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Hawaii CO2 Concentration Statistical Analysis Affected by Major Events Based on Bayesian Inference

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ABSTRACT This report analyzes the CO2 concentration in Hawaii increasing rate changes during two major events: The Fall of Berlin Wall and COVID-19 Pandemic. Ideally, the visualization gives a relatively sufficient evidence that The Fall of Berlin Wall reduces the increasing speed of CO2 concentration. Further investigation on derivatives, forecasts, and random effects based on generalized additive model using Bayesian inference explicitly proves that COVID-19 Pandemic reduces the CO2 concentration in Hawaii as well.

1 Introduction

The emission of CO2 has been a significant issue between all countries. This report will illustrate a discussion within the data of Hawaii scraped from the Scripps CO2 Program from 1960 to recent years. In this report, we are going to analyze two major events, that are **The Fall of Berlin Wall** and **COVID-19 Pandemic**. These two events are somehow affect the emission of CO2 based on statistical evidence.

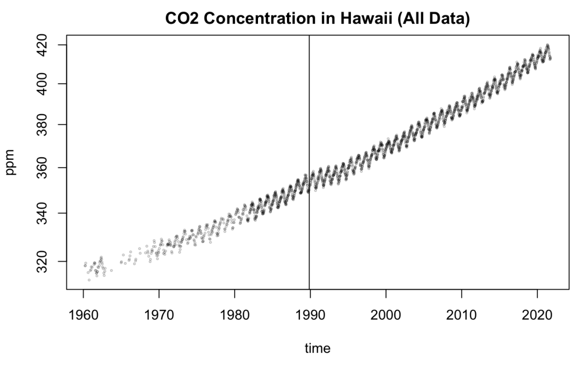

Figure 1.1: CO2 Concentration for All Data

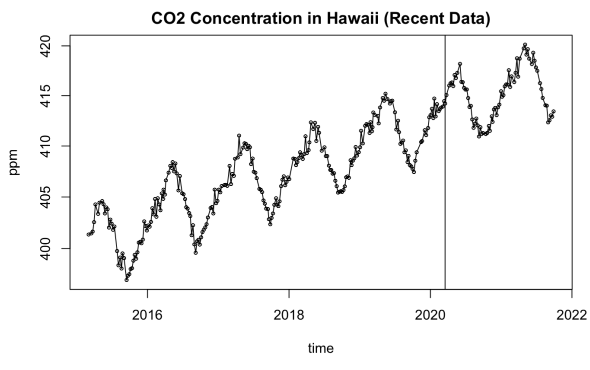

Figure 1.2: CO2 Concentration for Recent Data

As Figure 1.1 and Figure 1.2 show that the concentration of CO2 in Hawaii has an increasing trend and a seasonal effect. However, the increasing rate around 1989 was subdued, and the trend around 2020 is quite irregular as well. Based on the visual evidence, we will need to further investigate if it is a coincidence or statistical evidence behind the scene by using Bayesian inference for statistical modelling.

2 Statistical Analysis

2.1 Seasonal Effect

Obviously, there is a seasonal effect for the CO2 concentration data in Hawaii where the sinusoidal functions can be explained by

$$f(t) = \alpha \cos(2\pi t/365.25 + \phi) \tag{2.1}$$

such that we assume the cosine goes through one cycle every 12 months. α denotes the amplitude, and ϕ is the phase where $1/365.25$ is the frequency with one cycle has 365.25 days. Note that we can construct an equation to deal with the seasonal effect, that is

$$\beta_1 \cos(2\pi t/365.25) + \beta_2 \sin(2\pi t/365.25) = \sqrt{\beta_1^2 + \beta_2^2} \cos(2\pi t/365.25 + \arctan(-\frac{\beta_2}{\beta_1})) \tag{2.2}$$

such that $X_{i1} = \cos(2\pi t_i/365.25)$ and $X_{i2} = \sin(2\pi t_i/365.25)$ are linear covariates. t_i is the time in month of time observation i. Based on this idea, we can create some seasonal de-trending covariates before identifying the statistical model:

$$\begin{cases} X_{i1} = \cos(2\pi t/365.25) \\ X_{i2} = \sin(2\pi t/365.25) \\ X_{i3} = \cos(2\pi t/182.625) \\ X_{i4} = \sin(2\pi t/182.625) \end{cases} \tag{2.3}$$

We use a 12-month cycle and a 6-month cycle frequency to deal with the seasonal effect, then apply them to the model.

2.2 Statistical Modelling

With four covariates term, we can construct the model using Bayesian inference as

$$Y_i \sim N(\mu_i, \tau^2) \tag{2.4}$$

where

$$\mu_i = X_i \beta + U(t_i) \tag{2.5}$$

with the random effect of second order random walk with penalized term

$$[U_1, ...U_T]^T \sim \text{RW2}(\sigma_U^2) \tag{2.6}$$

such that

$$P(\sigma_U > 0.001) = 0.05 \tag{2.7}$$

Notations In the model, Y_i denotes the CO2 concentration that it follows normal distribution, with the expected mean μ_i. X_i is the covariates term represents four assumed sinusoidal-like cycles (seasonal effect). The random effect $U(t_i)$ follows a second order random walk with the standard deviation σ_U, that the probability of the standard deviation is greater than 0.001 is 0.05.

3 Results

Based on the generalized additive model we constructed with the random walk term, we can forecast the CO2 concentration in Hawaii for several years. According to figure 2.1 and figure 2.2, the trend has a likely decreasing signal after 2020. That is, we say the COVID-19 pandemic has an effect on the CO2 concentration with negative correlation. Moreover, a more forecasted figure shows a relatively stable

trend than previous years. Thus, we conclude that COVID-19 event is impacting the $CO2$ concentration in Hawaii negatively.
According to figure 3.3 and figure 3.4, the major event of fall of Berlin Wall decreases the increasing rate based on the visualization around 1990. The line around 1990 represents the event of Fall of Berlin Wall. Before this, the seasonal trend has a stable signal, but the derivatives start to fall after this event. This means the $CO2$ increasing rate is reduced. Furthermore, as figure 3.4 shows that after the line of COVID-19 outbreak, the derivative starts to behave like a sinusoidal function as it starts to slow down. Therefore, based on these two evidences, we say these two events impact the $CO2$ concentration in Hawaii.

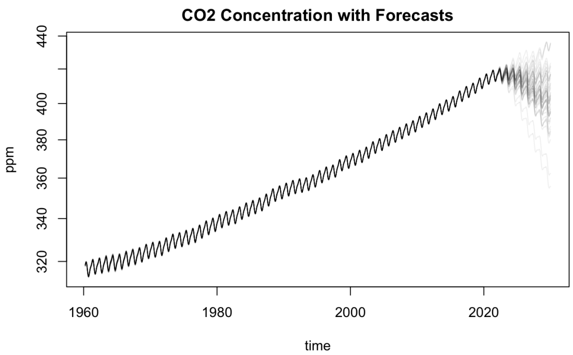

Figure 3.1: CO2 Concentration with Forecasts

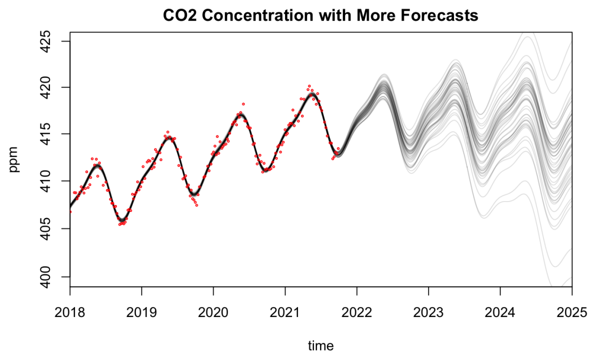

Figure 3.2: CO2 Concentration with More Forecasts

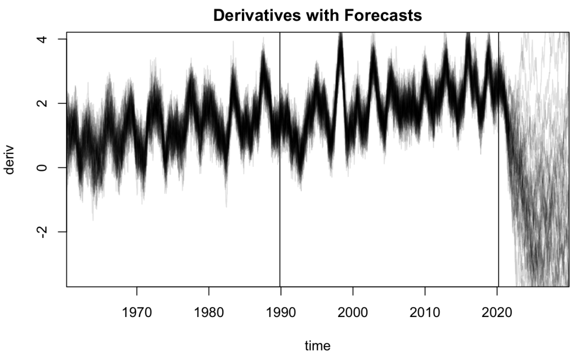

Figure 3.3: CO2 Derivatives with Forecasts

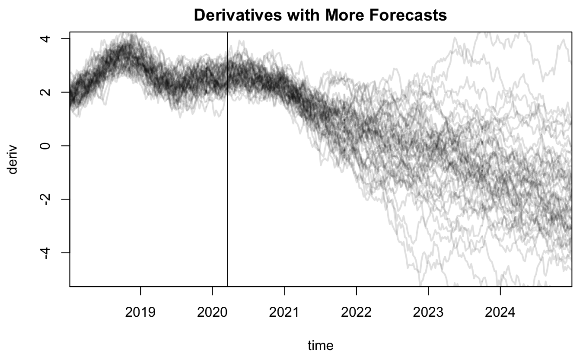

Figure 3.4: CO2 Derivatives with more Forecasts

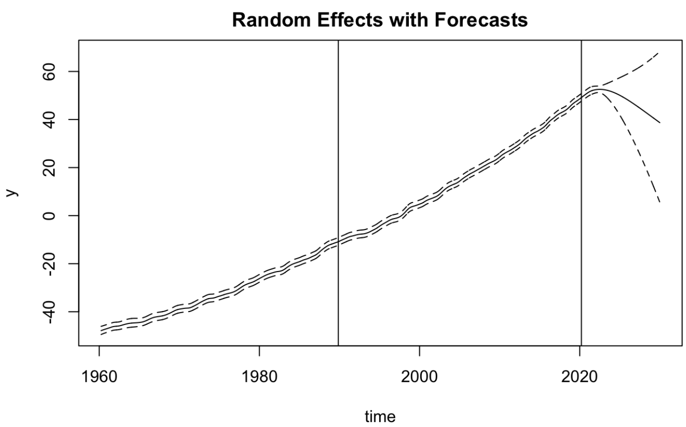

Figure 3.5: CO2 Random Effects with Forecasts

Table 3.1: Quantiles on Coefficients of CO2 Model

	0.5 Quantile	0.25 Quantile	0.975 Quantile
SD for Gaussian	0.6079	0.6252	0.5911
SD for Observations	0.0056	0.0051	0.0061

Table 3.1 shows 95% confidence interval for standard deviations of the model and observations. The number of standard deviation for observations are quite high, because the slope change is big enough with the times of around 0.006 per day.
However, in order to strengthen our conclusion, we are going to analyze the random effects during these two timelines. As shown in figure 3.5, the random effects around 1990 when Berlin Wall falls start to reduce the increasing trend. Around 2020 when COVID-19 outbreaks, the predicted random effects do not have a significant increasing trend, and the 95% confidence interval does not have a strong increasing prediction as well. Thus, we conclude CO2 concentration in Hawaii is impacted by these two events.