Dynamic modal characterization of musical instruments using digital holography

Nazif Demoli and Ivan Demoli

Institute of Physics, Bijenicka c. 46, PO Box 304, 10001 Zagreb, Croatia
demoli@ifs.hr

Abstract: This study shows that a dynamic modal characterization of musical instruments with membrane can be carried out using a low-cost device and that the obtained very informative results can be presented as a movie. The proposed device is based on a digital holography technique using the quasi-Fourier configuration and time-average principle. Its practical realization with a commercial digital camera and large plane mirrors allows relatively simple analyzing of big vibration surfaces. The experimental measurements given for a percussion instrument are supported by the mathematical formulation of the problem.

OCIS codes: (090.1760) Computer holography; (090.2880) Holographic interferometry

References and links

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1. Introduction

Dynamic modal characteristics (DMCs) of a musical instrument with membrane are defined by its physical properties such as: (i) the resonator structure, (ii) the membrane size, shape, and material, (iii) the membrane tension, mass density, and propagation velocity, and particularly (iv) the membrane inhomogeneities. For an ideal membrane system, these characteristics can be studied applying numerical modeling, but for a real musical instrument they must be measured experimentally. The DMCs reveal the sound quality of a particular instrument and may have considerable influence on the instrument design.

The holography based techniques such as the time-averaged holographic interferometry, classical [1] and digital [2], and the electronic speckle pattern interferometry [3], are

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recognized as particularly well suited for studying vibrating surfaces of musical instruments [4], since they are full-field, noncontact, and possess adequate sensitivity. Although highly utilized (for example: on the string instruments [5, 6], percussion instruments [7], or wind instruments [8]), these classical techniques are time and means consuming since they use either complicated procedures or expensive equipment. We propose the use of a digital holography technique by which we avoid wet processing and other complications, retaining at the same time the measuring simplicity and the processing possibilities inherent to handling with numerical data. Drawbecks of digital holography are related to the recording conditions of low numerical aperture which leads to large speckle noise and overlapping of zero- and first-order reconstruction terms. These conditions limit the size of investigated objects. An effective solution to overcome these difficulties named subtraction digital holography was reported [9] and demonstrated through several applications [10, 11]. We applied that technique in this work to achieve clean reconstructions of digital holograms.

The aim of this study was to present the DMCs of a membrane system as a movie showing changes of vibration patterns in a given frequency range. The movie should contain information about the measured resonant frequencies, the corresponding modal structure, the transient vibration structure, the phase shifts, and regularity of the vibration patterns. Any real membrane system would thus be possible to characterize uniquely. We use two approaches, theoretical, to numerically describe an ideal membrane system and, experimental, to measure the DMCs of a real instrument. For experimental measurements we constructed a low-cost device based on digital holography technique with compacted architecture and equipped with commercial elements.

Section 2 contains theoretical work including description of the quasi-Fourier digital holography technique as well as the calculation of the natural frequencies and DMCs for circular membranes. Numerical and experimental results for a percussion instrument are presented in section 3, while the conclusions are drawn in section 4.

2. Theory

2.1 Time-averaged quasi-Fourier digital holography

Digital holography is a holographic technique based on optical hologram recording by an array photodetector and numerical reconstruction. In quasi-Fourier setup, both a reference point source and an object are placed at the input plane P_1 [coordinates (x_1, y_1)], while the detector is placed at the hologram plane P_2 [coordinates (x_2, y_2)]. For vibration study, we describe input as

$$U(x_1, y_1, t) = \delta(x_1 - X, y_1 - Y) + s(x_1, y_1, t), \tag{1}$$

where (X, Y) is the position of the point source. We assume that only out-of-plane harmonic vibrations of the object occur and that the illumination is normal to the object surface. Then, the object wave front is in the form: $s(x_1, y_1, t) = s(x_1, y_1)\exp\left[i(4\pi/\lambda)h(x_1, y_1)\sin 2\pi ft\right]$, where $s(x_1, y_1)$ denotes the static wave front, $h(x_1, y_1)$ the vibration amplitude (the maximum deviation from the surface equilibrium), λ the wavelength, and f the vibration frequency. The diffracted field at the plane P_2 distanced d from the input plane P_1 can be calculated according to Fresnel approximation as

$$U(x_2, y_2, t) \propto \exp\left[i(\pi/\lambda d)\left(x_2^2 + y_2^2\right)\right] F\left\{U(x_1, y_1, t)\exp\left[i(\pi/\lambda d)\left(x_1^2 + y_1^2\right)\right]\right\}, \tag{2}$$

where F denotes the Fourier transform operation and the constant terms are omitted. Thus,

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$$U(x_2,y_2,t)\propto \exp\left\{i(\pi/\lambda d)\left[(x_2-X)^2+(y_2-Y)^2\right]\right\}+\exp\left[i(\pi/\lambda d)\left(x_2^2+y_2^2\right)\right]$$
$$\times F\left\{s(x_1,y_1)\exp\left[i(4\pi/\lambda)h(x_1,y_1)\sin 2\pi ft+i(\pi/\lambda d)\left(x_1^2+y_1^2\right)\right]\right\}. \qquad (3)$$

The exposure recorded by the detector can be described as an integral over the time τ: $E(x_2,y_2)=\int_0^\tau \left|U(x_2,y_2,t)\right|^2 dt$. From the condition $\tau \gg f$ one of additive terms becomes

$$E_1(x_2,y_2)\propto \exp\left[-i(2\pi/\lambda d)(x_2X+y_2Y)\right]$$
$$\times F\left\{s(x_1,y_1)J_0\left[(4\pi/\lambda)h(x_1,y_1)\right]\exp\left[i(\pi/\lambda d)\left(x_1^2+y_1^2\right)\right]\right\} \qquad (4)$$

where J_0 is the zero-order Bessel function of the first kind. Modulus of the inverse Fourier transform of relation (4) yields the reconstruction

$$\left|U'(x_1,y_1)\right|\propto \left|s(x_1-X,y_1-Y)\right|\left|J_0\left[(4\pi/\lambda)h(x_1-X,y_1-Y)\right]\right|. \qquad (5)$$

Relation (5) shows that the object reconstruction is modulated by the fringes depending on the vibration amplitude $h(x_1,y_1)$. The mathematical interpretation of fringes is that each fringe represents a line of constant displacement. The positions with zero amplitude define nodes and the positions of local maxima are antinodes.

2.2 Circular membrane: natural motion

The motion of a perfectly compliant circular membrane fixed at the radius *a* can be described in cylindrical coordinates (r,φ) by the wave Eq.

$$\left[\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right)-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]h(r,\varphi,t)=0 \qquad (6)$$

with the boundary condition: $h(a,\varphi,t)=0$. The parameter v denotes the propagation velocity on the membrane, $v=(F_t/\sigma)^{1/2}$, where F_t is the membrane tension (in N/m) and σ is the areal mass density of the membrane (in kg/m^2). Differential Eq. (6) can be solved using variable separation method. Thus, for $h(r,\varphi,t)=R(r)\Phi(\varphi)T(t)$ and the separation constants $-k^2$ and m^2, the solution is

$$h_{mn}(r,\varphi,t)=A_{mn}J_m(x_{mn}r/a)\cos(m\varphi-\varphi_0)\cos(\omega_{mn}t-\delta), \qquad (7)$$

where $J_m(x)$ is the *m*-th order Bessel function of the first kind with the *n*-th zero x_{mn} and where ω_{mn} is the membrane resonant frequency, $\omega_{mn}=vk_{mn}=v\,x_{mn}/a$ ($m=0,1,2,\ldots$, $n=1,2,3,\ldots$). The coefficients A_{mn} and the phase shifts φ_0 and δ are determined by the initial and boundary conditions. The complete solution for the motion of the circular membrane is

$$h(r,\varphi,t)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}h_{mn}(r,\varphi,t). \qquad (8)$$

Calculated without a driving force, these solutions show the natural behavior of an idealized membrane system differing generally from the behavior of a real membrane with the same parameters.

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2.3 Circular membrane: dynamic modal characteristics

The DMCs are obtained by determining the two-dimensional arrays of vibration amplitude values within a given range of frequencies ω: $\omega_{\min} \le \omega \le \omega_{\max}$. In practice, the membrane is set into vibration with the frequency ω and its motion can be described by the time dependent inhomogeneous differential Eq.

$$\frac{d^2T(t)}{dt^2} + \gamma\frac{dT(t)}{dt} + \omega_{mn}^2 T(t) = F(t), \tag{9}$$

where the force $F(t)$ is harmonic, i.e. $F(t) = P_0 \cos(\omega t + \vartheta_0)$. In the case of applying a sound pressure, the constant P_0 is proportional to the intensity of a sound source. The force $F(t)$ is compensated by the parameter γ that describes the energy loss and the resulting damping of the membrane. Generally, the membrane damping is small compared to ω and usually decreases as ω increases [4].

The steady-state solution for Eq. (9) is actually a particular solution which can be obtained for a function $T_p(t) = A\exp\left[i(\omega t + \vartheta_0)\right]$. Inserting $T_p(t)$ in Eq. (9) and calculating the constant *A*, it follows: $T_p(t) = \left\{P_0 \Big/ \left[\left(\omega_{mn}^2 - \omega^2\right) + i\gamma\omega\right]\right\}\exp\left[i(\omega t + \vartheta_0)\right]$. The amplitude of this function,

$$\left|T_p(\omega)\right| = P_0 \Big/ \left[\left(\omega_{mn}^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]^{1/2}, \tag{10}$$

shows the dependence of the vibration amplitude on the frequency ω. This amplitude reaches its maxima at the frequencies $\omega = \left(\omega_{mn}^2 - \gamma^2/2\right)^{1/2}$, which is (due to $\gamma \ll \omega_{mn}$ for every *m*, *n*) approximately equal to the resonant frequencies ω_{mn}.

Although the DMCs of a real membrane system ought to be determined experimentally due to the specific physical properties and particularly imperfections of the system parts, the insight into the modal properties of the analog ideal membrane system can be obtained from the solution given by Eq. (7). We replace the temporal term of Eq. (7) by the right-hand side of Eq. (10) and we take the values: $A_{mn} = 1$ and $\varphi_0 = 0$. Thus, the qualitative solution for the vibration amplitude is given by:

$$h(r,\varphi,\omega) = P_0 \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left\{1 \Big/ \left[\left(\omega_{mn}^2 - \omega^2\right)^2 + \gamma^2\omega^2\right]^{1/2}\right\} J_m\left(x_{mn} r/a\right)\cos m\varphi, \tag{11}$$

and the DMCs of a membrane with radius *a*, velocity *v*, and damping γ can be presented as a movie where the frames change with ω from ω_{min} to ω_{max}.

3. Results

We determined the DMCs for a 6-inch drumhead with parameters: $a = 0.0762\,m$ and $v = 82.6\,m/s$, and in the frequency range from $\omega_{\min} = 2724\,Hz$ ($f_{\min} = 434\,Hz$) to $\omega_{\max} = 11284\,Hz$ ($f_{\max} = 1796\,Hz$). The resonant frequencies are given in Table 1. The damping of the membrane γ and the corresponding Q-factors were not investigated in this study. We synthesized two movies, one from the experimentally measured data and the other numerically. The duration of each movie is 27 seconds.

Table 1. Resonant frequencies of the circular membrane used in this work

(m,n)	(0,1)	(1,1)	(2,1)	(0,2)	(3,1)	(1,2)	(4,1)	(2,2)	(0.3)	(5,1)	(3,2)	(6,1)
x_{mn}	2.40	3.83	5.14	5.52	6.38	7.02	7.59	8.42	8.65	8.77	9.76	9.94
ω_{mn} (Hz)	2724	4348	5835	6266	7242	7969	8616	9558	9819	9955	11079	11284
f_{mn} (Hz)	434	692	929	997	1153	1268	1371	1521	1563	1584	1763	1796

3.1 Experimental

Figure 1 shows the scheme (left) and the photograph (right) of the experimental setup. It is composed of a laser, a variable beam splitter, a beam collimator, an array sensor, and various lenses and mirrors. We used a He-Ne laser (λ = 632.8 nm) as a light source and a color digital camera (Olympus E-1, objective lens removed, sensor: 2560 x 1920 pixels of a size 6.8 μm x 6.8 μm, primary RGB filter) as an array photo-detector. The beam emerging from the laser is split into an object and a reference beam. These beams are expanded and steered by mirrors, as depicted in Fig. 1, to form the interference pattern at the detector plane. The membrane is excited externally with the help of a loudspeaker whose frequency and intensity are controlled by a computer. To shorten the total length of the device, we placed two large plane mirrors between the object and the sensor. Thus, digital holograms are recorded at a distance of 6 m (3 x 2 m) from the object, using the exposure time of 1 second and the beam intensity ratio (reference vs. object) between 2.5 and 3.5.

The movie was synthesized by the following procedure: (i) the colored images (digital holograms) were transformed into the 8-bit gray scale images, (ii) these images were then Fourier-transformed, (iii) each obtained image was cut and we saved only one reconstruction, and (iv) the movie was composed from the sequence of these reconstructions. An illustration of a raw colored digital hologram with its magnified portion is given in Fig. 2. Figure 3 shows the first frame of the movie composed from the experimental data. The movie contains the relevant information. For example, the resonant frequencies estimated at the maxima described by Eq. (10) are shown in Table 2, where Δf_{mn} denotes the difference between the calculated and measured values. Other data, such as the vibration structure, phase shifts, or pattern regularity can be easily extracted from the movie.

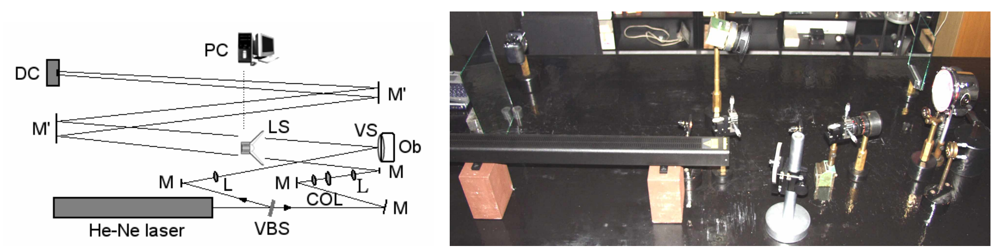

Fig. 1. Experimental setup. M and M' denote mirrors, L lenses, VBS variable beam splitter, COL collimator, LS loudspeaker, VS vibration surface, Ob object, DC digital camera, and PC personal computer.

Table 2. Resonant frequencies measured experimentally

(m,n)	(0,1)	(1,1)	(2,1)	(0,2)	(3,1)	(1,2)	(4,1)	(2,2)	(0.3)	(5,1)	(3,2)	(6,1)
f_{mn} (Hz)	520	690	945	1010	1180	1265	1410	1515	1570	1620	1745	1795
Δf_{mn} (Hz)	-86	2	-16	-13	-27	3	-39	6	-7	-36	18	1

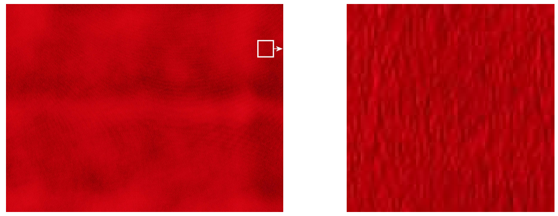

Fig. 2. An example of the colored digital hologram (left) and its portion (right).

3.1 Numerical

To synthesize numerical movie, we made a program in Mathcad and calculated the vibration amplitudes according to Eq. (11). We have chosen a graphic representation in which the lines of equal amplitude are constant in number. Thus, the transformation between the resonant modes is clearly visible and the relative amplitudes between frames (which depend on the unknown parameter γ) are irrelevant. Figure 4 shows the first frame of the numerical movie.

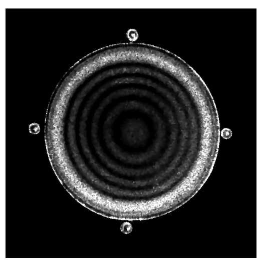

Fig. 3. A movie composed from experimentally obtained data for modal characteristics of a drum.

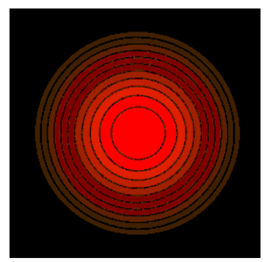

Fig. 4. A movie obtained numerically for a membrane analog to the drum.

4. Discussion and conclusions

We constructed a low-cost device that is: (i) based on the quasi-Fourier digital holography technique, (ii) equipped with a commercial color digital camera (objective removed), and (iii) compacted with large plane mirrors. We tested the device in a study of dynamic modal characterization of musical instruments with a membrane. We noticed minor degradation in the hologram recordings compared to using a professional monochromatic CCD sensor and long setup without the large mirrors (these mirrors introduce additional fringes in hologram recordings). These effects, however, do not disturb the measured DMCs. By showing the DMCs as a movie, it is demonstrated that such very informative measurements can be presented in a compact form.

We measured the DMCs for a percussion instrument. The obtained results are compared with the results of numerical analysis that was supported by the mathematical formulation of the problem. As expected, the DMCs measured experimentally differ from the ones obtained numerically. These differences, occurring in the resonant frequency values and the regularity of the vibration structures, are clearly pronounced using our approach.

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