Recite 2: Review of Basic Statistics

Introduction to Econometrics,Fall 2018

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Population,Parameters and Random Sampling

Population,Sample and i.i.d

A population is a collection of people, items, or events about which you want to make inferences.

- Population always have a probability distribution.

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- Infinite population
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- With replacement
- Without replacement: when the population size N is very large,compared with the sample size n, then we could say that they are nearly independent.

Definition

- The r.v.s are called a random sample of size n from the population f(x) if $X_1, ..., X_n$ are mutually independent and have the same p.d.f/p.m.f f(x). Alternatively, $X_1, ..., X_n$ are called independent,and identically distributed random variable with p.d.f/p.m.f ,commonly abbreviated to i.i.d. r.v.s.

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- A statistic is only a function of the sample.

Sample Mean and Sample Variance

Definition

- The sample average or sample mean X of the n observations $X_1, ..., X_n$ is

$$\bar{X} = \frac{1}{n}(X_1 + X_2 + ... + X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i$$

The sample variance is the statistic defined by

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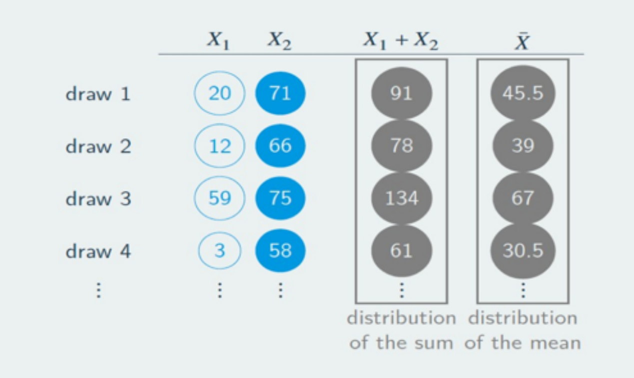
- we could assume that the sample mean has some certain probability functions to describe its distributions
- what is the expectation, variance or p.d.f./c.d.f of this distribution?

A simple case of sample mean



- Let $X_n \in [1, 100]$, assume n = 2, thus only X_1 and X_2

Large-Sample Approximations to Sampling Distributions

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   - The Law of Large Numbers(L.L.N.):when the sample size is large,X will be close to μ_Y the population mean with very high probability
   - The Central Limit Theorem(C.L.T.): when the sample size is large,the sampling distribution of the standardized sample average, $\frac{(\bar{Y}-\mu_Y)}{\sigma_Y}$ is approximately normal.

Convergence in probability

Definition

- Let $X_1, ..., X_n$ be an random variables or sequence, is said to converge in probability to a value b if for every $\varepsilon > 0$,

$$P(\|X_n - b\| > \varepsilon) \to 0$$

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- it is similar to the concept of a limitation in a probability way.

the Law of Large Numbers

Theorem

- Let $X_1, ..., X_n$ be an i.i.d draws from a distribution with mean μ and finite variance σ (a population) $\bar{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is the sample mean then

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- Intuition:the distribution of ??collapses?? $\bar{X}$ on μ

A simple case

Example

- Suppose X has a Bernoulli distribution if it have a binary values $X \varepsilon 0, 1$ and its probability mass function is

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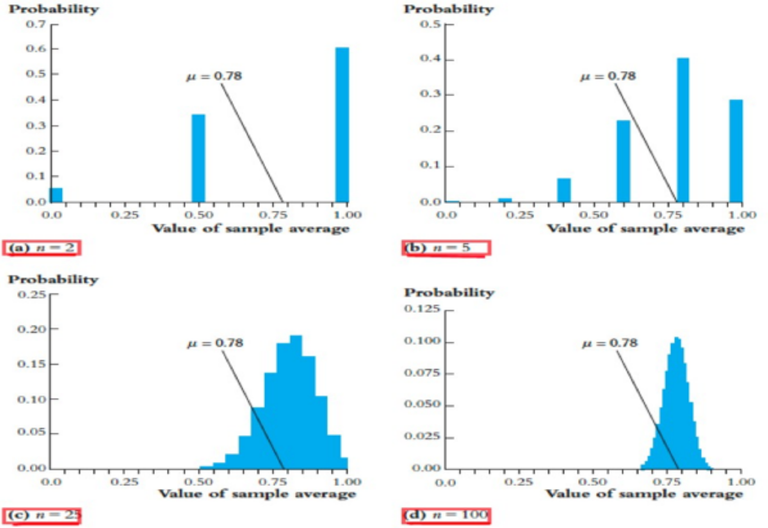
- Then E(X)=p=0.78 and Var(X)=p(1-p)=0.1716

Convergence in Distribution

FIGURE 2.8 Sampling Distribution of the Sample Average of n Bernoulli Random Variables



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- Let $X_1, ..., X_n$ be a sequence of r.v.s, and for n = 1,2,... let $F_n(x)$ be the c.d.f of X_n.Then it is said that $X_1, X_2, ...$ converges in distribution to r.v.W with c.d.f, F_W if

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- Common to standardize a r.v. by subtracting its expectation and dividing by its standard deviation

$$Z = \frac{X - E[X]}{Var[X]}$$

The Central Limit Theorem

Theorem

- Let $X_1, ..., Xn$ be an i.i.d draws from a distribution with sample size n with mean μ and $0 < \sigma^2 < \infty$, then

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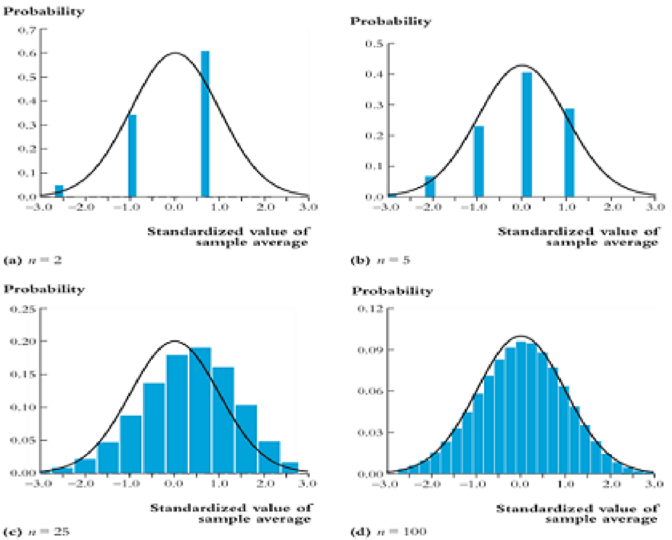
- Because we don??t have to make specific assumption about the distribution of X_i, so whatever the distribution of X_i, when n is big the standardized $\bar{X_n} \sim N(\mu, \frac{\sigma^2}{n})$

FIGURE 2.9 Distribution of the Standardized Sample Average of n Bernoulli Random Variables with $p = 0.78$



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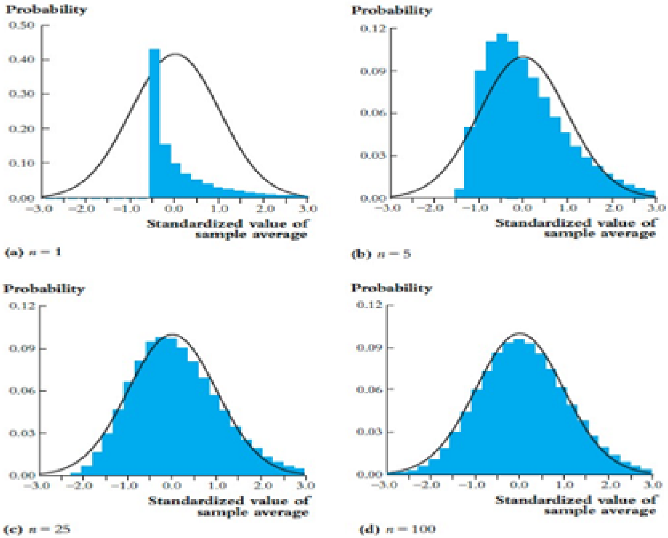
   - if Y_i themselves have a distribution that is far from normal,then this approximation can require $n = 30$ or even more.

FIGURE 2.10 Distribution of the Standardized Sample Average of n Draws from a Skewed Distribution



Statistical Inference: Estimation, Confident Intervals and Testing

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- we use simple difference in sample meansor the post-stratification estimator, where we estimate the estimate the difference among two subsets of the data (male and female,for instance) and then take the weighted average of the two variable

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- two ways: parametric and Non-parametric

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Estimator and Estimate

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Given a random sample$\{Y_1, Y_2, ..., Y_n\}$ drawn from a population distribution that depends on an unknown parameter θ, and an estimator $\hat{\theta}$ is a function of the sample: thus $\hat{\theta}_n = h(Y_1, Y_2, ..., Y_n)$

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- Question: what is the difference between an estimator and an statistic?

Estimator and Estimate

Definition

An estimate is the numerical value of the estimator when it is actually computed using data from a specific sample. Thus if we have the actual data $\{y_1, y_2, ..., y_n\}$,then $\hat{\theta} = h(y_1, y_2, ..., y_n)$

Three Characteristics of an Estimator

Let $\hat{\mu}_Y$ denote some estimator of μ_Y and $E(\hat{\mu}_Y)$ is the mean of the sampling distribution of $\hat{\mu}_Y$,

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3. **Efficiency:** Let $\tilde{\mu}_Y$ be another estimator of μ_Y and suppose that both $\tilde{\mu}_Y$ and $\hat{\mu}_Y$ are unbiaesd.Then $\hat{\mu}_Y$ is said to be more efficient than $\tilde{\mu}_Y$

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- Comparing variances is difficult if we do not restrict our attention to unbiased estimators because we could always use a trivial estimator with variance zero that is biased.

Properties of the sample mean

Let μ_Y and σ_Y^2 denote the mean and variance of Y, then

$$E(\bar{Y}) = \frac{1}{n}\sum_{i=1}^{N} E(Y_i) = \mu_Y$$

so Y is an **unbiased** estimator of μ_Y

- Based on the L.L.N.,$\bar{Y} \rightarrow \mu_Y$ so $\bar{Y}$ is also consistent.

The standard deviation of the sample mean is $\sigma_{\bar{Y}} = \frac{\sigma_Y}{\sqrt{n}}$

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- The variance of sample mean

$$Var(\bar{Y}) = var(\frac{1}{n}\sum_{i=1}^{N} Y_i) = \frac{1}{n^2}\sum_{i=1}^{N} Var(Y_i) = \frac{\sigma_Y^2}{n}$$

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Properties of the sample mean

Because efficiency entails a comparison of estimators,we need to specify the estimator or estimators to which Y is to be compared.

- Let $\widetilde{Y} = \frac{1}{n}(\frac{1}{2}Y_1 + \frac{3}{2}Y_2 + \frac{1}{2}Y_3 + \frac{3}{2}Y_4 + ... + \frac{1}{2}Y_{n-1} + \frac{3}{2}Y_n)$

Thus $\bar{Y}$ is more efficient than $\widetilde{Y}$.

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- $Var(\widetilde{Y}) = 1.25\frac{\sigma_Y^2}{n} > \frac{\sigma_Y^2}{n} = Var(\bar{Y})$

Thus $\bar{Y}$ is more efficient than $\widetilde{Y}$.

Confidence Interval and Interval Estimation

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- A different (complementary) approach to estimation is to produce a range of values that will contain the truth with some fixed probability.

What is a Confidence Interval?

Definition

A $100(1-\alpha)\%$ confidence interval for a population parameter θ is an interval $C_n = (a, b)$,where $a = a(Y_1, Y_2, ..., Y_n)$ and $b = b(Y_1, Y_2, ..., Y_n)$ are functions of the data such that

$$P(a < \theta < b) = 1 - \alpha$$

- In general,this confidence level is $1-\alpha$; where α is called **significance level**.

Interval Estimation and Condence Intervals

Suppose the population has a normal distribution $N(\mu, \sigma^2)$ and let $Y_1, Y_2, ..., Y_n$ be a random sample from the population.

- Then the sample mean has a normal distribution: $\bar{Y} \sim N(\mu, \frac{\sigma^2}{n})$

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- The standardized sample mean $\bar{Z}$ is given by: $\bar{Z} = \frac{\bar{Y}-\mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$

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- then $\theta = \bar{Z}$,then $P(a < \theta < b) = 1 - \alpha$ turns into

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- The random interval contains the population mean with a probability $1 - \alpha$

Interval Estimation and Condence Intervals

Two cases:σ is known and unknown - When σ is known, for example, $\sigma = 1$,thus $Y \sim N(\mu, 1)$ then $\bar{Y} \sim N(\mu, \frac{\sigma^2}{n} = \frac{1}{n})$

- From this we can standardize $\bar{Y}$,and because the standardized version of $\bar{Y}$ has a standard normal distribution, and we let $\alpha = 0.05$,then we have

$$P(-1.96 < \frac{\bar{Y} - \mu}{\frac{1}{\sqrt{n}}} < 1.96) = 1 - 0.05$$

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- So $P(\bar{Y} - \frac{1.96}{\sqrt{n}} < \mu < \bar{Y} + \frac{1.96}{\sqrt{n}}) = 0.95$

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- The interval estimate of μ may be written as $(\bar{Y} - \frac{1.96}{\sqrt{n}}, \bar{Y} + \frac{1.96}{\sqrt{n}})$

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- This could not work because S is not a constant but a r.v.

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where $c_{\frac{\alpha}{2}}$ is the critical value of the t distribution.

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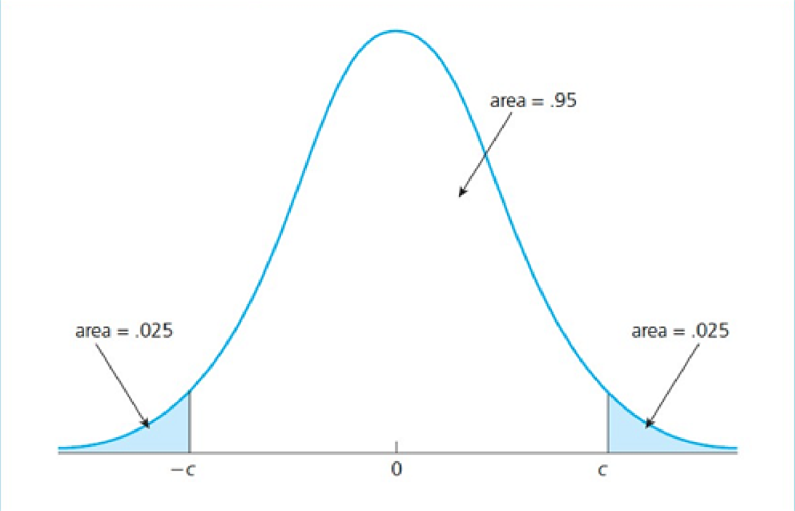
FIGURE C.4 The 97.5th percentile, c, in a t distribution.



A simple rule of thumb for a 95% confidence interval

Caution! An often recited, but incorrect interpretation of a confidence interval is the following:

- "I calculated a 95% confidence interval of $[0.05, 0.13]$, which means that there is a 95% chance that the true means is in that interval.??

The probabilistic interpretation comes from the fact that for 95% of all random samples,the constructed confidence interval will contain μ.

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Caution! An often recited, but incorrect interpretation of a confidence interval is the following:

- "I calculated a 95% confidence interval of $[0.05, 0.13]$, which means that there is a 95% chance that the true means is in that interval.??
- This is **WRONG**. actually μ either is or is not in the interval.

The probabilistic interpretation comes from the fact that for 95% of all random samples,the constructed confidence interval will contain μ.

TABLE 3.1 Trends in Hourly Earnings in the United States of Working College Graduates, Ages 25–34, 1992 to 2008, in 2008 Dollars

	Men			Women			Difference, Men vs. Women		
Year	$\overline{Y}_m$	s_m	n_m	$\overline{Y}_w$	s_w	n_w	$\overline{Y}_m - \overline{Y}_w$	$SE(\overline{Y}_m - \overline{Y}_w)$	95% Confidence Interval for d
1992	23.27	10.17	1594	20.05	7.87	1368	3.22**	0.33	2.58–3.88
1996	22.48	10.10	1379	18.98	7.95	1230	3.50**	0.35	2.80–4.19
2000	24.88	11.60	1303	20.74	9.36	1181	4.14**	0.42	3.32–4.97
2004	25.12	12.01	1894	21.02	9.36	1735	4.10**	0.36	3.40–4.80
2008	24.98	11.78	1838	20.87	9.66	1871	4.11**	0.35	3.41–4.80

These estimates are computed using data on all full-time workers aged 25–34 surveyed in the Current Population Survey conducted in March of the next year (for example, the data for 2008 were collected in March 2009). The difference is significantly different from zero at the **1% significance level.

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- If the value μ_0 lie within the calculated confidence interval,then we **fail to reject** the null hypothesis.

General framework

- A hypothesis test chooses whether or not to reject the null hypothesis based on the data we observe.

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Two Type Errors

In both cases, there is a certain risk that our conclusion is wrong.

Type I Error

- A Type I error is when we reject the null hypothesis when it is in fact true.(??left-wing??)

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P-value

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We can consider the **p-value** of a test

- Calculate the t-statistic
- The largest significance level at which we would fail to reject H_0 is the significance level associated with using t as our critical value.

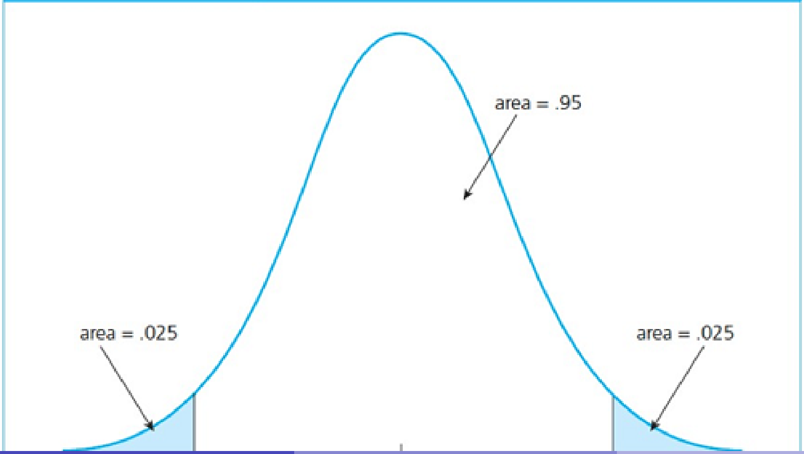
$$p - value = 1 - \Phi(t)$$

where denotes the standard normal c.d.f.(we assume that n is large enough)

- Suppose that $t = 1.52$, then we can find the largest significance level at which we would fail to reject H_0

$$p - value = P(T > 1.52|H_0) = 1 - \Phi(1.52) = 0.065$$

FIGURE C.4 The 97.5[th] percentile, c, in a t distribution.



For example,I said that I shot very well, and I can hit an average of 8 rings. How can I verify it? Let me do a few shots to see how my level is. First you choose to believe me, assuming I didn't brag, my score is near the 8 rings (this is the null hypothesis). As an 8-ring level player, the number of rings in the shot should be subject to a distribution with an average of 8

Comparing Means from Different Populations

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- Do recent male and female college graduates earn the same amount on average? This question involves comparing the means of two different population distributions.

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- Under randomization, difference-in-means is a good estimate for the ATE.

Hypothesis Tests for the Difference Between Two Means

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- Consider the null hypothesis that mean earnings for these two populations differ by a certain amount, say d_0. The null hypothesis that men and women in these populations have the same mean earnings corresponds to $H_0 : d_0 = \mu_m - \mu_w = 0$

The Difference Between Two Means

- Suppose we have samples of n_m men and n_w women drawn at random from their populations.Let the sample average annual earnings be $\bar{Y}_m$ for men and for $\bar{Y}_w$ for women.Then an estimator of $\mu_m - \mu_w$ is $\bar{Y}_m - \bar{Y}_w$

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- if σ_m^2 and σ_w^2 are known, then the this approximate normal distribution can be used to compute p-values for the test of the null hypothesis.In practice,however, these population variances are typically unknown so they must be estimated.
- Thus the standard error of $\bar{Y_m} - \bar{Y_w}$ is

$$SE(\bar{Y_m} - \bar{Y_w}) = \sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}$$

The Difference Between Two Means

- The t-statistic for testing the null hypothesis is constructed analogously to the t-statistic for testing a hypothesis about a single population mean,thus t-statistic for comparing two means is

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- If both n_m and n_w are large, then this t-statistic has a standard normal distribution when the null hypothesis is true.

Confidence Intervals for the Difference Between Two Population Means

the 95% two-sided confidence interval for d consists of those values of d within ± 1.96 standard errors of $\bar{Y_m} - \bar{Y_w}$ thus $d = \mu_m - \mu_w$ is:

$$(\bar{Y_m} - \bar{Y_w}) \pm 1.96 SE(\bar{Y_m} - \bar{Y_w})$$

Thank you