

2.3 PARTITIONING DEMOS (INC. IMPROVEMENTS)



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- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*



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2.3 PARTITIONING DEMOS

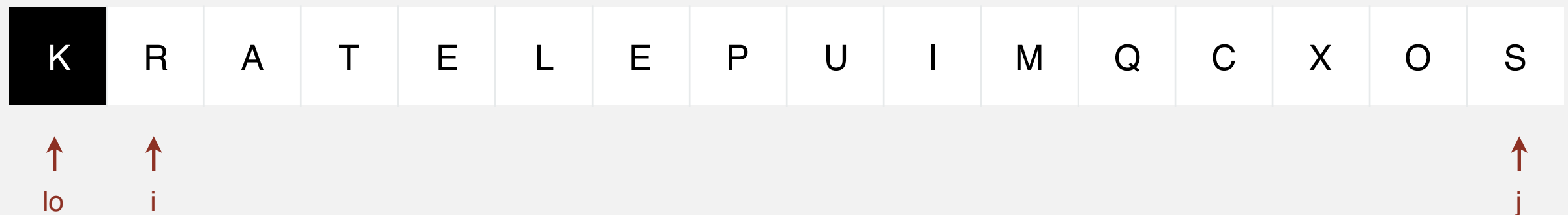
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Quicksort partitioning demo

Let v (pivot) = $a[\text{lo}]$

Repeat until i and j pointers cross.

- Scan i from left to right so long as $(a[i] < v)$.
- Scan j from right to left so long as $(a[j] > v)$.
- Exchange $a[i]$ with $a[j]$.



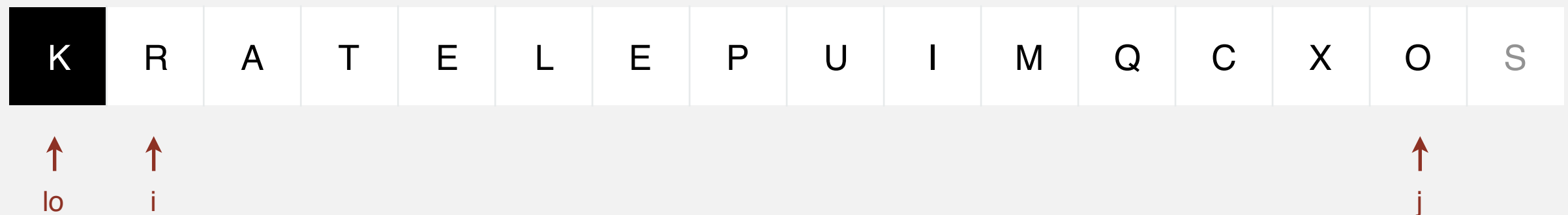
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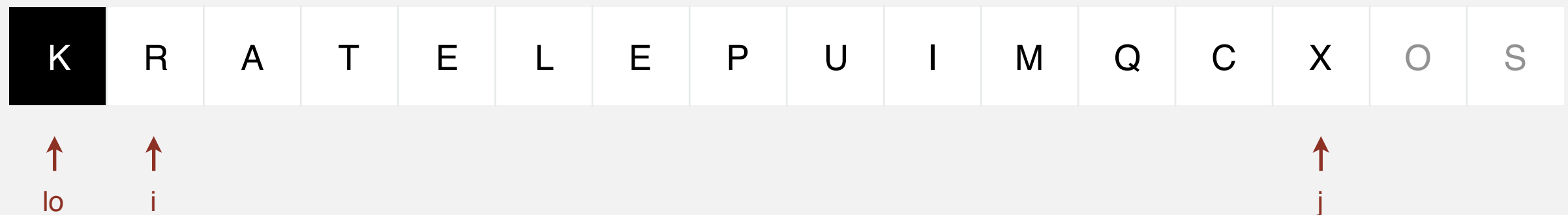


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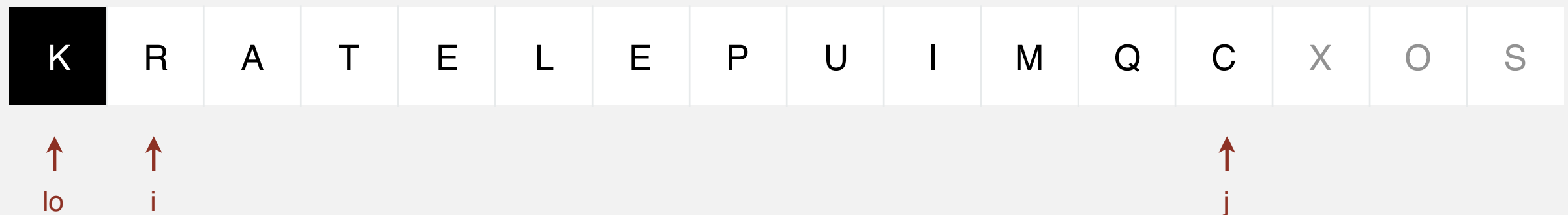
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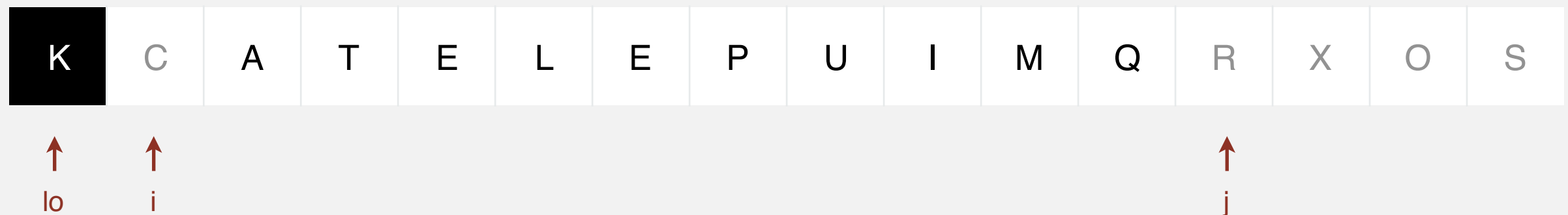


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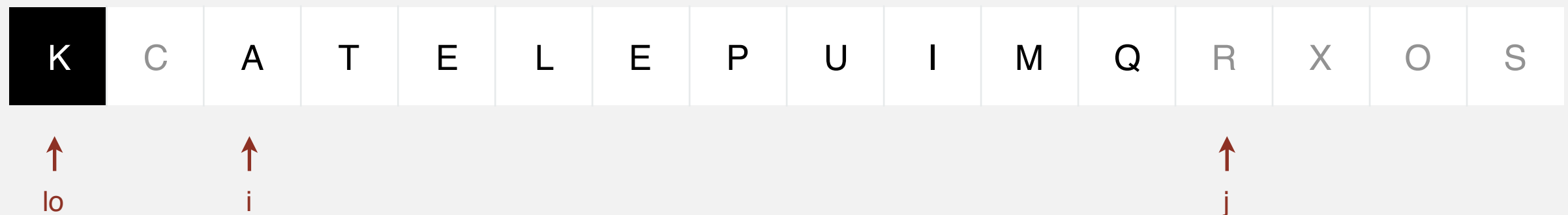
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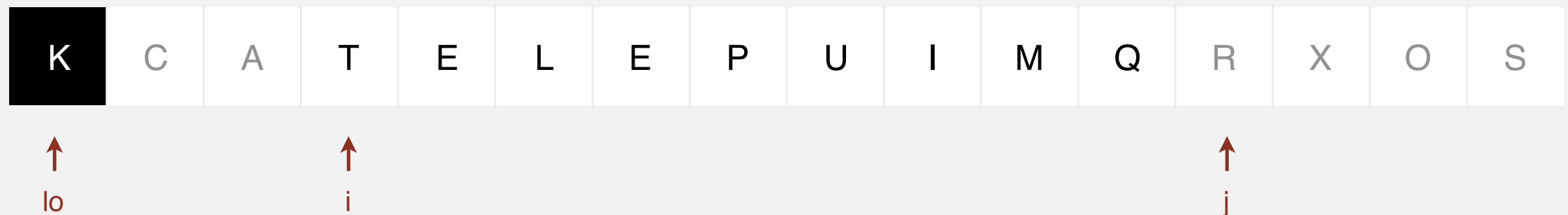
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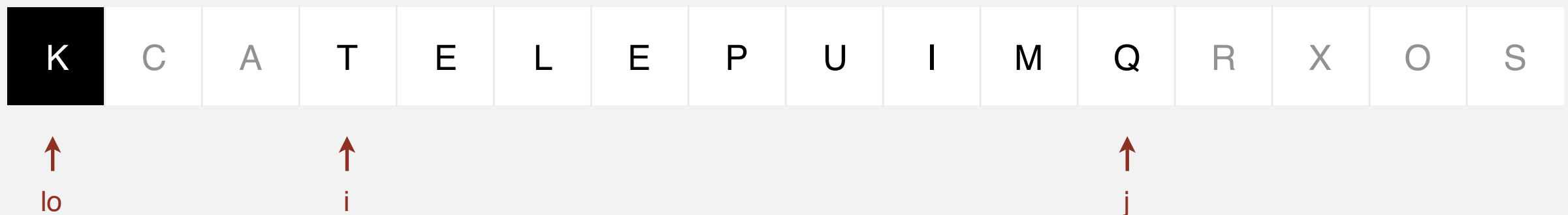


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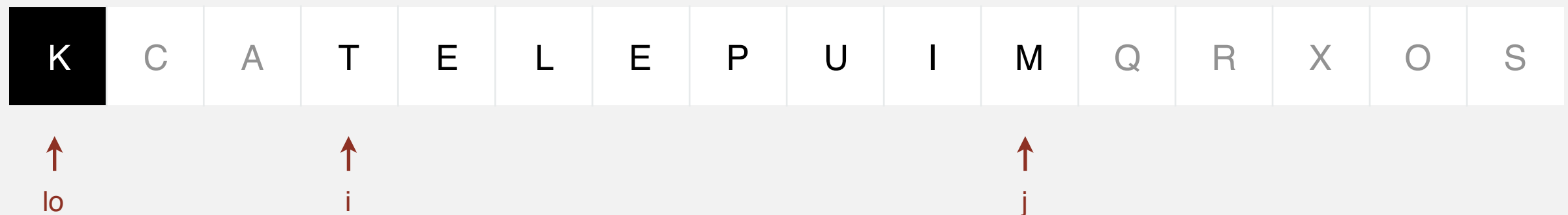
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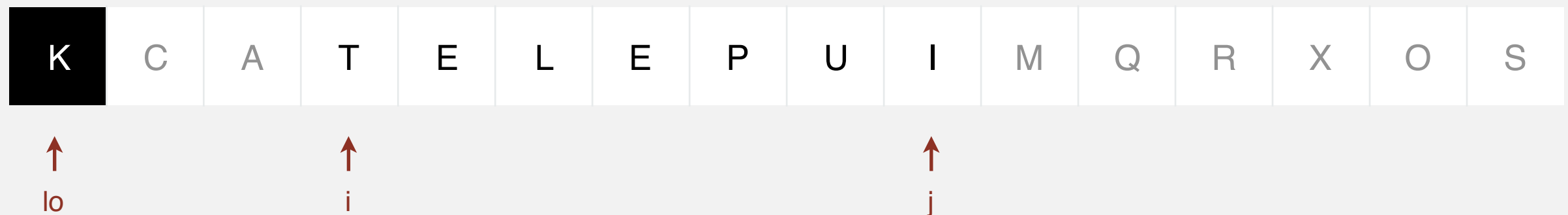
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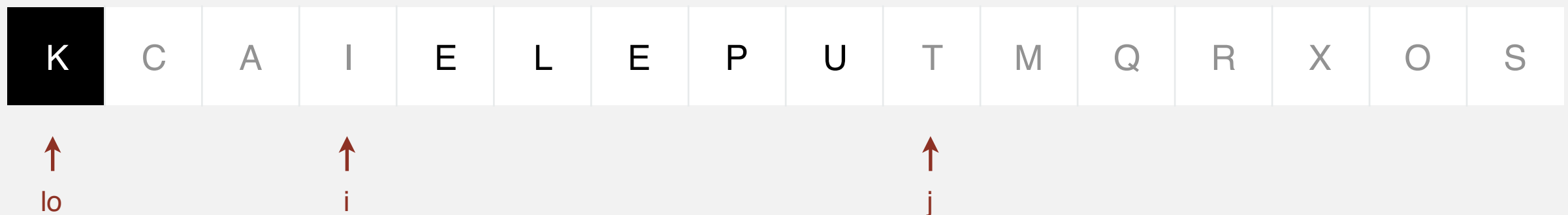


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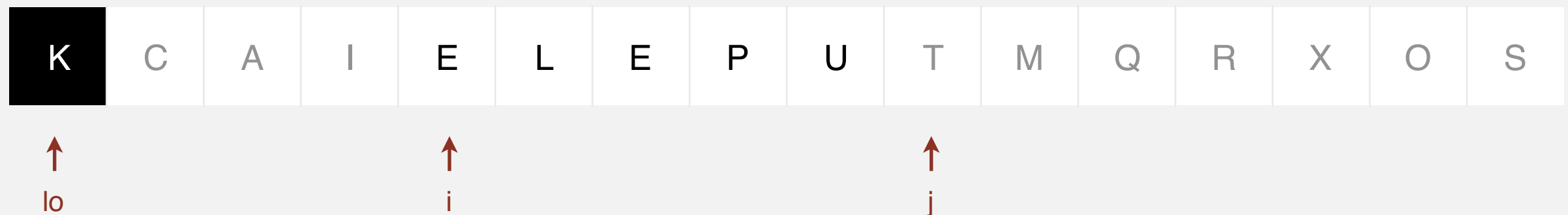
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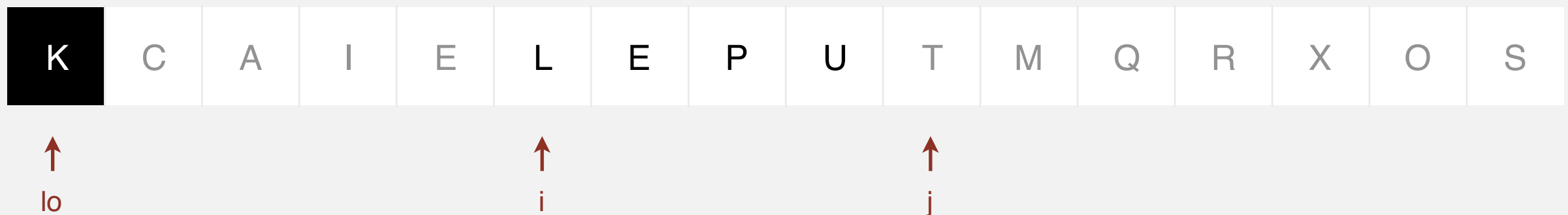
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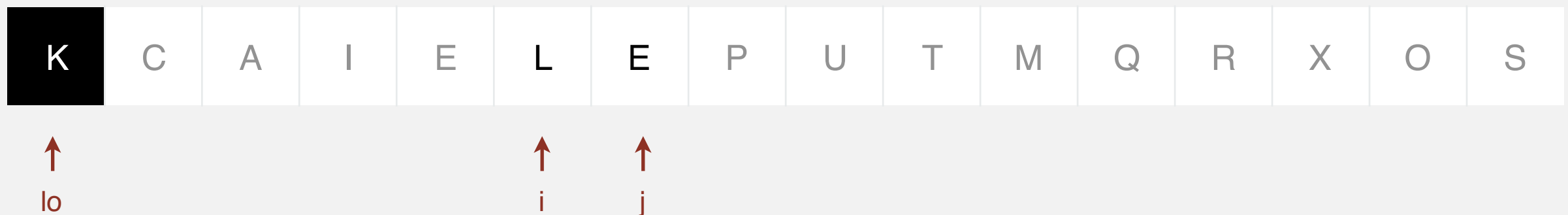
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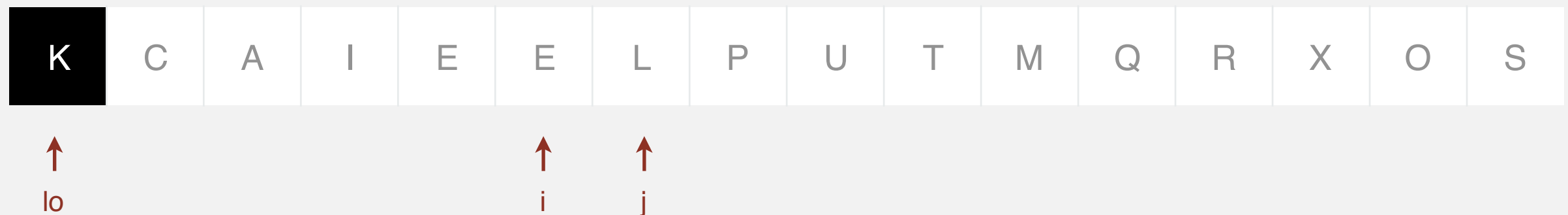


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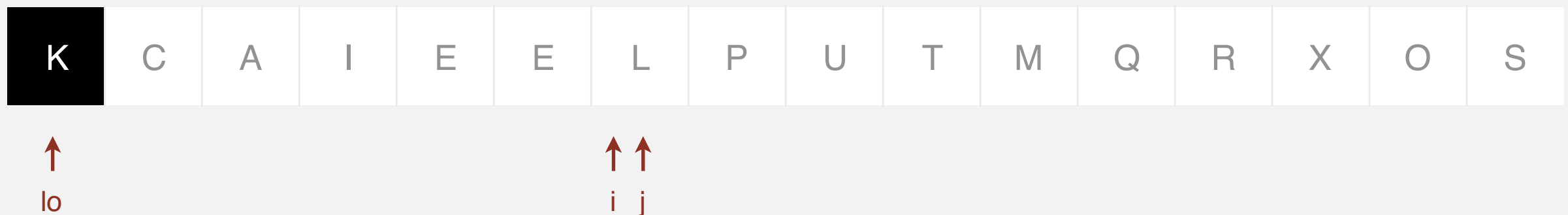
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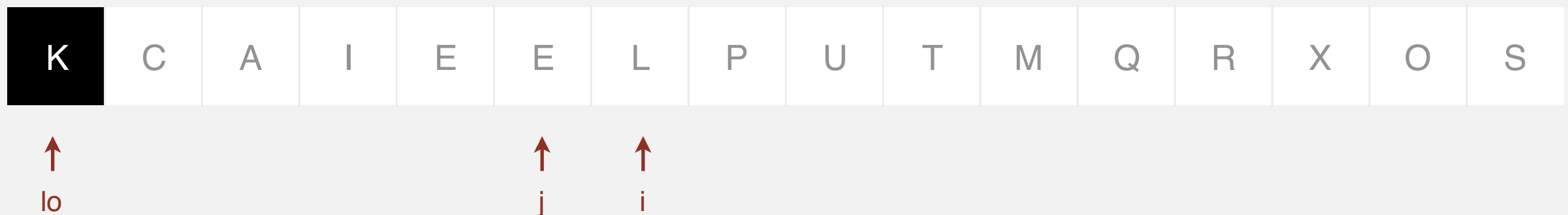


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stop j scan because $a[j] \leq v$

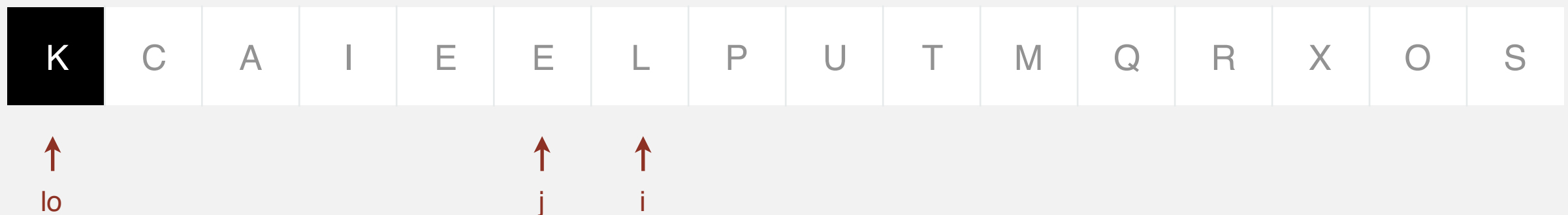
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- Scan i from left to right so long as $(a[i] < v)$.
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When pointers cross.

- Exchange $a[lo]$ with $a[j]$.



pointers cross: exchange $a[lo]$ with $a[j]$

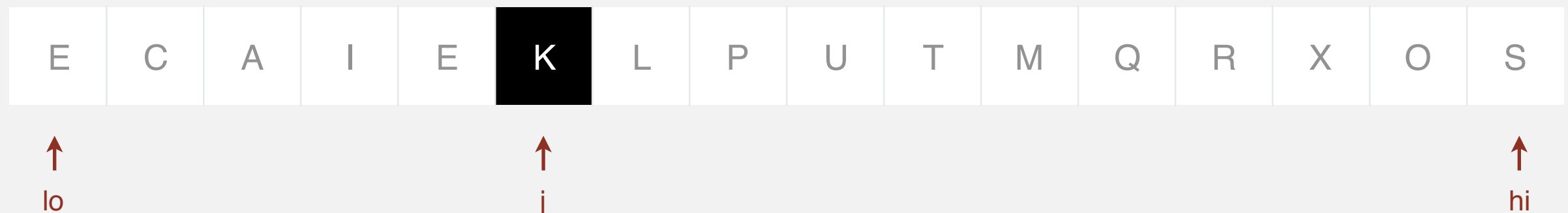
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partitioned!

Partition Algorithm: Java

```
private static int partition(Comparable[] a, int lo, int hi)
{ // Partition into a[lo..i-1], a[i], a[i+1..hi].
  int i = lo, j = hi+1;           // left and right scan indices
  Comparable v = a[lo];           // partitioning item
  while (true)
  { // Scan right, scan left, check for scan complete, and exchange.
    while (less(a[++i], v)) if (i == hi) break;
    while (less(v, a[--j])) if (j == lo) break;
    if (i >= j) break;
    exch(a, i, j);
  }
  exch(a, lo, j);                 // Put v = a[j] into position
  return j;                       // with a[lo..j-1] <= a[j] <= a[j+1..hi].
}
```



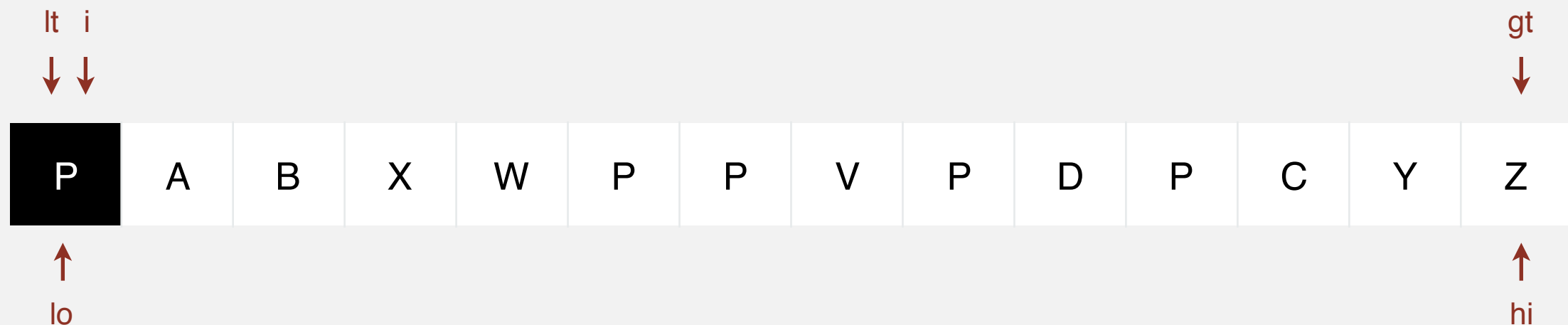

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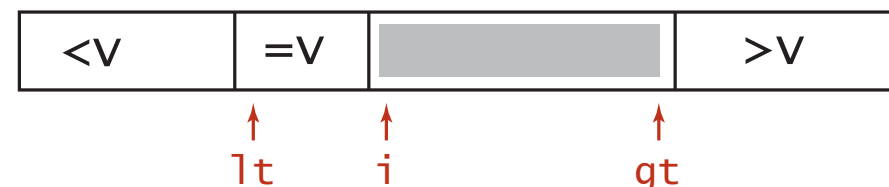
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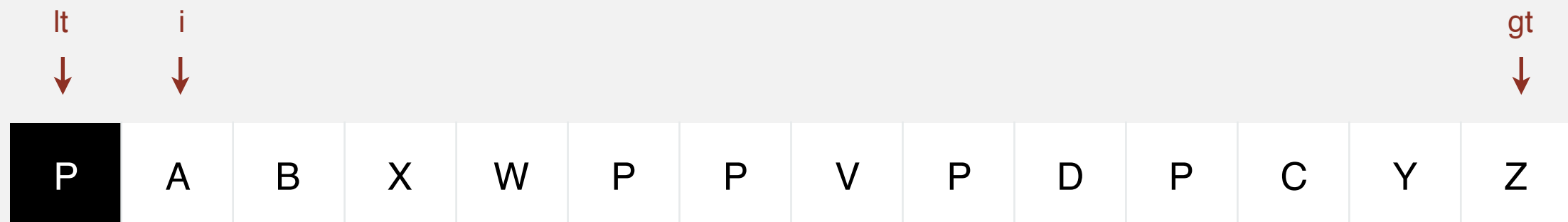


invariant

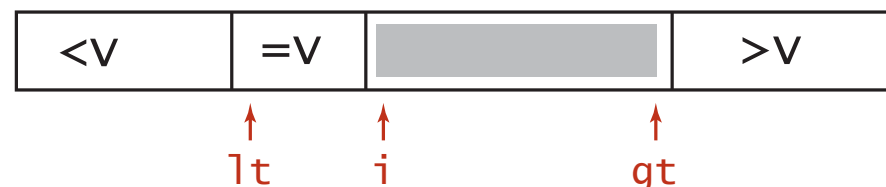


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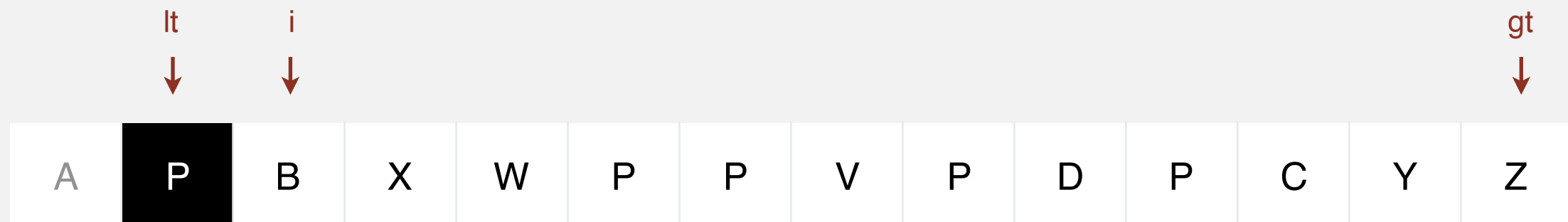


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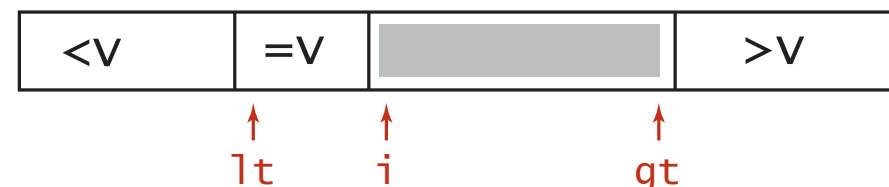


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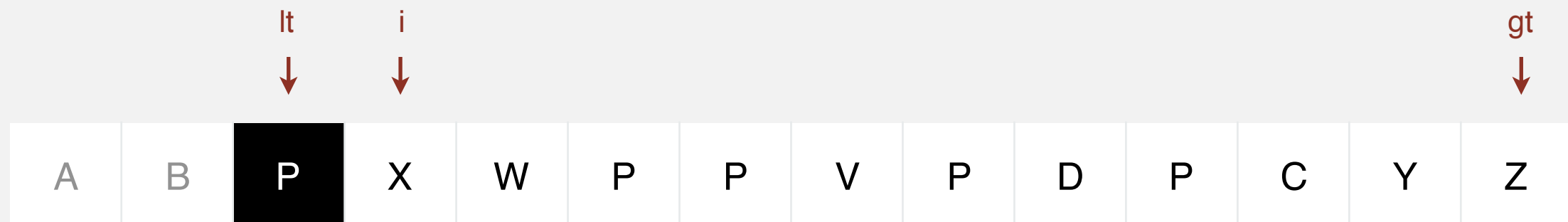


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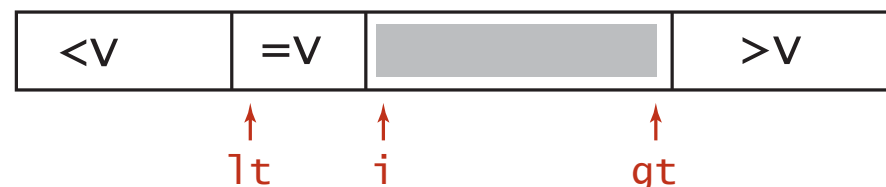


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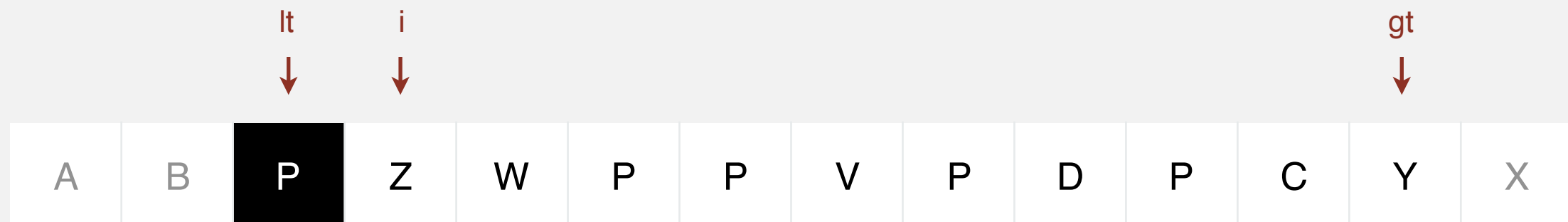


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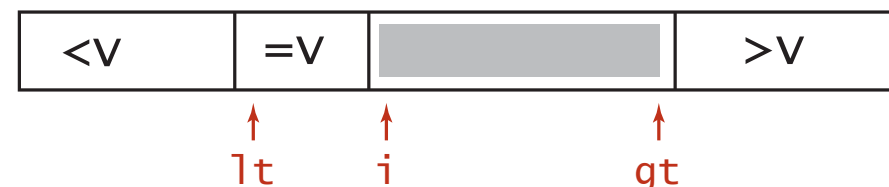


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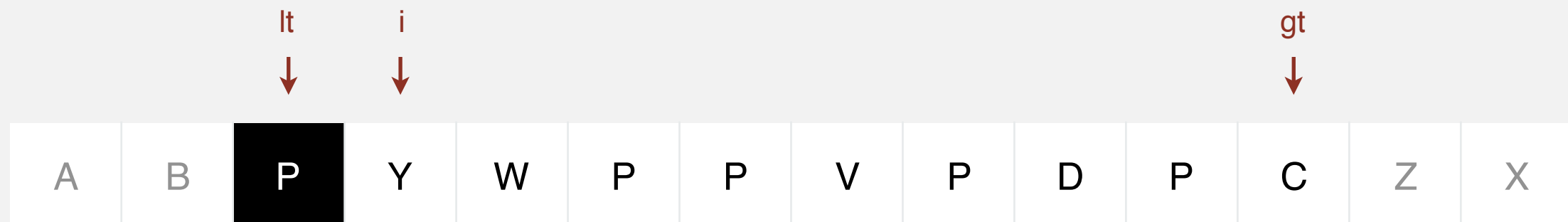


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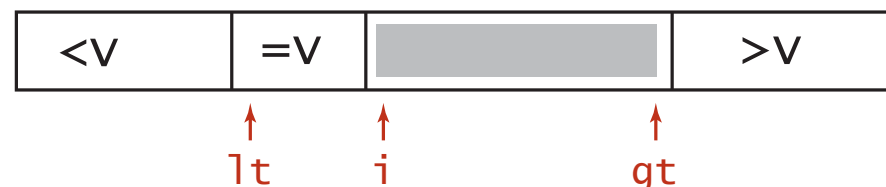


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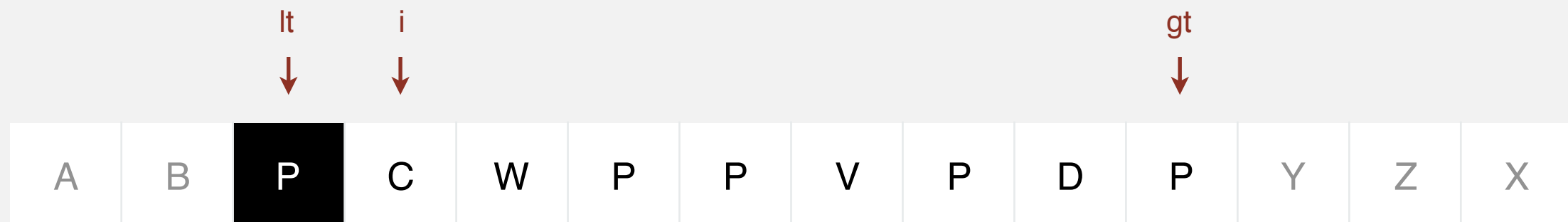


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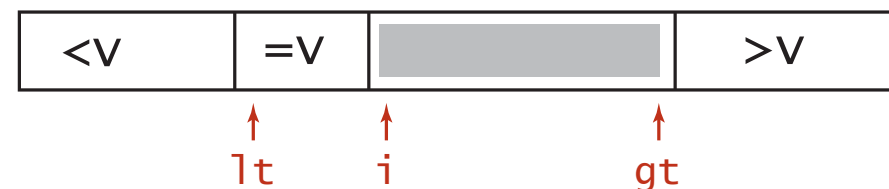


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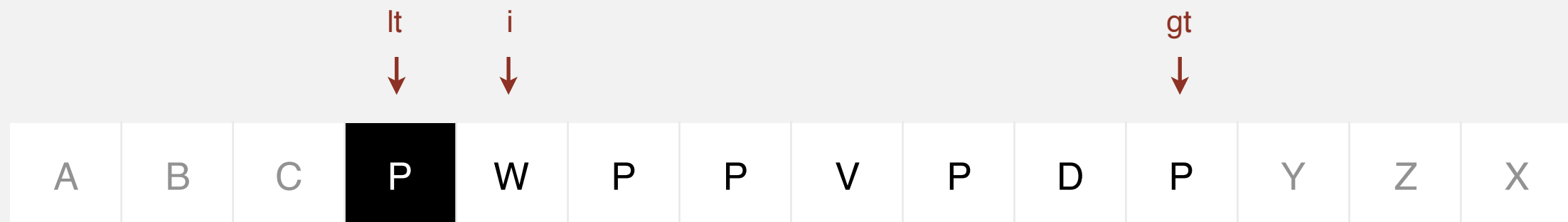


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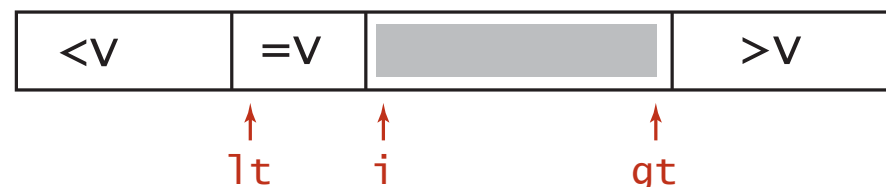


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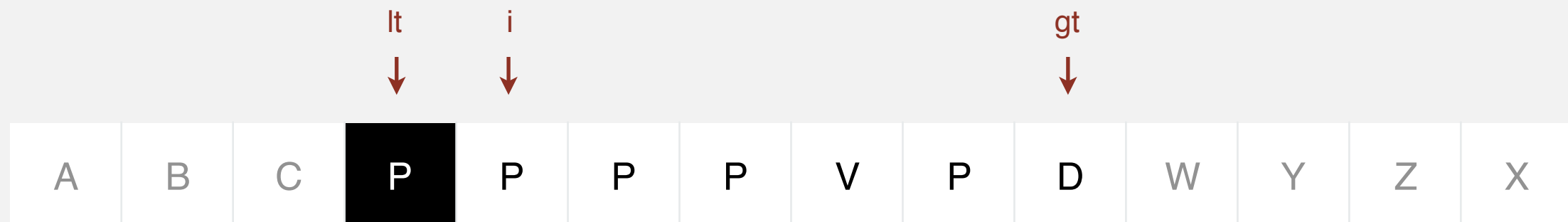


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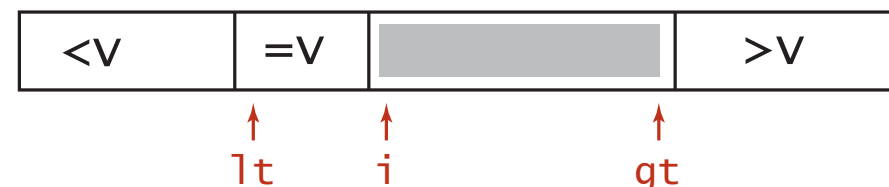


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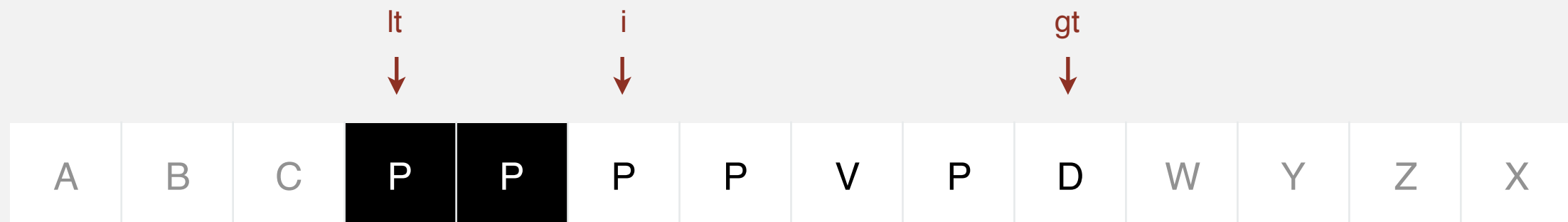


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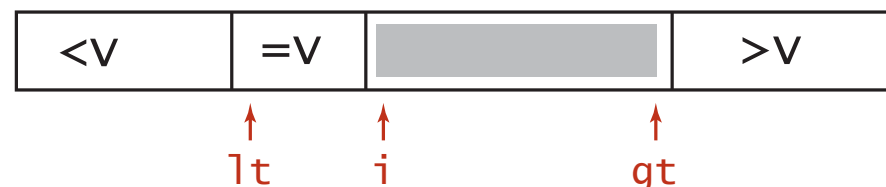


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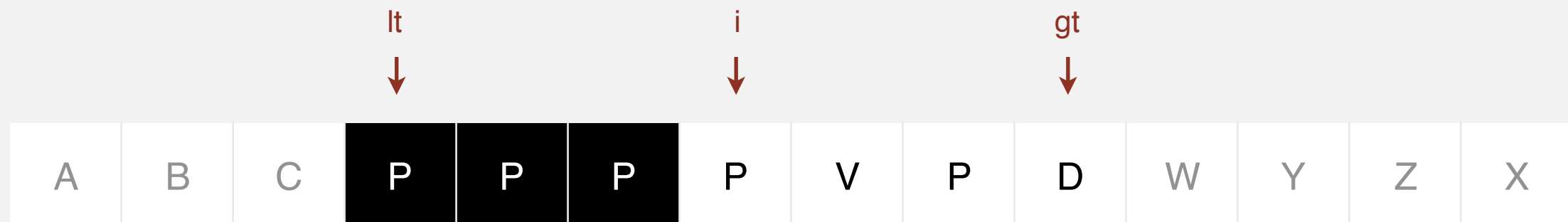


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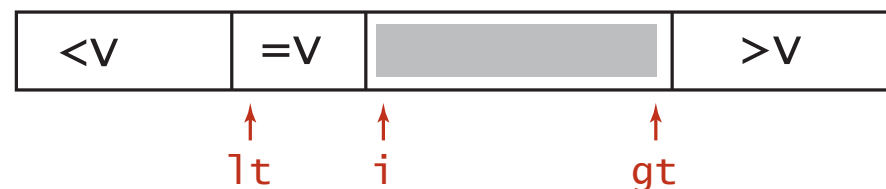


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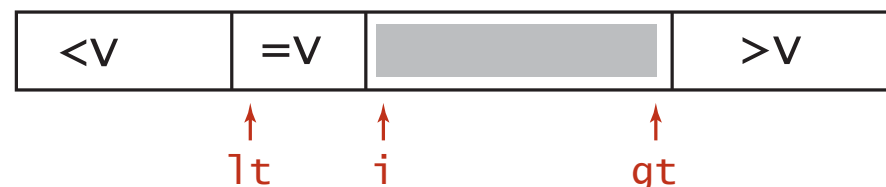


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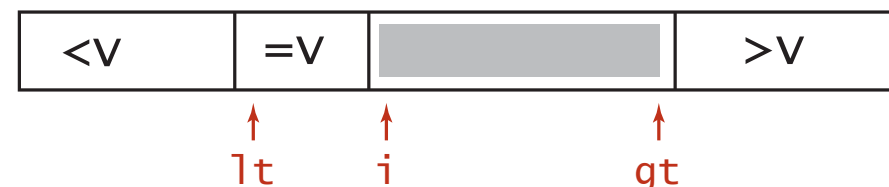


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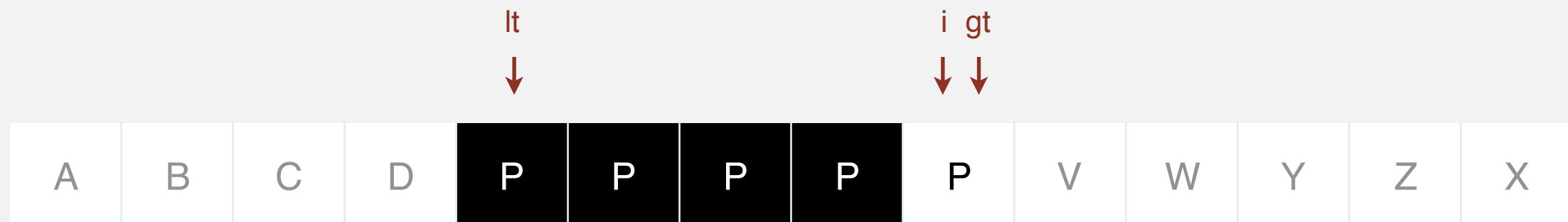


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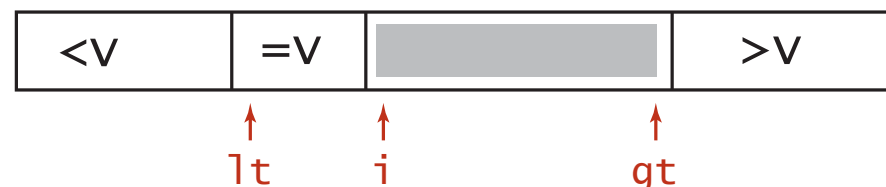


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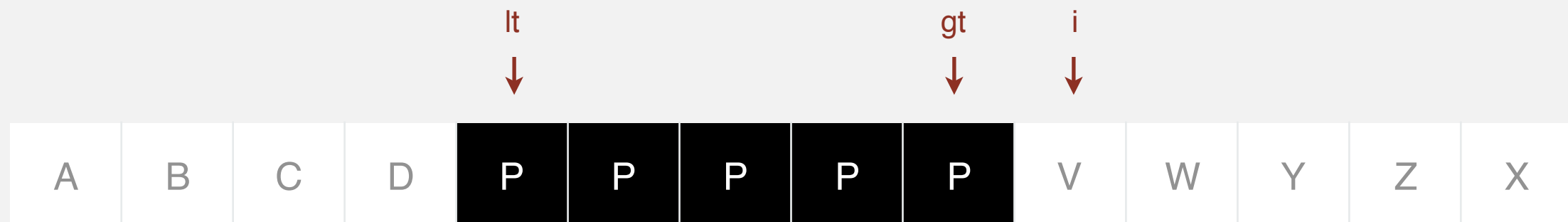


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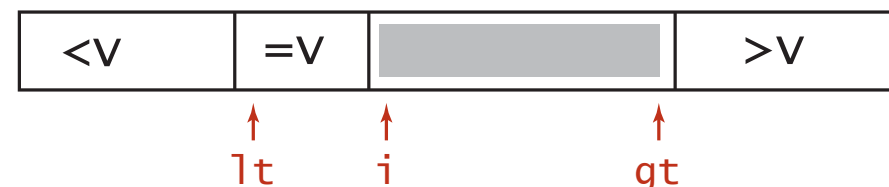


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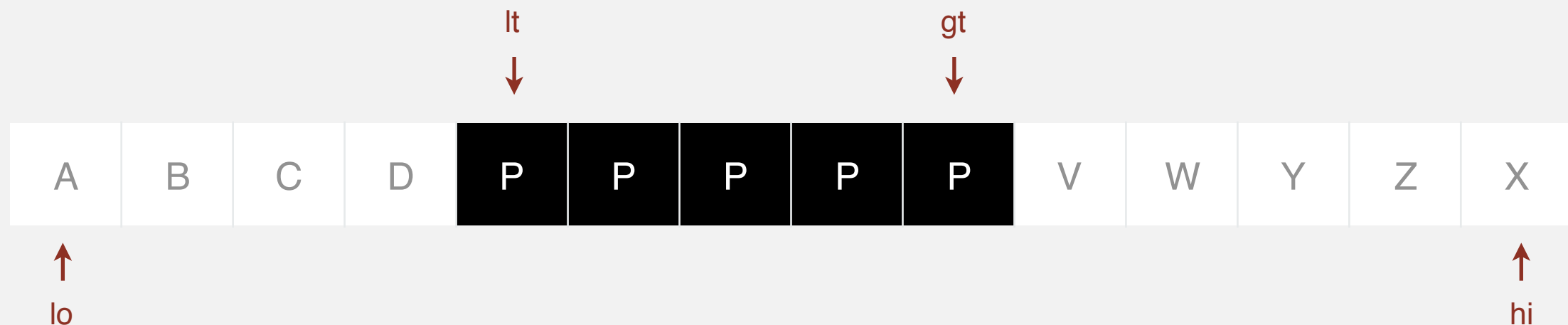


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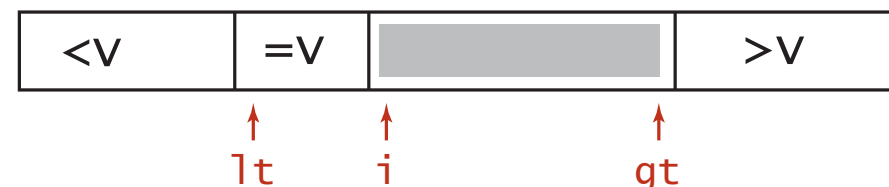


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invariant





<http://algs4.cs.princeton.edu>

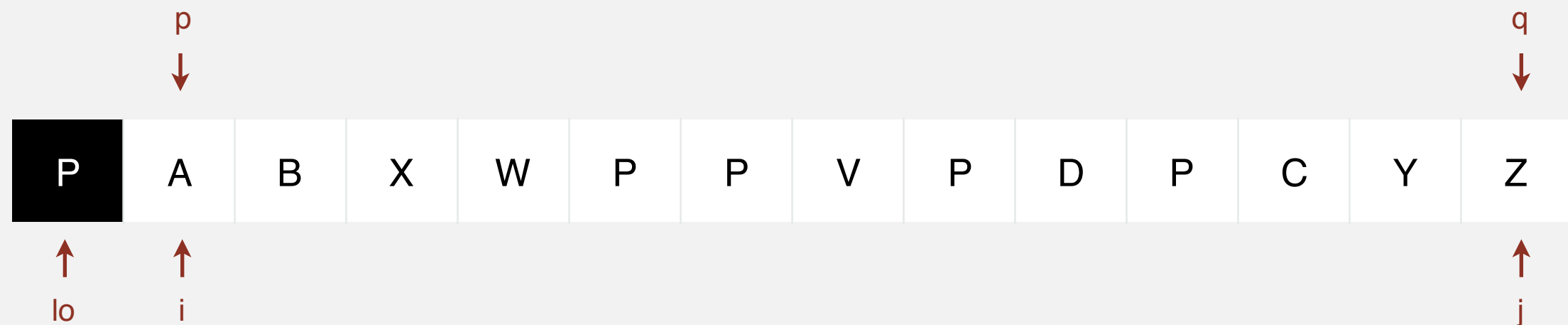
2.3 PARTITIONING DEMOS

- *Sedgewick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

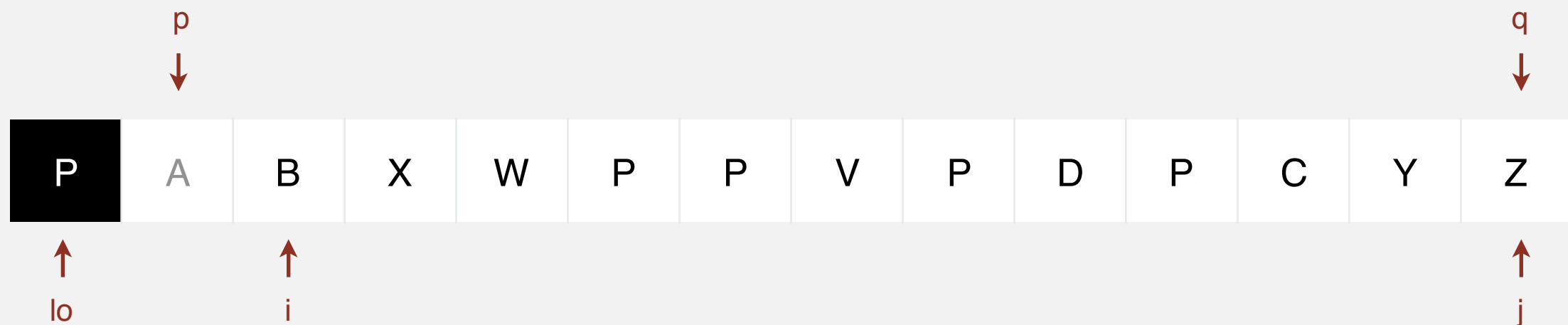
- Scan i from left to right so long as $(a[i] < a[lo])$.
- Scan j from right to left so long as $(a[j] > a[lo])$.
- Exchange $a[i]$ with $a[j]$.
- If $(a[i] == a[lo])$, exchange $a[i]$ with $a[p]$ and increment p .
- If $(a[j] == a[lo])$, exchange $a[j]$ with $a[q]$ and decrement q .



Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

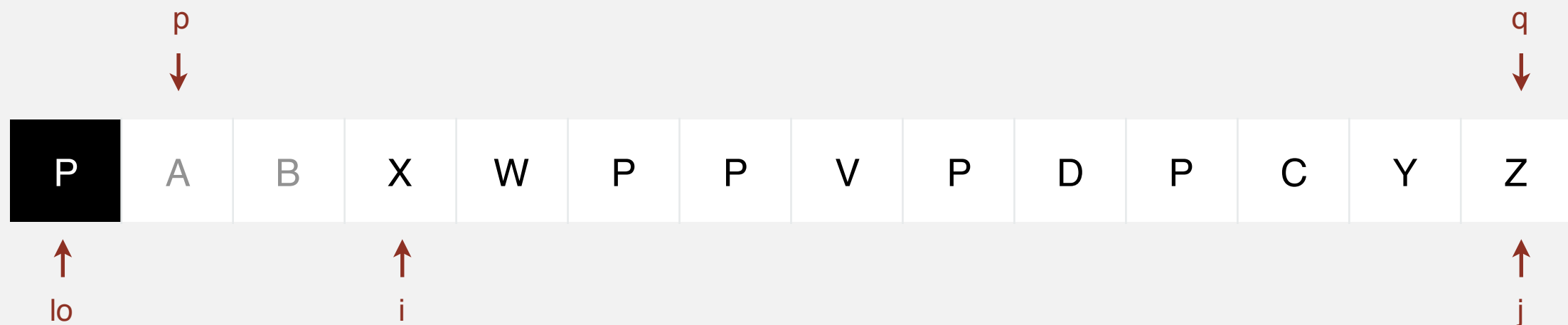
- Scan i from left to right so long as $(a[i] < a[lo])$.
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- Exchange $a[i]$ with $a[j]$.
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Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

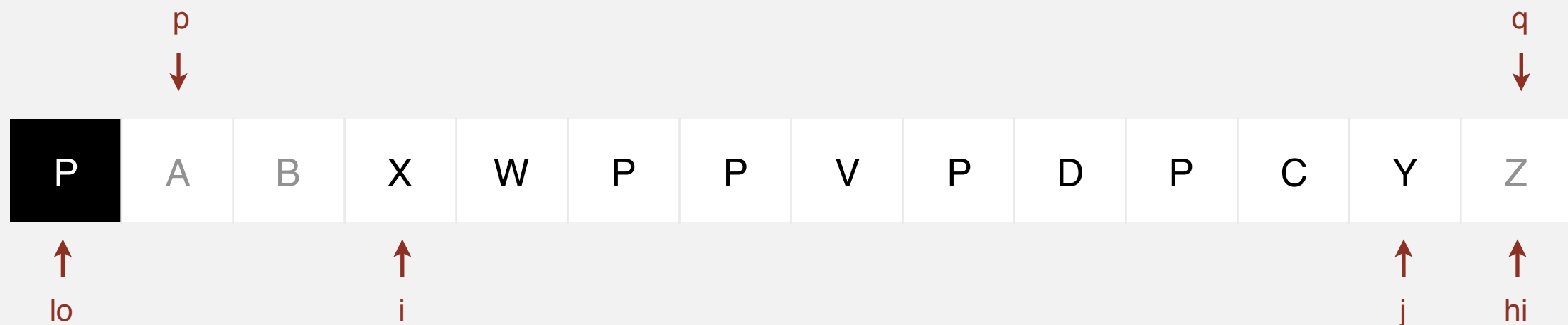
- Scan i from left to right so long as $(a[i] < a[lo])$.
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Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

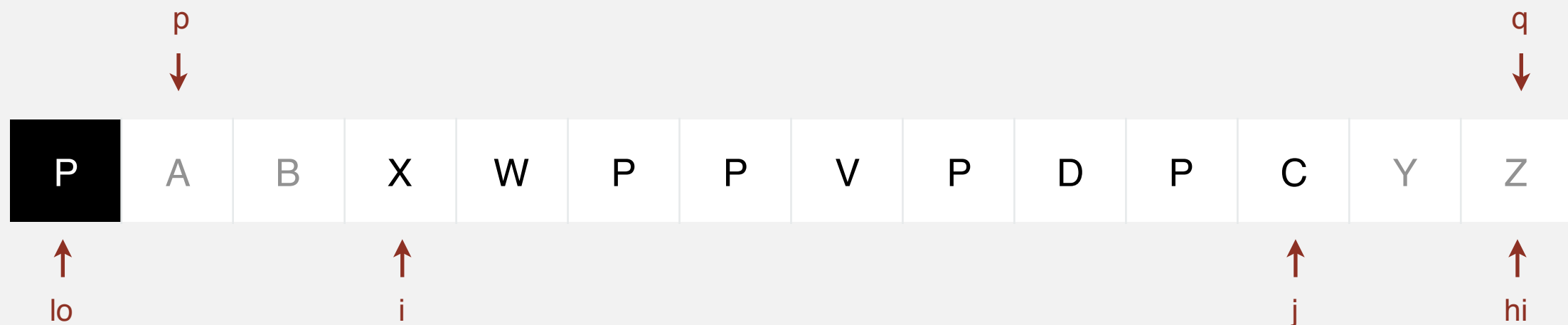
- Scan i from left to right so long as $(a[i] < a[lo])$.
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Bentley-McIlroy 3-way partitioning demo

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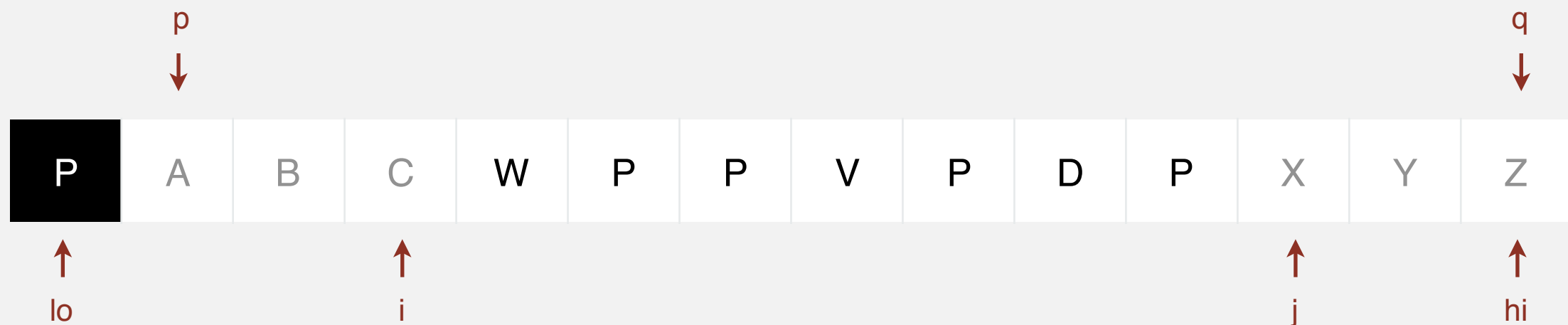


exchange $a[i]$ with $a[j]$

Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

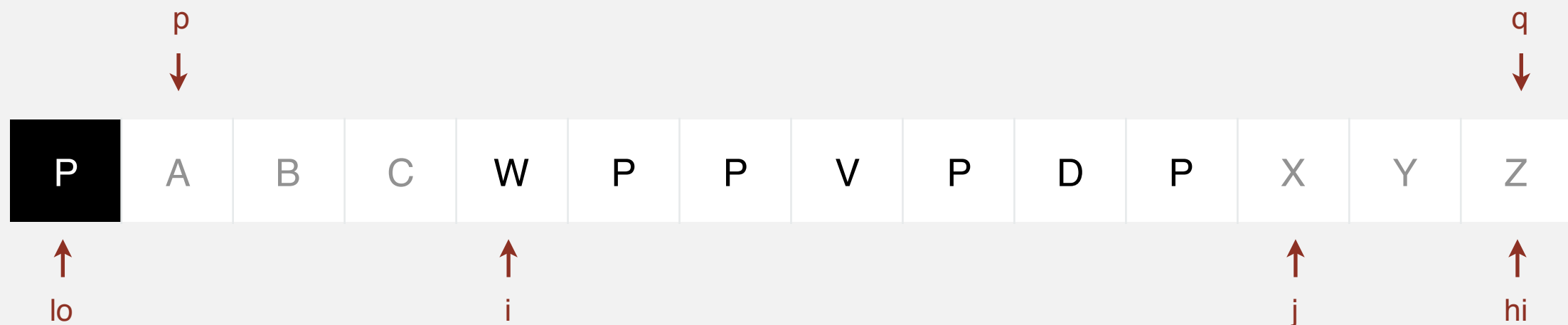
- Scan i from left to right so long as $(a[i] < a[lo])$.
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Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

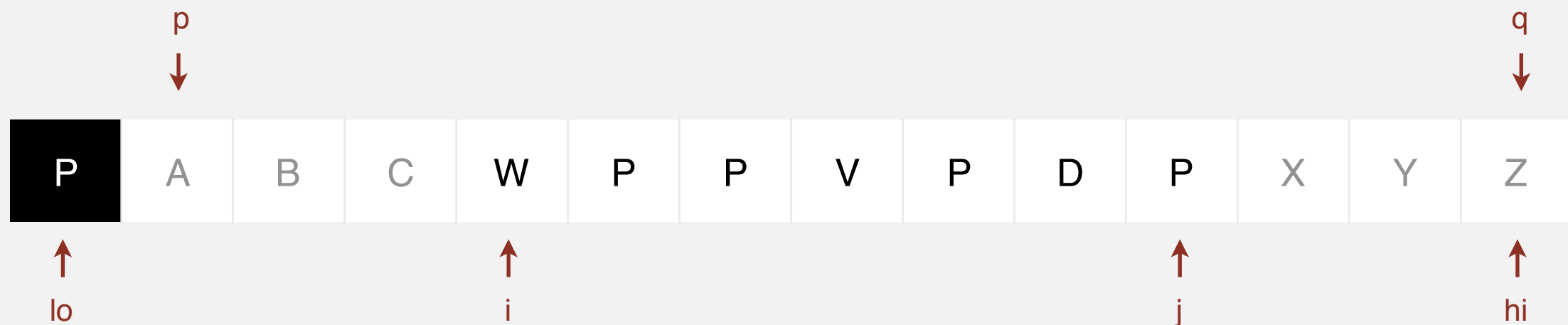
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Bentley-McIlroy 3-way partitioning demo

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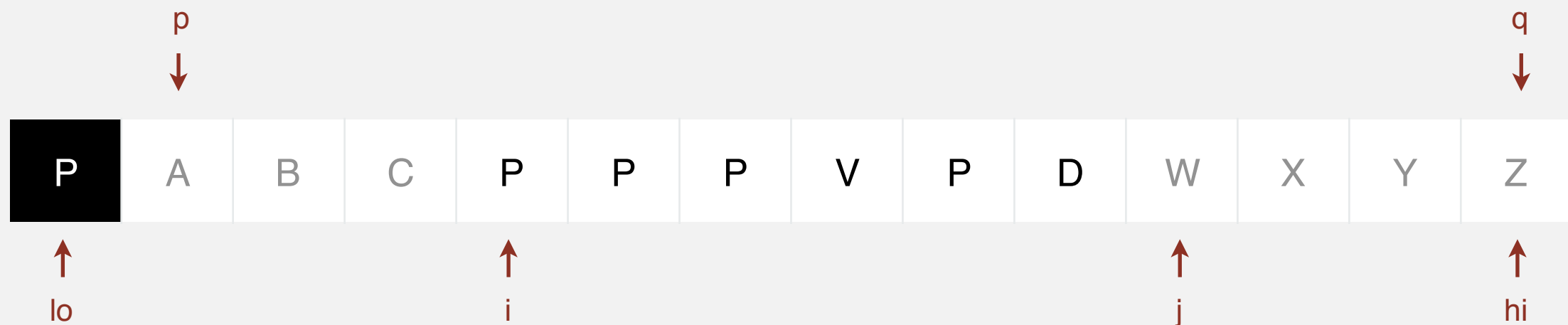


exchange $a[i]$ with $a[j]$

Bentley-McIlroy 3-way partitioning demo

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exchange $a[i]$ with $a[p]$ and increment p

Bentley-McIlroy 3-way partitioning demo

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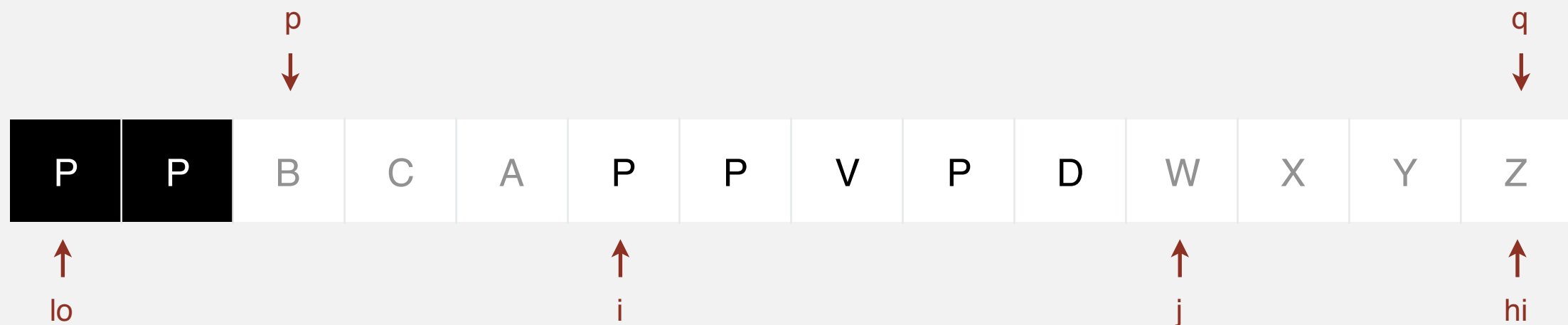
- Scan i from left to right so long as $(a[i] < a[lo])$.
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Bentley-McIlroy 3-way partitioning demo

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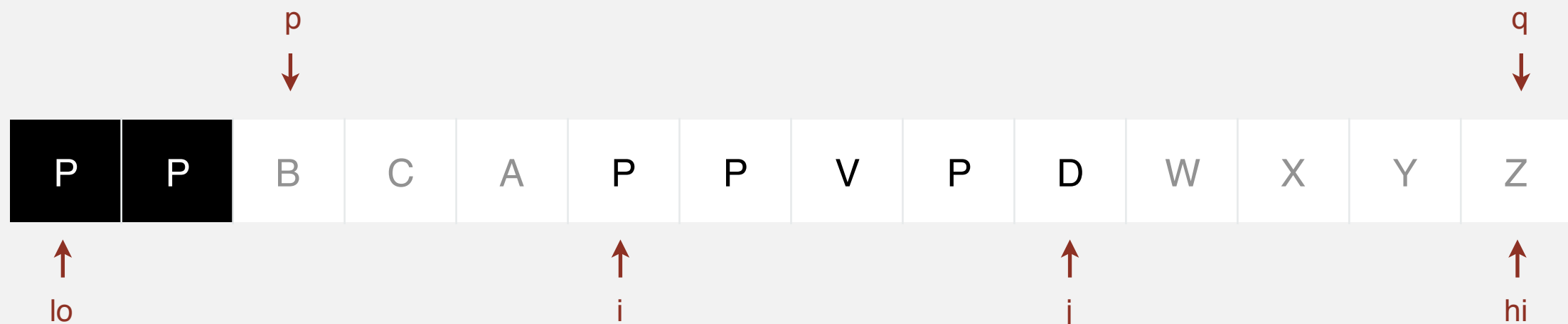
- Scan i from left to right so long as $(a[i] < a[lo])$.
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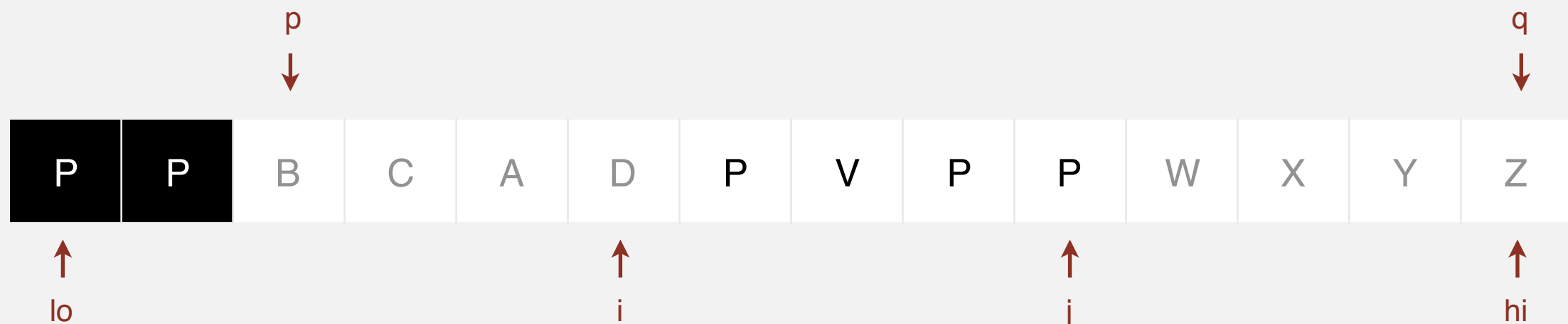


exchange $a[i]$ with $a[j]$

Bentley-McIlroy 3-way partitioning demo

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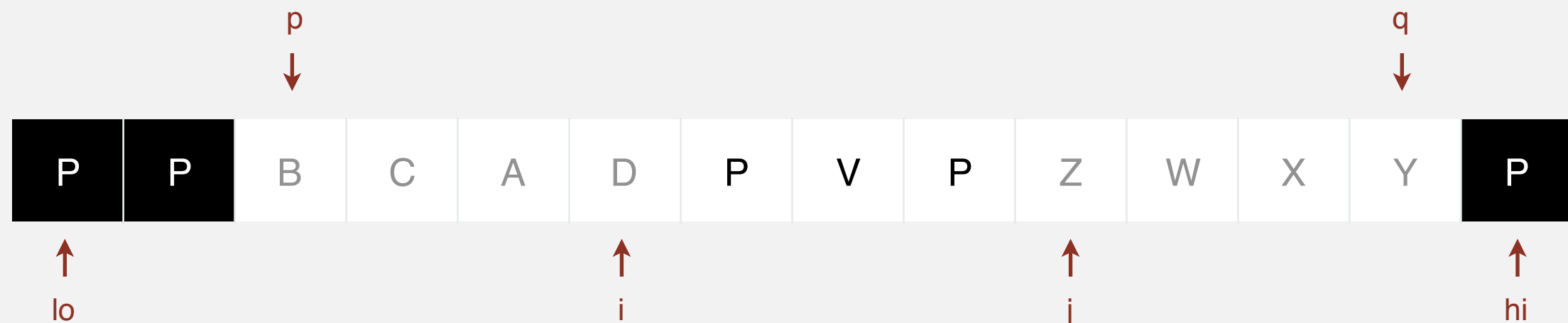


exchange $a[j]$ with $a[q]$ and decrement q

Bentley-McIlroy 3-way partitioning demo

Phase I. Repeat until i and j pointers cross.

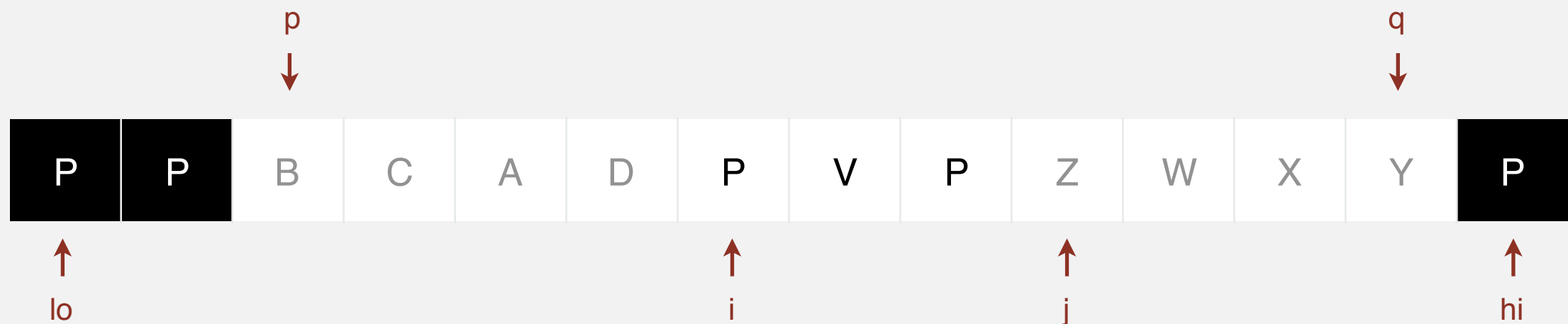
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Bentley-McIlroy 3-way partitioning demo

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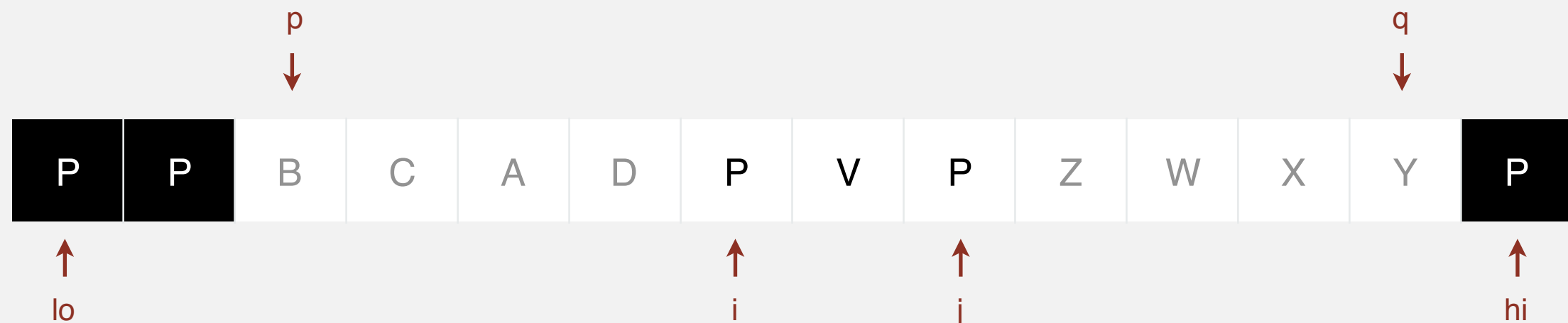
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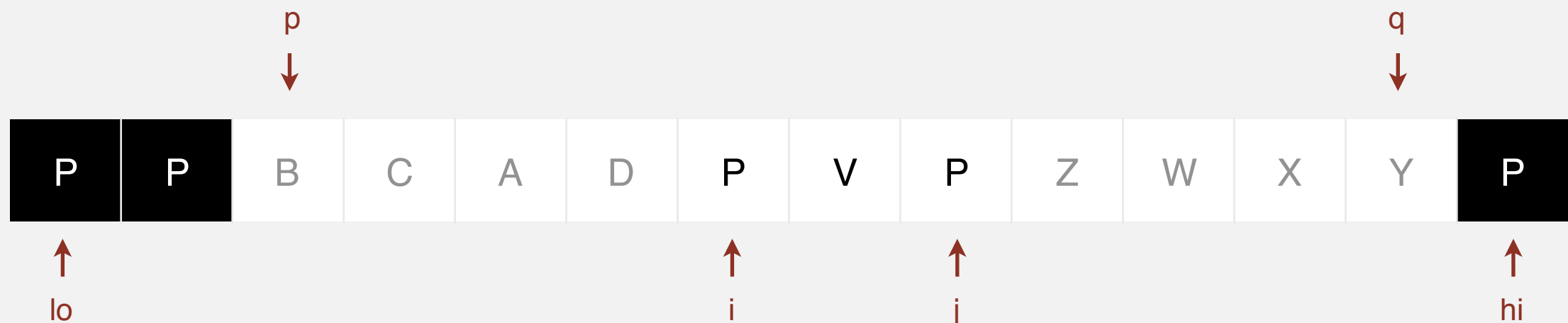


exchange $a[i]$ with $a[j]$

Bentley-McIlroy 3-way partitioning demo

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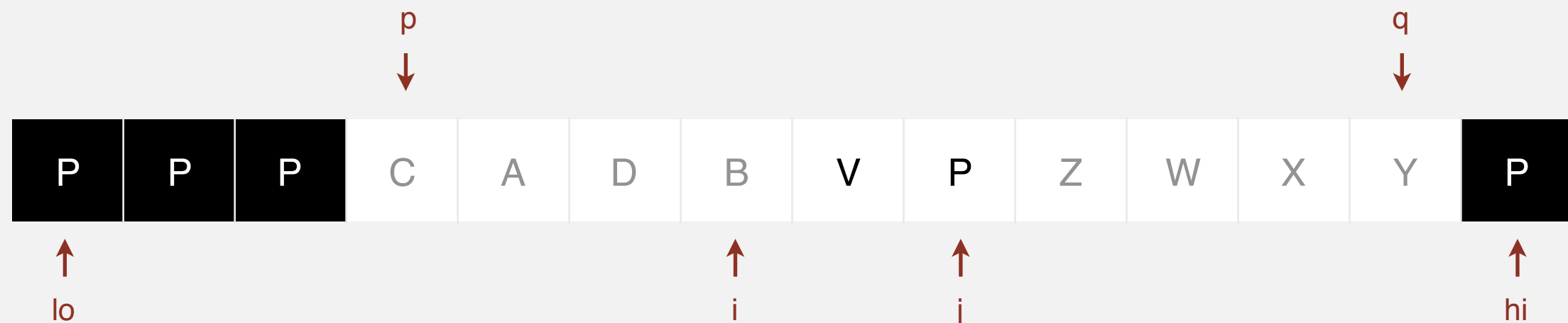


exchange $a[i]$ with $a[p]$ and increment p

Bentley-McIlroy 3-way partitioning demo

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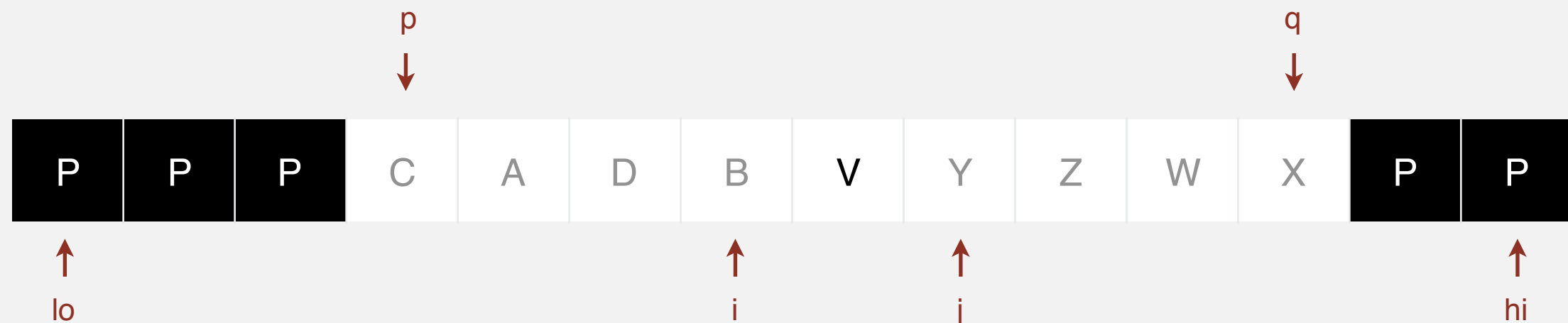


exchange $a[j]$ with $a[q]$ and decrement q

Bentley-McIlroy 3-way partitioning demo

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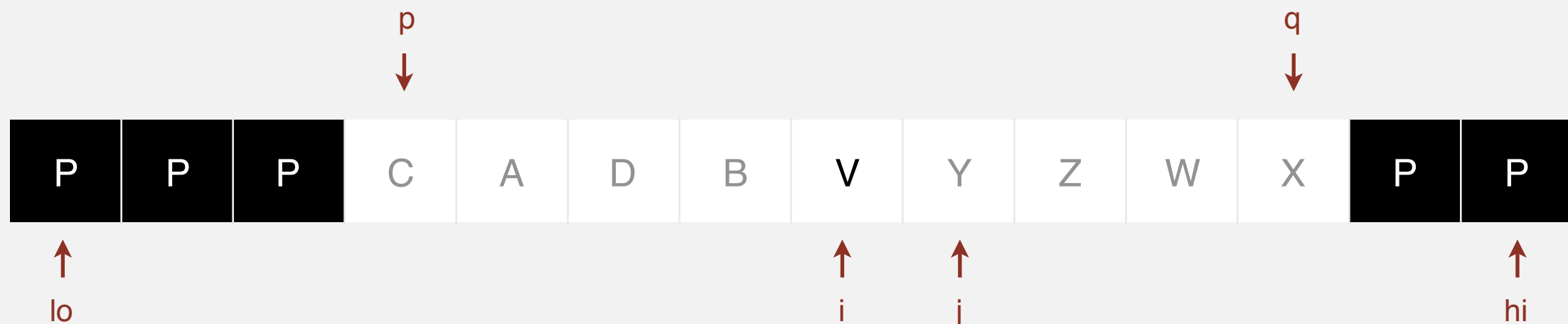
- Scan i from left to right so long as $(a[i] < a[lo])$.
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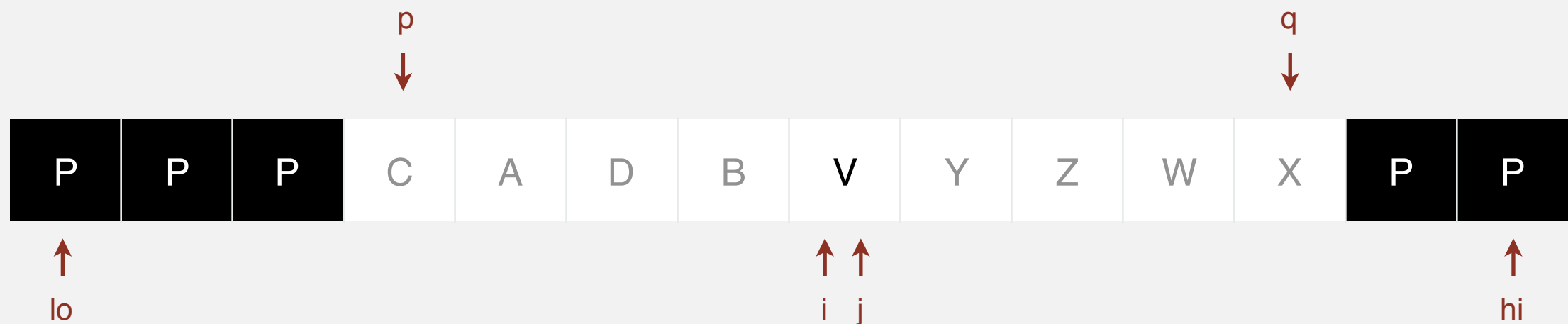
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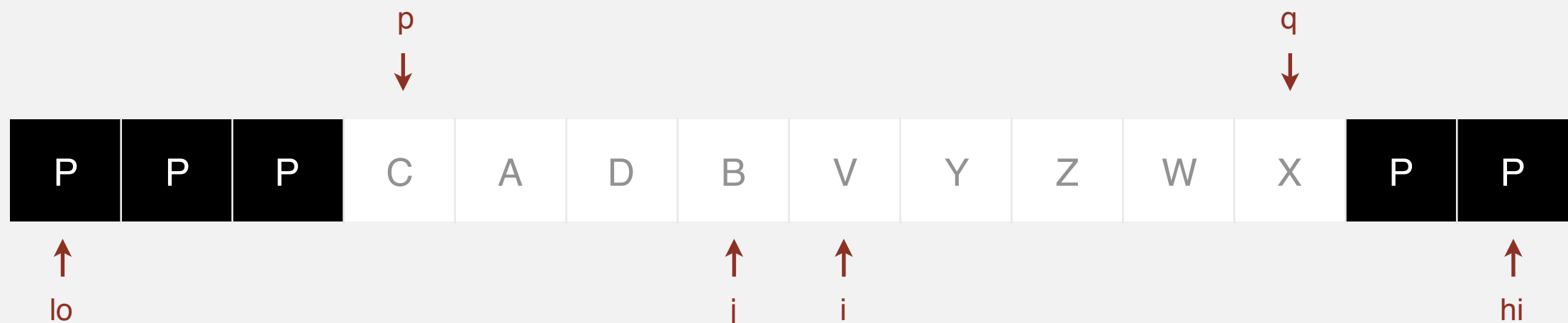
- Scan i from left to right so long as $(a[i] < a[lo])$.
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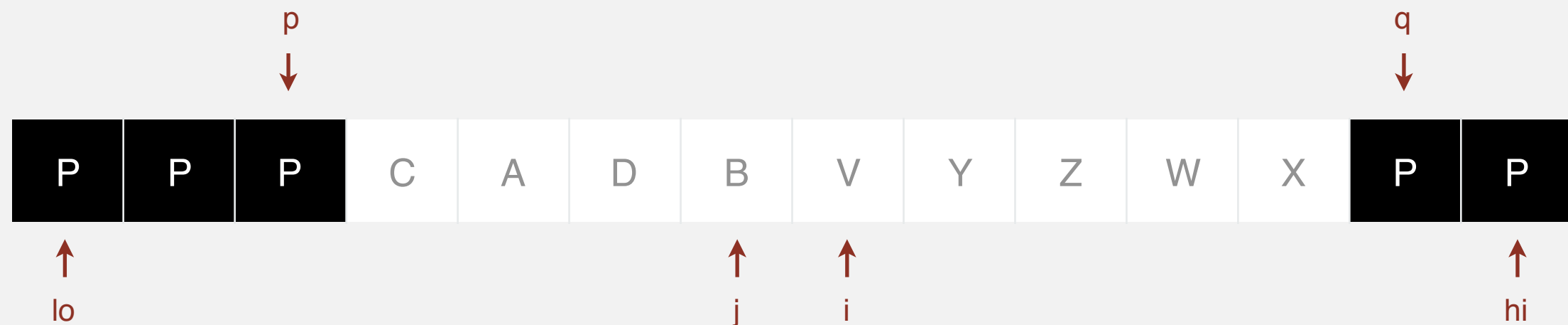


pointers cross

Bentley-McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan j and p from right to left and exchange $a[j]$ with $a[p]$.
- Scan i and q from left to right and exchange $a[i]$ with $a[q]$.

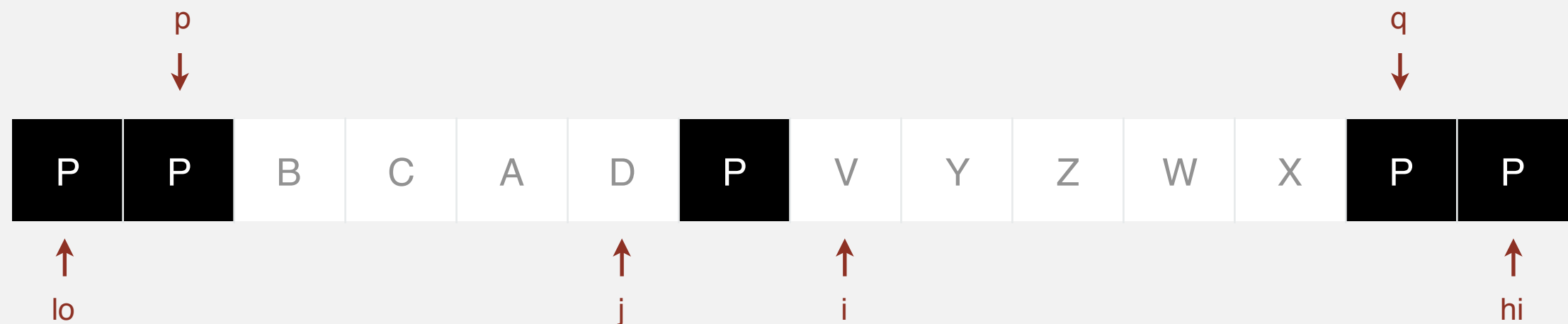


exchange $a[j]$ with $a[p]$

Bentley-McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan j and p from right to left and exchange $a[j]$ with $a[p]$.
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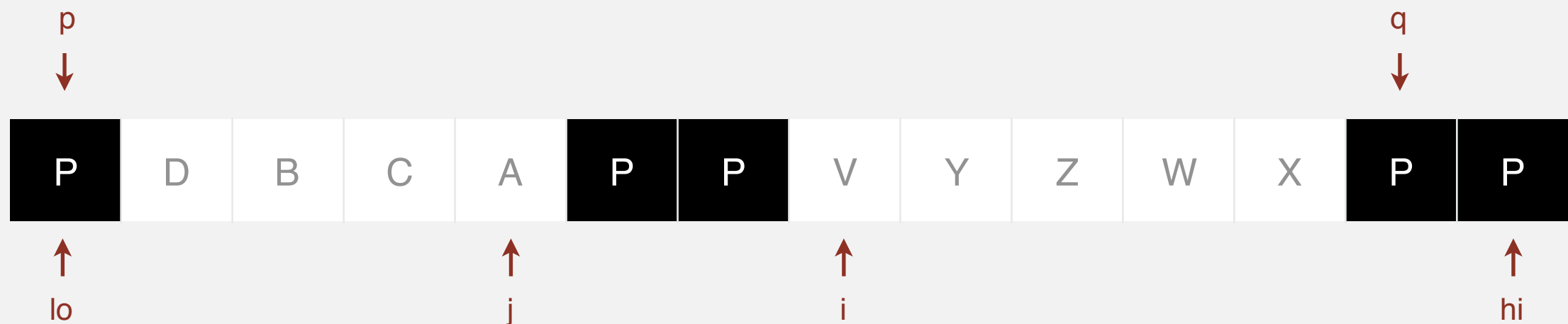


exchange $a[j]$ with $a[p]$

Bentley-McIlroy 3-way partitioning demo

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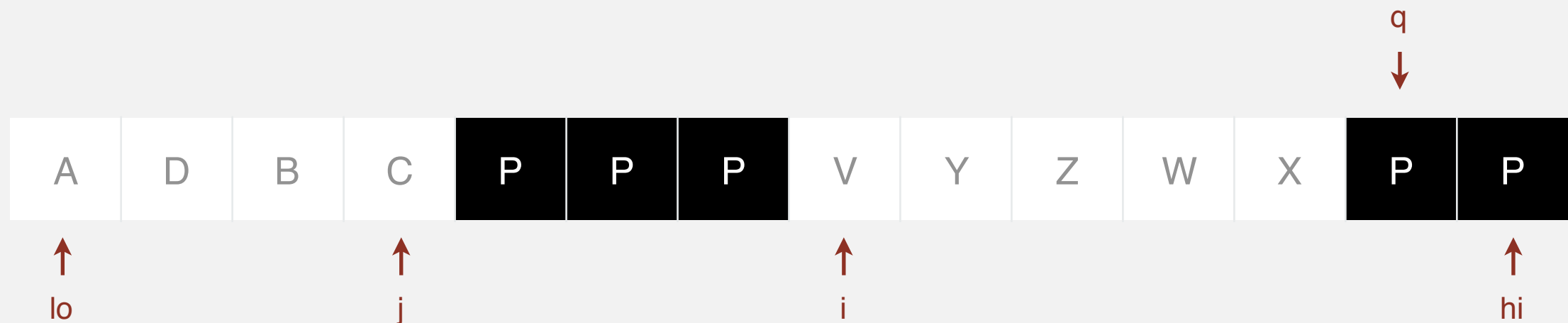


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Bentley-McIlroy 3-way partitioning demo

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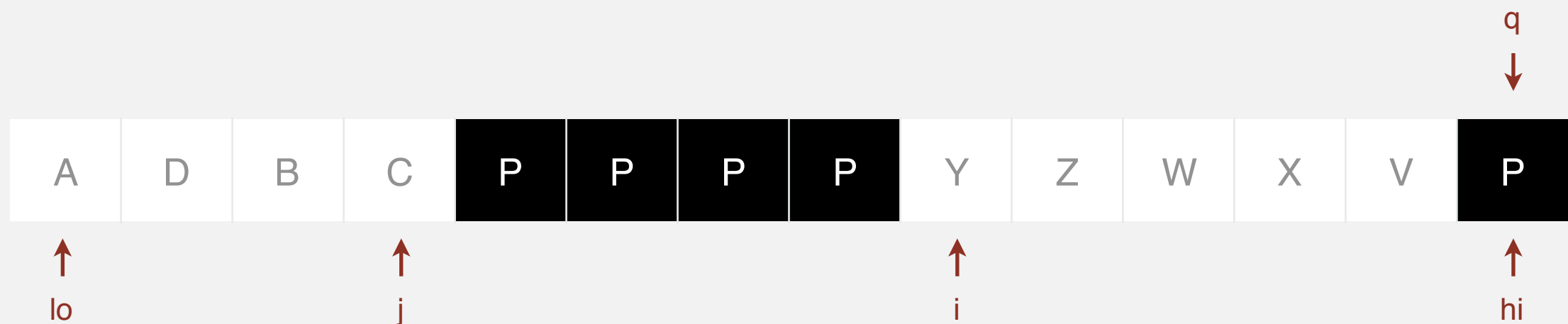


exchange $a[i]$ with $a[q]$

Bentley-McIlroy 3-way partitioning demo

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Bentley-McIlroy 3-way partitioning demo

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3-way partitioned



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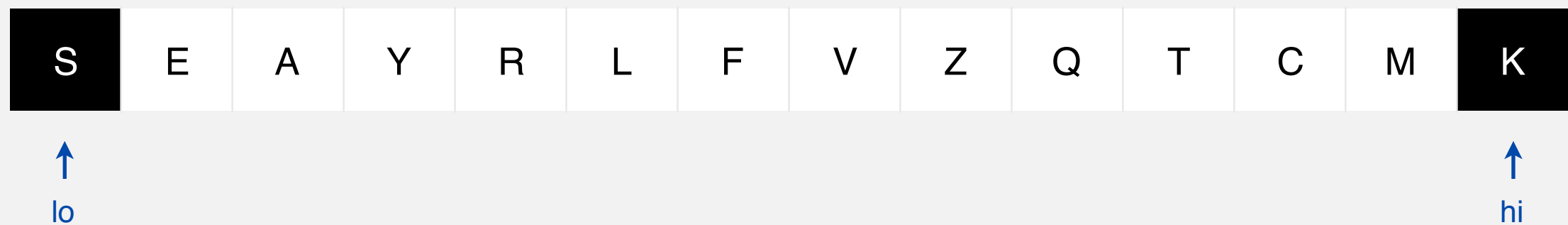
2.3 PARTITIONING DEMOS

- *Sedgwick 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley-McIlroy 3-way partitioning*
- *dual-pivot partitioning*

Dual-pivot partitioning demo

Initialization.

- Choose $a[\text{lo}]$ and $a[\text{hi}]$ as partitioning items.
- Exchange if necessary to ensure $a[\text{lo}] \leq a[\text{hi}]$.

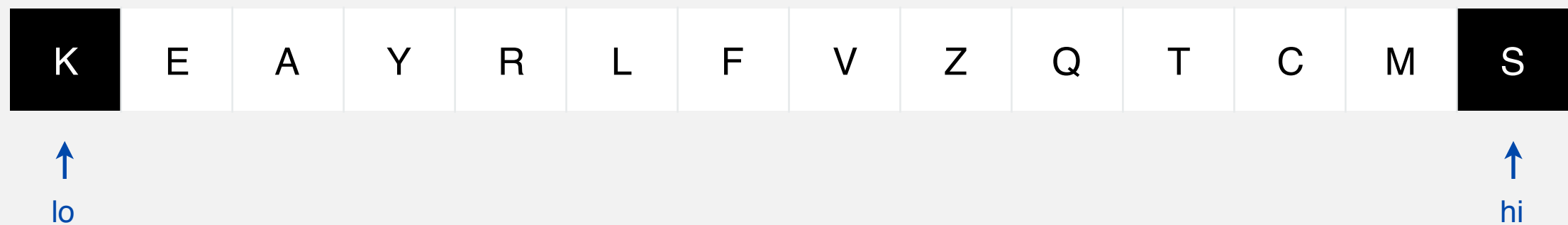


exchange $a[\text{lo}]$ and $a[\text{hi}]$

Dual-pivot partitioning demo

Initialization.

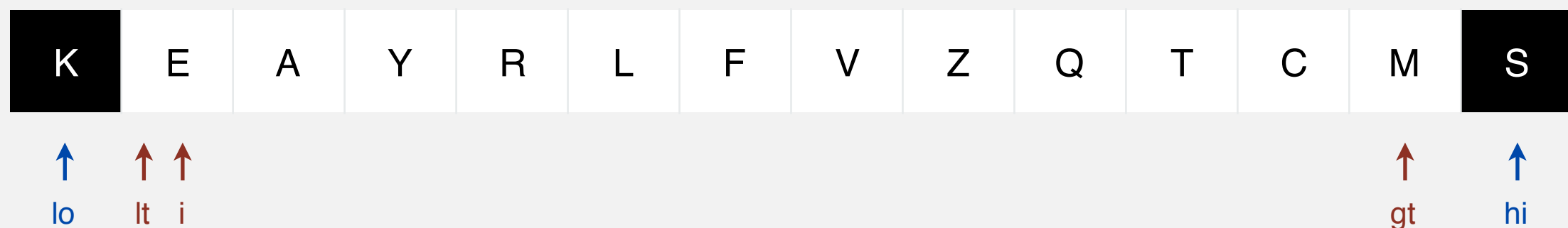
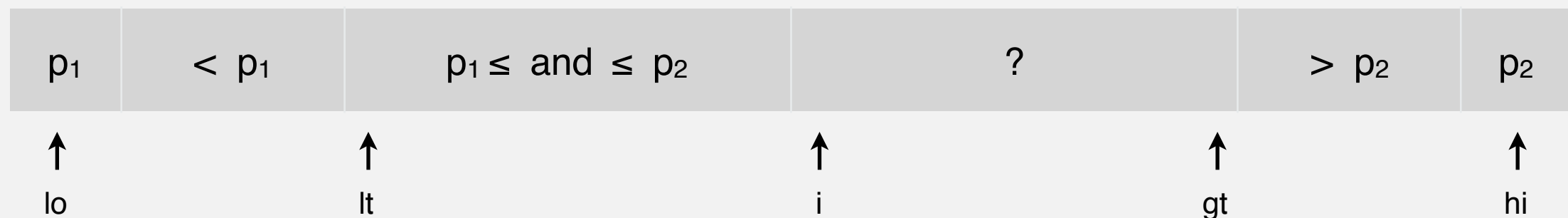
- Choose $a[\text{lo}]$ and $a[\text{hi}]$ as partitioning items.
- Exchange if necessary to ensure $a[\text{lo}] \leq a[\text{hi}]$.



Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If $(a[i] < a[lo])$, exchange $a[i]$ with $a[lt]$ and increment lt and i .
- Else if $(a[i] > a[hi])$, exchange $a[i]$ with $a[gt]$ and decrement gt .
- Else, increment i .

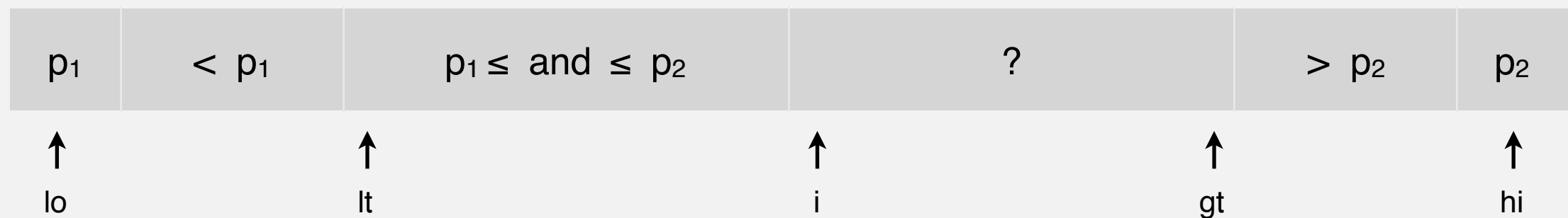


exchange $a[i]$ and $a[lt]$; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If $(a[i] < a[lo])$, exchange $a[i]$ with $a[lt]$ and increment lt and i .
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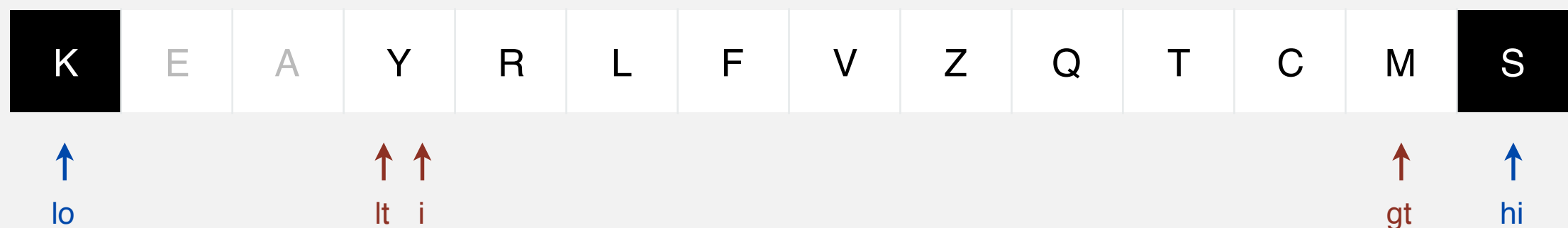
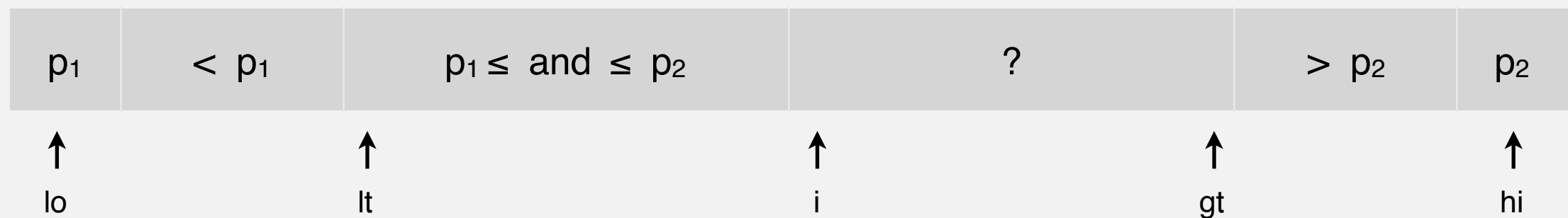


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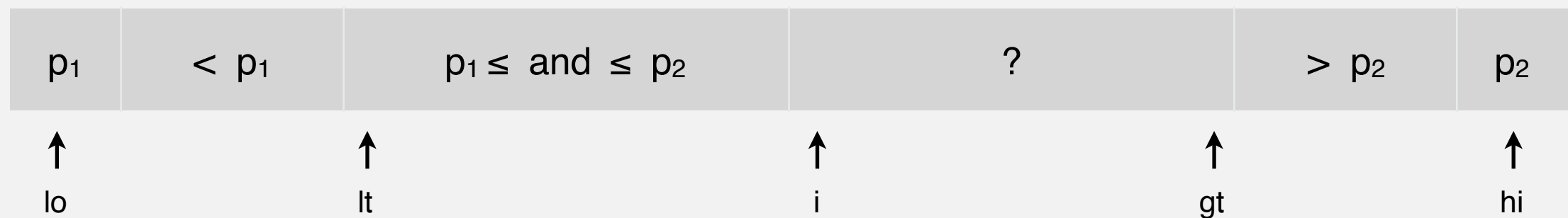


exchange $a[i]$ and $a[gt]$; decrement gt

Dual-pivot partitioning demo

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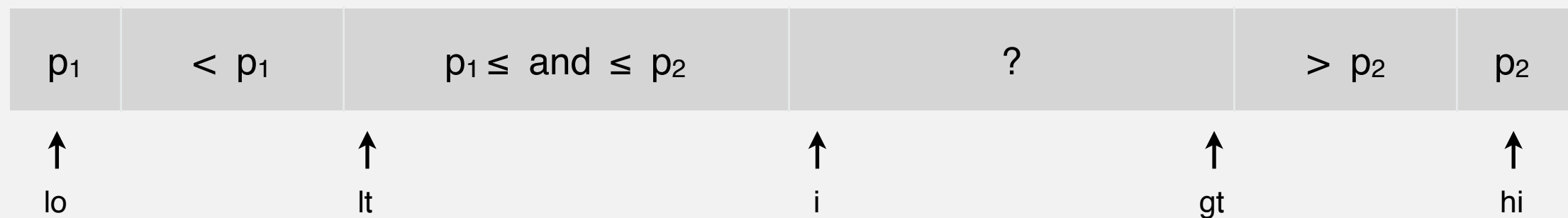


increment i

Dual-pivot partitioning demo

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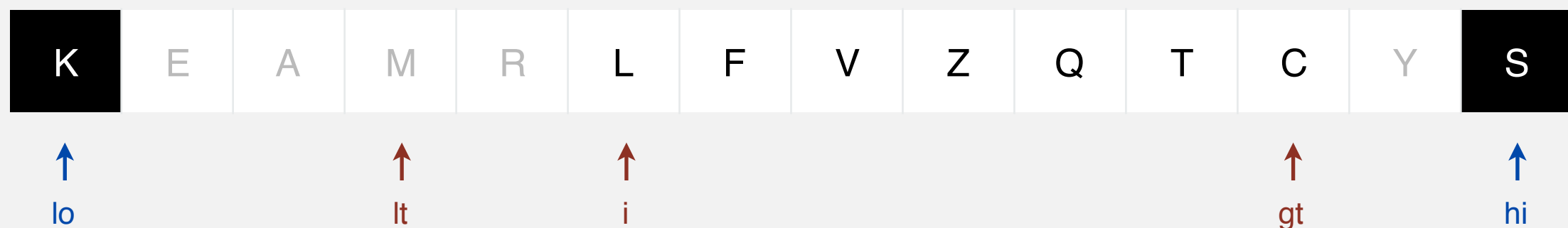
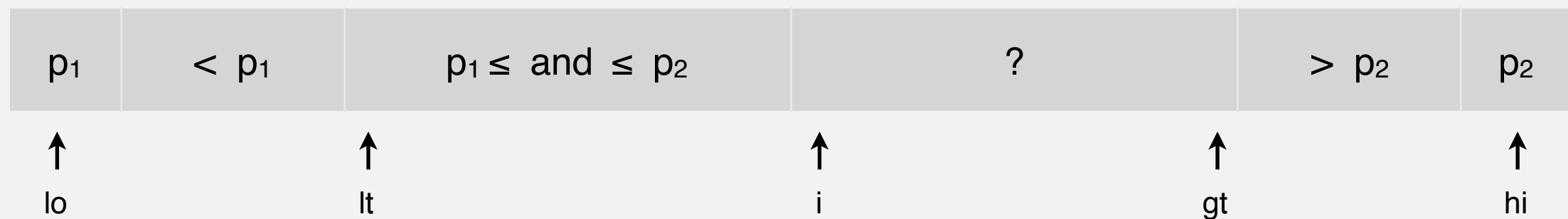


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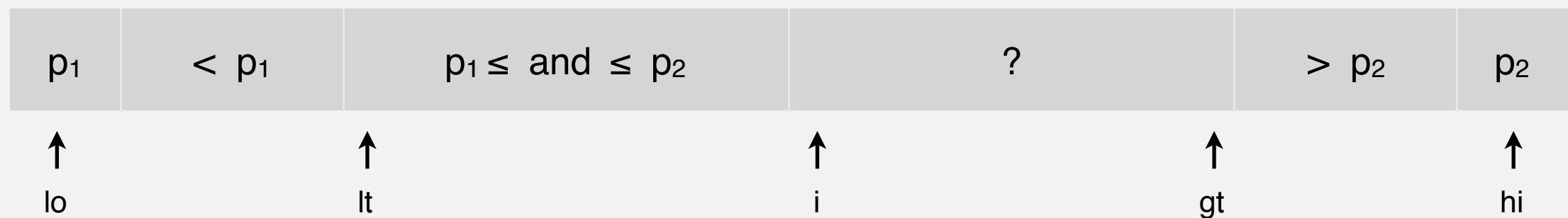


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- Else, increment i .

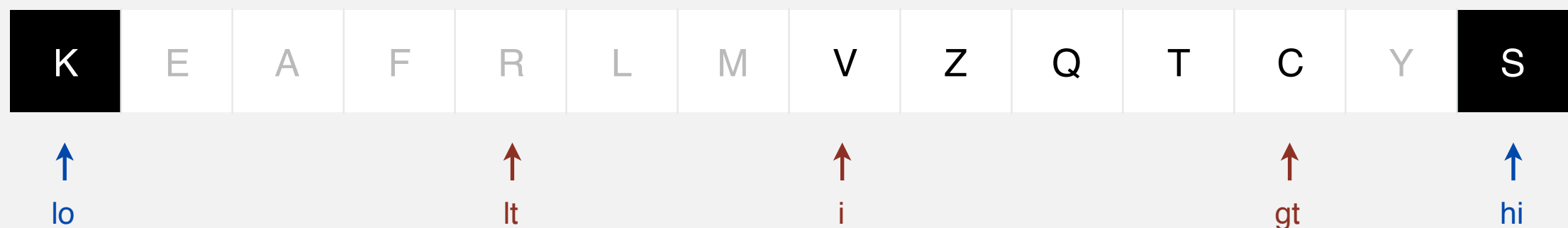
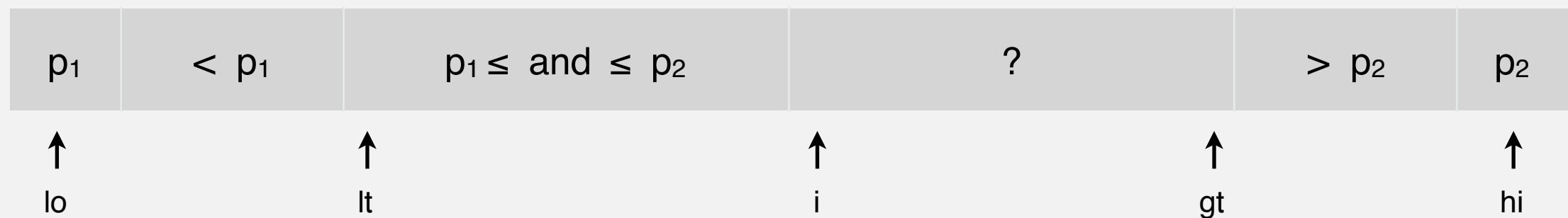


exchange $a[i]$ and $a[lt]$; increment lt and i

Dual-pivot partitioning demo

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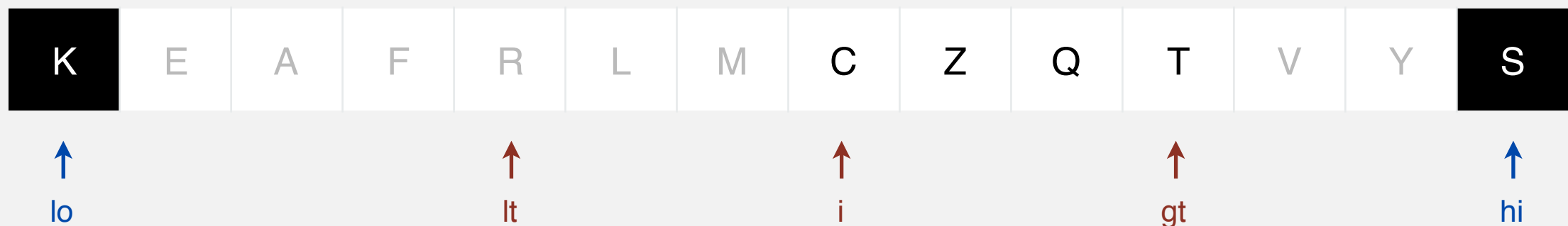


exchange $a[i]$ and $a[gt]$; decrement gt

Dual-pivot partitioning demo

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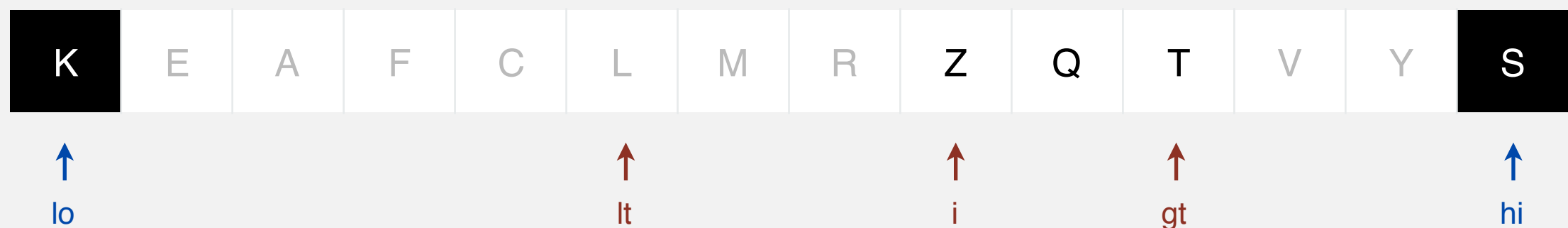
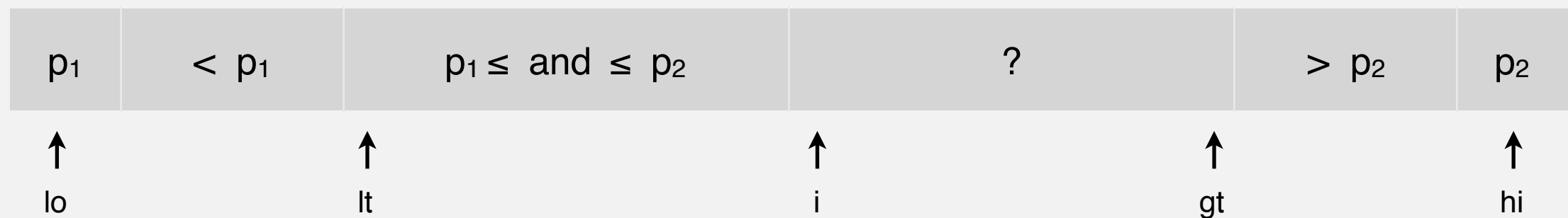


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Dual-pivot partitioning demo

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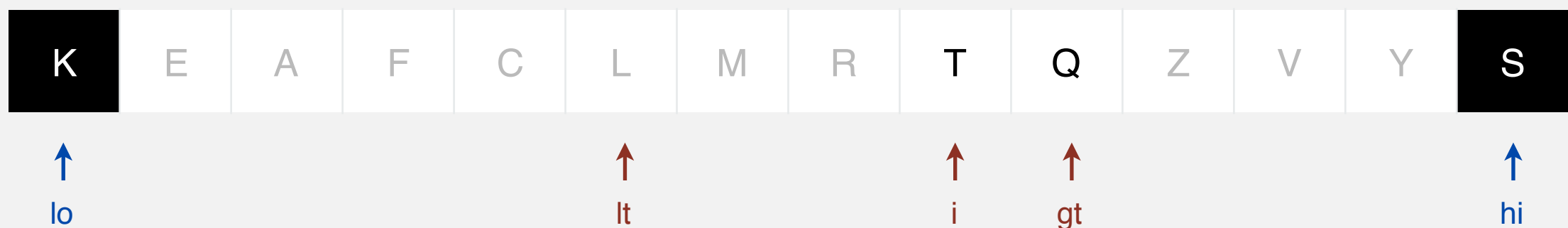
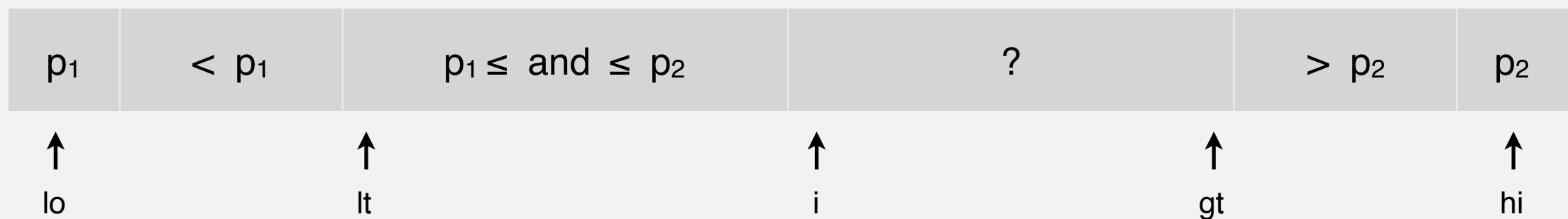


exchange $a[i]$ and $a[gt]$; decrement gt

Dual-pivot partitioning demo

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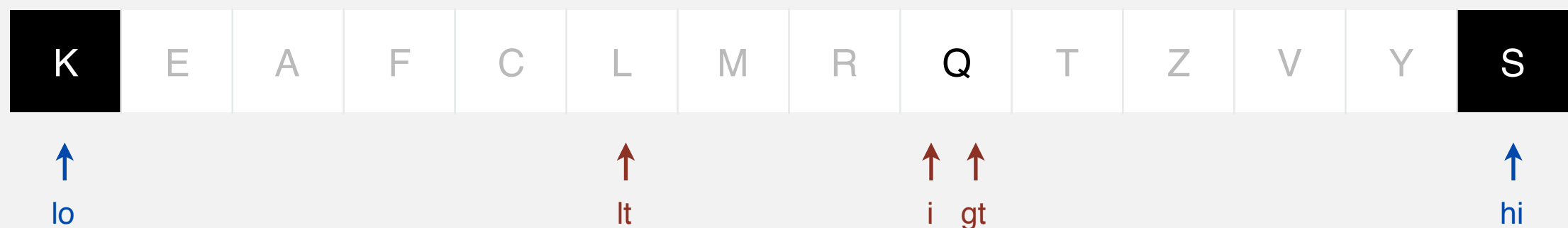
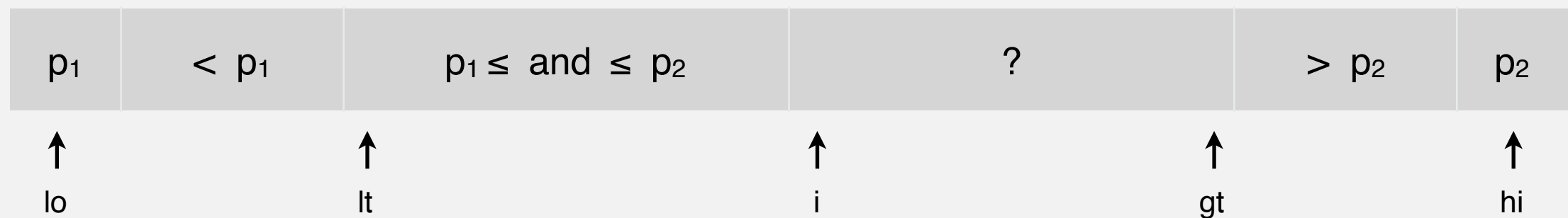


exchange $a[i]$ and $a[gt]$; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

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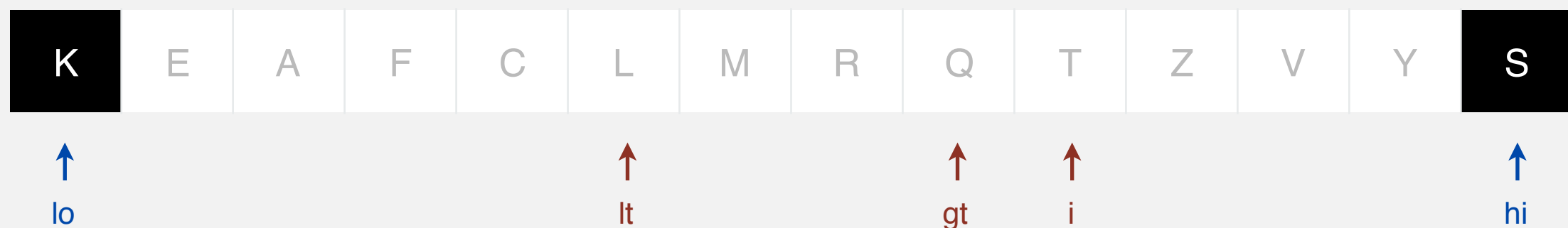
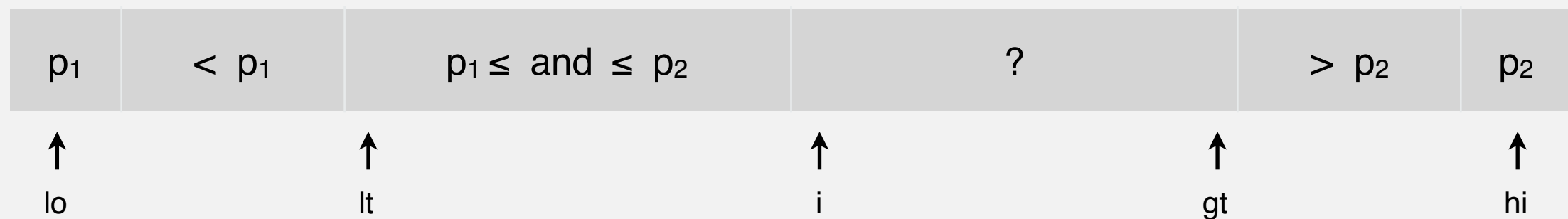


increment i

Dual-pivot partitioning demo

Main loop. Repeat until i and gt pointers cross.

- If $(a[i] < a[lo])$, exchange $a[i]$ with $a[lt]$ and increment lt and i .
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- Else, increment i .

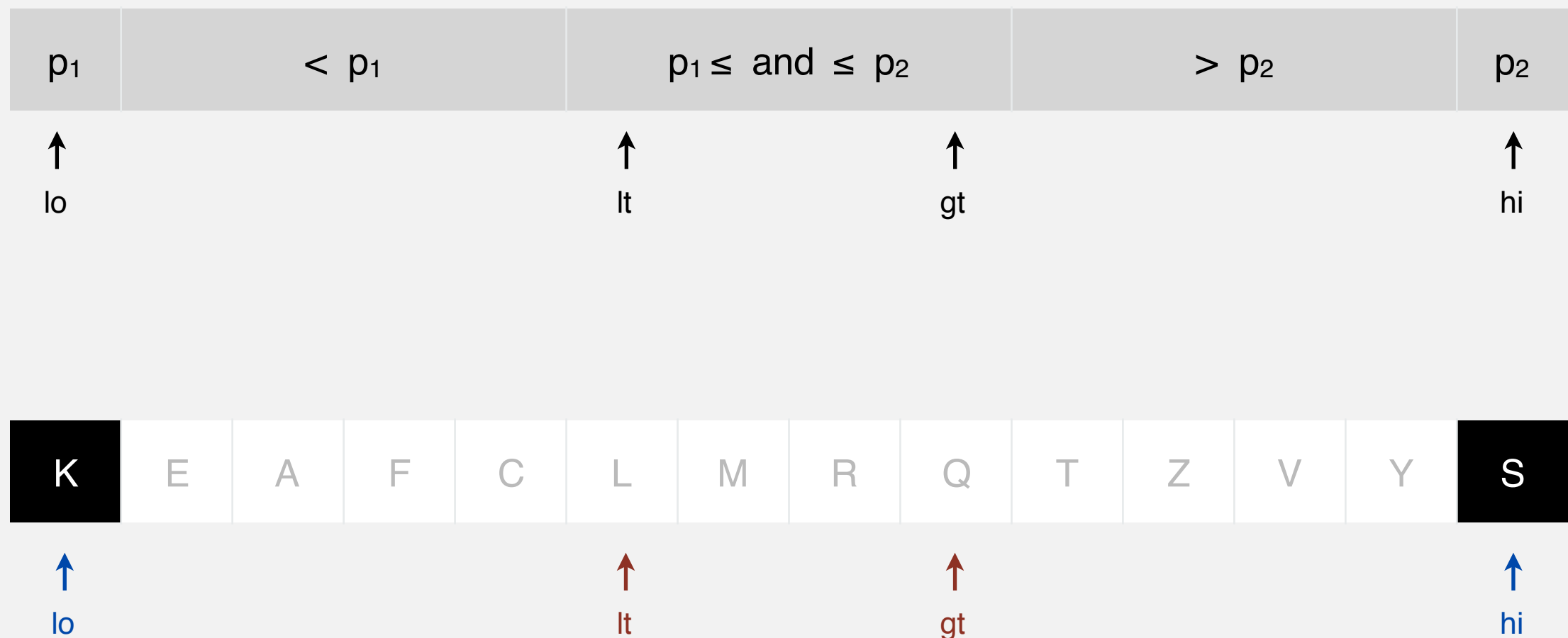


stop when pointers cross

Dual-pivot partitioning demo

Finalize.

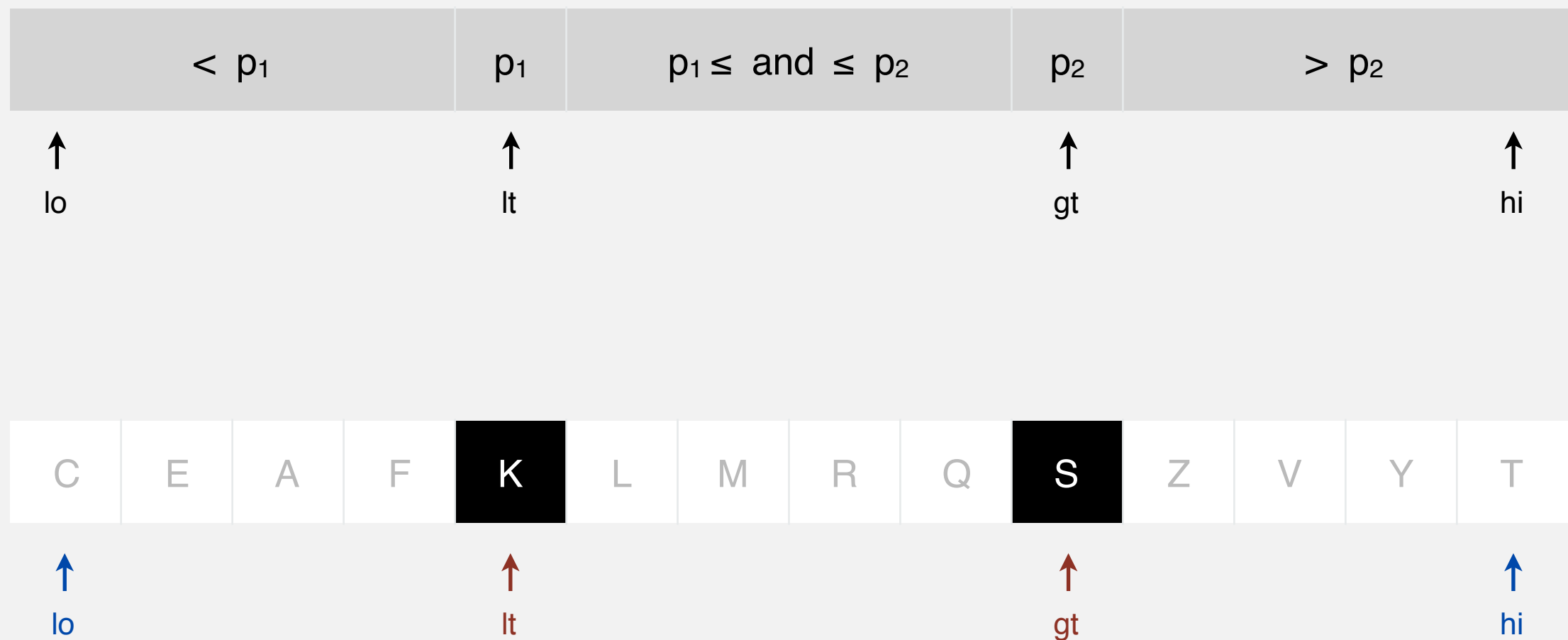
- Exchange $a[lo]$ with $a[--lt]$.
- Exchange $a[hi]$ with $a[++gt]$.



Dual-pivot partitioning demo

Finalize.

- Exchange $a[lo]$ with $a[--lt]$.
- Exchange $a[hi]$ with $a[++gt]$.



3-way partitioned