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The Epidemiological Analysis of Incubation of COVID-19 Based on the MCMC for Stochastic SIR Model

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ABSTRACT With the global pandemic of COVID-19 continuously affect people's daily life, the concern of delay of reporting cases has been a major problem of disease transmission. Using the MCMC to simulate the disease density with no incubation, 2 days and 5 days incubation will generally compare the difference of the effect of delays. It results that delays will increase the density of infection to certain dates, and increase the transmission rate as well as the removal rate.

1 INTRODUCTION

Some may be aware that the wave of COVID-19 may not reflect the real case of infections, that is, there are some infections are not detected in the beginning of the pandemic. This causes a delay of testing procedure, a huge delay may be harmful that the virus can start transmitting without any notice. We will use the method of Markov Chain Monte Carlo to simulate the stochastic SIR model with no delays, 2 days delay and 5 days delay. The simulation of infection can reflect how the curve will behave based on different incubation.

2 METHOD

2.1 STOCHASTIC SIR MODEL

We have the intensity function of $\lambda(t)$ where the intensity of all events, such as Infections or Removals, is $T = \{t_{(1)}...t_{(N)}\}$. The SIR Model is based on the intensity function:

$$\lambda(t) = \beta S(t) I(t) + \gamma I(t) \tag{2.1}$$

where β is the rate of infection, and γ is the rate of removals. We have the probability of events $\pi(D|\theta)$:

$$\pi(D|\theta) = \prod_{i;\delta_{(i)}=1} \beta S(t_{(i-1)}) I(t_{(i-1)}) \prod_{i;\delta_{(i)}=1} \gamma I(t_{(i-1)}) \exp(-\int_0^T \lambda(u)\,du) \tag{2.2}$$

We propose that new infections arrived from outside with rate τ infections per person per space of population density. The first infection is established as

$$P(\text{infection in } [t, t+dt] \approx [\beta I(t) + \tau] S(t) dt \tag{2.3}$$

Before the simulation, we need to identify the prior as the stochastic SIR model relies on the Bayesian inference. Using the parameter we got from the Delta wave of Maharashtra State of India, we can construct the prior for parameters:

$$\frac{1}{\sqrt{\text{shape}}} = \frac{sd(x)}{E(x)} \implies \beta \sim \text{Gamma}(0.002, 0.005^2) \tag{2.4}$$

$$\tau \sim \text{Gamma}(0.01, 0.001) \tag{2.5}$$

The prior for the intensity function and the recovery rate parameter is proposed as

$$\lambda(t_{(i)}) \sim \exp(1)$$

$$\gamma \sim \exp(1)$$

The prior for incubation is proposed as following

$$\beta \sim \text{Gamma}(0.002, 0.007^2) \tag{2.6}$$

$$\tau \sim \text{Gamma}(0.01, 0.02^2) \tag{2.7}$$

The incubation periods ρ_i^{INC} has the prior of

$$\rho_i^{INC} \sim \exp(\gamma^{INC})$$

and the delay periods ρ_i^{DEL} has the prior of

$$\rho_i^{DEL} \sim \exp(\gamma^{DEL})$$

Now we can do the simulation with the transmission rate of 0.001765, and the recovery rate of 0.0967 with the spark term of 0.001. The spark term indicates that for 1000 people, there will be 1 person of spontaneous infection.

3 Results

Doing 500 times of simulation will generate the output as shown in Figure 3.1 and Figure 3.2. Red dots represent that someone infected at this time period and we can see the left tail and right tail of infection periods are relatively equally distributed.

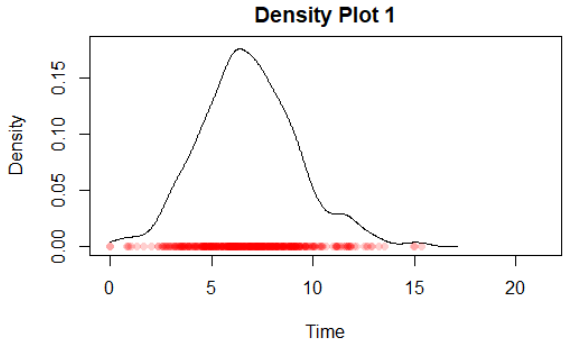

Figure 3.1: Simulation 1 Without Incubation

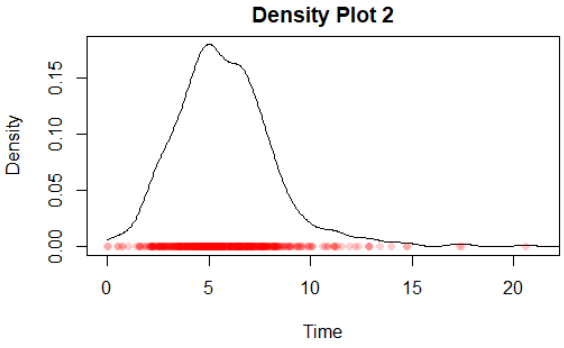

Figure 3.2: Simulation 2 Without Incubation

The trace plot of the simulation is shown in Figure 3.3 and Figure 3.4 where the trace behaves well.

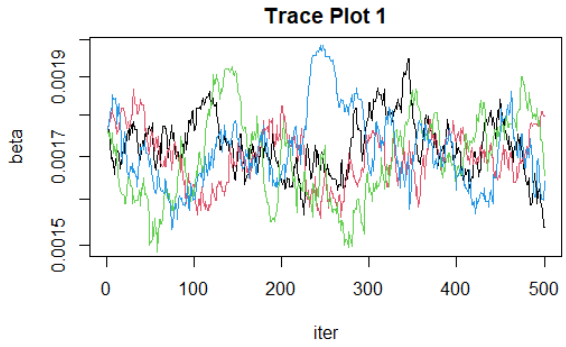

Figure 3.3: Infection MCMC Without Incubation

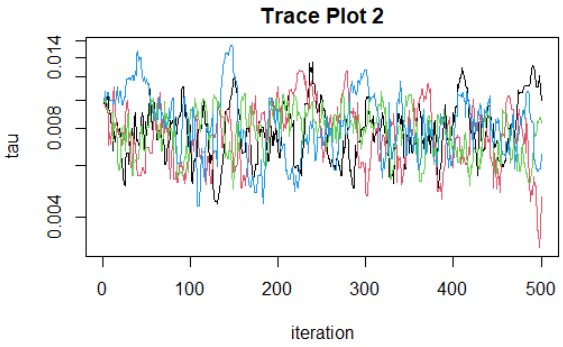

Figure 3.4: Spark MCMC Without Incubation

The acceptance rate for the simulation of no incubation is practicing normally as well as shown in Table 3.1.

Table 3.1: Acceptance Rate of Simulation Without Incubation

	Infection	Spark
Chain 1	0.8918	0.7916
Chain 2	0.9098	0.8016
Chain 3	0.8978	0.7976
Chain 4	0.8898	0.7715

Moreover, the simulation results of the credible interval of β and τ are shown in the Figure 3.5 and Figure 3.6. The infectious rate is between around 0.0016 to 0.0019. The removal rate is between around 0.004 to around 0.012.

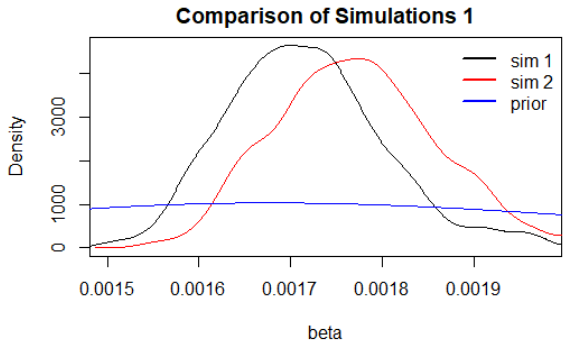

Figure 3.5: Infection Rate Credible Interval

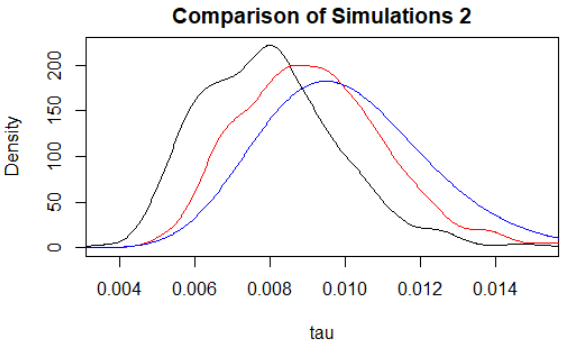

Figure 3.6: Spark Term Credible Interval

Now, we are interested how the incubation will affect the infection curve. By doing so, we will compare the original simulation and simulations with incubation of 2 days and 5 days.
The infection with incubation will be more dense on certain dates, and there will be no more or much less infections before or after certain dates. While the incubation periods increase, the effect will be stronger as shown in Figure 3.7 to Figure 3.10.

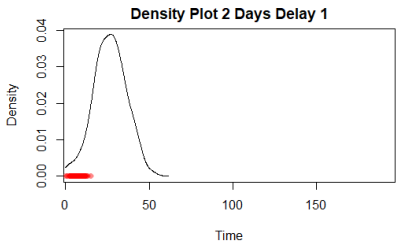

Figure 3.7: 2 Days Incubation Density 1

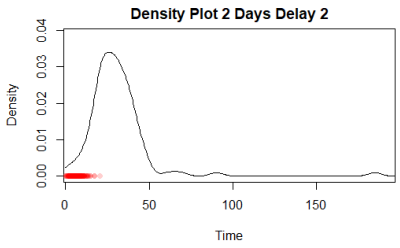

Figure 3.8: 2 Days Incubation Density 2

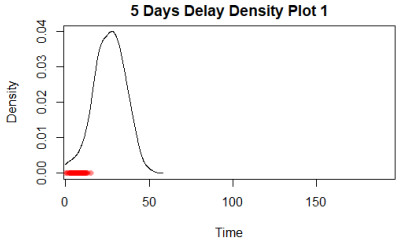

Figure 3.9: 5 Days Incubation Density 1

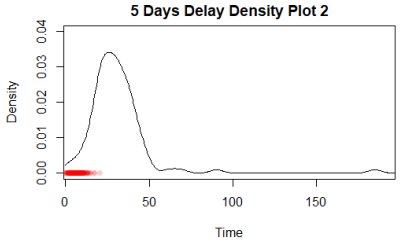

Figure 3.10: 5 Days Incubation Density 2

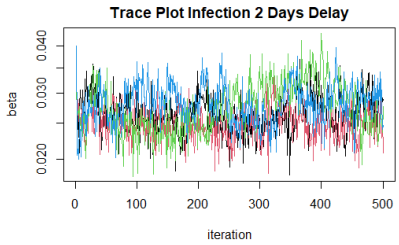

Figure 3.11: 2 Days Incubation Infection Trace

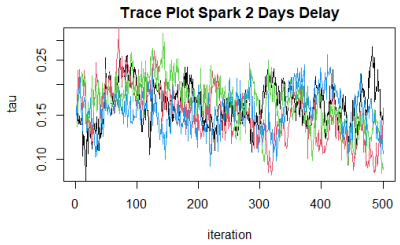

Figure 3.12: 2 Days Incubation Spark Trace

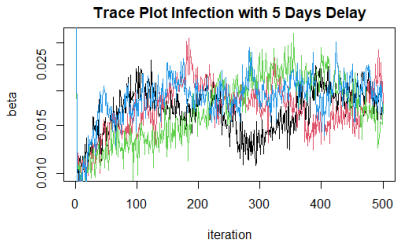

Figure 3.13: 5 Days Incubation Infection Trace

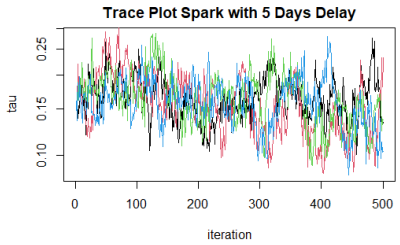

Figure 3.14: 5 Days Incubation Spark Trace

The trace plot of infection and spark shows the MCMC behavior, which seems pretty good as it shows a fuzzy trend. The acceptance rate for the simulation of 2 days incubation and 5 days incubation are considered well shown in Table 3.2.
The credible intervals for β and τ of incubation periods applied are shown in Figure 3.15 to Figure 3.18.

The higher incubation seems to have higher infection parameters, and higher removal parameters. The graph shows the τ for 5 days incubation is slightly higher than 2 days incubation. Both of the circumstances of incubation have much higher infection and removal rate.

4 Conclusion

According to the simulation results of no delay, 2 days delay, and 5 days delay based on the data of Delta wave in Maharashtra State, we conclude that higher periods of incubation or delay will result a

Table 3.2: Acceptance Rate of Simulation With Incubation

	Infection (2 Days)	Spark (2 Days)	Infection (5 Days)	Spark (5 Days)
Chain 1	0.8756	0.8978	0.8076	0.8176
Chain 2	0.8617	0.9038	0.7916	0.8417
Chain 3	0.8818	0.9138	0.8236	0.8277
Chain 4	0.8517	0.8858	0.8016	0.7635

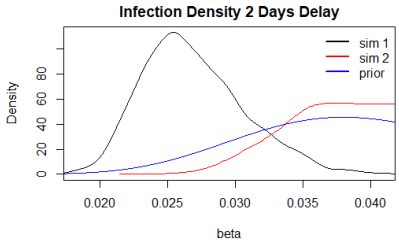

Figure 3.15: 2 Days Incubation Infection CI

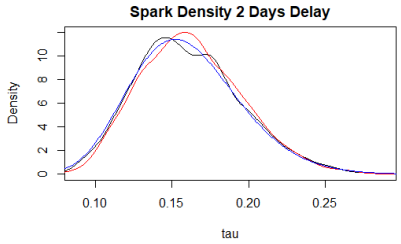

Figure 3.16: 2 Days Incubation Spark CI

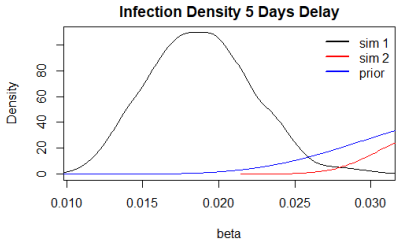

Figure 3.17: 5 Days Incubation Infection CI

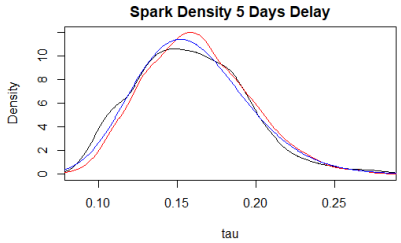

Figure 3.18: 5 Days Incubation Spark CI

rapid increases of infection. This may affect the hospitalization that healthcare system may not be capable when there is a high volume of infections. More delays will result a higher rate of infection and removal, while the removal rate is not significant when the incubation periods increase but it is significant comparing to the simulation without delay periods.

5 Limitation and Discussion

Due to the software limitations we can only do the simulation for 500 iterations with less population proposed. There are differences between different population densities. Thus, the results may not be helpful for all regions. For future studies, there may be a better solution to combine the MCMC results with the time series analysis.