Acoustics of snare drums

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Modes of vibration, of a snare drum, as observed by holographic interferometry, impulsive modal analysis, and scanning with accelerometers, are related to vibrations of the drum heads, the shell, and the support stand. A simple two-mass model correctly describes the lowest modes of vibration, but at the higher frequencies the drum heads vibrate more or less independently. Each mode of vibration radiates sound in a characteristic pattern which may have dipole, quadrupole, and higher multipole components. The snares move in a complex way, being in contact with the snare head during part of its cycle but losing contact at a later time and returning to strike the head.

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INTRODUCTION

Drums are practically as old as the human race. They have played an important role in nearly every musical culture. A detailed history is given by Blades.[1] As vibrating systems, drums can be divided into three categories: those consisting of a single membrane coupled to an enclosed air cavity (e.g., timpani[2,3] and tabla[4,5]); those consisting of a single membrane stretched over an open shell (e.g., tom-toms[6,7] and congas); and those consisting of two membranes coupled by an enclosed air cavity (e.g., bass drums[8,9] snare drums, o-daiko and turi-daiko of Japan[10]).

Snare drums or side drums, which belong to the family of untuned membranophones, have evolved over several centuries. The orchestral snare drum is a two-headed instrument about 35 cm (14 in.) in diameter and 13–20 cm (5–8 in.) deep. When the upper or *batter* head is struck, the lower or *snare* head vibrates against strands or cables of wire or gut (the snares). Alternately, the snares can be moved away from the lower head.

In this paper, we report the results of studies on the vibrational behavior and sound radiation from a snare drum.

I. THEORETICAL CONSIDERATIONS

A. Vibrations of ideal, real, and coupled membranes

The wave equation for an ideal circular membrane with tension T and area density σ has solutions of the type,

$$Z(r,\phi) = A_m J_m(kr) e^{\pm jm\phi} e^{j\omega t},$$

where $k = \omega\sqrt{\sigma/T}$. The nth zero of $J_m(kr)$ gives the frequency of the (m,n) mode, which has m nodal diameters and n nodal circles (including one at the boundary). In the fundamental (0,1) mode, the entire membrane moves in a single phase.

The normal mode frequencies of real membranes may be quite different from those of an ideal membrane. The principle effects in a membrane acting to change the mode frequencies are air mass loading, bending stiffness, and stiffness to shear. In general, air mass loading lowers the modal frequencies, while the other two effects tend to raise them. In thin membranes, air loading is usually the dominant effect.

If the motion of the heads in a vibrational mode results in a change in the volume of air inside the drum, the effective stiffness of the enclosed air will raise the frequency of that mode. The (0,1) mode in a kettledrum, for example, may be raised by as much as 50%, depending upon the tension in the head.[11,12] A smaller increase is noted in the axisymmetric (0,2) and (0,3) modes, while the nonaxisymmetric modes, such as (1,1) and (2,1), experience no such effect, since the net volume of air displaced is zero.

In a drum with two heads, there is appreciable coupling between the two heads, especially at low frequency.[9,10] This coupling may take place acoustically through the enclosed air or mechanically through the shell.

The in-phase (0,1) motion of the two heads can be reasonably well described by the simple two-mass model shown in Fig. 1. Each head is represented by a mass and a spring, and the enclosed air constitutes a third spring with constant K_c connecting the masses. Such a system is known to have two modes of vibration whose frequencies are given by

$$\omega^2 = \tfrac{1}{2}\{\omega_b^2 + \omega_s^2 + \omega_{cb}^2 + \omega_{cs}^2 \pm \sqrt{[(\omega_b^2 + \omega_{cb}^2) - (\omega_s^2 + \omega_{cs}^2)]^2 + 4\omega_{cb}^2\omega_{cs}^2}\}, \quad (1)$$

where

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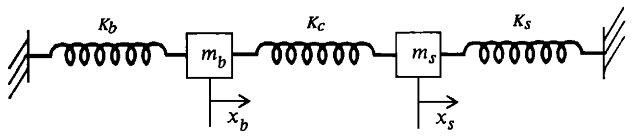

FIG. 1. Two mass model for describing the coupling of the (0,1) modes in the batter head (K_b, m_b) and snare head (K_s,m_s). K_c represents the spring constant of the air enclosed in the drum.

$$\omega_b^2 = \frac{K_b}{m_b}, \quad \omega_s^2 = \frac{K_s}{m_s}, \quad \omega_{cb}^2 = \frac{K_c}{m_b}, \quad \omega_{cs}^2 = \frac{K_c}{m_s}.$$

If the snare head (or batter head) is damped so that it cannot vibrate, we obtain a single frequency for the batter head ω_b' or snare head ω_s':

$$\omega_b' = \sqrt{\omega_b^2 + \omega_{cb}^2}\,, \quad \omega_s' = \sqrt{\omega_s^2 + \omega_{cs}^2}\,. \tag{2}$$

It is more convenient to measure ω_b' and ω_s' experimentally than ω_b and ω_s. K_c can be estimated from thermodynamic considerations, so ω_b and ω_s can be calculated.

B. Sound radiation from drumheads

Sound radiation from two coupled air-loaded membranes is rather difficult to calculate. The sound field can be approximated, however, by superimposing solutions obtained for two uncoupled membranes in infinite baffles.[11,13] When such a membrane vibrates in its (0,1) mode, for example, the sound field has a monopole character, whereas the (1,1) and (2,1) modes tend to produce fields with dipole and quadrupole characteristics, respectively. This has been demonstrated experimentally for the case of a kettledrum with a single head.[14]

Sound radiation from drumheads vibrating their (1,1) or higher mode tends to be quite inefficient due to the fact that areas moving in opposite phase are separated by less than one-half the sound wavelength in air. This means that air mainly flows back and forth next to the membrane with relatively little energy going into sound radiation. In spite of the inherent inefficiency of the radiation process, however, drums tend to radiate strongly, because the volume of air moved by their drumheads is large.

If both heads of a snare drum vibrate in their fundamental (0,1) mode, the sound field can have either a dipolar or monopolar character depending upon whether the drumheads move in the same or in opposite directions. Similarly, motion of the two heads in their (1,1) modes can produce sound radiation with either quadrupolar or dipolar character depending upon whether the heads move in the same or in opposite directions.

II. EXPERIMENT

A. Measuring tension

If a force F, uniformly distributed over a circle of radius b, is applied to a circular membrane of radius a with tension T, the deflection y of the membrane is given by: $y = (F/2\pi T)\ln(a/b)$, as long as the deflection is small. The tension depends upon the deflection, however, so to determine tension experimentally, we need to measure y as a function of F and extrapolate the resulting curve to $y = 0$.

The technique we used was to add mass in steps of 20 g to circular discs with different radius b centered on the membrane and to measure the deflection by means of a Gaertner micrometer slide, noting the height at which a sharp needle makes electrical contact with the metal disc. The curves of y vs F were fitted to polynomials of the type $y = cF + dF^2$ to obtain the best value of the initial slope from which the tension could be determined.

B. Determining modes of vibration

Modes of vibration were observed separately in the batter head, the snare head, the cylindrical drum shell, and the complete drum. Three methods were used: scanning the nearfield sound, holographic interferometry, and modal analysis with impact excitation. Each of these methods has certain advantages and disadvantages.

1. The batter head and snare head

It is difficult to measure the modes of vibration of the heads completely free of the rest of the drum. Therefore, we choose to study each head attached to the drum, with the other head and the drum shell damped by means of sand bags. The modes thus measured are those of the heads loaded by the mass of the air on each side and acted on by the restoring force of the air enclosed in the shell.

There appears to be no standard practice amongst performers for tuning the two heads of a snare drum. Most drummers set the thinner snare head to a slightly lower tension than the batter head, although its modes may still be slightly higher in frequency because of its smaller mass. For these studies, then, the important tuning criterion was merely that each head have as uniform a tension as possible. This was achieved, first of all, by listening to the pitches of the fundamental and the first overtone [from the lower (1,1) mode] as the head is struck lightly at points around its circumference, then by measuring these frequencies with a stroboscopic tuner. Finally, the uniformity of tension was checked by observing the circular node in the (0,2) mode of vibration.

In order to excite the different modes, a small (0.22 g) samarium–cobalt magnet was attached to the membrane at one of two different positions: at the center and at half the radius. A sinusoidal magnetic field was supplied by a small coil driven by an audio amplifier in order to provide a sinusoidal force. The nearfield sound was scanned by an electret condenser microphone (5 mm in diameter) about 3–5 mm from the drum head, and the locations of the nodal lines were carefully noted for each mode of vibration.

2. The shell

A shell of the same composite material as the drum shell was supported by two rubber bands and driven radially by one or two magnetic drivers of the type just described. The mode shapes were determined by scanning with a microphone, using the same procedure as was used for the modes of the heads.

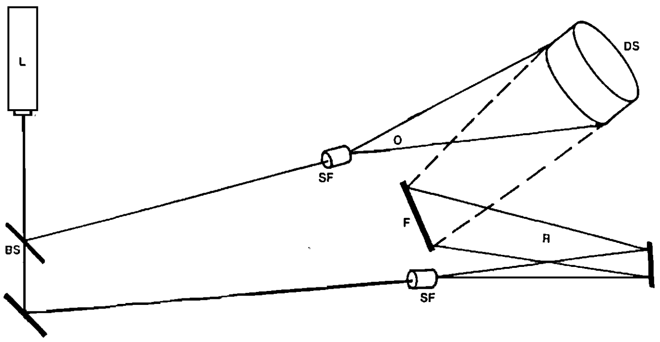

FIG. 2. Basic arrangement for holographic interferometry: L is the laser; BS is the beam splitter; O is the object beam; R is the reference beam; SF is the spatial filter; DS is the drum shell; F is the photographic film.

The vibrational modes of the shell were also observed by means of holographic interferometry.[15] In this case, it was necessary to support the shell by means of small pencils positioned under the shell at three or four of the nodes. The shell was coated with a white powder to enhance the holographic image. Modal frequencies with the shell supported on pencils at the nodes were very nearly the same as when the shell was supported by rubber bands.

The basic optical arrangement used for holographic interferometry is shown in Fig. 2. Light from a HeNe laser L is divided into an object beam O and a reference beam R by means of a beam splitter BS. After passing through a spatial filter SF, the reference beam is directed to the photographic film F. The object beam illuminates the drumshell DS, so that the diffusely reflected light (dashed lines) also reaches the film. Thus, at every point on the film, an interference pattern between the two beams is recorded. In order to illuminate both the inside and outside surfaces of the shell coherently, it was necessary to use two object beams with slightly different lengths which required a second beam splitter.[16] The entire apparatus is mounted on a vibration-isolating table having alternate layers of concrete, foam rubber, and steel.

3. The complete drum

All three methods were used to observe the modes in the complete drum. First, the drum was suspended by rubber bands and a magnet was attached to each of the heads, in turn, and also to the shell; scanning of the near-field sound was done with a microphone, as previously described. Second, holographic interferometry was used to record modes of vibration with the drum supported on sandbags.

The drum was also studied by means of modal analysis using impact excitation. An accelerometer was attached to the rim of the drum, and it was struck at 226 selected points with a PCB 86C80 miniature force hammer. The force hammer was used with a solenoid in order to deliver a reproducible blow to the structure. Four blows were averaged at each point of excitation. An Ono Sokki CF-930 fast Fourier transform (FFT) analyzer computed the 226 transfer functions using the preamplified signals from the accelerometer and force transducer. Normal modes were determined using SMS Model 3.0 software on a Hewlett-Packard 332 computer. The global curvefitting routine was employed.

4. Shell motion

In order to understand the role of shell motion in coupling vibrational motion of the two heads and also to ascertain how much sound might be radiated by the shell, the motion of the shell was studied by both holographic interferometry and by attaching accelerometers to the shell. Holographic interferometry was used to observe radial motion of the shell, whereas accelerometers were used to determine both radial and vertical (axial) motion of the drum.

C. Sound spectra and decay rates

Sound spectra obtained by striking the free shell and the complete drum were recorded by means of a Nicolet 660A FFT analyzer. In the case of the drum, the microphone was about 60 cm from the center of the struck head; in the case of the shell it was about 40 cm from the center of the shell. Although each mode of vibration has a different radiation pattern, individual peaks in the "room-averaged" sound spectra changed by only a few decibels when the microphone position was varied.

In order to determine decay rates for the various modes the initial spectrum was recorded on channel A of the FFT analyzer and a delayed spectrum on channel B. The delay time was varied from 0.02 to 1.0 s. From the series of spectra, sound decay rates could be determined at any desired frequency.

Decay rates in several 1/3-octave bands were measured by means of an Ivie IE-30A spectrum analyzer with an IE-17A microprocessor. Third-octave decay rates were sometimes affected by beats between radiated sound from two modes that are close in frequency, however.

D. Sound radiation

Sound radiation patterns were determined by suspending the drum with rubber bands on a rotatable platform in a small anechoic room. A sinusoidal force was applied to the batter head, and the sound pressure was measured by an electret condenser microphone one meter away from the center of the drum as the drum was rotated both around a diameter and about an axis normal to the plane of the head.

E. Motion of the snares

The velocities of the snare head and the snares were recorded by means of two electromagnetic velocity sensors consisting of a B & K MM0002 electromagnetic transducer. Waveforms and spectra were recorded for various values of snare tension and strengths of blow applied to the batter head. The snare tension was determined by measuring the force necessary to pull a few snare springs away from the snare head.

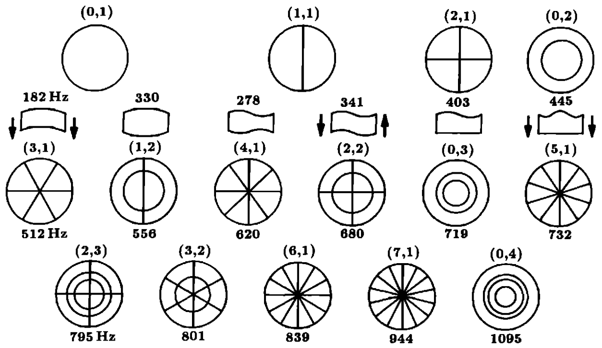

FIG. 3. Modes of vibration in a snare drum. Mode designation (m,n) are given above the mode shapes and modal frequencies below.

III. RESULTS

A. Head tension

Even in a large kettledrum, it is difficult to obtain a precise measurement of the tension. In a small drum it is even more difficult since the deflections must be measured to hundredths of a millimeter. The tension of the batter head was determined to be $T_b = 3200 \pm 200$ N/m and that of the snare head $T_s = 2070 \pm 100$ N/m. These are in reasonably good agreement with the tensions calculated from the modal frequencies.

TABLE I. Mode frequencies observed in the batter head, the snare head, and the complete drum. For the modal analysis, the drum was freely supported by an elastic rope. In the case of the sinusoidal driving force, the batter head frequencies were measured with the snare head and the drum shell damped with sand bags, and the snare head frequencies with the batter head similarly damped; the completely coupled drum modes were measured with the drum freely supported on rubber bands. All frequencies are in hertz.

	Batter head		Snare head		Coupled drum modes	
Mode	Modal analysis	Sine drive	Modal analysis	Sine drive	Modal analysis	Sine drive
01		224		271	183	182
01					327	330
11		280		310	275	278
11					344	341
02	451	445	595	616		
21	397,405	403	507	485		
31	514	512	687	674		
41	624	620	850	859		
51	730	732				
61	840	839				
71	948	944				
81		1011				
03		719		1044		
04	993	1095				
12	557	556				
22	687	680	993			
32	808	801				
42		907				

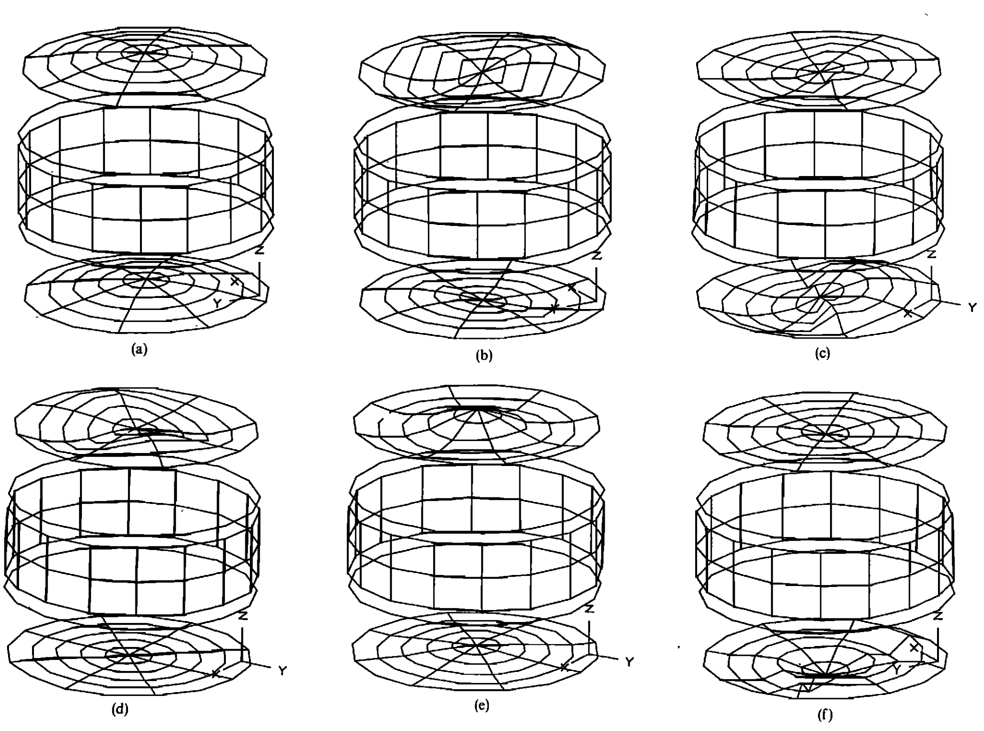

FIG. 4. Modal shapes of a snare drum as determined by modal analysis with impact excitation: (a) Lowest (0,1) mode, 183 Hz; (b, c) pair of (1,1) modes, 275 and 344 Hz; (d) (2,1) mode in batter head, 405 Hz; (3) (0.2) mode in batter head, 451 Hz; (f) (0,2) mode in snare head, 595 Hz.

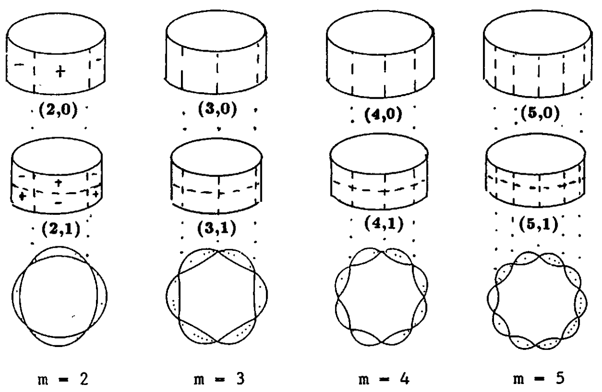

FIG. 5. Vibrational modes of a cylindrical drum shell.

B. Modes of vibration

Mode shapes and mode frequencies for a number of vibrational modes observed in the freely suspended drum are shown in Fig. 3. Each mode is labeled (m,n), where m gives the number of nodal diameters and n the number of nodal circles (including the one at the circumference). Note that in the case of the (0,1) and (1,1) modes, strong coupling between the heads leads to mode doublets. In the lower (parallel) (0,1) mode, the shell moves up and down opposite to the direction of the heads; the amplitude of this shell motion is about 1% of the amplitude of the batter head. In the upper (0,1) mode and the lower (1,1) mode, in which the heads move in opposite directions, there is little shell motion. In the upper (1,1) mode, the shell of a freely suspended drum rocks slightly, although the shell amplitude is only 0.2% of the batter head amplitude. In the case of the (0,2) modes, a weak coupling takes place, which apparently leads to a doublet with small frequency separation, although mode identification is not certain.

Mode frequencies measured in the batter head, the snare head, and the complete drum are given in Table I. Above the (0,2) mode, there appears to be little coupling between the two heads, so the modes observed in the freely suspended drum are the same as the modes of the separate heads. In most cases, the mode frequencies observed with sinusoidal excitation agree closely to those observed by modal analysis with impact excitation. The former are typically 1%–2% lower due to the mass of the attached magnet.

Modal shapes of several modes, as obtained from modal analysis with impact excitation, are shown in Fig. 4. In the lowest (0,1) mode at 183 Hz, the two heads move in phase and radiate strongly. In the lower (1,1) mode at 275 Hz the heads move in opposite directions, so air "sloshes" back and forth from one side to the other inside the drum shell. In the higher (1,1) mode at 344 Hz, the two heads move in the same phase, so air is displaced only slightly in the axial direction; the lower effective air mass results in a higher modal frequency.

At 405 Hz, the batter head shows (2,1) motion, with two nodal diameters, as shown in Fig. 5(d), but the snare head moves very little. Actually, there are two (2,1) modes at 397 and 405 Hz, differing only in the orientation of the two nodal diameters. (0,2) motion of the batter head occurs at 451 Hz, as shown in Fig. 5(e), while (0,2) motion of the snare head is shown in Fig. 5(f) at 595 Hz.

C. Motion of the shell

The lowest modes of the free drum shell are the cylindrical shell modes having m nodes parallel to the axis and n circular nodes, as shown in Fig. 5. Holographic interferograms of three $(m,0)$ modes and three $(m,1)$ modes are shown in Fig. 6. The outside and inside surfaces were recorded simultaneously by using two object beams.[16]

The modal frequencies for the $(m,0)$ modes in the drum shell are given in Table II along with the decay rates. Also given in Table II are the frequencies of modes in the complete drum in which the shell tends to vibrate radially with $2m$ nodes. These are interpreted as being due to the motion of the shell coupled to the rim, the heads, and the attached hardware. Note that this motion in the complete drum generally occurs at higher frequencies than the corresponding modal frequencies in the free shell, as expected. Holographic interferograms illustrating several modes of vibration of the drum that are mainly shell modes are shown in Fig. 7.

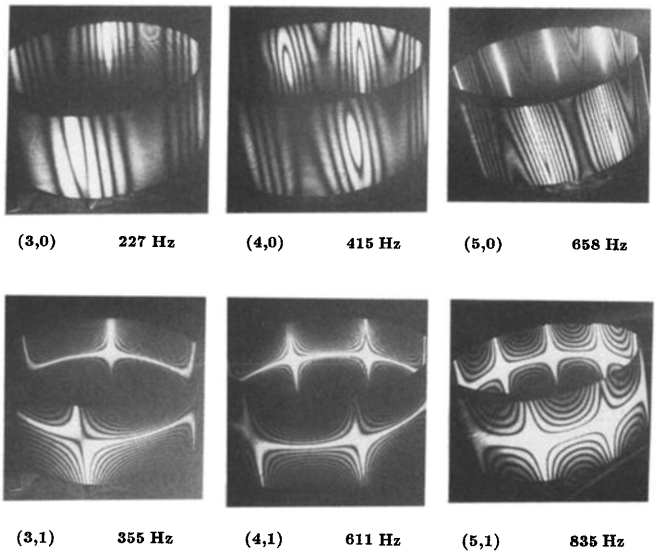

FIG. 6. Holographic interferograms of a snare drum shell without the heads.

TABLE II. Modal frequencies and decay rates of $(m,0)$ modes in a free drum shell and frequencies at which similar motion is observed in a complete drum.

	Free shell		Drum
Mode	Frequency (Hz)	Decay rate (dB/s)	Frequency (Hz)
(2,0)	80	55	221
(3,0)	225	114	421
(4,0)	425	175	874
(5,0)	670	195	1064
(6,0)	950	250	1271
(7,0)	1260	300	1408
(8,0)	1595	330	

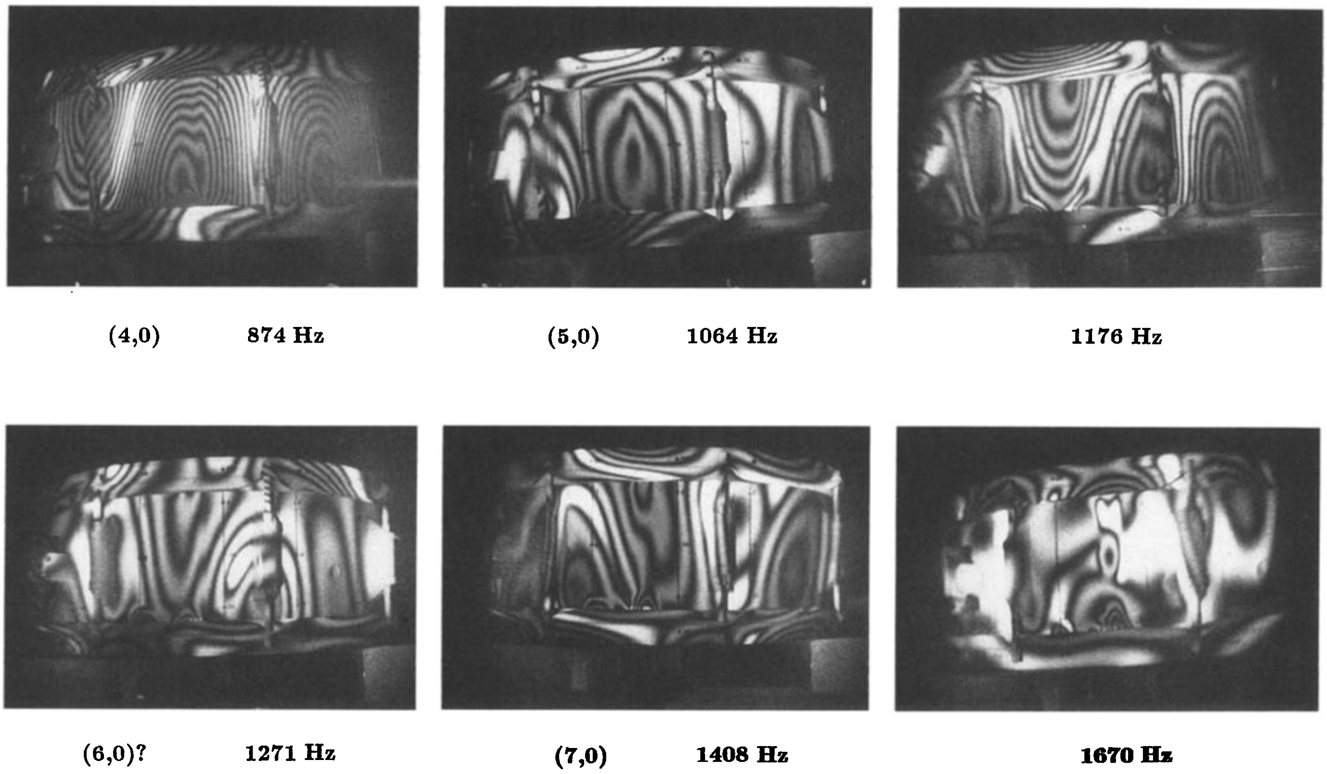

FIG. 7. Holographic interferograms of the complete drum showing modes of vibration that are mainly modes of the drum shell. (a) (4,0) mode, 874 Hz; (b) (5,0) mode, 1064 Hz; (c) (5,0) + (6,0) modes, 1176 Hz; (d) (6,0) mode, 1271 Hz; (e) (7,0) mode, 1408 Hz; (f) possibly (8,0) mode, 1670 Hz.

The vertical motion of the drum, when freely suspended on rubber bands, was determined by attaching an accelerometer at eight different locations around the circumference of the ring while the drum was driven at the resonance frequencies of several modes. The acceleration amplitudes were compared to those of the batter head. The following results were obtained:

(1) In the parallel (0,1) mode (181 Hz), the shell moved oppositely to the heads with an amplitude approximately 1% of that of the batter head;

(2) In the antiparallel (0,1) mode (322 Hz), the shell amplitude was less than 0.2% of that of the batter head;

(3) In the parallel (1,1) mode (340 Hz), the shell "rocked" with an amplitude approximately 1% that of the center of the adjacent half of the batter head;

(4) In the antiparallel (1,1) mode (272 Hz), the shell rocking amplitude was about 0.3% that of the batter head, and it moved in the same direction as the batter head (opposite to the snare head);

(5) In the (0,2) mode [batter head moves in (0,2) mode while snare head moves slightly in (0,1) mode; see Fig. 5(e)], the shell amplitude is about 0.5% of the center of the batter head and in the opposite direction; and

(6) In other modes, the shell motion was less than 0.2% that of the batter head.

D. Sound radiation from the drum

Sound radiation patterns in the vertical plane for the four lowest modes of the drum are shown in Fig. 8. The pattern for the lowest mode [the parallel (0,1) mode] has a strong dipole component, as expected, with maximum radiation along the center axis of the drum. The sound pressure level in front of the batter head is about 2 dB greater than in front of the snare head but 13 dB greater than at the midplane of the drum. The radiation from the other (0,1) mode [the antiparallel (0,1) or "breathing" mode] is considerably weaker (for the same driving force) and is nearly isotropic.

The radiation pattern for the lower (1,1) mode, in which the heads move essentially in opposite directions, also has a relatively strong dipole component with minima in the direction of the nodal line, as seen in Fig. 8(c). The pattern for the higher (1,1) mode, in which the heads move in the same direction, has four maxima (due to a quadrupole component) when viewed in the vertical plane, as shown in Fig. 8(d).

Sound radiation patterns in the horizontal plane for the two (1,1) modes are shown in Fig. 9. The radiation pattern for the lower (1,1) mode again shows a strong dipole component as it did in Fig. 8, while that of the parallel (1,1) mode (which showed four maxima in the vertical plane, see Fig. 8(d)] is nearly isotropic in the horizontal plane.

E. Modal decay rates

The modal decay rates for several modes in a drum tuned to a low tension (about 2400 N/m) are given in Table III. It is clear that the lowest (0,1) mode and the (0,2) mode decay considerably faster when the drum is supported on a drum stand than when it is freely supported on elastic cords. These modes incorporate an appreciable amount of vertical motion of the shell. Decay rates of other modes depend much less on the type of support.

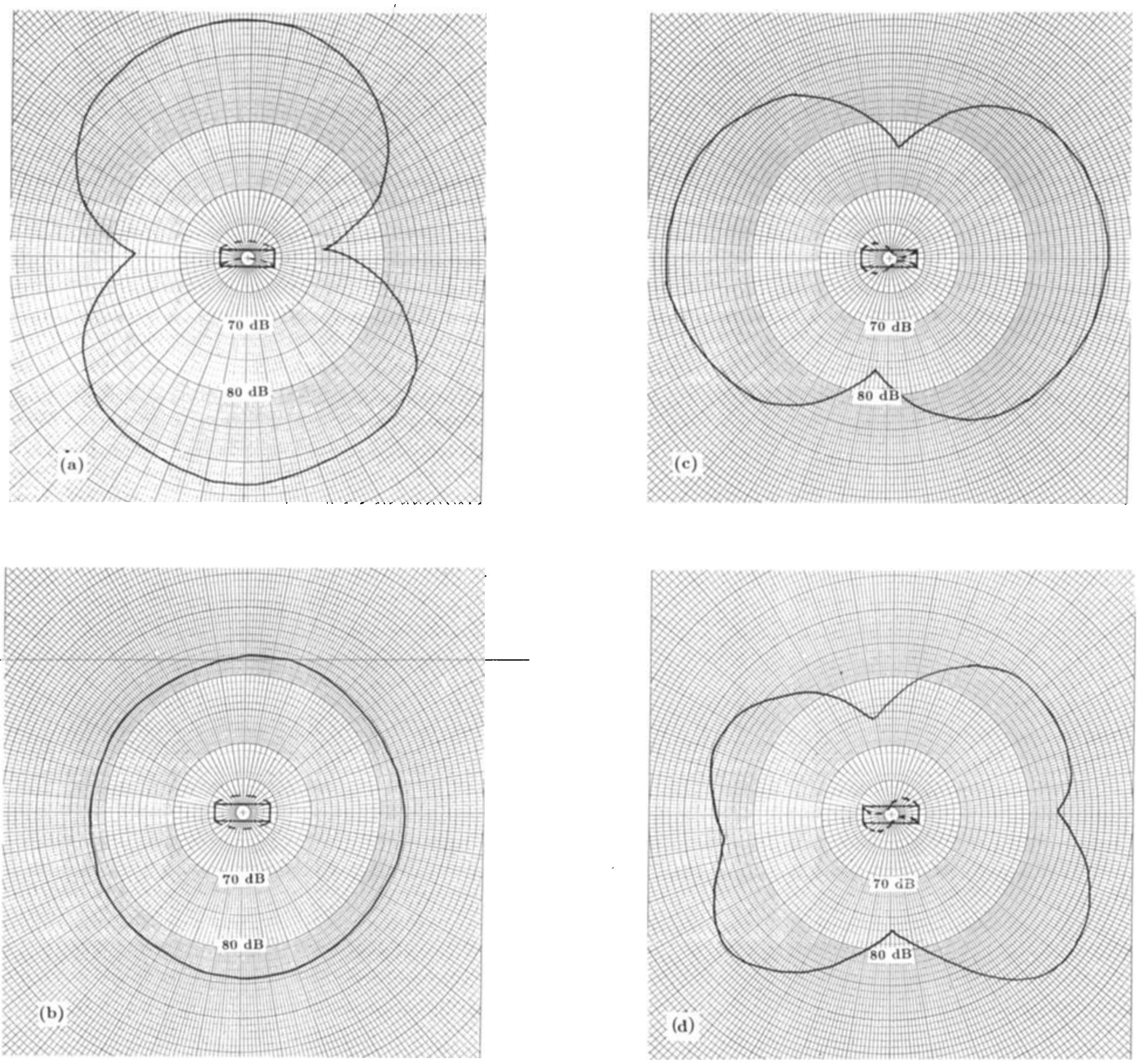

FIG. 8. Sound radiation patterns in the vertical plane for a snare drum vibrating in its four lowest modes: (a),(b) (0,1) modes at 182 and 330 Hz; (c),(d) modes at 278 and 341 Hz.

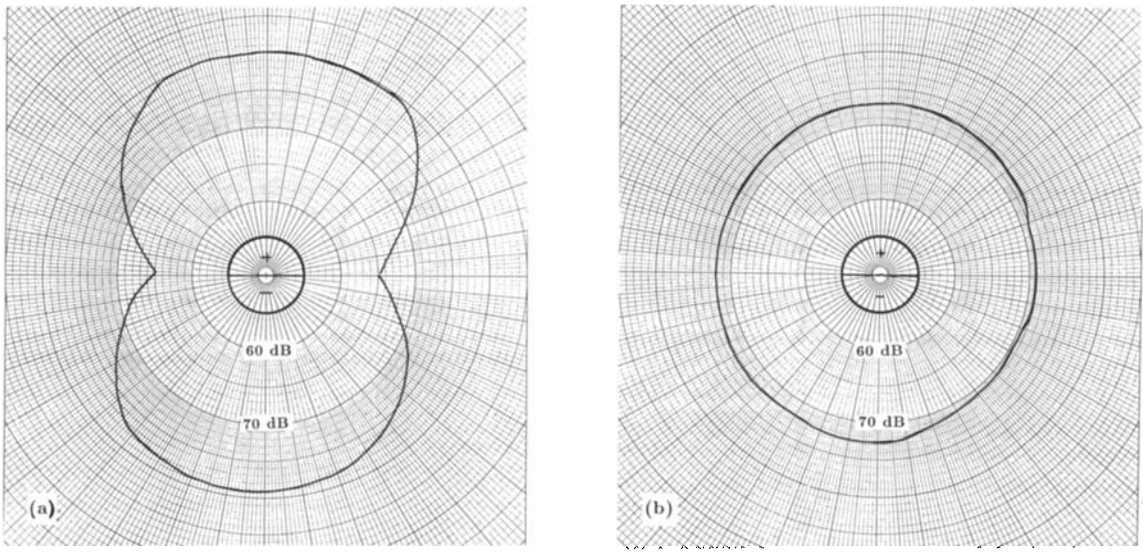

FIG. 9. Sound radiation patterns in the horizontal plane for a snare drum vibrating in its two (1,1) modes: (a) 278 Hz; (b) 341 Hz.

TABLE III. Modal decay rates (in dB/s) of a snare drum freely supported by elastic cords and supported on a drum stand. The drum was struck at the center of the batter head (columns 1 and 3) or half way between the center and the rim (columns 2 and 4).

	Elastic cords		Drum stand	
Mode	Center	Off-center	Center	Off-center
01	35	38	70	60
11	40	30	40	30
11	38	49	47	...
01	45	25	47	30
21	...	...	24	30
02	40	30	65	50
31	22	32	...	35
41	47	45	58	65

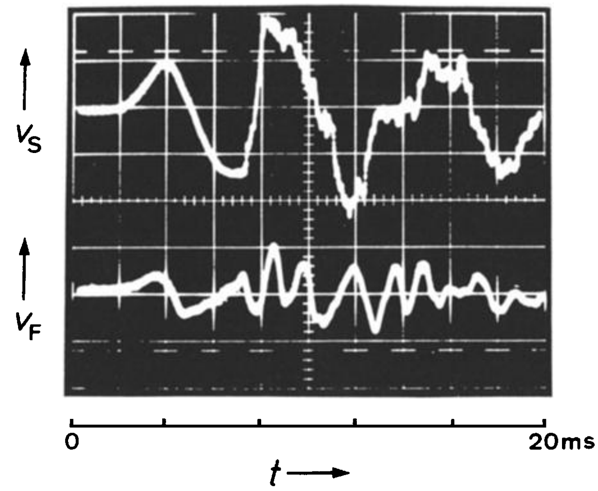

FIG. 10. Velocities of snares (above) and snare head (below) as functions of time after a blow to the batter head. The head velocity is measured at the center of the head next to the snares.

F. Motion of the snares

The velocities of the snares and the snare head are shown as functions of time in Fig. 10. The snare velocity initially resembles a smooth sine curve whose period is somewhat greater than that of the snare head velocity. Therefore, the return point of the head ($v_h = 0$) is reached sooner and the snares lose contact with the head. The sinusoidal head velocity curve is interrupted at the moment t_c when the snares, vibrating back ($v_s < 0$) contact the head, which is already moving in the opposite direction ($v_h < 0$). Through the impact, higher modes of vibration are excited in the snares as well as in the snare head.

Figure 11 shows the spectrum of the snare head velocity, without and with the snares in contact and also the snare velocity spectrum. Damping by the snares broadens the fundamental resonance at 175 Hz. In the snare velocity spectrum, this head resonance has a maximum velocity only slightly below that of the snare fundamental resonance at 90 Hz. Conversely, the snare resonance is noted in the head velocity spectrum.

Sound spectra of a snare drum for four different values of snare tension with piano (p), mezzo-forte (mf), and forte (f) blow strengths are shown in Fig. 12. The drum was struck in the center with a wooden mallet driven by an electrically controlled striking device.

IV. DISCUSSION

A. Modal frequencies and modal shapes

Coupling between the uniphase (0,1) modes in the two heads leads to a doublet with a frequency ratio of about 1.8, somewhat smaller than the frequency ratio observed in a bass drum with much larger heads.[9] In a large o-daiko drum,[10] whose shell length is greater than the head diameter, the ratio was only 1.15. In a shallow turi-daiko[10] only the lower member of the pair was observed. We now compare the frequency ratio to that predicted by the two-mass model discussed in Sec. I.

In theory, we can calculate K_b and K_s from the measured head tensions, and we can calculate K_c from thermodynamic considerations. In practice, this leads to rather large uncertainty, since the drum heads do not behave as ideal membranes. Rather, we determine K_b and K_s from the measured frequencies of the (0,1) mode in each head with the other head damped [ω_b' and ω_s' in Eq. (2)]. Then we

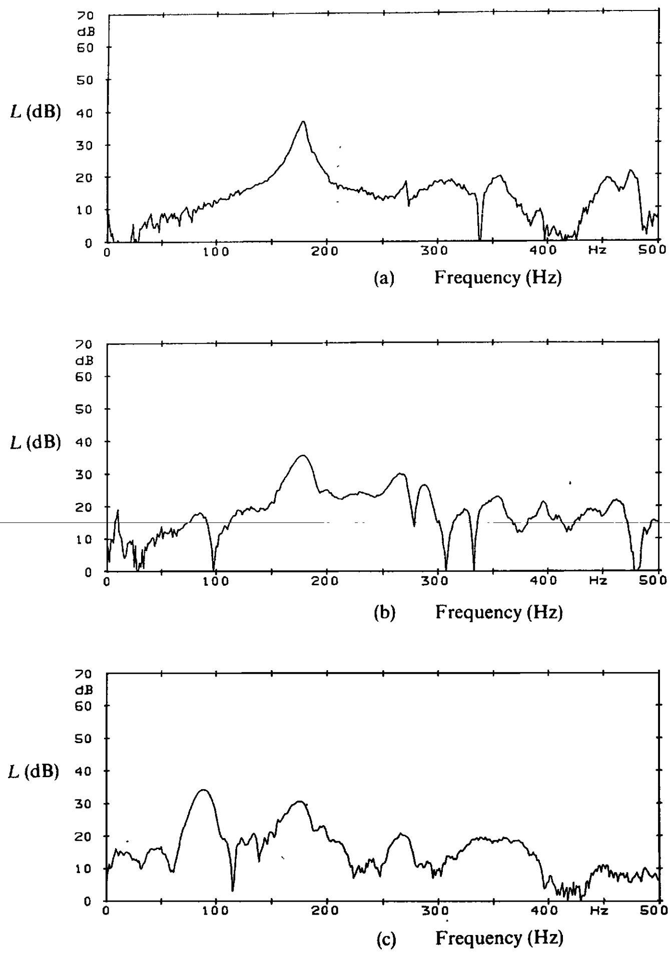

FIG. 11. Velocity spectra of the snare head and snares. (a) Snare head without snares; (b) snare head with snares at low tension; (c) snares at low tension.

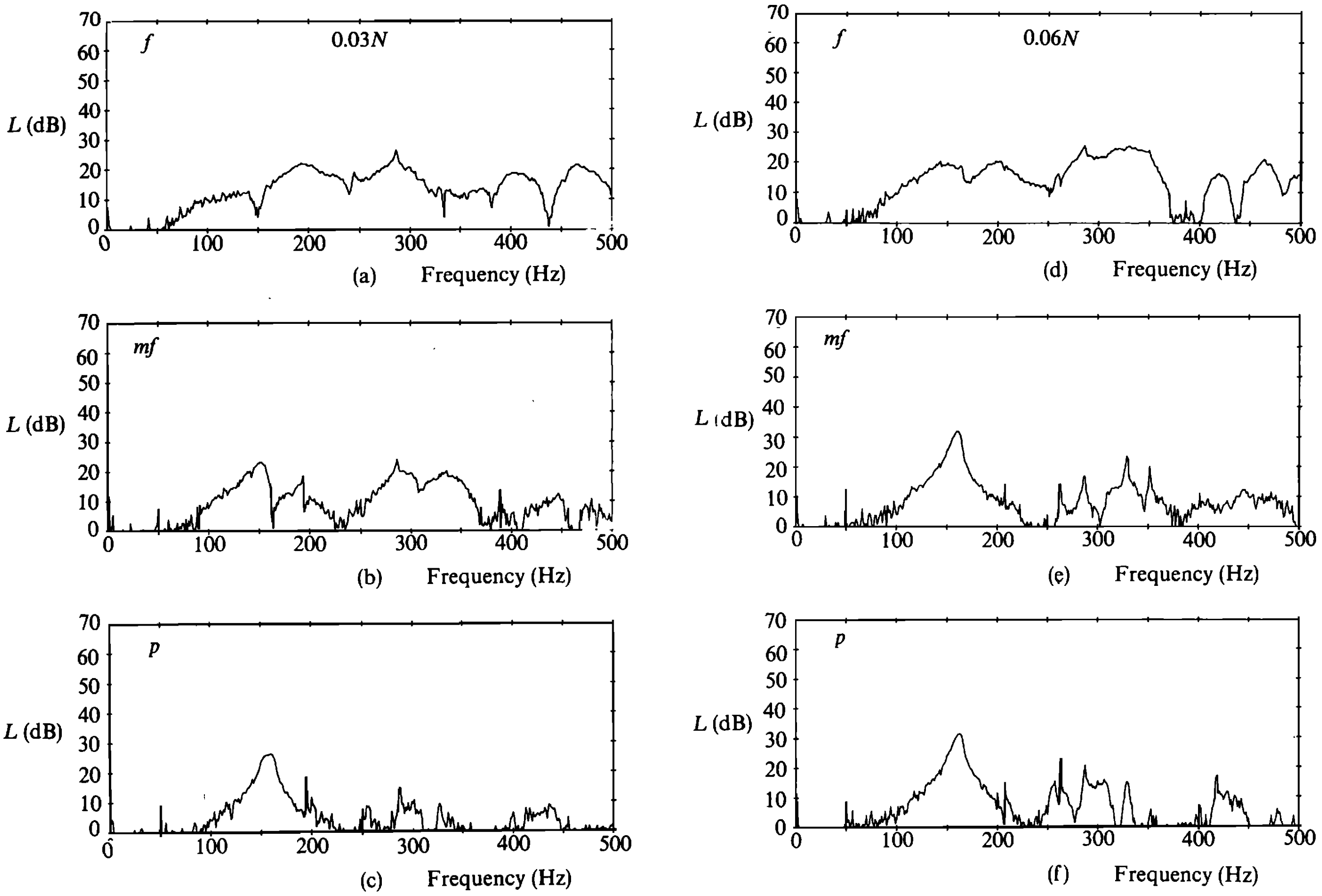

FIG. 12. Sound spectra of a snare drum for three blow strengths (p, mf, f) and four values of snare tension (0, 0.03N, 0.06N, and 0.09N).

only need to calculate K_c, the effective stiffness of the confined air.

If we consider the compression to be adiabatic, the spring constant K_c can be estimated as

$$K_c = \frac{\Delta F}{y_{\text{ave}}} = -qA^2 \frac{\Delta p}{\Delta V} + q \frac{\gamma p_0 A}{H},$$

where p_0 is the average pressure, A is the area of the head, H is the height of the drum, and q is a factor that relates the average displacement y_{ave} to the displacement $y_{\max}$ at the center of the head:

$$q = \frac{y_{\text{ave}}}{y_{\max}} = \frac{\int_0^a J_0(kr) 2\pi r\, dr}{\pi a} = 0.433.$$

Using the measured frequencies for f_b and f_s, leads to the following values for spring constants: $K_c = 4.37 \times 10^4$ N/m, $K_b = 2.73 \times 10^4$ N/m, and $K_s = 2.51 \times 10^4$ N/m. These values predict (0,1) mode frequencies of 155 and 342 Hz compared to measured values of 182 and 330 Hz. At first glance, this seems to be rather poor agreement at the lower frequency until we note the rather sensitive dependence of the doublet splitting on the factor q. Lowering q by 10%, for example, raises f_1 by about 8%, while raising f_{cb} by 10% raises f_1 by about 10%.

Coupling between the (1,1) modes also leads to a doublet with a frequency ratio of 1.2. A qualitative explanation of this coupling is straightforward: when the heads move in opposite directions, air "sloshes" back and forth inside the drum, leading to considerable mass loading of the heads. When they move in the same direction, as in the (1,1) mode of higher frequency, air moves only a short distance in the axial direction, so the effective mass loading is diminished. A quantitative model that correctly predicts the frequencies of these modes has not yet been developed.

One way to estimate the effect of air mass loading (or the stiffness of the confined air) is to calculate the wave speed at each modal frequency. This is done by dividing the observed modal frequency by the appropriate Bessel function root. The results are shown in Fig. 13. Note that the $(0,n)$, $(m,1)$, and $(m,2)$ families appear to lie along slightly different curves (showing a different air mass loading effect), but they all approach $v = 95.5$ m/s, the velocity calculated for an unloaded membrane with a tension of 3200 N/m and area density of 0.3505 kg/m^2. The effective stiffness of the enclosed air gives the upper (0,1) mode a high wave velocity (152 m/s), above the top of the graph, as predicted by the two-mass model. The upper (1,1) mode also appears to be raised by the effective stiffness of the enclosed air, although no quantitative model to explain its behavior is available at the present time. Some air stiffness effect is noted in the lower (0,1) mode (due to slightly different air displacement by the batter and snare heads even though they move in the same direction) and in the (0,2) mode (due to a small net air displacement by the batter head vibrating in this mode).

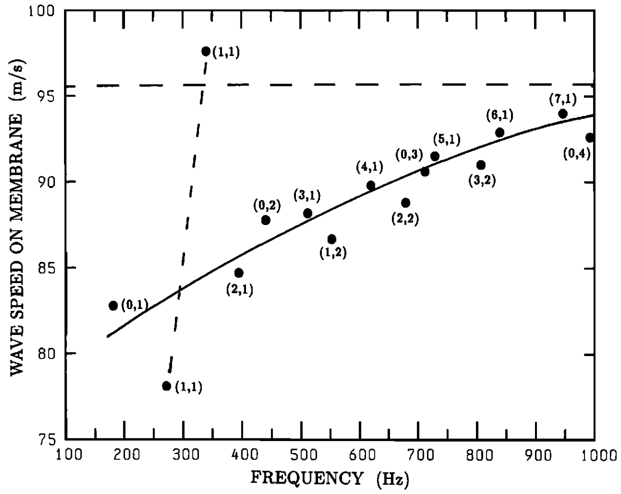

FIG. 13. Wave speeds determined for each mode by dividing the observed modal frequency by the argument of the appropriate Bessel function at its zero. The dashed curve is the calculated wave speed for an unloaded membrane with $T = 3200$ N/m and $\sigma = 0.3505$ kg/m^2.

B. Motion of the shell

The shell has a much greater mass than the heads, and thus it is not surprising that the shell moves very little when the heads vibrate. Nevertheless, modal analysis revealed some rather strong resonances resulting from strongly coupled motion of the heads and the shell. Modes of this type at 198, 204, 220, and 323 Hz were found to radiate quite strongly. Further examination revealed that although an appreciable amount of energy may be associated with the vibrational motion of the shell, most of the sound is radiated by the heads.

Earlier studies with a tom tom (whose shell has slightly more mass than a snare drum) attached to a drum stand revealed several prominent modes resulting from strong coupling between the drum and the stand.[6] This coupled motion, which depends on the mass and stiffness of the drum shell as well as the stand, is an important factor in determining the decay rates of the drum heads at their various modal frequencies.

When the freely suspended drum vibrated in the parallel (0,1) mode, the shell moved up and down with about 1% of the amplitude at the center of the batter head in order to keep the center of mass stationary. [The combined masses of the heads (54 g) is about 1.7% of the total drum mass (3160 g), but the "average" amplitude of the heads is less than the maximum amplitude.]

The vibrational motion of the drum shell itself, which is largely in the radial direction, is strongly related to the radial modes observed in the free shell, although the stiffening effect of the metal rim and other attached hardware cause these modes to occur at higher frequencies in the drum, as indicated in Table II.

Decay rates in the free shell of composite material were found to be very large, especially at the higher frequencies. We have not tested free shells of metal and wood, but no doubt the decay rates would be substantially less. Nevertheless, because the coupling from the heads to radial modes in the shell is minimal, damping in the shell probably has little effect on the sound decay rate.

C. Sound radiation and decay rates

Sound spectra show that the fundamental partial radiated by the lower (0,1) mode is about 20 dB stronger than the next strongest partials when the snares are detached. If the drum is supported on a drum stand, this mode may transfer a considerable amount of energy to the stand, since the shell moves in the axial direction. When the drum is freely supported, however, the decay rate of this mode is no greater than that of the other modes (see Table III). A drum supported on a solid stand is generally judged to have a different timbre than a drum carried or placed on a resilient support.

For the same reason, the mass of the drum shell has appreciable effect on the decay rate of the low-frequency modes and thus on the timbre of the drum. This explains the preference of some drummers for heavy cast bronze shells.

The directional dependence of the sound field radiated by nearly all of the modes makes the sound spectrum strongly dependent on direction. This is an important fact to consider when a drum is played outside away from reflecting walls (as in a marching band) or microphones are placed near the drum in the direct sound field.

D. Motion of the snares

For the snares to sound, the mutual phase condition of the snare head and the snares must be correct. At a given snare-head tension, this can be adjusted by the snare tension, which is optimum when both the head and the snares are moving in opposite directions at the time of contact. In this case, the impulse of the snares on the snare head is greatest.

Figure 12(a)–(c) shows that for the lowest snare tension ($F = 0$), the snare noise predominates for every blow strength; the fundamental head resonance hardly appears in the sound spectra. With increasing tension, the resonance in the curve for a light blow (p) becomes narrower, which results in a clearly perceptible fundamental in the sound.

The same series of measurements reported here were carried out in a drum with a different snare mechanism (HCD 580 Bubinga). In this drum, the snares are not tensioned with cords but by parallel shifts of a frame to which the snares are attached. They extend out over the rim so that the effective vibrating length is greater. In this case, the resonance at low tension becomes sharper indicating a smaller head damping by the snares for the same snare tension.

V. CONCLUSION

Even with its snares disengaged, the modern snare drum vibrates in a complex way when the batter head is struck by a drum stick. A substantial portion of the energy imparted by the blow is transmitted to the snare head, and smaller amounts go to the shell and to the drum stand or other support. This energy transfer is efficiently carried out at the resonance frequencies of the entire drum or those of various components.

The vibratory motion of the drum can be described in terms of normal modes of vibration. Each mode of vibration has its own characteristic radiation pattern, and so the timbre of the drum will be quite different in different directions. Decay rates of different modes are also different, so the timbre changes with time. Modal decay rates depend rather strongly on how the drum is supported and also on the mass of the drum shell.

ACKNOWLEDGMENTS

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[1] J. Blades, *Percussion Instruments and Their History* (Faber and Faber, London, 1975), Chap. 15.

[2] T. D. Rossing, "The physics of kettledrums," Sci. Am. **247** (5), 172–178 (1982).

[3] R. S. Christian, R. S. Davis, A. Tubis, C. A. Anderson, R. I. Mills, and T. D. Rossing, "Effects of air loading on timpani membrane vibrations," J. Acoust. Soc. Am. **76**, 1336–45 (1984).

[4] C. V. Raman, "The Indian musical drum," Proc. Indian Acad. Sci. **A1**, 179–88 (1934).

[5] T. D. Rossing and W. A. Sykes, "Acoustics of Indian drums," Percussive Notes **19** (3), 58–67 (1982).

[6] I. Bork, "Entwicklung von akustischen Optimierungsverfahren für Stabspiele und Membraninstrumente," Report No. 5267, PTB, Braunschweig (1983).

[7] C. Rose, "A New Drumhead Design: An Analysis of the Nonlinear Behavior of a Compound Membrane," M.S. thesis, Northern Illinois University, DeKalb, Illinois (1978).

[8] H. Fletcher and I. Bassett, "Some experiments with the bass drum," J. Acoust. Soc. Am. **64**, 1570–76 (1978).

[9] T. D. Rossing, "Acoustical behavior of a bass drum," J. Acoust. Soc. Am. Suppl. 1 **82**, S69 (1987).

[10] J. Obata and T. Tesima, "Experimental Studies on the Sound and Vibration of Drums," J. Acoust. Soc. Am. **6**, 267–274 (1935).

[11] P. M. Morse, *Vibration and Sound*, 2nd ed. (McGraw Hill, New York, 1948) (reprinted by Acoust. Soc. Am., 1981).

[12] C. A. Anderson, "The Acoustics of Timpani: An Analysis of Vibrating Circular Membranes," M.S. thesis, Northern Illinois University, DeKalb, Illinois (1978).

[13] N. H. Fletcher and T. D. Rossing, *The Physics of Musical Instruments* (Springer-Verlag, New York, 1991).

[14] H. Fleischer, "Die Pauke: Mechanischer Schwinger und akustiche Strahler," Univ. der Bundeswehr, München, 1988.

[15] R. L. Powell and K. A. Stetson, "Interferometric vibration analysis by wavefront reconstruction," J. Opt. Soc. Am. **55**, 1593–1598 (1965).

[16] D. O. Fystrom and T. D. Rossing, "Increasing the depth of field in holographic interferometry," submitted to Am. J. Phys.

[17] H. Zhao, "Acoustics of Snare Drums: An Experimental Study of the Modes of Vibration, Mode Coupling and Sound Radiation Pattern," M.S. thesis, Northern Illinois University, 1990.