# Assessing Bridge Condition in New York State Using Multiple Linear Regression\*

Age, Municipality, and Ownership as Factors Influencing Bridge Condition Ratings

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# **Table of contents**

| T | intro | Dauction                          | • |
|---|-------|-----------------------------------|---|
| 2 | Dat   | a 2                               |   |
|   | 2.1   | Overview                          | - |
|   | 2.2   | Measurement                       |   |
|   | 2.3   | Outcome Variable                  |   |
|   |       | 2.3.1 Condition                   |   |
|   | 2.4   | Predictor variables               |   |
|   |       | 2.4.1 Age of Bridge at Inspection |   |
|   |       | 2.4.2 Bridge Located Municipality |   |
|   |       | 2.4.3 Ownership of Bridge         |   |
| 3 | Mod   | lel 9                             |   |
|   | 3.1   | Model Setup                       |   |
|   | 3.2   | Model Justification               |   |
|   |       | 3.2.1 Variable Choice             |   |
|   |       | 3.2.2 Model Choice                |   |
|   |       | 3.2.3 Limitations                 |   |
|   | 3.3   | Alternative model                 |   |

<sup>\*</sup>Code and data are available at: https://github.com/Jiaqi-Xing/NYS\_Bridge\_Condition\_Analysis.

| 4  | Results                                  | 13       |
|----|--|----------|
| 5  | Discussion  5.1 First discussion point   | 14<br>14 |
| Αŗ | Appendix                                 | 15       |
| Α  | A Additional data details                | 15       |
| В  | B.1 Linearity and Homoscedasticity Check |          |
| Re | References                               | 18       |

# 1 Introduction

Overview paragraph

Estimand paragraph

Results paragraph

Why it matters paragraph

Telegraphing paragraph: The remainder of this paper is structured as follows. Section 2....

# 2 Data

#### 2.1 Overview

The dataset used in this analysis, titled "New\_York\_Bridges\_2016", was accessed from the Data And Story Library (DASL) (Data And Story Library (DASL) 2024), which archives historical data originally compiled by the New York State Open Data platform (New York State Open Data 2020). The original dataset includes contributions from county-level data provided by the New York State Department of Transportation (NYSDOT) (New York State Department of Transportation (NYSDOT) 2016).

This dataset was chosen over the updated 2020 version from New York State Open Data due to the change in how bridge condition data is recorded. In the 2016 dataset, the condition

variable is a continuous variable measured on a seven-point scale (1 = "Deficient Condition" to 7 = "Good Condition"). However, in the 2020 dataset, NYSDOT replaced this scale with a binary Poor/OK variable, recorded as Poor01 (1 = Poor, 0 = OK). This change limits the analytical potential of the dataset, as the binary variable lacks the granularity required for linear regression that relies on continuous variables to detect nuanced trends or incremental differences. By using the 2016 dataset, this study retains the detailed numerical condition ratings, enabling to analysis impact of factors such as age, location, and owners on bridge conditions.

The raw dataset contains 17,502 rows and 13 columns, providing structural, geographical, and condition-related data on bridges across New York State. After data cleaning, four key variables were selected for analysis: the response variable "Condition" and three predictor variables, "AgeAtInspection", "Located\_Municipality", and "Owner\_Group". The variables "Condition" and "AgeAtInspection" are numeric and were directly retained from the raw data. The categorical variables "Located\_Municipality" and "Owner\_Group" were newly constructed by categorizing the original variables "Municipality" and "Owner", respectively.

To conduct the analysis for this paper, we utilized the statistical programming language R (R Core Team 2023). The workflow was supported by several packages, including tidyverse (Wickham et al. 2019) for data cleaning and manipulation, arrow (Richardson et al. 2023) for efficient data storage and retrieval, validate (van der Loo and de Jonge 2021) for data testing and validation, knitr(Xie 2023) for creating tables and ggplot2 (Wickham 2016) and broom(Robinson, Hayes, and Couch 2023) for creating visualizations.

#### 2.2 Measurement

In New York State, bridge inspectors from the State Department of Transportation (NYSDOT) conduct evaluations of non-toll bridges every two years to ensure their safety and structural integrity. Their assessments are performed on a span-by-span basis, meaning that each section of the bridge between its supporting points is inspected individually. For every bridge, inspectors examine up to 47 structural elements, including 25 span-specific components such as the deck, beams, and supports. Each component is assigned a condition score, ranging from 1 (severely deficient) to 7 (new or excellent condition), which reflects its structural health at the time of inspection. Inspectors also note any defects, such as cracks, corrosion, or wear, and document associated quantities, such as the length, area, or number of affected elements, to capture the extent of the damage.

In addition to these detailed evaluations, inspectors assign federal ratings based on the overall average condition of a bridge's major components. These federal ratings correspond to the values of the **Condition** variable used in our paper and analysis. Bridges with condition scores below 5 are classified as being in "poor" condition and are further categorized as either Structurally Deficient (SD) or Functionally Obsolete (FO). All findings from these assessments are

recorded in inspection reports, providing essential data to guide maintenance, rehabilitation, or replacement efforts.(New York State Department of Transportation (NYSDOT) 2024)

#### 2.3 Outcome Variable

#### 2.3.1 Condition

The **Condition** variable reflects the overall state of a bridge, encompassing both its structural integrity and functional adequacy as determined during inspections. It is measured on a continuous scale from 1 to 7, with higher scores indicating better condition. Bridges with condition scores above 5 are considered to be in good standing, while scores below 5 indicate that a bridge is classified as either Structurally Deficient (SD) or Functionally Obsolete (FO).

Table 1: Summary Statistics of NYS Bridge Condition Ratings

| Min  | Max | Mean | Median | Standard Deviation | Count |
|------|-----|------|--------|--------------------|-------|
| 1.93 | 7   | 5.38 | 5.3    | 0.84               | 15855 |

Table 1 presents for the condition ratings of bridges in New York State, based on a clenaed dataset of 15,855 observations. The mean condition rating of 5.38 indicates that, on average, bridges are in fair to good condition. The median rating of 5.30 suggests that half of the bridges have condition ratings below or equal to 5.30. The close alignment of the mean and median reflects a relatively symmetrical distribution. While the minimum condition rating is 1.93, and the maximum is 7, the standard deviation of 0.83 suggests that most bridges cluster around the mean, with only a few outliers at the extreme ends of the scale. These statistics indicate that the majority of bridges are maintained at a stable and adequate level, with relatively few in very poor or excellent condition.

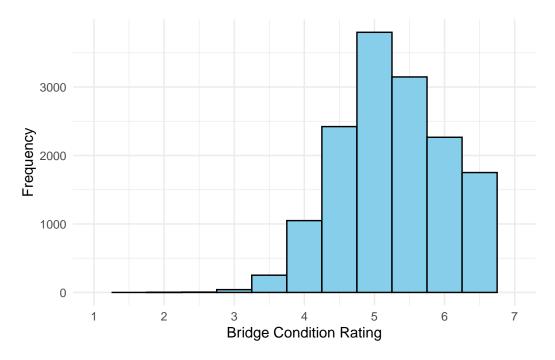


Figure 1: Distribution of Bridge Condition Ratings

Figure 1 The histogram displays the distribution of bridge condition ratings in New York State. The majority of bridges are clustered around the middle to upper end of the condition scale, with relatively few bridges at either the lower or higher ends. The left tail, representing bridges in severely poor condition, is thin, indicating that such bridges are uncommon. Similarly, the right tail, corresponding to bridges in excellent condition, is sparse, suggesting that high-quality, newly constructed bridges are not predominant. The distribution highlights a concentration of bridges in moderate condition, with a gradual decrease in frequency toward both ends, reflecting the natural aging process and variability in maintenance practices across the state.

## 2.4 Predictor variables

## 2.4.1 Age of Bridge at Inspection

The **AgeAtInspection** variable is a continuous measure that represents the age of a bridge in years. It is calculated as the time elapsed from the year the bridge was originally built or entirely replaced to its most recent inspection date.

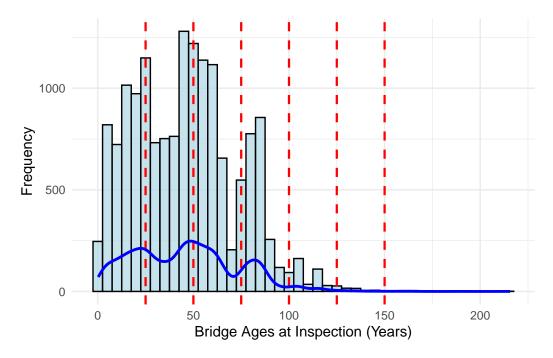


Figure 2: Distribution of Bridge Ages at the Time of Inspection. The red dashed lines mark intervals of 25 years and the blue density curve provides a smooth visualization of the distribution

Figure 2 shows distribution of bridge ages at the time of inspection, overlaid with red dashed lines and a blue density curve. The red dashed lines mark intervals of 25 years and align with clear peaks in the histogram, suggesting periodic patterns in bridge construction or major renovations. Prominent clusters are observed around 25, 50, and 75 years of age, indicating that many bridges were likely built or reconstructed during these timeframes. The blue density curve provides a smooth visualization of the distribution, confirming the periodic modes while highlighting the overall trends. Beyond 100 years, the frequency drops significantly, with very few bridges older than this still in operation, reflecting the typical lifecycle of infrastructure. The concentration of bridges between 20 and 80 years old suggests that a significant portion of bridges are mid-life and may soon require maintenance or replacement. It highlights both the historical patterns of bridge construction and the current age distribution of bridges in the dataset.

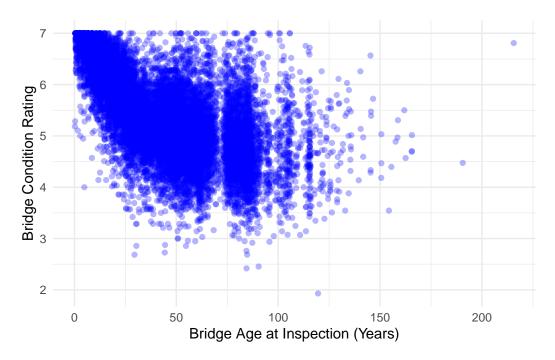


Figure 3: Relationship Between Bridge Age at Inspection and Bridge Condition Rating. Each blue dot represents a bridge, showing its age at the time of inspection and the corresponding condition rating obtained from that inspection.

Figure 3 illustrates the relationship between bridge age at inspection and their condition ratings. Most points are concentrated in the upper-left quadrant, showing that younger bridges tend to have higher condition ratings (close to 7). As bridge age increases, condition ratings generally decline, reflecting the expected deterioration over time. However, beyond approximately 80 to 100 years, the relationship becomes less defined, with condition ratings displaying a wide range. This variability may be influenced by differences in maintenance practices, varying levels of transportation load, or disparities in construction quality among older bridges. Overall, the pattern indicates that bridge age is a key factor in determining condition, particularly for younger and midlife bridges.

#### 2.4.2 Bridge Located Municipality

The **Located\_Municipality** variable categorizes bridges based on the type of municipality where they are located. The three categories are City, Town, and Village.

Table 2: Proportion of Bridges by Municipality. The first column lists the type of municipality where the bridges are located, including City, Town, and Village, while the second column shows the proportion of bridges in each category.

| Bridge Located Municipality | Percentage Account |
|-----------------------------|--------------------|
| City                        | 9.15%              |
| Town                        | 82.11%             |
| Village                     | 8.75%              |

Table 2 shows the distribution of bridges across different municipality types. Towns account for the majority of the bridges, representing 82.11% of the dataset, while cities and villages make up much smaller proportions, at 9.15% and 8.75%, respectively. This indicates that most bridges are located in towns, likely due to their larger geographic coverage.

Table 3: Average Bridge Condition Rating by Municipality. The first column lists the type of municipality where the bridges are located, including City, Town, and Village, and the second column provides the average condition rating for bridges in each category.

| Bridge Located Municipality | Average Condition Rating |
|-----------------------------|--------------------------|
| City                        | 5.16                     |
| Town                        | 5.41                     |
| Village                     | 5.33                     |

Table 3 presents the average condition ratings of bridges by municipality type. Bridges in towns have the highest average condition rating of 5.41, followed by villages at 5.33 and cities at 5.16. The variation in average condition ratings may reflect differences in maintenance practices, traffic loads, or resource allocation among the municipality types.

#### 2.4.3 Ownership of Bridge

The Owner\_Group variable categorizes bridges based on their ownership. This variable is designed to estimate potential differences in bridge conditions that may arise from varying ownership responsibilities. The categories include Municipalities, NYSDOT, and Other. The Other category combines bridges owned by private entities, authorities, commissions, and similar groups, as each accounts for a small portion of the data, making it practical to group them together

Table 4: Proportion of Bridges by Ownership. The first column lists the type of bridge ownership, including NYSDOT, Municipality, and Other (private entities, authorities, commissions), while the second column shows the proportion of bridges in each category.

| Ownership of Bridge | Percentage Account |
|---------------------|--------------------|
| Municipalities      | 49.83%             |
| NYSDOT              | 43.05%             |
| Other               | 7.13%              |

Table 4 summarizes the ownership distribution of bridges. The two largest ownership groups are municipalities and NYSDOT, which account for 49.83% and 43.05% of the dataset, respectively. The remaining 7.13% of bridges fall under the "Other" category, which includes private, authority, and commission ownerships. The high percentage of bridges owned by municipalities and NYSDOT reflects their dominant role in managing public infrastructure.

Table 5: Average Bridge Condition Rating by Ownership. The first column lists the type of bridge ownership, including NYSDOT, Municipality, and Other (private entities, authorities, commissions), and the second column provides the average condition rating for bridges in each category.

| Ownership of Bridge         | Average Condition Rating |
|-----------------------------|--------------------------|
| Municipalities NYSDOT Other | 5.45<br>5.33<br>5.10     |

Table 5 shows the average condition ratings of bridges grouped by ownership. Bridges owned by municipalities have the highest average condition rating of 5.45, while NYSDOT-owned bridges have a slightly lower rating of 5.33. Bridges in the "Other" ownership category have the lowest average condition rating of 5.10, which may reflect less consistent maintenance or differing priorities among private or non-government entities.

## 3 Model

The goal of our modeling strategy is examine the relationship between **bridge condition** ratings(condition) and key predictors, including the type of municipality where the bridge is located (Located\_Municipality), the age of the bridge at the time of inspection (AgeAtInspection), and the ownership of the bridge (Owner\_Group). By constructing a multiple linear regression model, we aim to quantify the impact of these factors

on changes in bridge conditions. This analysis will provide a foundation for estimating the time and budget required for future bridge repair projects, supporting effective infrastructure maintenance planning.

Here, we present a multiple linear regression model, explaining its setup, assumptions, and justification. Details of model diagnostics and validation can be found in **?@sec-model-details**.

## 3.1 Model Setup

The Multiple linear regression model, **Bridge Condition model**, is expressed as:

$$\begin{aligned} \text{Condition}_i &= \beta_0 + \beta_1 \cdot \text{Age}_i + \beta_2 \cdot I(\text{Municipality} = \text{City}) + \beta_3 \cdot I(\text{Municipality} = \text{Village}) \\ &+ \beta_4 \cdot I(\text{Owner} = \text{NYSDOT}) + \beta_5 \cdot I(\text{Owner} = \text{Other}) + \epsilon_i \end{aligned}$$

where:

- Condition<sub>i</sub>: Condition rating of bridge i (response variable).
- Age: Age of bridge i at the time of inspection.
- I(Municipality = City): Indicator variable for bridges located in a City.
- I(Municipality = Village): Indicator variable for bridges located in a Village.
  - Reference category: **Town**.
- I(Owner = NYSDOT): Indicator variable for bridges owned by **NYSDOT**.
- I(Owner = Other): Indicator variable for bridges owned by **Other entities**.
  - Reference category: Municipalities.
- $\beta_0$ : Intercept, representing the average condition rating for a Town-owned bridge with zero age.
- $\beta_1$ : Coefficient for the effect of Age on condition rating.
- $\beta_2, \beta_3$ : Coefficients for the effect of municipality type (City or Village, relative to Town).
- $\beta_4, \beta_5$ : Coefficients for the effect of ownership type (NYSDOT or Other, relative to Municipality).
- $\epsilon_i$ : Error term, capturing unexplained variation.

The model assumes that the relationship between predictors and the response variable is linear, and that the residuals are normally distributed with constant variance.

We implemented the multiple linear regression model in R using the lm() function, which estimates coefficients through ordinary least squares.

#### 3.2 Model Justification

#### 3.2.1 Variable Choice

The selection of predictors in the model reflects key aspects discussed in the data section, ensuring a comprehensive understanding of bridge condition ratings. Age at Inspection was included as a continuous variable because Figure 3 demonstrated a clear relationship between bridge age and condition ratings. Using age as a continuous predictor avoids the loss of information that would occur if age were categorized into groups, allowing the model to capture subtle variations in condition ratings as bridges age. Additionally, this choice aligns with the dataset's structure, where age is measured on a continuous scale.

Municipality Type (City, Town, Village) was treated as a categorical variable with Town as the reference category. The data revealed that condition ratings vary systematically between municipality types, likely due to differences in traffic loads, maintenance resources, and environmental stressors. Modeling Municipality Type as levels rather than using dummy variables for individual municipalities ensures clarity and interpretability, highlighting differences between City, Town, and Village bridges.

Ownership Type (Municipalities, NYSDOT, Other) was similarly treated as a categorical variable, allowing the model to reflect systematic differences in bridge conditions based on ownership responsibilities. The inclusion of ownership accounts for varying maintenance schedules and resource allocation. The reference category (Municipalities) was chosen because it comprises the largest proportion of bridges in the dataset, providing a stable baseline for comparison.

Together, these features were selected to balance the need for interpretability and explanatory power while reflecting meaningful patterns observed in the data section.

#### 3.2.2 Model Choice

A multiple linear regression model was chosen for its simplicity, interpretability, and suitability for quantifying the influence of individual predictors while controlling for others. This model effectively supports the research goal of understanding how Age, Municipality Type, and Ownership Type affect bridge condition ratings.

The model assumes a linear relationship between the predictors and the response variable (Condition Rating). This assumption is reasonable given observed trends, including the clear negative association between Age and Condition, and systematic differences between categories of Municipality and Ownership. While this assumption aligns with the data, it may not fully capture non-linear relationships or interactions between predictors. Nevertheless, the simplicity of linear regression is appropriate for this study's goals, balancing clarity with analytical rigor.

Other key assumptions include Normality of Residuals and homoscedasticity (constant variance of residuals). Diagnostic tools were used to validate these assumptions, with details provided in Appendix B.

#### 3.2.3 Limitations

Several limitations must be acknowledged. Due to data constraints, the model excludes potentially influential factors such as transport load, weather conditions, and environmental influences, introducing the risk of omitted variable bias. This limitation reduces the explanatory power of the model and may skew the estimated effects of included predictors. Additionally, the model does not account for interaction effects, such as how age and municipality might jointly influence condition ratings. Incorporating such interactions could enhance the model's explanatory ability but would increase its complexity. Finally, the linearity assumption, while reasonable, may oversimplify more nuanced relationships in the data.

Despite these limitations, the model's moderate complexity ensures it is neither overly simplistic nor unnecessarily complicated, making it appropriate for addressing the research question.

#### 3.3 Alternative model

In addition to the multiple linear regression model, we considered a generalized additive model (GAM), which unique strengths but were ultimately deemed less suitable for this analysis compared to multiple linear regression.

A GAM could have been used to capture potential non-linear relationships between Age and Condition Rating. This flexibility is particularly beneficial when predictors exhibit non-linear effects, which might better reflect the natural deterioration of bridges over time. However, while GAMs provide excellent predictive performance, they sacrifice interpretability, as the model outputs are more challenging to communicate to non-technical audiences. Given the project's emphasis on explaining the relationship between predictors and condition ratings in a transparent manner, this complexity outweighed the benefits.

# 4 Results

The linear regression model predicting bridge condition ratings estimates how factors the bridge age at inspection, the located municipality and the ownership influence the condition of bridges. The model result is shown in Table 6

Table 6: Summary Table of Bridge Condition Model

| Variables              | Coefficients |
|------------------------|--------------|
| Intercept              | 6.15         |
| Age                    | -0.02        |
| Municipality = Town    | 0.17         |
| Municipality = Village | 0.19         |
| Owner = NYSDOT         | -0.09        |
| Owner = Other          | -0.16        |

The intercept, estimated at 6.15, represents the baseline condition rating for a bridge located in a city, owned by the municipality, and with an age of zero years. This is a useful reference point against which the effects of age, municipality, and ownership are compared.

The coefficient for Age is -0.02, indicating that for each additional year a bridge ages, its condition rating decreases by 0.02 points on average. This result reflects the natural deterioration of infrastructure over time, as older bridges are more likely to experience wear and tear. Figure 3 presented in the Data section illustrates this trend, showing a clear negative relationship between bridge age and condition. This visual evidence aligns with the regression model, where the Age coefficient (-0.02) confirms this downward trend. Although the effect of age on condition appears small on an annual basis, the cumulative impact over several decades can significantly compromise a bridge's integrity, highlights the importance of regular inspections and maintenance programs to mitigate the impact of aging on bridge conditions.

The location of the bridge, represented by the municipality variable, also plays a significant role in condition ratings. Bridges located in towns and villages have higher condition ratings compared to those in cities, as indicated by the positive coefficients for Municipality = Town (0.17) and Municipality = Village (0.19). These findings suggest that urban bridges are more prone to deterioration, possibly due to heavier traffic loads or more complex structural demands. In contrast, bridges in towns and villages may face fewer stressors, resulting in better overall condition ratings.

Ownership of the bridge is another significant factor affecting its condition. The model uses Municipality-owned bridges as the reference category. The results show that bridges owned by Municipality have the highest condition rating, NYSDOT has condition ratings that are 0.09 points lower, while those owned by Other entities have ratings 0.16 points lower, on average. These differences may reflect variations in maintenance practices, funding availability,

or operational priorities across ownership groups. Bridges owned by municipalities might benefit from closer proximity to local resources and management, whereas NYSDOT and other entities may face challenges such as broader jurisdictional responsibilities or limited funding for specific assets.

Taken together, these results highlight how a combination of aging, geographic context, and administrative oversight influences bridge condition ratings. Bridges in cities or under the management of entities other than municipalities tend to have lower ratings, while those in towns, villages, or managed by municipalities are generally better maintained. This underscores the importance of tailoring maintenance strategies to address the specific challenges faced by different ownership groups and geographic locations. Future studies could enhance this analysis by exploring interactions between municipality and ownership or incorporating additional variables such as traffic volume, funding levels, or structural design characteristics to provide a deeper understanding of the factors driving bridge condition outcomes.

# 5 Discussion

# 5.1 First discussion point

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# 5.2 Second discussion point

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## 5.3 Third discussion point

# 5.4 Weaknesses and next steps

Weaknesses and next steps should also be included.

# **Appendix**

## A Additional data details

# **B Model Diagnostics**

# **B.1 Linearity and Homoscedasticity Check**

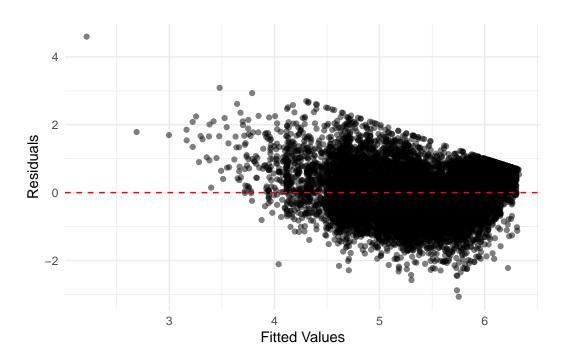


Figure 4: Residuals vs. Fitted Values for Bridge Condition rating Model. The residuals (differences between observed and predicted bridge condition ratings) are on the y-axis and the fitted values (predicted condition ratings from the model) are on the x-axis. Each dot represents a bridge, showing the magnitude and direction of its residual. The red dashed line indicates the zero residual line, representing where residuals would lie if the predictions were perfect.

Figure 4 provides evidence supporting the assumptions of linearity and homoscedasticity for the model. First, the random scatter of residuals around the horizontal line at zero indicates no systematic patterns, suggesting that the relationship between the predictors and the response variable is approximately linear. This random distribution implies that the model captures the linear relationships effectively without significant curvature or deviations.

Second, while the upper residuals show a declining trend due to the bounded nature of the response variable (maximum rating of 7), the lower residuals and the majority of residuals across the range of fitted values exhibit relatively constant variance. For bridges with lower condition ratings (fitted values below approximately 4), residuals display more variability and are predominantly negative, indicating that the model overpredicts for poorly maintained bridges. Despite these deviations, the overall spread of residuals remains consistent for the majority of the data, suggesting that the assumption of homoscedasticity is largely satisfied. Therefore, the model is deemed appropriate for analyzing the dataset, though its limitations in predicting low-condition bridges should be noted.

## **B.2 Nomality of Residuals**

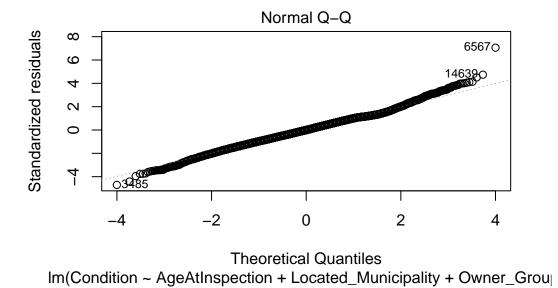


Figure 5: Q-Q Plot of Standardized Residuals for Bridge Condition Model. The x-axis represents the theoretical quantiles from a standard normal distribution, and the y-axis represents the standardized residuals of the Bridge Condition model. Each dot represents a bridge, showing how its residual compares to the expected normal distribution. The black dotted line represents the theoretical relationship the residuals would follow if they were perfectly normally distribute.

Figure 5 indicates that the residuals of the model generally align with the theoretical quantiles of a normal distribution, as most points lie close to the reference line. While there is slight deviation in the extreme tails, this is not unusual for larger datasets and does not significantly

impact the overall assumption of normality. The observed residual pattern supports the suitability of the model for the data, as minor deviations in the tails are unlikely to undermine the reliability of hypothesis testing or predictions. Therefore, the model can be considered appropriate for analysis.

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