Title: Sudoku Challenges

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Group members: Jiaqi Lin

Method that I used

I used the linear programming (LP) optimization algorithm and I used the PuLP python library (https://coin-or.github.io/pulp/) to solve the LP problem.

Let X_{ijk}indicate the event that the (i, j) element of the Sudoku grid contains number k. If it is true, $X_{ijk} = 1$, otherwise, $X_{ijk} = 0$.

The Constraints in the linear programming model are shown below:

- Constraint to ensure that values from 1 to 9 is filled only once in a column, $\sum_{i=1}^{9} X_{ijk} = 1$, for $j, k\epsilon\ [1, 9]$
- Constraint to ensure that values from 1 to 9 is filled only once in a row, $\sum_{j=1}^{9} X_{ijk} = 1$, for $i, k\epsilon\ [1, 9]$
- Constraint to ensure that values from 1 to 9 is filled only once in the 3x3 grid, $\sum_{j=3p-2}^{3p} \sum_{i=3q-2}^{3q} X_{ijk} = 1$, for $k\epsilon\ [1, 9]$ and $p, q\epsilon\ [1, 3]$
- Constraint to ensure only one value is filled for a cell, $\sum_{k=1}^{9} X_{ijk} = 1$, for $i, j\epsilon\ [1, 9]$

I don’t specify an objective function for the LP model because the Sudoku solving is only a feasibility problem, i.e., finding the values of X_{ijk} that satisfy the constrains and the Sudoku is solved. We do not need to maximize or minimize any objective functions. Thus, in my python code, I just give value 0 as the objective function.

Without an objective function, the model will keep searching until it satisfies all the constraints. Advantage is that the model will always find the solution. Disadvantage is that it might be bit slower than using objective functions in some cases, because sometimes an objective function can guide the model to find values that satisfy the constraints faster.

Success rate

The success rates on data sets A, B are shown in the table below.

Iterations	Data set A	Data set B
Iteration 1	100%	100%
Iteration 2	100%	100%
Iteration 3	100%	100%

As shown in the table, I run the sudoku_solver.py three times and each time we randomly select 1000 puzzles from each data set. The success rate is all 100%. As explained in the above section, without using an objective function, the model will always find the solution.