

Heterogeneous Susceptibility Infection Rate Assumption Based on Stochastic SIR Model with Simulation

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ABSTRACT This report studies on the disease transmission rate that can cause epidemics that infect at most 10% of population. We use stochastic SIR model to simulate the infection. The result shows higher infection rate may have smaller time interval for the wave, and it implies to the infection may be only 10% of the population.

1 INTRODUCTION

The COVID-19 continuously wreaks havoc with more and more people being infected around the world. People's fears towards this disease has been rigorously high. The demand on some scientific progress has been increasing. This report will study about the infectious disease special characteristics. Some scientists have assumptions that infectious disease may only infect 10% of population. We will use stochastic SIR model to simulate the infectious disease.

2 METHODS

2.1 STOCHASTIC SIR MODEL

The discrete-time Markov Chain SIR model is defined as

$$P(s,i) \rightarrow (s+ds,i+di)(\Delta t) = \begin{cases} (\beta si/N)\Delta t & (ds, di) = (-1, 1) \\ \gamma i \Delta t & (ds, di) = (0, -1) \\ 1 - (\beta si/N)\Delta t - \gamma i \Delta t & (ds, di) = (0, 0) \end{cases}$$

Note that Δt is sufficiently small such that at most one change occurs during a time interval Δt . The continuous-time Markov Chain SIR model is defined as

$$P(s,i) \rightarrow (s+ds,i+di)(\Delta t) = \begin{cases} (\beta si/N)\Delta t + o(\Delta t) & (ds, di) = (-1, 1) \\ \gamma i \Delta t + o(\Delta t) & (ds, di) = (0, -1) \\ 1 - (\beta si/N)\Delta t - \gamma i \Delta t + o(\Delta t) & (ds, di) = (0, 0) \\ o(\Delta t) & \text{otherwise} \end{cases}$$

In above equations, β is the average rate of infection for the content, γ is the removal rate. We have $N = S + I + R$, and S, I, R are all constants. The system of the ODE is written as:

$$\begin{cases} S' = -\beta si/N \\ I' = \beta si/N - \gamma i \\ R' = \gamma i \end{cases}$$

and $o(\Delta t)$ is the offset term for continuous time interval where $o(\Delta t) \ll \Delta t$.

2.2 SIMULATION HETEROGENEOUS SIR MODEL

We use the following heterogeneous SIR model for the simulation:

$$P_i(t) = 1 - \exp\{-[S_i(\sum_{j \in I(t)} \tau) + \epsilon]\} \quad (2.1)$$

where S_i is the susceptibility for person i and higher S_i indicates more susceptible, and we assume there is a constant transmissibility $T_j = \tau$. τ is the transmission parameter, infected people are transmitting disease to others with rate τ . The model has a constant spark term ϵ .

Suppose that

$$S_i \sim \text{Gamma}(1/5, 1/5)$$

and

$$S_i = X_i \beta$$

by creating one column matrix with entries

$$X_{i1} \sim \text{Gamma}$$

and setting $\beta = 1$ and $\beta = 2$ to test the simulation.

3 RESULTS

Based on the stochastic SIR model, we generated simulations of infections over time and cumulative infections.

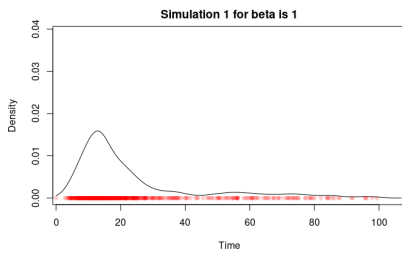


Figure 3.1: Simulation 1 when $\beta = 1$

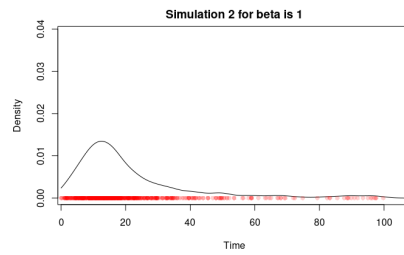


Figure 3.2: Simulation 2 when $\beta = 1$

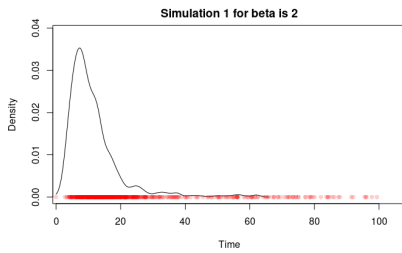


Figure 3.3: Simulation 1 when $\beta = 2$

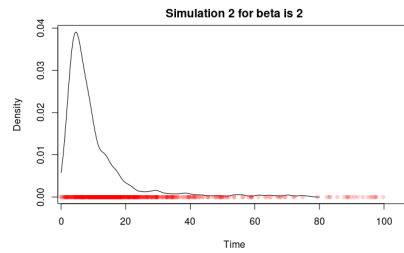


Figure 3.4: Simulation 2 when $\beta = 2$

According to Figure 3.1 and Figure 3.2, when $\beta = 1$ the infection period is longer than when $\beta = 2$ as shown in Figure 3.3 and Figure 3.4. Even though the infection rate is high, the simulation does not show a longer infection period. The graph may seem sharp for $\beta = 2$ but the interval of the wave is half shorter than $\beta = 1$.

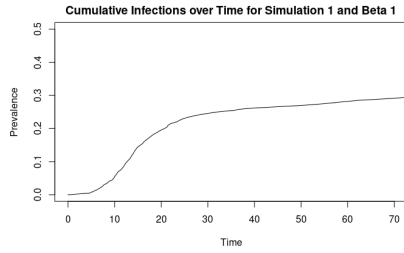


Figure 3.5: Cumulative Infection 1 when $\beta = 1$

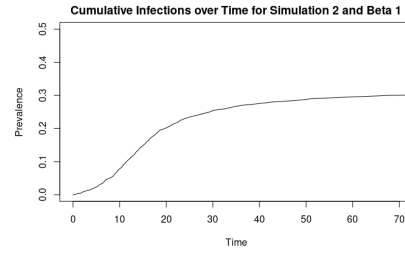


Figure 3.6: Cumulative Infection 2 when $\beta = 1$

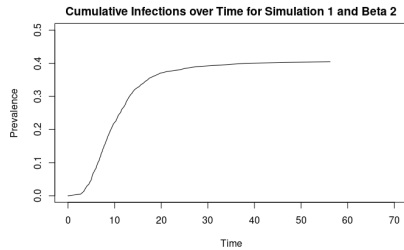


Figure 3.7: Cumulative Infection 1 when $\beta = 2$

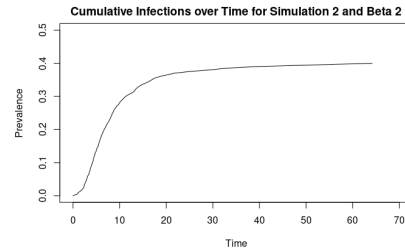


Figure 3.8: Cumulative Infection 2 when $\beta = 2$

As shown in Figure 3.5 to Figure 3.8, we have done the simulation for cumulative infection. Based on these figures, we found that when the infection rate is small, the lasting period for the disease is relatively longer than when the infection rate is large. Figure 3.7 and Figure 3.8 both have a ending point of the disease but the infection rate is 2 while Figure 3.5 and 3.6 show the simulation of when infection rate is 1.

4 CONCLUSION

Simulations of different infection rates have shown a significant result that the total infected population may not exceed 10% since higher infection rate has smaller time interval for the disease transmission. The cumulative infection does not show a significant difference based on total infected population. Thus, we conclude that the infection can cause at most 10% of population.