mortgage Decisions

| Variable | Definition | Sample Average |
|---|---|---|
| **Financial Variables** | | |
| *P/I ratio* | Ratio of total monthly debt payments to total monthly income | 0.331 |
| *housing expense-to-income ratio* | Ratio of monthly housing expenses to total monthly income | 0.255 |
| *loan-to-value ratio* | Ratio of size of loan to assessed value of property | 0.738 |
| *consumer credit score* | 1 if no "slow" payments or delinquencies<br>2 if one or two slow payments or delinquencies<br>3 if more than two slow payments<br>4 if insufficient credit history for determination<br>5 if delinquent credit history with payments 60 days overdue<br>6 if delinquent credit history with payments 90 days overdue | 2.1 |
| *mortgage credit score* | 1 if no late mortgage payments<br>2 if no mortgage payment history<br>3 if one or two late mortgage payments<br>4 if more than two late mortgage payments | 1.7 |
| *public bad credit record* | 1 if any public record of credit problems (bankruptcy, charge-offs, collection actions)<br>0 otherwise | 0.074 |

图 1: pic

# Example: Mortgage Applications(long regression)

**Additional Applicant Characteristics**

| | | |
|---|---|---|
| *denied mortgage insurance* | 1 if applicant applied for mortgage insurance and was denied, 0 otherwise | 0.020 |
| *self-employed* | 1 if self-employed, 0 otherwise | 0.116 |
| *single* | 1 if applicant reported being single, 0 otherwise | 0.393 |
| *high school diploma* | 1 if applicant graduated from high school, 0 otherwise | 0.984 |
| *unemployment rate* | 1989 Massachusetts unemployment rate in the applicant's industry | 3.8 |
| *condominium* | 1 if unit is a condominium, 0 otherwise | 0.288 |
| *black* | 1 if applicant is black, 0 if white | 0.142 |
| *deny* | 1 if mortgage application denied, 0 otherwise | 0.120 |

图 **2:** pic

# Example: Mortgage Applications(long regression)

**TABLE 11.2** Mortgage Denial Regressions Using the Boston HMDA Data

**Dependent variable: *deny* = 1 if mortgage application is denied, = 0 if accepted; 2380 observations.**

| *Regression Model* | *LPM* | *Logit* | *Probit* | *Probit* | *Probit* | *Probit* |
|---|---|---|---|---|---|---|
| **Regressor** | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** |
| *black* | 0.084** (0.023) | 0.688** (0.182) | 0.389** (0.098) | 0.371** (0.099) | 0.363** (0.100) | 0.246 (0.448) |
| *P/I ratio* | 0.449** (0.114) | 4.76** (1.33) | 2.44** (0.61) | 2.46** (0.60) | 2.62** (0.61) | 2.57** (0.66) |
| *housing expense-to-income ratio* | −0.048 (0.110) | −0.11 (1.29) | −0.18 (0.68) | −0.30 (0.68) | −0.50 (0.70) | −0.54 (0.74) |
| *medium loan-to-value ratio* ($0.80 \leq$ *loan-value ratio* $\leq 0.95$) | 0.031* (0.013) | 0.46** (0.16) | 0.21** (0.08) | 0.22** (0.08) | 0.22** (0.08) | 0.22** (0.08) |
| *high loan-to-value ratio* (*loan-value ratio* $> 0.95$) | 0.189** (0.050) | 1.49** (0.32) | 0.79** (0.18) | 0.79** (0.18) | 0.84** (0.18) | 0.79** (0.18) |
| *consumer credit score* | 0.031** (0.005) | 0.29** (0.04) | 0.15** (0.02) | 0.16** (0.02) | 0.34** (0.11) | 0.16** (0.02) |
| *mortgage credit score* | 0.021 (0.011) | 0.28* (0.14) | 0.15* (0.07) | 0.11 (0.08) | 0.16 (0.10) | 0.11 (0.08) |
| *public bad credit record* | 0.197** (0.035) | 1.23** (0.20) | 0.70** (0.12) | 0.70** (0.12) | 0.72** (0.12) | 0.70** (0.12) |
| *denied mortgage insurance* | 0.702** (0.045) | 4.55** (0.57) | 2.56** (0.30) | 2.59** (0.29) | 2.59** (0.30) | 2.59** (0.29) |
| *self-employed* | 0.060** (0.021) | 0.67** (0.21) | 0.36** (0.11) | 0.35** (0.11) | 0.34** (0.11) | 0.35** (0.11) |

# Example: Mortgage Applications(long regression)

| | | | | | | |
|---|---|---|---|---|---|---|
| *single* | | | | 0.23** (0.08) | 0.23** (0.08) | 0.23** (0.08) |
| *high school diploma* | | | | –0.61** (0.23) | –0.60* (0.24) | –0.62** (0.23) |
| *unemployment rate* | | | | 0.03 (0.02) | 0.03 (0.02) | 0.03 (0.02) |
| *condominium* | | | | | –0.05 (0.09) | |
| *black* × *P/I ratio* | | | | | | –0.58 (1.47) |
| *black* × *housing expense-to-income ratio* | | | | | | 1.23 (1.69) |
| *additional credit rating indicator variables* | no | no | no | no | yes | no |
| *constant* | –0.183** (0.028) | –5.71** (0.48) | –3.04** (0.23) | –2.57** (0.34) | –2.90** (0.39) | –2.54** (0.35) |

# Example: Mortgage Applications(long regression)

*(Table 11.2 continued)*

**F-Statistics and p-Values Testing Exclusion of Groups of Variables**

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| *applicant single; high school diploma; industry unemployment rate* | | | | 5.85 ($< 0.001$) | 5.22 (0.001) | 5.79 ($< 0.001$) |
| *additional credit rating indicator variables* | | | | | 1.22 (0.291) | |
| *race interactions and black* | | | | | | 4.96 (0.002) |
| *race interactions only* | | | | | | 0.27 (0.766) |
| *difference in predicted probability of denial, white vs. black (percentage points)* | 8.4% | 6.0% | 7.1% | 6.6% | 6.3% | 6.5% |

These regressions were estimated using the $n = 2380$ observations in the Boston HMDA data set described in Appendix 11.1. The linear probability model was estimated by OLS, and probit and logit regressions were estimated by maximum likelihood. Standard errors are given in parentheses under the coefficients, and $p$-values are given in parentheses under the $F$-statistics. The change in predicted probability in the final row was computed for a hypothetical applicant whose values of the regressors, other than race, equal the sample mean. Individual coefficients are statistically significant at the *5% or **1% level.

# the last evaluation

- Both for the LPM models as for the Probit & Logit models we have to consider threats to the **Internal validity**
  - Is there omitted variable bias?
  - Is the functional form correct?
  - Is there measurement error?
  - Is there sample selection bias?
  - is there a problem of simultaneous causality?

# More Extension: Categoried and Limited Dependent Variables families

- Binary outcomes: LPM, logit and probit
- Mulitnomial outcomes: No order, sucha as (multi-logit,mprobit)
- Ordered outcomes: Ordered Response Models(order probit and logit)
- Count outcomes: The outcomes is a nonnegative integer or a count. (possion model)
- Limited Dependent Varaible(Censored, Tobit and Selection Models)
- Time: (Duration Model)